Inside the Stars, IAU Colloquium 137
ASP Conference Series, Vol. 40, 1993
Werner W. Weiss and Annie Baglin (eds.)

INITIAL ASTEROSEISMIC INVERSIONS

D.O.GOUGH[1] AND A.G.KOSOVICHEV[2]
Institute of Astronomy, Madingley Road, Cambridge CB3 0HA, England

INTRODUCTION

Accurate measurement of the frequencies of low-degree acoustic oscillations of sun-like stars is imminent. We report on our first calculations with proxy data, aimed at assessing the kind of physical information that is likely to be acquired from seismic analysis and the precision with which the frequencies must be measured in order to obtain that information. The results will have an important bearing on future observing strategies, for the duration of observation should be determined primarily by the precision required of the frequency measurements. Our inversions are of eigenfrequencies of modes of an evolved main-sequence star of mass $1.1M_\odot$. The modes are of degree 0, 1 and 2, with frequencies in the range 1–3 mHz. Thus, by analogy with solar oscillations, they are modes that one should expect to observe in stars similar to the sun. Figure I*a* depicts an idealized spectrum of stellar acoustic oscillations as one might expect from intensity variations such as those that could be measured from the proposed ESA spacecraft PRISMA. We report on the extent to which we have found it possible to determine the mass and radius of the stars, and on the seismic evidence for evolution having taken place in the core.

INVERSION TECHNIQUE

The frequencies ω_i can be expressed in terms of their relative small difference $\delta\omega_i^2/\omega_i^2$ from those of a standard reference model of similar mass and radius according to the linearized expression

$$\delta\omega_i^2/\omega_i^2 = \int_0^1 \left(K_{f,Y}^i \frac{\delta f}{f} + K_{Y,f}^i \delta Y\right) dx - I_q^i \delta q,$$

where $x = r/R$, $q = M/R^3$, Y is the helium abundance and f is any function of p and ρ; M and R are stellar mass and radius, in solar units, $K_{f,Y}$ and $K_{Y,f}$ are appropriate kernels and I_q is an integral over the reference model. These constraints can provide localized averages of $\delta \ln f$ and estimates of δY and δq of the kind

$$\overline{\delta \ln f} \equiv \int_0^1 \sum_i a_i(x_0) K_{f,Y}^i \delta \ln f\, dx \equiv \int_0^1 A_{f,Y}(x, x_0) \delta \ln f\, dx = \sum_i a_i(x_0) \frac{\delta\omega_i^2}{\omega_i^2}$$

[1]and Department of Applied Mathematics and Theoretical Physics, University of Cambridge

[2]and Crimean Astrophysical Observatory

https://doi.org/10.1017/S0252921100018388 Published online by Cambridge University Press

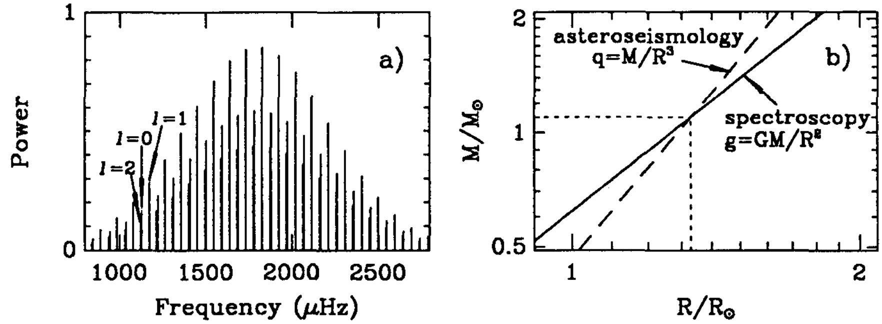

FIGURE I a) A sample of anticipated PRISMA data of observations of a star of $M = 1.1M_{\odot}, R = 1.33R_{\odot}$: p modes of l=0-2, n=6-28; b) Measuring stellar mass and radius: $M - R$ diagram

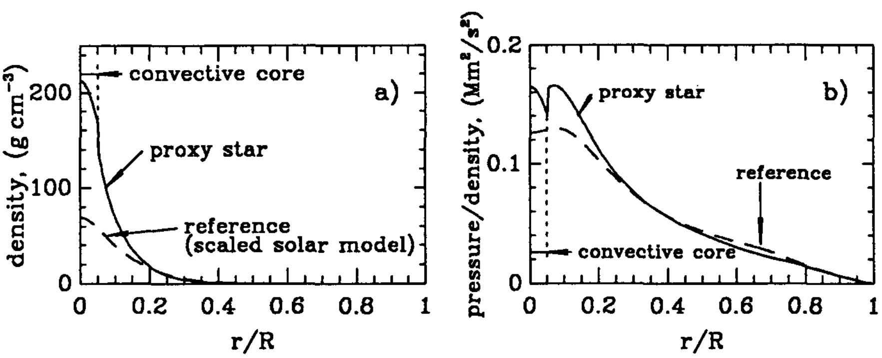

FIGURE II Parameters of the proxy star ($M = 1.1M_{\odot}, R = 1.33R_{\odot}$) and the reference model: a) ρ and b) $u \equiv p/\rho \simeq \mathcal{R}T/\mu$

