

HIGH INFLATION, SEASONAL COMMODITIES, AND ANNUAL INDEX NUMBERS

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This paper studies the problems of measuring economic growth under conditions of high inflation. Traditional bilateral index number theory implicitly assumes that variations in the price of a commodity within a period can be ignored. To justify this assumption under conditions of high inflation, the accounting period must be shortened to a quarter, a month, or possibly a week. However, once the accounting period is less than a year, the problem of seasonal commodities is encountered; i.e., in some subannual periods, many seasonal commodities will be unavailable and hence the usual bilateral index number theory cannot be applied. The paper systematically reviews the problems of index number construction when there are seasonal commodities and high inflation. Various index number formulas are justified from the viewpoint of the economic approach to index number theory by making separability assumptions on consumers' intertemporal preferences. We find that accurate economic measurement under conditions of high inflation is very complex.

Keywords: Aggregation of Commodities, Consumer Theory, Index Numbers, Inflation, Seasonal Adjustment

1. INTRODUCTION

Ever since the German hyperinflation of the 1920's, accountants have noted that high inflation causes historical cost accounting measures of income and wealth to become virtually useless. One way to restore credibility to business accounts would be to deflate current values by appropriate price indexes. However, the construction of price indexes is not straightforward under conditions of high inflation, particularly when seasonal commodities are present. Recently, Hill (1996) addressed some of these problems in the context of adapting the United Nations (1993) system of national accounts to high-inflation situations. This paper can be regarded as an extension of Hill's contributions, taking account of seasonal commodities.

Before describing the contents of the paper, we address some preliminary questions.

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What are seasonal commodities? They are commodities that either (1) are not available during certain seasons or (2) are always available but there are fluctuations in prices or quantities that are synchronized with the season or time of year.¹

What are the sources of seasonal fluctuations in prices or quantities? There are two main sources²: climate and custom. In the first category, fluctuations in temperature, precipitation, wind, and hours of daylight cause fluctuations in demand for things such as ice skates, fuel oil, umbrellas, snow tires, seasonal clothing, and electricity. In more generic terms, climatic changes cause fluctuations in energy demands, recreational activities, and food consumption patterns. In fact, seasonal fluctuations are present in almost all sectors of most economies.

What are the implications of seasonality for index number theory? If we break the year up into M seasons (e.g., $M = 4$ if the season is a quarter or $M = 12$ if the season is a month), then the existence of type-1 seasonal commodities in the set of goods that we are aggregating over means the dimension of the commodity space will not be constant. Thus it will be impossible to apply the usual bilateral index number theory.³

Even if all commodities were available in all seasons, the existence of type-2 seasonal commodities may mean that bilateral indexes that are exact for an underlying utility function cannot be justified. The economic approach assumes that the seasonal aggregator function is the same in each season being compared, which is not a reasonable assumption if climate and customs interact with tastes. This suggests that type-2 seasonal commodities should be classified further into subtypes a and b.

A type-2 seasonal commodity is defined to be of subtype a if its seasonal-quantity fluctuations can be rationalized by utility-maximizing behavior over a set of seasons where the prices fluctuate but the utility aggregator function remains unchanged, and of type 2b if its quantity fluctuations cannot be rationalized by maximizing an unchanging utility function over the periods in question.⁴ An example may be helpful. As harvest conditions vary, the price of potatoes in my local supermarket varies and I purchase more potatoes as the price falls and fewer as it rises. On the other hand, the price of beer remains quite constant throughout the year but my consumption increases greatly during the summer. Weather shifts my seasonal demand function for beer which is a type-2b seasonal commodity and but not for potatoes which are a type-2a seasonal commodity. The usual economic approach to index number theory can be applied to type-2a seasonal commodities but not to type-2b ones.

