Vector Map Data Compression with Wavelets

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Wavelets and wavelet transforms can be used for vector-map data compression. The choice of wavelet, the level of decomposition, the method of thresholding, the height of the threshold, relative CPU times and file sizes, and reconstructed map appearance were investigated using the Wavelet Toolbox of MATLAB. Quantitative error measures were obtained. For two test vector-map data sets consisting of longitude and latitude points, compressions of 35 to 50 percent (1·5:1 to 2:1) were obtained with root-mean-square errors less than 0·003 to 0·01° longitude/latitude for wavelet packet decompositions using selected wavelets.

KEY WORDS

1. Maps/Charts. 2. Wavelets. 3. Data Compression.

1. INTRODUCTION.

Storage and transmission of vector-map data can be improved if the data can be compressed, but the compression must not delete any critical information. For example, the speed of transmission of vector-map data from a ground transmitter station to a receiver in the cockpit of an aircraft can be increased if a compressed version of the data is available.

A relatively new compression technique uses wavelet transforms. The Wavelet Toolbox of MATLAB™ was implemented to study vector-map data compression using wavelet transforms and wavelet packet transforms. Two test vector-map data sets consisting of 1024 and 2039 points of longitude and latitude values, respectively, were analysed. This paper describes the techniques and the results achieved.

2. BACKGROUND – WAVELET THEORY.

A function $y(x)$ may be expanded in a linear combination of basis functions:

$$y(x) = \Sigma a_{\mathrm{sk}}\, w_{\mathrm{sk}}(x),$$

where, the a_{sk} are the amplitudes of the individual basis functions w_{sk}. An example is the expansion that uses sines and cosines as basis functions, producing the familiar Fourier series coefficients. Wavelets are relatively new basis functions with

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mathematically desirable features (Daubechies, 1992; Strang and Nguyen, 1996; Mallat, 1998). With wavelet basis functions, the two parameters s and k describe the scaling and shifting of the mother wavelet.

Some desirable properties of a basis are: fast computation of inner products with the basis functions, to keep the expansion complexity low; easy superposition of the basis functions, to keep the reconstruction complexity low; good spatial localisation, to identify the portion of a signal that contributes a large component; regularity, or good frequency localization, to identify oscillations in the signal; and independence, so that not too many basis elements match the same portion of the signal (Wickerhauser, 1994). Perhaps the simplest and easiest wavelet to visualize is the Haar (Haar, 1910). Daubechies wavelets have a number of good qualities and are often used (Daubechies, 1992). Biorthogonal wavelets relax the orthogonality condition (Strang and Nguyen, 1996). Three examples of wavelets (the Haar, the Daubechies 4, and the Biorthogonal 4.4) are shown in Figure 1.

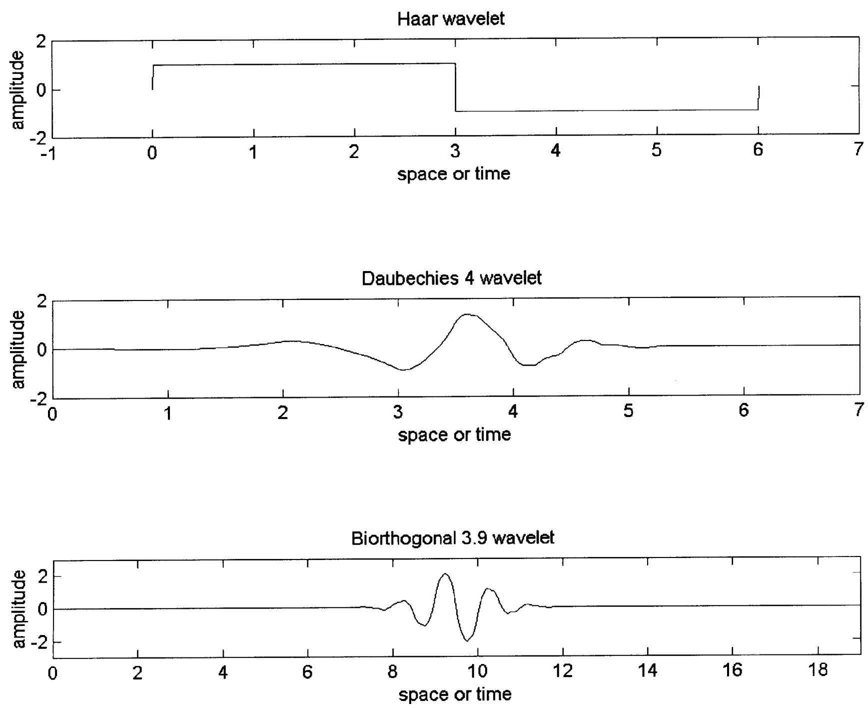

Figure 1. Three wavelets. (a) Haar, (b) Daubechies 4, (c) Biorthogonal 3.9.

In wavelet decomposition of a signal, the signal is split using highpass and lowpass filters into an approximation and a detail. The approximation is then itself split again into an approximation and a detail. This process is repeated until no further splitting is possible or until a specified level is reached. In wavelet packet decomposition, the signal is similarly split into an approximation and a detail, but then both the detail as well as the approximation are filtered into approximations and details. Thus there are more possible divisions of the signal. Wavelet packets are the basis functions behind sub-band coding, which has been used in both speech and image coding and compression (Wickerhauser, 1994). More than five levels of decomposition are rarely needed (Strang and Nguyen, 1996).

