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A NOTE ON INCREASING HAZARD FUNCTIONS AND THE MONETARY TRANSMISSION MECHANISM

Fang Yao
Reserve Bank of New Zealand
and
University of Erlangen-Nuremberg

This note studies the implications of the price reset hazard function for the monetary transmission mechanism of sticky price models. I first document some general analytical results that highlight the central role of the price (accumulative) distribution in linking the hazard function and the impulse response function. I find that nominal rigidity underlying increasing hazard functions is more successful than real rigidity in replicating realistic macro persistence. In addition, numerical simulations show that the interaction between the increasing hazard function and the real rigidity provides a powerful propagation mechanism for monetary shocks.

Keywords: Price Reset Hazard Function, Nominal Rigidity, Real Rigidity, Monetary Policy

1. INTRODUCTION

The purpose of this note is to investigate how the shape of the price reset hazard function affects the monetary transmission mechanism of sticky price models. This is interesting because of advances in both empirical and theoretical research on sticky prices. On one hand, recent empirical studies based on micro-level price data widely conclude that the probability of price adjustments depends on the duration of prices.[1] This evidence raises serious doubts about the predictions of the popular New Keynesian models based on the Calvo sticky price assumption [Calvo (1983)]. On the other hand, recent research on micro-founded pricing models [e.g., Costain and Nakov (2011)] shows that a realistic sticky price model should feature a smoothly rising hazard function, which is closer to the Calvo model than the fixed menu cost model. However, because of technical challenges post by

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state-dependent pricing models, implications of nonconstant hazard functions for macro dynamics have not been fully explored in the literature.

To fill this gap, this note applies a tractable generalized time-dependent pricing model, set forth by Wolman (1999), in which the probability of price adjustments is allowed to vary with respect to the time elapsed since the last price revision. As a first pass, I solve a simplified version of the model analytically and document some general results regarding the role of the price (accumulative) distribution in bridging the price reset hazard function at the micro level and equilibrium dynamics at the macro level. Next, I solve the full model numerically and investigate how increasing hazard functions interact with other important features of modern monetary models, such as strategic complementarity in price setting, nonzero steady-state inflation, and two kinds of monetary policy rules.

The main analytical results show that, after a money level shock, both the impulse responses of the output and the aggregate price are determined by the accumulative distribution of price durations, whereas the inflation response is driven by the distribution of price durations. Consequently, changing the shape of the hazard function generates different macro dynamics through its effects on the price distributions. To illustrate this result, I compare three models based on increasing, constant, and decreasing hazard functions. The increasing-hazard model predicts more persistent responses in inflation and output than the other two models. Similarly, after a persistent money growth shock, the increasing-hazard model predicts a hump-shaped response of inflation, but a less persistent output response than the constant-hazard model.

Next, I extend my analysis to a more realistic setting. To this end, I solve the model combining nominal rigidity underlying increasing hazard functions with "real rigidity" [Ball and Romer (1990)] and positive steady-state inflation. I also test the robustness of my quantitative results under two kinds of monetary policy rules, namely a money growth rule and an interest rate rule [Taylor (1993)]. The numerical results show that the effects of the increasing hazard function, real rigidity, and nonzero steady-state inflation can reinforce each other, and thus the interaction between them gives rise to a powerful propagation mechanism for monetary shocks. The economic intuition behind this insight is that, on one hand, the increasing hazard function postpones the timing of price adjustments; on the other hand, strategic complementarity in price setting makes adjusting firms opt for optimal prices that are close to the average level of prices in the economy. As a result, strategic complementarity amplifies the delayed effect of the increasing hazard function. The presence of positive inflation accelerates relative price dispersion, which in turn enhances the interaction between strategic complementarity and the increasing hazard function further. All quantitative results hold under both monetary policy rules.