The problem of index number construction when there are seasonal commodities has a long history.⁵ However, what has been missing is an exposition of the assumptions on the consumer's utility function⁶ that are required to justify a particular formula. We systematically list separability assumptions on intertemporal preferences that can be used to justify various seasonal index number formulas from the viewpoint of the economic approach to index number theory.⁷

We now set out the general model of consumer behavior that we will specialize in subsequent sections. Suppose that there are M seasons in the year and the Statistical

Agency has collected price and quantity data on the consumer's purchases for $1 + T$ years. Suppose further that the dimension of the commodity space in each season remains constant over the $T + 1$ years; i.e., season m has N_m commodities for $m = 1, \dots, M$. For season m of year t , we denote the vector of positive prices facing the consumer by $\mathbf{p}^{tm} \equiv [p_1^{tm}, p_2^{tm}, \dots, p_{N_m}^{tm}]$ and the vector of commodities consumed by $\mathbf{q}^{tm} \equiv [q_1^{tm}, q_2^{tm}, \dots, q_{N_m}^{tm}]$. It is convenient to have notation for the annual price and quantity vectors, and so, we define these by

$$\mathbf{p}^t \equiv [\mathbf{p}^{t1}, \mathbf{p}^{t2}, \dots, \mathbf{p}^{tM}]; \quad \mathbf{q}^t \equiv [\mathbf{q}^{t1}, \mathbf{q}^{t2}, \dots, \mathbf{q}^{tM}]; \quad t = 0, 1, \dots, T. \quad (1)$$

To apply the economic approach to index number theory, it is necessary to assume that the observed quantities of q_n^{tm} are a solution to an optimization problem involving the observed prices p_n^{tm} . We assume that the intertemporal quantity vector $[\mathbf{q}^0, \mathbf{q}^1, \dots, \mathbf{q}^T]$ is a solution to the following intertemporal utility maximization problem:

$$\max_{\mathbf{x}^0, \mathbf{x}^1, \dots, \mathbf{x}^T} \left\{ U(\mathbf{x}^0, \mathbf{x}^1, \dots, \mathbf{x}^T) : \sum_{t=0}^T \delta_t \mathbf{p}^t \cdot \mathbf{x}^t = W \right\}, \quad (2)$$

where $\mathbf{x}^t \equiv [x^{t1}, x^{t2}, \dots, x^{tm}]$ and each seasonal quantity vector \mathbf{x}^{tm} has the dimensionality of \mathbf{q}^{tm} , $\mathbf{p}^t \cdot \mathbf{x}^t \equiv \sum_{m=1}^M \mathbf{p}^{tm} \cdot \mathbf{x}^{tm}$ and $\mathbf{p}^{tm} \cdot \mathbf{x}^{tm} \equiv \sum_{n=1}^{N_m} p_n^{tm} x_n^{tm}$, U is the consumer's intertemporal preference function (assumed to be continuous and increasing), $\delta_t > 0$ is an annual discount factor and wealth W is the consumer's current and expected future discounted income viewed from the perspective of the beginning of year 0. If the consumer can borrow and lend at a constant annual nominal interest rate r , then $\delta_0 \equiv 1$ and

$$\delta_t = 1/(1+r)^t, \quad t = 1, 2, \dots, T. \quad (3)$$

Because we are assuming that the quantity vector $[\mathbf{q}^0, \mathbf{q}^1, \dots, \mathbf{q}^T]$ is a solution to (2), it must satisfy the intertemporal budget constraint in (2), and so, we can replace W by⁸

$$W \equiv \sum_{t=0}^T \delta_t \mathbf{p}^t \cdot \mathbf{q}^t. \quad (4)$$

The economic approach to index numbers requires strong assumptions. Some advantages of this approach are: (i) It allows for substitution in response to changes in the prices, (ii) it provides a concrete framework that can be used to assess operational alternatives that occur when a Statistical Agency constructs an index number, and (iii) it leads to definite recommendations about the choice of functional forms for index number formulas which then can be evaluated from other perspectives, such as the test approach.

Having made our basic economic assumptions [namely, that the observed sequence of annual quantity vectors $[\mathbf{q}^0, \mathbf{q}^1, \dots, \mathbf{q}^T]$ solves (2) with W defined by (4)], now make additional assumptions on the structure of the intertemporal utility function U .

In Section 2, we show how equation (2) can be specialized to yield the annual indexes first proposed by Mudgett (1955, p. 97) and Stone (1956, pp. 74–75). In Section 3, we note that our Hicksian intertemporal utility maximization problem (2) needs to be modified when inflation is high. The annual discount factors δ_t that appear in (2) and (4) do not provide an adequate approximation to the consumer's intertemporal problem with even moderate inflation between seasons. We need to introduce between-season intrayear discount rates as well. In Section 4, we show that when there are seasonal commodities, the use of annual sums of seasonal quantities and the corresponding annual unit values are unsatisfactory as annual quantity and price aggregates. Section 5 concludes.