3. THRESHOLDS AND COMPRESSION. The choice of which of the decomposition coefficients to use for reconstruction of the signal is an important decision. Obviously, signals may be perfectly reconstructed if all the basis functions w_{sk} and all the coefficients a_{sk} are used. However, by using only selected transform coefficients, and/or modifying them, desirable results such as noise removal or compression may be obtained. For example, lowpass filtering of Fourier transform coefficients (setting high frequency coefficients to zero) often removes some of the noise, which is usually more dominant at higher frequencies than the signal of interest.

The 'best basis' for wavelet decomposition is the optimal decomposition with respect to a convenient criterion, computable by an efficient algorithm (Misiti *et al.*, 1997). The best basis means keeping only selected approximation and detail coefficients and not all of them. It is called a 'hedge' by Wickerhauser (1994) and a 'besttree' by MATLAB™ (Misiti *et al.*, 1997), which uses the entropy as the information cost function to be minimised. MATLAB™ calculates the threshold entropy for each of the two possible splittings at a given level. The threshold entropy is the number of coefficients whose absolute value is greater than the threshold. If the sum of the entropy from the two splittings is less than the entropy of the original level, the level is split (Misiti *et al.*, 1997).

After the optimal wavelet decomposition tree and coefficients are determined, instead of lowpass filtering the coefficients, they are more often thresholded; that is, coefficients whose absolute value is below a selected threshold are set equal to zero (Strang and Nguyen, 1996; Mallat, 1998; Vidakovic, 1999). Threshold methods include hard, soft, semisoft, nn-garrote, and hyperbole (Vidakovic, 1999). A hard threshold means simply that any coefficients whose absolute value is less than the threshold are set to zero. A soft threshold sets to zero those coefficients whose absolute values are less than the threshold and also decreases the amplitude of the remaining coefficients above the threshold by the amount of the threshold (Strang and Nguyen, 1996; Mallat, 1998). This procedure is also called wavelet shrinkage (Vidakovic, 1999). A soft threshold was chosen for the test data set studied in this work. The best threshold for compression is computed by MATLAB™ to be the median of the absolute values of the detail coefficients at level 1 (Misiti *et al.*, 1997). Many wavelet coefficients are zero after decomposition and thresholding. These coefficients can be grouped so that an entropy coder can take full advantage of long strings of zeros. Run length or Huffman coding or a combination can be used for further compression.

4. RUN LENGTH AND HUFFMAN CODING. Data compression can be considered to be 'lossy' or 'lossless' (Nelson, 1991). In lossy compression, some accuracy is given up in exchange for greater compression. Compression obtained by setting to zero small amplitude wavelet coefficients (which are often due to noise) is therefore lossy. The uncompressed or reconstructed data are acceptable since the noise has been reduced. In lossless compression, the uncompressed data are identical to the original. Lossless compression is used when omission of even a single bit of information would be destructive.

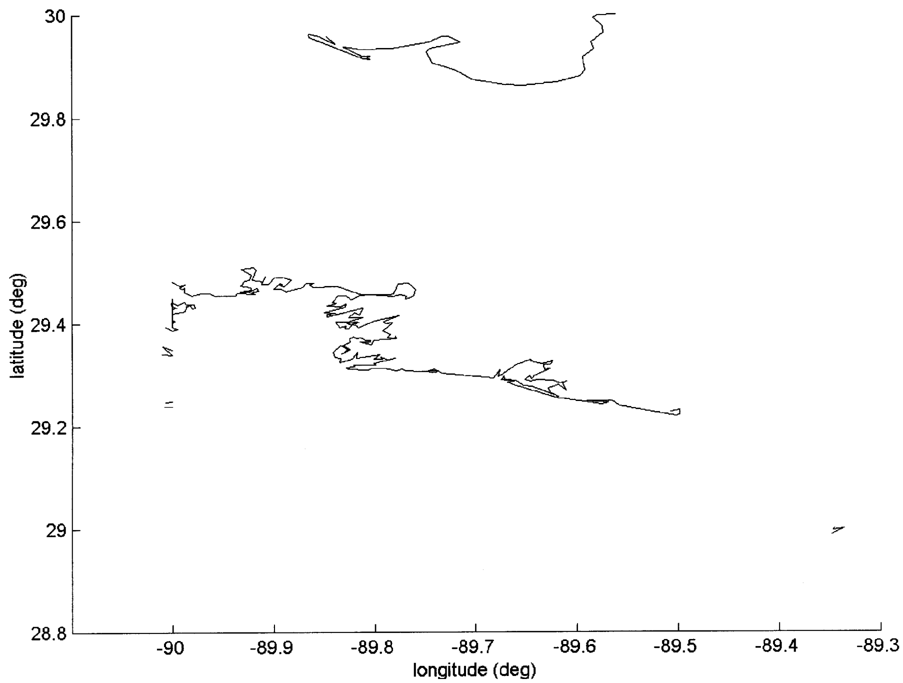

Figure 2. Test vector-map data, consisting of 1024 longitude and latitude points giving the beginning and ending locations of 256 line segments. The line segments outline a portion of the Louisiana coastline.