In the literature, the implications of nonconstant hazard functions have been studied in a number of papers. Wolman (1999) raises the issue that inflation dynamics should be sensitive to hazard functions underlying different pricing rules. He demonstrates this result in a simple partial equilibrium analysis. Kiley (2002) compares the Calvo and Taylor staggered price setting in a general equilibrium setup and shows that the output dynamics resulting from monetary shocks is both quantitatively and qualitatively quite different across the two pricing specifications unless one assumes a substantial level of real rigidity in the economy. Mash (2004) also studies a sticky price model with the general hazard function. He derives the generalized New Keynesian Phillips curve log-linearized around a constant-inflation steady state and shows numerically that this model can reconcile the tension between evidence on aggregate inertia at the macro level and frequent price setting at the micro level. Whelan (2007) and Sheedy (2010) focus on the implication of the shape of hazard functions for generating intrinsic inflation persistence. They show that the shape of the hazard function affects the signs and magnitudes of the coefficients on lagged inflation in the theoretical Phillips curve. This note makes a theoretical contribution that extends the analysis of nonconstant hazard functions in a broad class of general equilibrium New Keynesian models.

The remainder of the note is organized as follows: in Section 2, I present the firms' pricing problem with the generalized time-dependent pricing scheme; Section 3 shows results regarding insights gained from relaxing the constant hazard function underlying the Calvo assumption; Section 4 contains some concluding remarks.

2. THE FIRMS' PRICING PROBLEM

The model that I use in my analysis is a standard New Keynesian model except for a more general setting for sticky prices. Following Wolman (1999), I set up the staggered price adjustment process, which is characterized by a price reset hazard function. In contrast to the Calvo sticky price model, the probability of adjusting a price is allowed to vary across price durations, so that it nests the Calvo price setting as a special case. In the following, I outline the firm's pricing behavior under the general hazard function.[2]

There is a continuum of monopolistically competitive firms, indexed on the unit interval by j. Firms adjust their prices according to a stochastic timing scheme. At each period, firms draw from a price-resetting lottery, whose winning probabilities depend on the ages of their prices. This price adjustment scheme is summarized in term of a price reset hazard function, defined as

$$h_i = P(\text{adjust at } i \mid \text{survival to } i-1). \tag{1}$$

The hazard function gives the probability of price adjustments i periods since the last revision conditional on the price having been fixed for $i-1$ periods, for $i = 0, \ldots, I$, where I is the maximum price duration. In addition, I assume $0 \leqslant h_i \leqslant 1$, for all i.

Given the hazard function, the survival function is defined as

$$S_i = \prod_{n=0}^{i} (1 - h_n), \quad \forall i = 0, 1, \ldots I - 1. \tag{2}$$

The survival function gives the probability of a price being fixed at least for i periods.

Note that the dynamics of this stochastic price adjustment process can be modeled as a Markov chain with the price durations as its states and hazard rates defining its transition matrix. We can derive the invariant distribution of the states by calculating the eigenvector associated with the unit eigenvalue:

$$\theta_i = \frac{S_i}{\sum\limits_{n=0}^{I-1} S_n}, \quad \forall i = 0, 1, \cdots I-1. \tag{3}$$

According to equation (3), this Markov chain eventually converges to the stationary price-duration distribution and, once the invariant distribution is reached, there is no need to track the dynamics of price distribution any more. In the rest of the note, I assume that the price vintage distribution has converged to the invariant distribution and thus I will use equation (3) to calculate the aggregate price index.[3]

In a given period, the optimal price chosen should reflect the possibility that it will not be revised in the near future. Consequently, adjusting firms choose an optimal price that maximizes the discounted sum of profits in the time horizon over which the new price is expected to be fixed. Based on the generalized sticky price assumption discussed earlier, the maximization problem for a price setter can be expressed as

$$\max_{P_{j,t}} \Pi_t = E_t \sum\nolimits_{i=0}^{I-1} S_i Q_{t,t+i} (P_{j,t} - \mathrm{MC}_{j,t+i}) Y_{j,t+i|t}, \tag{4}$$

where $Q_{t,t+i}$ is the stochastic discount factor appropriate for discounting profits from t to $t+i$. $\mathrm{MC}_{j,t}$ denotes the nominal marginal costs of the firm. The adjusting firm maximizes discounted profits subject to demand for intermediate goods in period $t+i$ given that the firm resets the price in period t. The first-order necessary condition for the optimal price is

$$P^*_{j,t} = \frac{\eta}{\eta - 1} \frac{E_t \sum_{i=0}^{I-1} S_i [Q_{t,t+i} Y_{t+i} P^{\eta}_{t+i} \mathrm{MC}_{j,t+i}]}{E_t \sum_{i=0}^{I-1} S_i [Q_{t,t+i} Y_{t+i} P^{\eta}_{t+i}]}. \tag{5}$$

Intuitively, the optimal price is equal to the markup multiplied by a weighted sum of all future marginal costs, where weights depend on the survival probabilities, the discount factor, and future market conditions. In the Calvo case, where $S_i = (1-h)^i$, this equation reduces to the Calvo optimal pricing condition.