2. CONSTRUCTION OF ANNUAL INDEXES UNDER CONDITIONS OF LOW INFLATION

In the Mudgett (1955, p. 97) and Stone (1956, pp. 74–75) approach to annual index numbers when there are seasonal commodities, we need to restrict the consumer's intertemporal utility function U as follows: there exist F and f such that

$$U(\mathbf{x}^0, \mathbf{x}^1, \dots, \mathbf{x}^T) = F[f(\mathbf{x}^0), f(\mathbf{x}^1), \dots, f(\mathbf{x}^T)], \quad (5)$$

where f is a linearly homogeneous, increasing, and concave annual utility function⁹ and F is an intertemporal utility function that is increasing and continuous in its $T + 1$ annual utility arguments. The annual utility function f is assumed to be unchanging over time.

If $\mathbf{q}^0, \mathbf{q}^1, \dots, \mathbf{q}^T$ solves (2) with W defined by (4) and U defined by (5), then it can be seen that \mathbf{q}^t , the observed annual consumption vector for year t , is a solution to the following year t utility maximization problem:

$$\max_{\mathbf{x}^t} \{f(\mathbf{x}^t) : \mathbf{p}^t \cdot \mathbf{x}^t = \mathbf{p}^t \cdot \mathbf{q}^t\} = f(\mathbf{q}^t); \quad t = 0, 1, \dots, T. \quad (6)$$

Now we are in a position to apply the theory of exact index numbers.¹⁰ Assume that the bilateral quantity index $Q(\mathbf{p}^s, \mathbf{p}^t, \mathbf{q}^s, \mathbf{q}^t)$ is exact for the linearly homogeneous aggregator function f . Then we have

$$f(\mathbf{q}^t)/f(\mathbf{q}^s) = Q(\mathbf{p}^s, \mathbf{p}^t, \mathbf{q}^s, \mathbf{q}^t); \quad 0 \leq s, t \leq T. \quad (7)$$

As an example of (7), suppose that the annual aggregator function f is $f(\mathbf{x}) \equiv (\mathbf{x} \cdot \mathbf{A}\mathbf{x})^{1/2}$, where \mathbf{A} is a symmetric $N^* \times N^*$ matrix of constants satisfying certain regularity conditions. This functional form is flexible; i.e., it can provide a second-order approximation to an arbitrary differentiable linearly homogeneous function. The quantity index that is exact for this functional form is the Fisher (1922) ideal quantity index Q_F :

$$Q_F(\mathbf{p}^s, \mathbf{p}^t, \mathbf{q}^s, \mathbf{q}^t) \equiv [\mathbf{p}^t \cdot \mathbf{q}^t \mathbf{p}^s \cdot \mathbf{q}^t / \mathbf{p}^t \cdot \mathbf{q}^s \mathbf{p}^s \cdot \mathbf{q}^s]^{1/2}. \quad (8)$$

Because Q_F is exact for a flexible functional form, it is a superlative index.

Given any bilateral quantity index Q , its associated price index P can be defined as follows using Fisher's (1911, p. 403) weak factor reversal test:

$$P(\mathbf{p}^s, \mathbf{p}^t, \mathbf{q}^s, \mathbf{q}^t) \equiv \mathbf{p}^t \cdot \mathbf{q}^t / \mathbf{p}^s \cdot \mathbf{q}^s Q(\mathbf{p}^s, \mathbf{p}^t, \mathbf{q}^s, \mathbf{q}^t). \quad (9)$$

Given any linearly homogeneous, increasing, and concave aggregator function f , its dual unit cost function can be defined for strictly positive prices $\mathbf{p} \gg \mathbf{0}_{N^*}$ as

$$c(\mathbf{p}) \equiv \min_{\mathbf{x}} \{\mathbf{p} \cdot \mathbf{x} : f(\mathbf{x}) = 1\}. \quad (10)$$

When the utility function f is linearly homogeneous, the Konüs (1939) price index between periods s and t reduces to the ratio of the unit cost functions evaluated at the period s and t prices, $c(\mathbf{p}^t)/c(\mathbf{p}^s)$. If the bilateral quantity index Q is exact for f , then its companion bilateral price index P defined by (9) is exact for the unit cost function c dual to f ; i.e., in addition to (7), we also have

$$c(\mathbf{p}^t)/c(\mathbf{p}^s) = P(\mathbf{p}^s, \mathbf{p}^t, \mathbf{q}^s, \mathbf{q}^t); \quad 0 \leq s, t \leq T. \quad (11)$$