A coefficient can be scaled from an 8 byte or 64 bit floating point value to a 2 byte or 16 bit value. Because the coefficient can be positive or negative, one bit is used as a sign bit. The remaining bits store the magnitude of the coefficient. The maximum number that 63 bits can represent is $2^{63} = 9{\cdot}2 \times 10^{18}$; 15 bits can represent $2^{15} = 32\,768$. By multiplying a given coefficient value of magnitude less than one by 32 768 and retaining only the integer part, the compression obtained is $64:16 = 4:1$. Since the original amplitudes have been rounded or approximated, this compression is lossy.

Run length coding (Nelson, 1991; Strang and Nguyen, 1996) is useful when there are strings or 'runs' of zeros in an integer coefficient list, such as is obtained when the coefficients are thresholded. It converts such a list of coefficients into a collection of three number sets: the number of preceding zeros in the original list (run-length), the storage space required to hold the following non-zero integer coefficient (size), and the actual value of the coefficient (amplitude). Run length coding is therefore lossless compression. It is obviously the most useful when there are strings of zeros in a coefficient list.

The output from run length coding can be compressed further using Huffman coding. Huffman coding (Huffman, 1952; Nelson, 1991) creates variable length codes that are an integral number of bits. A Huffman tree is constructed using the frequency or probability of a symbol's appearance. Symbols with higher probabilities get shorter codes. The symbol with the highest probability is assigned the fewest bits, and the symbol with the lowest probability is assigned the most bits.

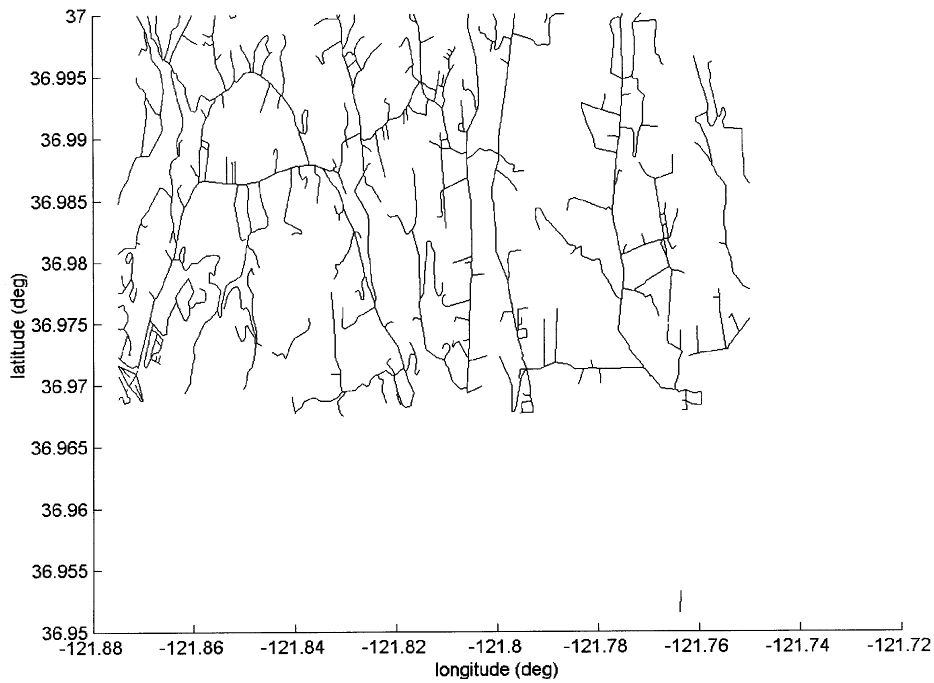

Figure 3. Test vector-map data, consisting of 2 039 longitude and latitude points, divided into 481 sets of longitude and latitude values defining a continuous line for each set. The line segments depict various roads in a small section of California.

5. ERROR MEASURES. Reconstructed maps can be visually compared to the original map to determine qualitatively the accuracy of the reconstruction. Quantitative error measures can also be employed. The root-mean-square (rms) error is the square root of the square of the difference between the reconstructed value and the original value. The average rms error can be computed as a function of a parameter such as the height of the threshold used with the wavelet coefficients.

The disbalance of energy is the uneven distribution of energy in a signal (Vidakovic, 1999). A Lorentz curve (Lorentz, 1905) is a graphical representation of disbalance, first used by economists for measuring inequalities of incomes or wealth in populations. The Lorentz curve for a more balanced distribution lies above that of a less balanced distribution. In signal processing, the energy of the signal is used for the measure instead of the economic value. Disbalance is desirable in signal processing since such a signal can be well described by only a few energetic components (Vidakovic, 1999).

6. TEST VECTOR-MAP DATA. The first test vector-map data set (Lohrenz, 1988) consisted of 1024 longitude and latitude coordinates at a map scale of 1:250000. Each set of four points gave the beginning and ending longitude and latitude locations of a line segment; thus there were 256 line segments. Figure 2 shows the test map data, a portion of the Louisiana coastline, produced by plotting the lines connecting the beginning and ending longitude and latitude coordinates. All of the segments were shorter than 7 km each.