Finally, given the invariant price distribution θ_i, the aggregate price index can be rewritten as a distributed sum of past reset prices. Let P^*_{t-i} be the aggregate optimal price set i periods ago. The aggregate price index is obtained as

$$P_t = \left(\sum\nolimits_{i=0}^{I-1} \theta_i P^{*1-\eta}_{t-i} \right)^{\frac{1}{1-\eta}}. \tag{6}$$

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3. ANALYSIS

In the analysis, I derive some analytical and numerical results to study how the shape of the price reset hazard function affects the monetary transmission mechanism of sticky price models.

3.1. A Simple Model

To deliver some economic intuitions, I first use a simple version of the model, which is a system of three equations,

$$\hat{p}_t^* = E_t\left[\sum_{i=0}^{I-1}\theta_i\left(\hat{y}_{t+i}+\hat{p}_{t+i}\right)\right], \tag{7}$$

$$\hat{p}_t = \sum_{i=0}^{I-1}\theta_i\,\hat{p}_{t-i}^*, \tag{8}$$

$$\hat{m}_t - \hat{p}_t = \hat{y}_t, \tag{9}$$

where all variables are expressed as percentage deviations from the deterministic steady state without inflation. Equation (7) is the log-linearized optimal price given by (5), equation (8) represents the log-linearized aggregate price, and equation (9) corresponds to a log-linearized aggregate demand condition.

In addition, I consider two different monetary regimes. First, I assume that $\hat{m}_t$ follows a first-order autoregressive process:

$$\hat{m}_t = \rho\hat{m}_{t-1} + \epsilon_t. \tag{10}$$

Second, I consider a first-order autoregressive money growth rule,

$$\hat{\mu}_t = \rho\hat{\mu}_{t-1} + \epsilon_t, \tag{11}$$

where $\hat{\mu}_t = \hat{m}_t - \hat{m}_{t-1}$. In both stochastic processes, innovations $\epsilon_t \backsim$ i.i.d.$(0, \sigma_m^2)$ and ρ is the persistence parameter, which lies between zero and one. In this model, the only structural parameters are the distribution of price durations θ_i and the persistence parameter of monetary shock processes ρ. In the following, I will derive how aggregate dynamics of prices and the output gap depend on these parameters.

Effects of the level shock.

PROPOSITION 1. *Assume that the log deviations of nominal money supply follow an $AR(1)$ process specified in (10). Under the sticky price model summarized by equations (7), (8), and (9), impulse response functions to a money level* *shock are given by*

$$\mathrm{IR}\left(\hat{p}^{*}_{t+n}\right) = \bar{F}\rho^{n}, \tag{12}$$

$$\mathrm{IR}(\hat{p}_{t+n}) = \bar{F}\sum_{i=0}^{n}\theta_{i}\rho^{n-i}, \tag{13}$$

$$\mathrm{IR}\left(\hat{y}_{t+n}\right) = \rho^{n}\left(1 - \bar{F}\sum_{i=0}^{n}\frac{\theta_{i}}{\rho^{i}}\right), \tag{14}$$

where

$$\bar{F} = \sum_{i=0}^{I-1}\theta_{i}\rho^{i}. \tag{15}$$

Proof. See Appendix A. ∎

Several interesting insights stand out from this proposition. First, the impulse response function of the optimal price equation (12)] is affected not only by the rate (ρ) at which the monetary disturbance decays over time, but also by a scaling factor $\bar{F}$, which captures the firm's forward-looking pricing behavior. As seen in equation (15), $\bar{F}$ is smaller than one when $\rho < 1$. The optimal price adjustment is scaled down, because firms expect that the money level shock will dissipate over time at the rate of ρ. When $\rho = 1$, implying a permanent rise in the money shock, the scaling factor is equal to one, and the optimal price will jump immediately to the new level that is one-to-one to the size of the money level shock.