As an example of (11), suppose that the annual aggregator function is the homogeneous quadratic aggregator $f(\mathbf{x}) \equiv (\mathbf{x} \cdot A\mathbf{x})^{1/2}$ and c is its unit cost dual function. Then (11) holds with $P = P_F$ where the Fisher ideal price index P_F is defined by

$$P_F(\mathbf{p}^s, \mathbf{p}^t, \mathbf{q}^s, \mathbf{q}^t) \equiv [\mathbf{p}^t \cdot \mathbf{q}^t \mathbf{p}^t \cdot \mathbf{q}^s / \mathbf{p}^s \cdot \mathbf{q}^t \mathbf{p}^s \cdot \mathbf{q}^s]^{1/2}. \quad (12)$$

The above analysis seems to indicate that the construction of annual price and quantity indexes when there are seasonal commodities is straightforward: simply regard each physical commodity in each season as a separate economic commodity and apply ordinary index number theory to the enlarged annual commodity space. However, this does not work when there is severe or even moderate inflation between seasons within the year.

3. CONSTRUCTION OF ANNUAL INDEXES UNDER CONDITIONS OF HIGH INFLATION

In Section 2, a discount rate δ_t was used to make the prices in year t comparable to the base-year prices. With low inflation, this is an acceptable approximation to the consumer's intertemporal choice problem. However, when inflation is high, we can no longer neglect interseasonal interest rates.

Consider the budget constraint in (2). We now interpret δ_t as the discount factor that makes one dollar at the beginning of year t equivalent to one dollar at the beginning of year 0. From the beginning of t to the middle of season m in t , another discount factor is required, e.g., ρ_{tm} , which will make a dollar at the beginning of

t equivalent to a dollar in the middle of season m of t . Thus the budget constraint in (2) must be replaced by the following intertemporal constraint:

$$\sum_{t=0}^T \sum_{m=1}^M \delta_t \rho_{tm} \mathbf{p}^{tm} \cdot \mathbf{x}^{tm} = W, \tag{13}$$

where \mathbf{p}^{tm} and \mathbf{q}^{tm} are the (spot) price and quantity vectors for season m of year t and \mathbf{x}^{tm} is a year t , season m , vector of decision variables. Similarly, definition (4) for wealth W is now

$$W \equiv \sum_{t=0}^T \sum_{m=1}^M \delta_t \rho_{tm} \mathbf{p}^{tm} \cdot \mathbf{q}^{tm}. \tag{14}$$

Making assumption (5) again, we now can derive the following counterparts to (6):

$$\begin{aligned} \max_{\mathbf{x}^{t1}, \dots, \mathbf{x}^{tM}} \left\{ f(\mathbf{x}^{t1}, \dots, \mathbf{x}^{tM}) : \sum_{m=1}^M \rho_{tm} \mathbf{p}^{tm} \cdot \mathbf{x}^{tm} = \sum_{m=1}^M \rho_{tm} \mathbf{p}^{tm} \cdot \mathbf{q}^{tm} \right\} \\ = f(\mathbf{q}^{t1}, \dots, \mathbf{q}^{tM}) \equiv f(\mathbf{q}^t); \quad t = 0, 1, \dots, T, \end{aligned} \tag{15}$$

where the annual year t observed quantity vector \mathbf{q}^t is equal to $[\mathbf{q}^{t1}, \dots, \mathbf{q}^{tM}]$ and \mathbf{q}^{tm} is the season m , year t observed quantity vector.

Note that the seasonal discount factors ρ_{tm} appear in the constraints of the annual utility maximization problems (15). Define the vector of year t , season m discounted (to the beginning of year t) prices \mathbf{p}^{tm*} as

$$\mathbf{p}^{tm*} \equiv \rho_{tm} \mathbf{p}^{tm}; \quad t = 0, 1, \dots, T; \quad m = 1, \dots, M. \tag{16}$$