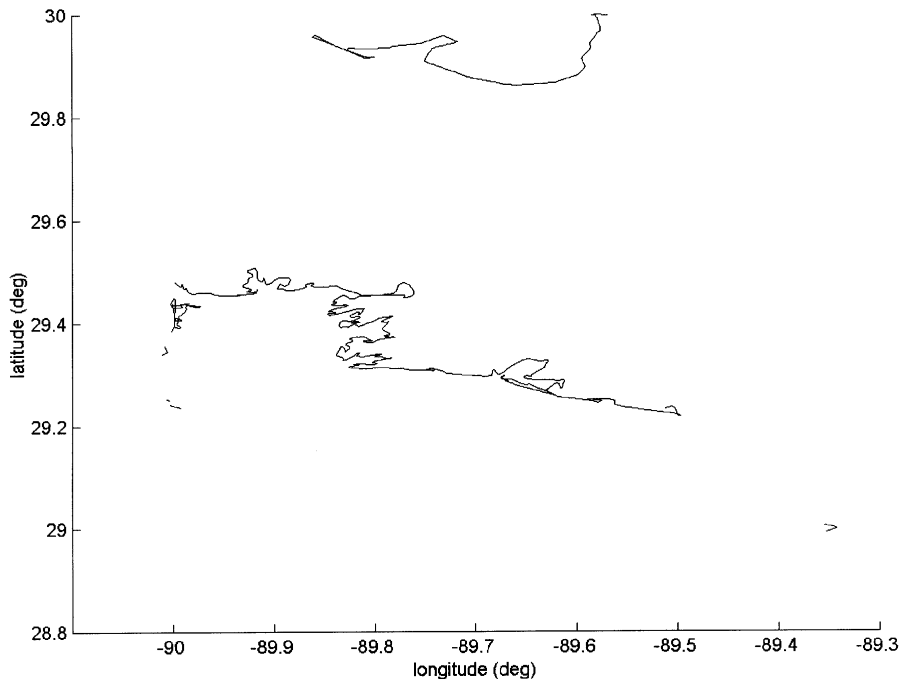

Figure 4. Reconstructed map for the test vector-map data of Figure 2 using a Biorthogonal 3.9 level 3 wavelet packet decomposition. The longitude compression was 56% (2·3:1) and the latitude compression was 54% (2·2:1).

The second test vector-map data set (Standards for Digital Line Graphs, 1998) contained 2039 points of latitude and longitude divided into 481 sets of longitude and latitude values that define a continuous line for each set. The map scale was 1:100000. Figure 3 is a plot of the second test map data set, which depicts various roads in a small area of California.

The longitude and latitude were processed separately in this work so that each could be considered as a 'time series' of equally spaced values, with the point number as the independent variable. Compression is difficult for data of this type as it is not harmonic, and there is large randomness, or entropy, or information. A test using 2048 points in the second test set did not show significantly shorter CPU times, which would have occurred with processing using the Fast Fourier Transform.

7. TEST VECTOR DATA COMPRESSION RESULTS. The choice of mother wavelet for wavelet packet decomposition depends on the data to be transformed. Both wavelet and wavelet packet decompositions were investigated, with wavelet packet decomposition giving better results. Wavelets studied included the Haar or Daubechies 1; Daubechies 2, 3, 4, 5, 6, and 10; symmetric 2, 3, 4, and 5; Biorthogonal 3.3, 3.5, 3.7, 3.9, 4.4, 5.5, and 6.8; Coiflet 3, 4, and 5; Morlet; Meyer; Mexican hat; and chirplet. Wavelet packet decomposition was performed with each wavelet to several levels. Various quantitative measures were computed for the longitude and the latitude using each wavelet at various levels; these included the percent compression for the longitude and the latitude, the average rms error for the longitude and latitude, the CPU time for the transmitter program and the