Second, the impulse response of the aggregate price is affected by the accumulative distribution of price durations. This is intuitive, because the accumulative distribution of price durations represents the fraction of firms that are allowed to react to the shock over time. Together with the size of the response, they determine the response of the aggregate price to a money level shock. An instructive case emerges when the shock is permanent ($\rho = 1$). In this case, the shape of the aggregate price response is determined solely by the accumulative distribution of price durations,

$$\mathrm{IR}\left(\hat{p}_{t+n}\right) = \sum_{i=0}^{n}\theta_{i}. \tag{16}$$

Furthermore, one can show that the impulse response of inflation to a permanent money level shock is equal to the distribution of price durations:

$$\begin{aligned}\mathrm{IR}(\hat{\pi}_{t+n}) &= \mathrm{IR}\left(\hat{p}_{t+n}\right) - \mathrm{IR}\left(\hat{p}_{t+n-1}\right)\\ &= \left[\sum_{i=0}^{n}\theta_{i} - \sum_{i=0}^{n-1}\theta_{i}\right]\\ &= \theta_{n}.\end{aligned} \tag{17}$$

Third, the real effect of the money level shock is just the flip side of its nominal effect. Again, the shape of the accumulative price distribution plays a central role in determining the output response to a money level shock. Using the example of

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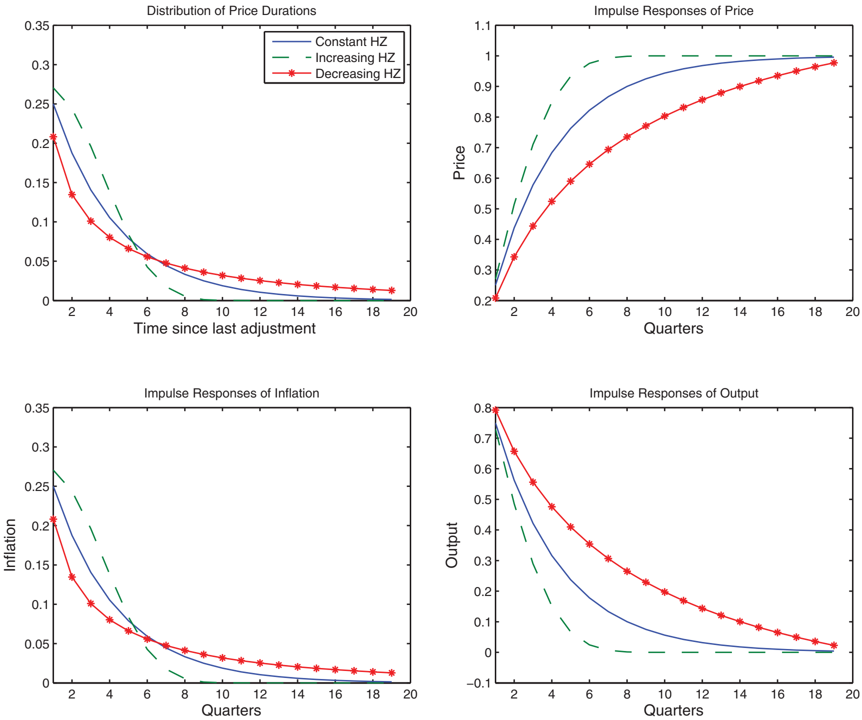

FIGURE 1. Effects of a permanent money level shock ($\rho = 1$).

a permanent shock again, the real effect of a money level shock can be expressed as

$$\mathrm{IR}\left(\hat{y}_{t+n}\right) = 1 - \sum_{i=0}^{n} \theta_i. \tag{18}$$

The analytical results shed light on the central role played by the price distributions in linking the shape of the price reset hazard function and impulse responses of aggregate variables. To illustrate this mechanism, in Figure 1, I plot price distributions and impulse response functions to a permanent money level shock as the result of three different hazard functions. The first hazard function is constant at a level of 25%, implying that the average price duration is four quarters (solid line). The second hazard function is linearly increasing in the time since last adjustment (dashed line), and the third hazard function is linearly decreasing in the time since last adjustment (starred line).[4]