The constraints in (15) now can be written as $\mathbf{p}^{t*} \cdot \mathbf{x}^t = \mathbf{p}^{t*} \cdot \mathbf{q}^t$, where the year t discounted price vector is defined as $\mathbf{p}^{t*} \equiv [\mathbf{p}^{t1*}, \mathbf{p}^{t2*}, \dots, \mathbf{p}^{tM*}]$. Now we can repeat the analysis in the preceding section associated with equations (7–12): We need only replace the year t spot price vectors \mathbf{p}^t by the year t discounted vectors \mathbf{p}^{t*} . In particular, assuming that the bilateral index number formula Q is exact for the homogeneous aggregator function f and its dual unit cost function c , we have the following counterparts to (7) and (11):

$$f(\mathbf{q}^t)/f(\mathbf{q}^s) = Q(\mathbf{p}^{s*}, \mathbf{p}^{t*}, \mathbf{q}^s, \mathbf{q}^t); \quad 0 \leq s, t \leq T, \tag{17}$$

$$c(\mathbf{p}^{t*})/c(\mathbf{p}^{s*}) = P(\mathbf{p}^{s*}, \mathbf{p}^{t*}, \mathbf{q}^s, \mathbf{q}^t); \quad 0 \leq s, t \leq T, \tag{18}$$

where P is the bilateral price index associated with the quantity index Q defined using the counterpart to (9) which replaces \mathbf{p}^s and \mathbf{p}^t by \mathbf{p}^{s*} and \mathbf{p}^{t*} .

Thus our approach to constructing annual index numbers when there are seasonal commodities and high inflation is to use the Mudgett-Stone annual indexes with

the year t season m spot prices \mathbf{p}^{tm} replaced by the within-year inflation-adjusted prices \mathbf{p}^{tm*} defined by (16).

To see why we must use inflation-adjusted prices in our annual index number formulas, consider the situation in which there is a hyperinflation and we are using the Fisher quantity index defined by (8). If the hyperinflation takes place only in season m of year t , then the Paasche part $\mathbf{p}^t \cdot \mathbf{q}^t / \mathbf{p}^t \cdot \mathbf{q}^s$ of the Fisher index will be approximately equal to $\mathbf{p}^{tm} \cdot \mathbf{q}^{tm} / \mathbf{p}^{tm} \cdot \mathbf{q}^{sm}$; i.e., only consumption in season m of year t , \mathbf{q}^{tm} , and consumption in season m of year s , \mathbf{q}^{sm} , will enter into the comparison between years s and t if spot prices \mathbf{p}^{tm} are used in place of the discounted prices \mathbf{p}^{tm*} . This is obviously undesirable.

Note that $\rho_{tm+1} / \rho_{tm} \equiv 1 + r_{tm}$ for $m = 1, 2, \dots, M - 1$, where r_{tm} is the average interest rate faced when borrowing or lending money from the middle of season m to $m + 1$ in t . If prices are expected to increase in $m + 1$ compared to m , then the nominal interest rate r_{tm} can be expected to increase too. Thus, if the discounted prices $\rho_{tm} \mathbf{p}_n^{tm}$ are used in place of the nominal prices \mathbf{p}_n^{tm} in an annual index number formula, the effects of high inflation in any season will be nullified by the discount rates ρ_{tm} .

The use of the seasonally discounted prices \mathbf{p}^{t*} in (17) and (18) in place of the nominal prices \mathbf{p}^t poses difficulties for economic statisticians. Not only must the Statistical Agency collect seasonal data on nominal prices and quantities, but data on season-to-season interest rates r_{tm} also must be collected to calculate the seasonal discount factors ρ_{tm} . In principle, the interest rate r_{tm} should be a weighted average of all interest rates that consumers face (both borrowing and lending rates) where the weights are proportional to the amounts of funds loaned out or borrowed by consumers during season m of year t . This is not a trivial task. Moreover, many statisticians will object to using discounted prices in constructing annual price and quantity indexes on the grounds that the Fisher (1930) and Hicks (1946) intertemporal consumer theory on which (17) and (18) are based is too unrealistic. Thus we consider some alternatives to the use of interest rates as discount factors in forming the seasonally deflated prices \mathbf{p}^{tm*} defined by (16).

A simple alternative is to use the price of a widely traded commodity as a discount factor. Thus if p_G^{tm} is the price of gold in season m of year t , then the *gold-standard* discount factors are

$$\rho_{tm}^G \equiv p_G^{t1} / p_G^{tm}; \quad t = 0, 1, \dots, T; \quad m = 1, \dots, M. \quad (19)$$

The gold-deflated prices $p_n^{tm*} \equiv \rho_{tm}^G p_n^{tm}$ could be used as the normalized prices in (17) and (18).