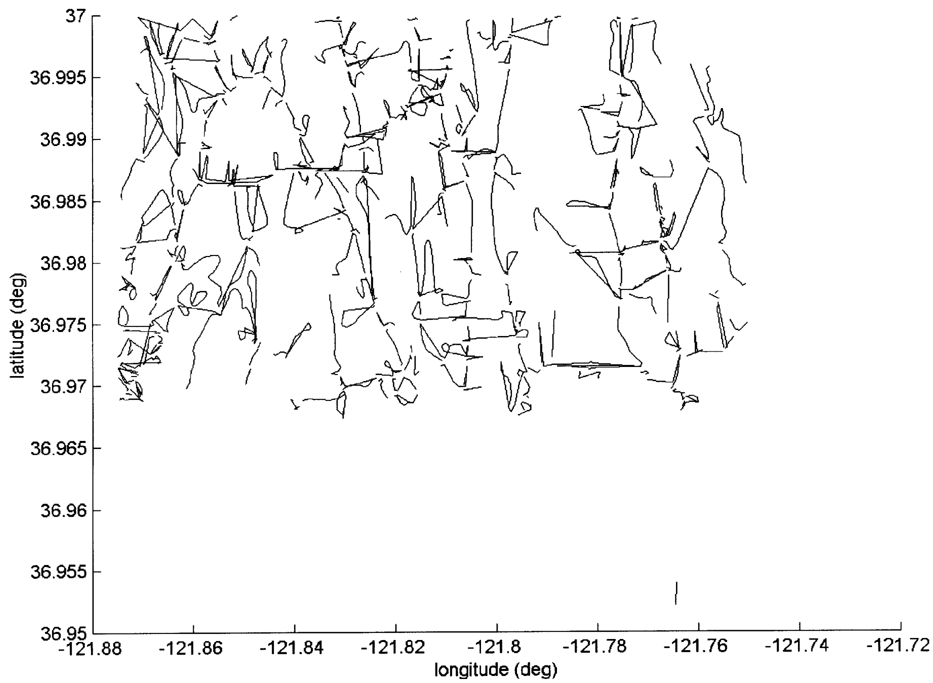

Figure 5. Reconstructed map for the test vector-map data of Figure 3 using a Biorthogonal 3.9 level 4 wavelet packet decomposition. The longitude compression was 25% (1·3:1) and the latitude compression was 38% (1·6:1).

receiver program, and the compressed data file size. No one wavelet performed the best for all measures for both data sets.

Results are presented below for several wavelets whose performance was approximately the same. The test vector data sets consisted of discrete line segments which when plotted together represented a coastline or roadmap. Gendron (1999) obtained good results for edge detection using the Haar wavelet. Prochazka *et al.* (1998) recommend the Haar wavelet as very effective in the case of step signal changes. Mallat (1998) states that Biorthogonal wavelets are often used in image compression because their quasi-orthogonality guarantees a good numerical stability and they offer a good trade-off between the support size, the number of vanishing moments, and the regularity.

Wavelet packet compression using a soft threshold was accomplished for the longitude and latitude separately in a 'transmitter' MATLAB™ program, representing the ground station sending a vector map to an aircraft. The input to this program was the test vector data set, the name of the wavelet to be used and the level of decomposition, and the name for the output file. The mean values of the longitude and the latitude were subtracted from the original values as the first processing step, thus restricting the data values to be small numbers around zero. The best tree structure for the wavelet decomposition was selected by the MATLAB™ entropy decision. The best threshold for compression was selected by MATLAB™ to be the median of the absolute values of the detail coefficients at level 1.

The output of the transmitter program was a file containing the name of the wavelet and the level of decomposition, the sizes of the tree structure and the data

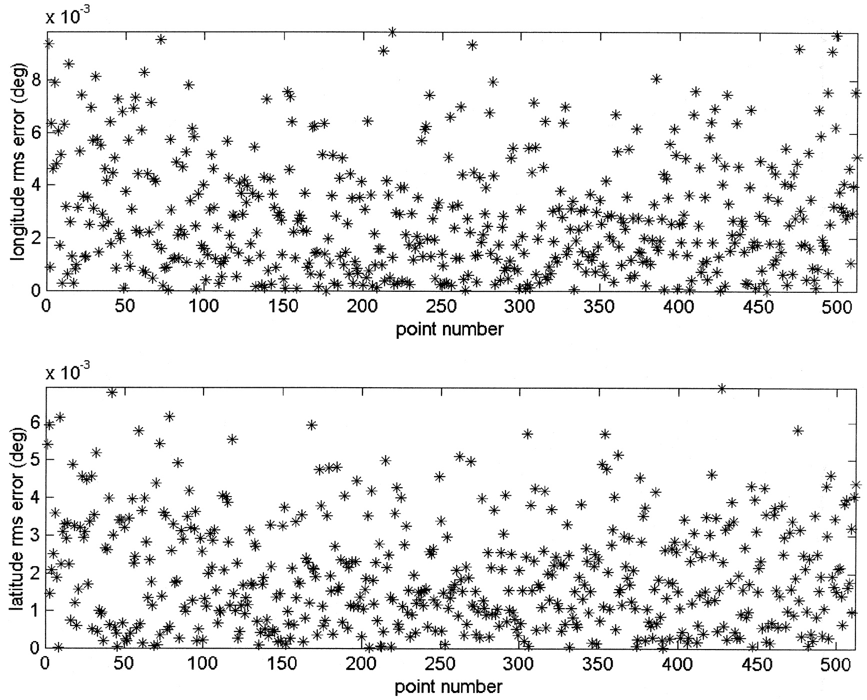

Figure 6. Root mean square error in degrees longitude or latitude for each point of the reconstruction of Figure 4 for (a) longitude, and (b) latitude. The average percent rms error was 0·00284° for the longitude and 0·00198° for the latitude.

structure of the decomposition, the mean values of the longitude and the latitude, the wavelet coefficient tree structure and wavelet coefficient data structure for the longitude, and the wavelet coefficient tree structure and wavelet coefficient data structure for the latitude. Compression appears because of the zeros replacing those coefficients that were less than the threshold in the coefficient data structures for the longitude and for the latitude. The amount of compression was the percentage of wavelet coefficients that were zero; i.e., they fell below the threshold.

Reconstruction was performed in a separate 'receiver' MATLAB™ program, representing the cockpit of the aircraft receiving the vector-map data from the ground transmitting station. The input to the receiver program was the output file written by the transmitter program. The longitude and latitude were reconstructed separately from their separate wavelet packet coefficients and the resulting coordinates used to plot the reconstruction map. This map was the output produced by the receiver program. Since there were a few large jumps from one area to another, each beginning to ending longitude and latitude difference was checked against a pre-set length and rejected if it exceeded that value. This improved the visual appearance of the reconstructed map. Different maximum lengths were used for the longitude and latitude for each test data set. A reconstructed map for a particular choice of parameters is shown in Figure 4 for the first data set and in Figure 5 for the second.