The top left figure depicts the distribution of price durations implied by different hazard functions. With the constant hazard function, the probability mass falls geometrically as the price duration gets longer. In contrast, the price distribution of the increasing hazard function drops more slowly at the shorter durations, but faster at the longer durations, whereas the price distribution of the decreasing hazard function follows the opposite pattern. According to equation (17), the

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shape of the price distribution determines the inflation response to the money level shock. In the top right panel, we see that responses of the aggregate price converge to the new level fastest in the increasing-hazard model, and slowest in the decreasing-hazard model. Because the output response is also determined by the accumulative distribution, as seen in the bottom right panel, the decreasing-hazard model predicts a more persistent response of output to a permanent money level shock than the other two models. The bottom left panel illustrates how the change in the slope of hazard functions affects the inflation dynamics through its effects on the accumulative distribution of price durations. The increasing-hazard model predicts a more persistent inflation response than the constant- and decreasing-hazard models.

Effects of the growth shock.

PROPOSITION 2. *Assume that nominal money supply growth rates follow an $AR(1)$ process as specified in (11). Under the sticky price model summarized by equations (7), (8), and (9), the impulse response function of inflation to a money growth disturbance is given by*

$$\mathrm{IR}(\pi_{t+n}) = \sum_{k=0}^{I-1} \theta_k \sum_{i=0}^{I-1} \theta_i \rho^{n+i-k}, \qquad \forall n+i-k \geq 0. \tag{19}$$

Proof. See Appendix B. ■

This proposition is intended to show how the price distribution affects inflation dynamics after a money growth shock. Again, it is instructive to study a permanent money growth shock first. In this case, the impulse response of inflation is equal to a complex summation of the price distribution. However, in the Calvo model, when $\theta_i = (1-\tau)\tau^i$, where $0 < \tau < 1$ denotes the nonadjustment rate, the impulse response function of inflation becomes

$$\mathrm{IR}(\pi_{t+n}) = (1-\tau)^2 \sum_{k=0}^{\infty} \tau^k \sum_{i=0}^{\infty} \tau^i = 1. \tag{20}$$

This implies that inflation reacts immediately to the permanent money growth shock and the size of the response is equal to the size of the money growth shock.

Figure 2 illustrates the transitional dynamics of disinflationary scenarios predicted by the increasing- and constant-hazard models. In this simulation, I assume that, before $t = 1$, output is at its steady state level, whereas inflation is equal to 2.5% per quarter. At $t = 1$, the central bank changes the growth rate of nominal money stock from 2.5% per quarter to zero and keep it constant permanently. In the left panel, the constant-hazard model predicts a costless disinflation; i.e., inflation drops immediately to the new level, whereas real output is not affected at all. This result recapitulates the famous criticism of the Calvo sticky price model [e.g., Ball (1994) and Mankiw (2001)]. In contrast, the increasing-hazard model predicts a recession lasting up to eight periods during the slow disinflationary process.

Figure 3 further compares the effects of the constant- and increasing-hazard functions after a persistent money growth shock ($\rho = 0.5$). The left figure

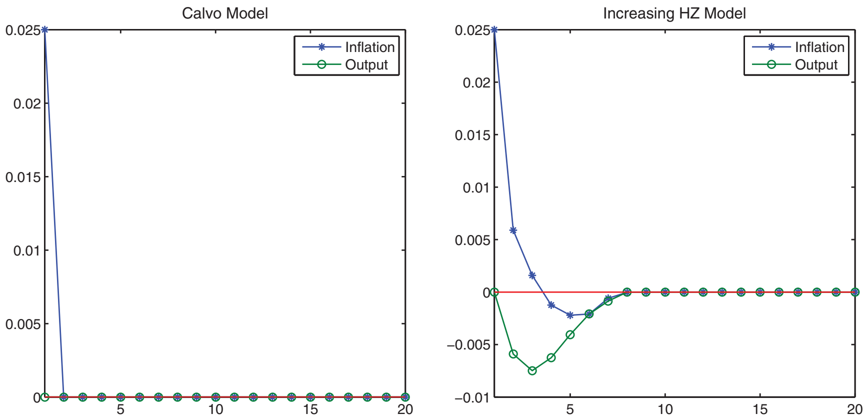

FIGURE 2. Effects of a permanent money growth shock ($\rho = 1$).