Another alternative is to convert nominal prices into prices expressed in terms of a stable currency. In this case, the *foreign-currency* discount factors ρ_{tm}^E are defined by

$$\rho_{tm}^E \equiv e_{tm} / e_{t1}; \quad t = 0, 1, \dots, T; \quad m = 1, \dots, M, \quad (20)$$

where e_{tm} is the average number of units of foreign currency required to buy one unit of domestic currency in season m of year t .

Instead of using the price of gold p_G^{tm} as the deflator in (19), we could use any price for any commodity that is traded during each season, or, instead of deflating by a single commodity price, the price or cost of a basket of non-seasonal and type-2a seasonal commodities might be used as the deflator. The season m year t price vector \mathbf{p}^{tm} could be divided into the vectors $[\tilde{\mathbf{p}}^{tm}, \hat{\mathbf{p}}^{tm}]$ where $\tilde{\mathbf{p}}^{tm} \equiv [\tilde{p}_1^{tm}, \tilde{p}_2^{tm}, \dots, \tilde{p}_K^{tm}]$ and each of the K commodities represented in $\tilde{\mathbf{p}}^{tm}$ is either a nonseasonal commodity or a type-2a seasonal commodity.¹¹ Let $\mathbf{b} \equiv [b_1, b_2, \dots, b_K]$ be a vector of appropriate commodity quantity weights. Then, the year t season m price of this basket of goods is $\tilde{\mathbf{p}}^{tm} \cdot \mathbf{b}$ and the *commodity-standard* discount factors are defined by

$$\rho_{tm}^B \equiv \tilde{\mathbf{p}}^{tm} \cdot \mathbf{b} / \tilde{\mathbf{p}}^{t1} \cdot \mathbf{b}; \quad t = 0, 1, \dots, T; \quad m = 1, \dots, M. \quad (21)$$

As a further refinement to (21), we could replace the fixed basket index $\tilde{\mathbf{p}}^{tm} \cdot \mathbf{b}$ with a general price index, $\tilde{P}(\tilde{\mathbf{p}}^{t1}, \tilde{\mathbf{p}}^{tm}, \tilde{\mathbf{q}}^{t1}, \tilde{\mathbf{q}}^{tm})$, which compares the prices of commodities (excluding type-1 and type-2b seasonal commodities) in season m of year t , $\tilde{\mathbf{p}}^{tm}$, to their prices in the base period, $\tilde{\mathbf{p}}^{t1}$. Now the *index number* discount factor is

$$\rho_{tm}^P \equiv 1 / \tilde{P}(\tilde{\mathbf{p}}^{t1}, \tilde{\mathbf{p}}^{tm}, \tilde{\mathbf{q}}^{t1}, \tilde{\mathbf{q}}^{tm}); \quad t = 0, 1, \dots, T; \quad m = 1, \dots, M. \quad (22)$$

Each of the choices for the seasonal discount factors ρ_{tm} represented by (19–22) has advantages and disadvantages. All of these choices seem somewhat arbitrary. However, each of these will lead to sensible index number comparisons in the presence of hyperinflation. If we make use of the observation that nominal rates of interest are approximately equal to real rates plus the rate of inflation, it can be seen that the inflation-rate choices that are imbedded in the discount-factor choices (19–22) will be approximately equal to the interest-rate choice for ρ_{tm} that we advocated originally, provided that the season-to-season real rates of return are small.

The important conclusion that we should draw from the analysis presented in this section is that when constructing annual quantity indexes in high-inflation situations, seasonal prices *must* be deflated for general inflation that occurred from season to season throughout the year. If this deflation is not done, the quantities corresponding to high-inflation seasons will receive undue weight in the annual quantity index.