The accuracy of the reconstruction was quantified by computing the root-mean-square (rms) error for each reconstructed longitude and latitude value as compared with the original value. The best rms errors were generally less than about 0·003° for the longitude and 0·0021° for the latitude for the first test set, and generally less than about 0·0003° for the longitude and 0·00021° for the latitude for the second test set. The rms errors for the longitude and latitude for the reconstruction of Figure 4 are shown in Figure 6, and in Figure 7 for the reconstruction of Figure 5.

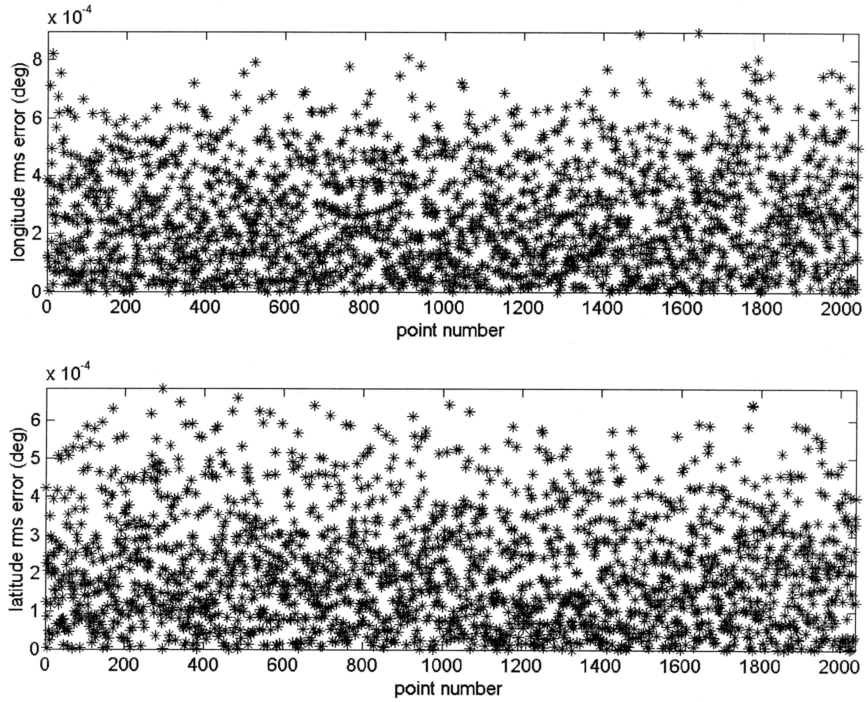

Figure 7. Root mean square error in degrees longitude or latitude for each point of the reconstruction of Figure 5 for (a) longitude, and (b) latitude. The average percent rms error was 0·000260° for the longitude and 0·000297° for the latitude.

To investigate the effect of threshold height, the compression and the average rms error for all points of a given reconstruction were computed for the longitude and the latitude and plotted versus threshold height. The percent compression increases with threshold height since fewer coefficients means more compression, as shown in Figure 8 for the first test data set. Figure 9 shows that the average rms error also increases with threshold height since fewer coefficients means more error in the reconstruction. Similar plots were obtained for the second test data set.

Figure 10 shows the Lorentz curves for the longitude and latitude data for the first test vector-map data and the wavelet decompositions using the Biorthogonal 3.9 wavelet at level 3 and the Coiflet 4 wavelet at level 4. It can be seen that the energies of the transform coefficients are more disbalanced than that of the original. The Lorentz curves for the Haar, Daubechies, and Biorthogonal wavelets were similar, reflecting the fact that the maps reconstructed using these wavelets resembled each other, with comparable rms error values and compressions. The Lorentz curves for the second test data set showed similar results.