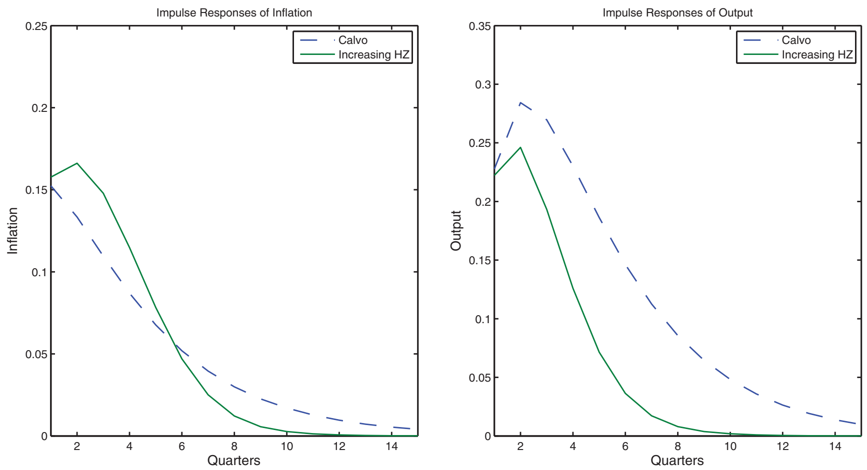

FIGURE 3. Effects of a persistent money growth shock ($\rho = 0.5$).

illustrates the impulse responses of inflation. Whereas the Calvo model predicts a monotonically declining response to the money growth shock (the dashed line), the increasing-hazard model generates a hump-shaped response of inflation (the solid line). Even though this hump is not very eminent, peaking only in the second period after the impact of the shock, it represents a qualitative improvement toward the empirically plausible shape of inflation response to a monetary shock. In the following sections, I will show that adding other features to the model will help enhance the delayed effect of the increasing hazard function. The right figure depicts the impulse response function of the output gap to a money growth shock. The Calvo model predicts a more persistent real effect of money growth shocks than the increasing-hazard model.

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To sum up the analytical results, I find that there is a close correspondence between the impulse responses of output and aggregate price, because both of them are mainly determined by the accumulative distribution of price durations. In contrast, there is a trade-off between persistent responses of output and inflation as the slope of the hazard function changes. Increasing-hazard functions enhance inflation inertia, but they weaken the persistent response of output.

3.2. Robustness Checks

To check the robustness of the analytical results, I solve the whole log-linearized model numerically. Specifically, I conduct robustness checks to study the interactions between the generalized nominal rigidity and (1) the strategic complementarity in price setting; (2) nonzero steady-state inflation; and (3) the Taylor-style interest rate rules. The main findings are summarized as follows.[5]

First, regarding to the interaction between the shape of the hazard function and the real rigidity, I find that, in separation, the nominal rigidity underlying increasing hazard functions provides a more powerful propagation mechanism in generating macro persistence than the real rigidity. When they are used used together, however, strategic complementarity amplifies the delayed effect of the increasing hazard function, leading to a more salient hump-shaped impulse response of inflation to a monetary shock.

Second, I study the effects of nonzero steady-state inflation in the increasing-hazard model. Even without strategic complementarity in price setting, positive steady-state inflation enhances the ability of the increasing-hazard model to generate persistent macro dynamics. Combining all three features, the increasing hazard function, a modest level of real rigidity, and 2% trend inflation, accounts for the realistic level of persistence in both inflation and output. The intuition for this result is that the presence of positive inflation accelerates relative price dispersion, which in turn enhances the interaction between strategic complementarity and the increasing hazard function.

Third, given the ubiquitous Taylor rule in the modern monetary policy research, I check the robustness of my quantitative results under a simple Taylor rule. The simulation results show that increasing hazard functions enhance the persistence of macro dynamics under the Taylor rule. The Calvo model with a Taylor rule has very weak inner propagation for persistence. By comparison, the sticky price model with features discussed in the preceding replicates the hump-shaped impulse response of inflation and predicts important real effects of money under the Taylor rule.