by minimizing amongst coefficients $a_i(x_0)$ the functional:

$$\int_0^1 A_{f,Y}^2(x,x_0)J_f dx + \lambda_1 \int_0^1 \left(\sum_i a_i K_{Y,f}^i\right)^2 J_Y dx + \lambda_2 \left(\sum_i a_i I_q^i\right)^2 + \alpha \sum_i a_i^2 \epsilon_i^2$$

for tradeoff parameters λ_1, λ_2 and α, where ϵ_i are standard relative errors in the data. Appropriate weight functions J depend on the averages sought. For determining δq, we set $J_f = 1, J_Y = 1, \lambda_1 \neq 0$ and $\lambda_2 = 0$;
for $\overline{\delta \ln f}(x_0)$ we set $J_f = (x - x_0)^2, J_Y = 1, \lambda_1, \lambda_2 \neq 0$;
for $\overline{\delta Y}$ we set $J_f = 1, J_Y = 1, \lambda_1, \lambda_2 \neq 0$.

Some optimally localized averaging kernels $A_{f,Y}, A_{Y,f}$, with $f = u = p/\rho$, are illustrated in Figure III*a*. With only high-order low-degree modes it is not possible to localize $A_{f,Y}$ beyond $x \simeq 0.3$; to do so would require modes of lower order. The most robust quantity that can be determined is the homology frequency-scaling parameter q, which, coupled with a knowledge of surface gravity from spectroscopic analysis, determines M and R separately (see Figure I*b*).

https://doi.org/10.1017/S0252921100018388 Published online by Cambridge University Press

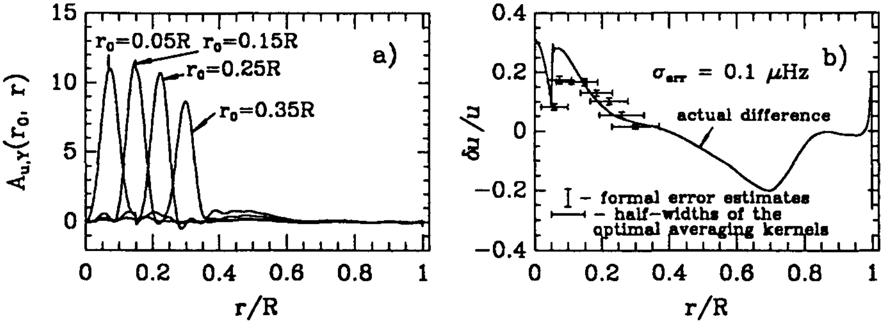

FIGURE III a) Optimal averaging kernels for $\delta u/u$; b) test inversion for relative differences in $u \equiv p/\rho$. Frequencies of the proxy star are perturbed by random errors with the standard deviation σ_{err}.

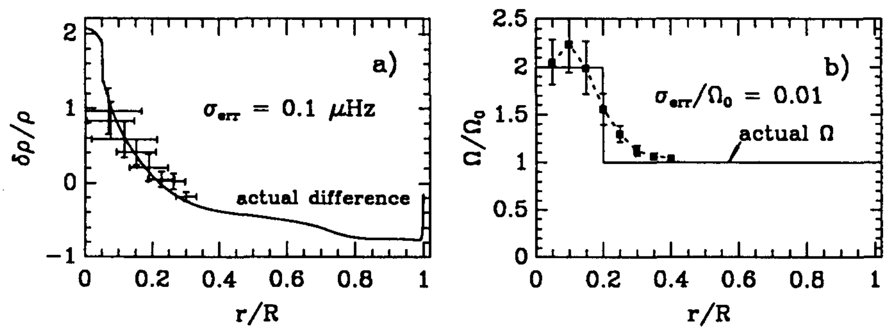

FIGURE IV Test inversions a) for relative differences in density ρ and b) for equatorial angular velocity Ω

INVERSION RESULTS

We have carried out a test inversion of artificial data obtained from a stellar model of $1.1 M_{\odot}$, evolved somewhat off the main sequence and with a convective core of radius $x_c = r_c/R \simeq 0.05$, and with $q = 0.468$. The reference was a solar model. The two models are illustrated in Figure II. The inversion yielded $q = 0.476$. Inversions for $\delta u/u$ and $\delta\rho/\rho$ are illustrated in Figures III*b* and IV*a* for assumed errors in the data with standard deviation 0.1 μHz. It should be appreciated that the relative difference between the models is not small, so the linearization may not have provided a good approximation. We presume that with the addition of some theoretical prejudice obtained by adopting more of the assumptions of stellar evolution theory, a closer reference model could be found and a better inversion obtained. We report that if errors with standard deviation 1 μHz are added to the data, the inversions are substantially degraded.

Finally, in Figure IV*b*, we illustrate a set of optimally localized averages of the angular velocity obtained from rotational splittings.