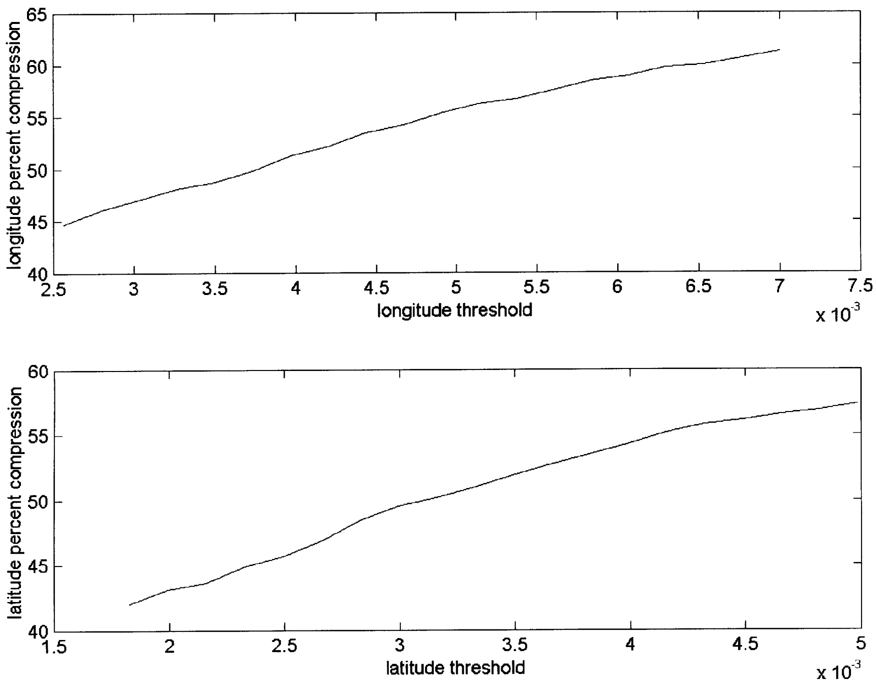

Figure 8. Percent compression versus threshold for the test data of Figure 2 using a Biorthogonal 3.9 level 3 wavelet packet decomposition for (a) longitude, and (b) latitude. The compression increases with threshold since more coefficients are set to zero.

Relative CPU times were obtained for both the transmitter program and the receiver program. The time for the receiver program was considered to be the more critical, as it represents the time needed in the aircraft for map reconstruction. The transmitter program could be run at an earlier, more convenient time when elapsed time is less critical. The shortest times for the first test data set were around 10 sec using a Gateway 2000 P5–120 PC. The shortest times for the second data set were around 70 sec on the same PC computer.

Relative file sizes for the compressed data files were obtained for each level tested for each wavelet tested. The smallest compressed data files for the first test data set were about 87 percent of the original, or a compression of 1·1:1. The smallest compressed data files for the second test data set were about 42 percent of the original, or a compression of 2·4:1.

The best tree for a particular wavelet packet decomposition was sometimes identical for several different scales or levels. For example, for the first test data set, with a Daubechies 4 wavelet, the best tree for both the longitude and the latitude for levels 3 through 6 was the same; i.e., the best tree ended at level 3. Consequently, for levels 3, 4, 5, and 6, the compressions were the same, the average errors were the same, the compressed file sizes were the same, and consequently the reconstruction/receiver CPU times were approximately the same. The compression/transmitter CPU times were longer according to the level, so the lowest level (level 3 for this example) was selected.

The chirp-type wavelets (Morlet, Meyer, Mexican hat, and chirplet) produced the greatest compressions for both test data sets of 60 to 85 percent zeros (2·5:1 to 6·7:1),

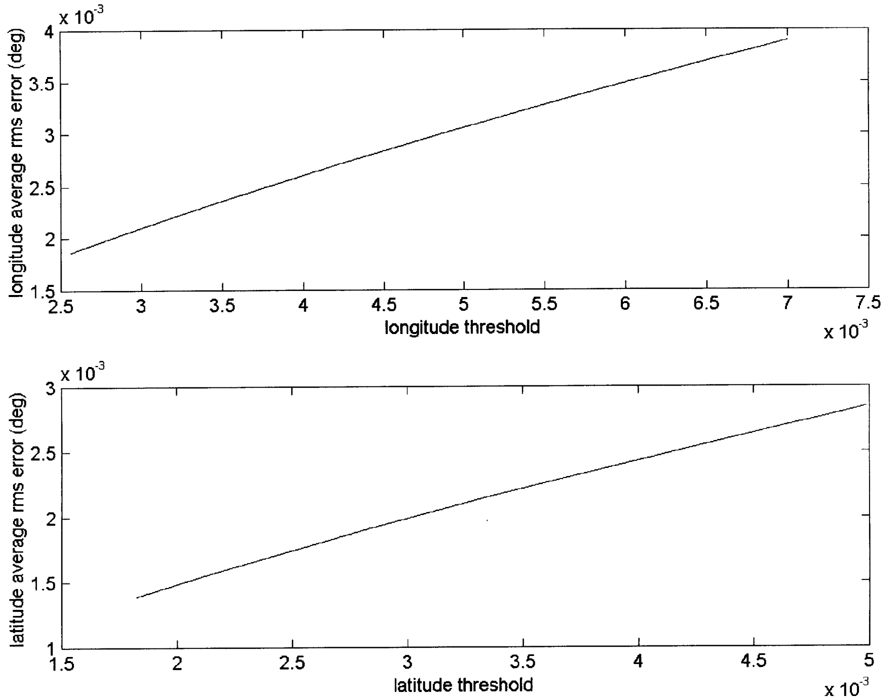

Figure 9. Average root mean square error versus threshold for the test data of Figure 2 using a Biorthogonal 3.9 level 3 wavelet packet decomposition for (a) longitude, and (b) latitude. The rms error increases with threshold since fewer coefficients are retained.

with the resulting output files the smallest at 20 to 80 percent of the original (5:1 to 1·25:1). The rms errors for the reconstructed map points for these wavelets were larger, however, ranging from 0·01 to over 0·08°, and the visual appearance of the reconstructed maps was unsatisfactory.