4. CONCLUSION

The central purpose of this note is to evaluate effects of the price reset hazard function on the monetary transmission mechanism of the sticky price model. I present new analytical and numerical results, regarding mechanisms through which the hazard function affects aggregate dynamics and interacts with other important features in the sticky price model. First, I show that the shape of the price distribution greatly affects the persistence of output and inflation. The distribution of price durations lies in the center of the propagation mechanism. Second, I show that nominal rigidity underlying the increasing hazard function is more successful than real rigidity in generating persistence of inflation and the output gap. Furthermore, the interaction between the increasing hazard function and strategic complementarity in price setting provides a powerful propagation mechanism for monetary shocks.

Admittedly, the model I use has weaknesses. Above all, the timing decision is not endogenous, so that important dynamics resulting from the extensive margin of price adjustments is completely missing in my analysis. To study this effect, one would need to work with a micro-founded state-dependent pricing model [e.g., Golosov and Lucas (2007)]. Although the state-dependent pricing models are superior in principle, because of their tractability challenges, one can only solve them at the expense of realism on other aspects of the economy. I view my current modeling choice as a reasonable compromise and reserve more micro-founded analysis for future research.

NOTES

1. See, e.g., Alvarez (2007), Klenow and Kryvtsov (2008), and Nakamura and Steinsson (2008).

2. The derivation of the whole model is available in an Online Appendix at https://sites.google.com/site/yao0fang/.

3. This feature of the time-dependent pricing model gives rise to the high tractability, but at the same time it suffers the deficiency of ignoring the extensive margin dynamics.

4. Here, I parameterize the hazard function in a parsimonious way. In particular, the functional form I apply is the hazard function of the distribution, which has two parameters:

$$h(j) = \frac{\tau}{\lambda}\left(\frac{j}{\lambda}\right)^{\tau-1}.$$

λ is the scale parameter, which controls the average duration of the price adjustment, whereas τ is the shape parameter, which determines the monotonic property of the hazard function. I choose λ so that it implies an average price duration of four quarters. The shape parameter is set at values in $\{1, 2, 0.5\}$ to represent constant, increasing, and decreasing hazard functions.

5. More detailed discussion of my calibration exercise and the numerical results is available in an Online Appendix at https://sites.google.com/site/yao0fang/.

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APPENDIX

A.1. PROOF OF PROPOSITION 1

First, based on the assumption that the nominal money supply follows an $AR(1)$ process,

$$\hat{m}_t = \rho \hat{m}_{t-1} + \epsilon_t, \text{ where } \epsilon_t \backsim \text{i.i.d.}(0, \sigma_m^2). \tag{A.1}$$

The impulse response function of the nominal money supply to an innovation ϵ_t is

$$\text{IR}\,(\hat{m}_{t+n}) = \rho^n. \tag{A.2}$$

Using the definition of impulse response function, I derive the impulse response function of the optimal price to a monetary level shock as

$$\begin{aligned} \text{IR}\left(\hat{p}^*_{t+n}\right)\epsilon_t &= E_t(\hat{p}^*_{t+n}) - E_{t-1}(\hat{p}^*_{t+n}) \\ &= \sum_{i=0}^{I-1} \theta_i \left[E_t\,(\hat{m}_{t+n+i}) - E_{t-1}\,(\hat{m}_{t+n+i})\right] \\ &= \sum_{i=0}^{I-1} \theta_i \text{IR}\,(\hat{m}_{t+n+i})\,\epsilon_t, \\ \text{IR}\left(\hat{p}^*_{t+n}\right) &= \rho^n \sum_{i=0}^{I-1} \theta_i \rho^i. \end{aligned} \tag{A.3}$$

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Following the aggregate price equation (8), I derive the impulse response function of the aggregate price as follows:

$$
\begin{aligned}
\mathrm{IR}\,(\hat{p}_{t+n})\,\epsilon_t &= E_t(\hat{p}_{t+n}) - E_{t-1}(\hat{p}_{t+n}) \\
&= \sum_{i=0}^{n} \theta_i E_t(\hat{p}^*_{t+n-i}) - \sum_{i=0}^{n} \theta_i E_{t-1}(\hat{p}^*_{t+n-i}) \\
&= \sum_{i=0}^{n} \theta_i \left[E_t\left(\hat{p}^*_{t+n-i}\right) - E_{t-1}(\hat{p}^*_{t+n-i}) \right] \\
&= \sum_{i=0}^{n} \theta_i \mathrm{IR}\left(\hat{p}^*_{t+n-i}\right)\epsilon_t, \\
\mathrm{IR}\,(\hat{p}_{t+n}) &= \sum_{i=0}^{n} \theta_i \mathrm{IR}\left(\hat{p}^*_{t+n-i}\right).
\end{aligned}
\tag{A.4}
$$

Plugging (A.3) into (A.4) yields

$$
\begin{aligned}
\mathrm{IR}\,(\hat{p}_{t+n}) &= \sum_{i=0}^{n} \theta_i \left[\rho^{n-i} \sum_{i=0}^{I-1} \theta_i \rho^i \right] \\
&= \underbrace{\sum_{i=0}^{I-1} \theta_i \rho^i}_{\bar{F}} \sum_{i=0}^{n} \theta_i \rho^{n-i} \\
&= \bar{F} \sum_{i=0}^{n} \theta_i \rho^{n-i}.
\end{aligned}
\tag{A.5}
$$

Finally, using equation (9), I derive the impulse response function of real output as follows:

$$
\begin{aligned}
\mathrm{IR}\,(\hat{y}_{t+n}) &= \mathrm{IR}\,(\hat{m}_{t+n}) - \mathrm{IR}\,(\hat{p}_{t+n}) \\
&= \rho^n - \bar{F} \sum_{i=0}^{n} \theta_i \rho^{n-i} \\
&= \rho^n \left(1 - \bar{F} \sum_{i=0}^{n} \frac{\theta_i}{\rho^i} \right).
\end{aligned}
\tag{A.6}
$$

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B.1. PROOF OF PROPOSITION 2

First, inflation, defined as $\hat{\pi}_t = \hat{p}_t - \hat{p}_{t-1}$, can be expressed as

$$\begin{aligned}\hat{\pi}_t &= \hat{p}_t - \hat{p}_{t-1} \\ &= \sum_{k=0}^{I-1} \theta_k \left(\hat{p}^*_{t-k} - \hat{p}^*_{t-k-1}\right)\end{aligned} \quad \textbf{(B.1)}$$

Then, substituting equation (7) for $\hat{p}^*_t$, I obtain

$$\begin{aligned}\hat{\pi}_t &= \sum_{k=0}^{I-1} \theta_k \left(\hat{p}^*_{t-k} - \hat{p}^*_{t-k-1}\right) \\ &= \sum_{k=0}^{I-1} \theta_k \sum_{i=0}^{I-1} \theta_i E_t \left(\hat{m}_{t+i-k} - \hat{m}_{t+i-k-1}\right) \\ &= \sum_{k=0}^{I-1} \theta_k \sum_{i=0}^{I-1} \theta_i E_t \left[\Delta \hat{m}_{t+i-k}\right].\end{aligned} \quad \textbf{(B.2)}$$

Based on the assumption that nominal money growth rate follows an $AR(1)$ process,

$$\Delta \hat{m}_t = \rho \Delta \hat{m}_{t-1} + \epsilon_t, \quad \text{where } \epsilon_t \backsim \text{i.i.d.}(0, \sigma_m^2),$$

we can derive the impulse response function of money growth shock to an innovation ϵ_t as

$$\text{IR}\left(\Delta \hat{m}_{t+n}\right) = \rho^n. \quad \textbf{(B.3)}$$

It follows that

$$\begin{aligned}\text{IR}\left(\hat{\pi}_{t+n}\right) &= \sum_{k=0}^{I-1} \theta_k \sum_{i=0}^{I-1} \theta_i \text{IR}\left(\Delta \hat{m}_{t+n+i-k}\right) \\ &= \sum_{k=0}^{I-1} \theta_k \sum_{i=0}^{I-1} \theta_i \rho^{n+i-k}, \qquad \forall n+i-k \geq 0.\end{aligned} \quad \textbf{(B.4)}$$

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