The wavelets used with the first test data set that produced the best maps with the lowest reconstruction rms errors were the Biorthogonal 3.9 level 3 with 0·00198° rms error, Biorthogonal 3.7 level 3 with 0·00204°, and Daubechies 4 level 3 with 0·00212°. These values were for the latitude points; the longitude points had slightly higher rms errors. The best compressions for the wavelets with less than 0·005° rms error were the Biorthogonal 3.9 level 3 with over 56% zeros (2·3:1), and the Coiflets 3 level 4, 4 level 4, and 5 level 3, all with over 55% zeros (2·2:1). These compressions were for the longitude; the latitude had slightly smaller compressions. CPU times for the reconstruction program were all around ten seconds or less for these wavelets, but longer (10 to 14 sec) for the chirp-type wavelets.

The wavelets used with the second test data set that produced the best results with the lowest reconstruction rms errors were the Symmetric 5 level 5, Daubechies 4 level 5, and Biorthogonal 3.9 level 4 wavelets, all with 0·000207° rms error. These values were for the latitude points; the longitude points had slightly higher rms errors. The best compressions with latitude rms errors of less than 0·00025° for wavelet packet decompositions were with the Biorthogonal 3.7 level 4 and 3.3 level 8 with over 39% zeros (1·6:1), and Biorthogonal 3.5 level 4 with over 38% zeros. These compressions were for the latitude; the longitude had slightly smaller compressions. CPU times for

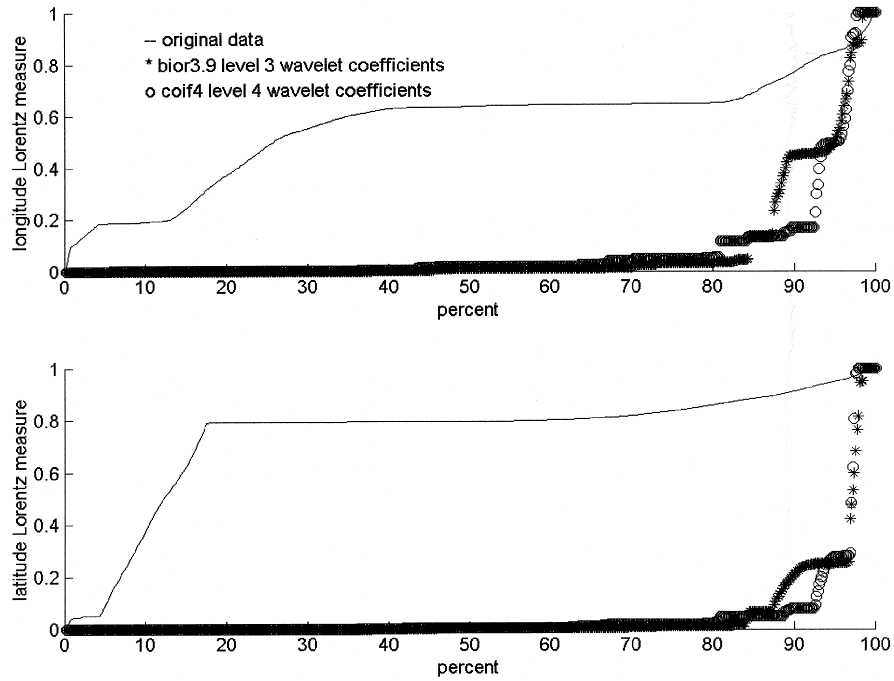

Figure 10. Lorentz curves for the original test data of Figure 2 (solid line), Biorthogonal 3.9 level 3 wavelet packet decomposition (asterisk *), and Coiflet 4 level 4 wavelet packet decomposition (circle o), for (a) longitude, and (b) latitude. The wavelet decompositions show more disbalance in the energy since the curves are below that of the original data.

the reconstruction/receiver program were longer for this data set since it was larger, around 50 to 80 sec. The compressed data file sizes were less than half of the original file size, with the best less than 42 percent of the original (2·4:1).

8. SUMMARY AND CONCLUSIONS. Wavelets and wavelet transforms can be used for vector-map data compression. The choice of wavelet, the level of decomposition, the method of thresholding, and the height of the threshold were investigated using the MATLAB™ Wavelet Toolbox. The MATLAB™ threshold entropy criterion was used to determine the decomposition tree structure. The threshold for the coefficients was computed by MATLAB™ to be the median of the level 1 detail coefficients. The amount of compression was found by calculating the percentage of wavelet coefficients that were set to zero; those that fell below the threshold. The reconstructed maps were visually compared to the original to determine the qualitative error in the reconstruction. Additional quantitative error measures of the root-mean-square error, average rms error, and Lorentz curves were obtained. For a test vector-map data set of 1 024 longitude and latitude points for coastline data, compressions of around 50 percent (2:1) were obtained with rms errors of less than 0·005° for wavelet packet decompositions using Daubechies and Biorthogonal wavelets. CPU times for the transmitter and receiver MATLAB™ programs were similar and averaged only around 10 seconds each for this test data set. The compressed data file sizes were less than 87 percent of the original file size (1·1:1). For

a second test vector-map data set of 2039 longitude and latitude points for roadmap data, compressions of 25 to 40 percent (1·3 : 1 to 1·7 : 1) were obtained with rms errors of less than 0·00027° for wavelet packet decompositions using Biorthogonal wavelets. CPU times were longer for this data set since it was larger. The compressed data file sizes were less than half of the original file size. Additional compression could be obtained using binary encoding, run length coding, and Huffman coding of the wavelet coefficient files.

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