We conclude this section by discussing the interpretation of the annual price index $P(\mathbf{p}^{s*}, \mathbf{p}^{t*}, \mathbf{q}^s, \mathbf{q}^t)$ in (18). We assume that the price and quantity indexes P and Q that appear in (17) and (18) satisfy the weak factor reversal test (9) with normalized prices p^{t*} used in place of nominal prices p^t . Thus, P and Q satisfy

$$\sum_{m=1}^M \mathbf{p}^{tm*} \cdot \mathbf{q}^{tm} / \sum_{m=1}^M \mathbf{p}^{sm*} \cdot \mathbf{q}^{sm} = P(\mathbf{p}^{s*}, \mathbf{p}^{t*}, \mathbf{q}^s, \mathbf{q}^t) Q(\mathbf{p}^{s*}, \mathbf{p}^{t*}, \mathbf{q}^s, \mathbf{q}^t); \quad (23)$$

i.e., using our original seasonal interest-rate discount factors ρ_{tm} , (23) says that the discounted (to the beginning of year t) sum of seasonal values $\sum_{m=1}^M p^{tm*} \cdot q^{tm}$ divided by the discounted (to the beginning of year s) sum of seasonal values $\sum_{m=1}^M p^{sm*} \cdot q^{sm}$ is decomposed into $P(p^{s*}, p^{t*}, q^s, q^t)$ times $Q(p^{s*}, p^{t*}, q^s, q^t)$. The price index $P(p^{s*}, p^{t*}, q^s, q^t)$ captures the change in discounted year t prices relative to discounted year s prices. The interpretation of $P(p^{s*}, p^{t*}, q^s, q^t)$ when the specific commodity discount factors defined by (19–22) are used is less clear. If we use the discount factors defined by (19), then the normalized prices in season 1 of each year t , p_n^{t1*} , are equal to the corresponding nominal prices p_n^{t1} , but the normalized prices for later seasons $m > 1$, p_n^{tm*} , are equal to the corresponding nominal prices, p_n^{tm} , divided by the year t , season m to 1, gold price relative, p_G^{tm} / p_G^{t1} . $P(p^{s*}, p^{t*}, q^s, q^t)$ is a measure of price-level change going from year s to t with the seasonal prices within each year stabilized in terms of season-1 prices using the price of gold as the deflator of post-season-1 prices. This index does not have a clear interpretation as a measure of the average level of nominal prices in year t versus year s .

In Section 4, we discuss the possible use of annual unit values as prices in the construction of annual price and quantity indexes.

4. ANNUAL UNIT VALUE INDEXES UNDER CONDITIONS OF HIGH INFLATION

The reader may well feel that the annual index number model that we developed in the preceding section is too complex. One simpler alternative is the following: Instead of distinguishing commodities by season, add up consumption of each physically distinct commodity over the seasons and use these annual total consumptions as the quantities to be inserted into an index number formula. The price corresponding to each such annual quantity would be the total annual value of expenditures on that physical commodity divided by the annual quantity—an annual unit value.

This is a reasonable proposal, particularly when we consider that at some stage of disaggregation, unit values must be used to aggregate individual transactions, if we want to apply bilateral index number theory.¹² However, an important characteristic of a unit value is the time period over which it is calculated. The time period should be short enough that individual variations of price within the period can be regarded as unimportant. In periods of rapid inflation or hyperinflation, nominal prices vary substantially between seasons. Seasonal values that correspond to high-inflation seasons will be weighted too heavily in the annual unit value.

The above argument does not rule out the use of annual unit values, provided that nominal prices p_n^{tm} are replaced by the within-year inflation-adjusted normalized prices p_n^{tm*} defined by (16), and provided that these prices are approximately constant across seasons m for each commodity n . This proviso will not be satisfied if there are seasonal commodities.

The problem with the use of (normalized) annual unit values when there are seasonal commodities can be illustrated as follows. Imagine two years, where in the second year, after transportation and storage improvements, a constant quantity of a seasonal fruit, e.g., bananas, is consumed at a constant price. In the first year, the same total annual quantity is consumed mostly in one season at a price slightly lower than the second-year constant price. In the other seasons of the first year, one banana is consumed at a very high price. The prices are such that the value of banana consumption is constant over the two years. The unit value for bananas also would be constant over the two years as would the corresponding total annual quantity index. However, most economists would feel that the utility of banana consumption is higher in the second year than in the first year and an index number comparison ought to show this. Given low seasonal real interest rates, under the above conditions the use of a Mudgett-Stone Fisher ideal quantity index would lead to a banana quantity index greater than 1. Thus there generally will be real biases in using annual (normalized) unit value indexes if there are substantial seasonal fluctuations in quantities and (normalized) prices.

To compare more formally the use of annual unit value indexes using normalized prices with the Mudgett-Stone annual indexes in the preceding section, we make the simplifying assumption that there are no type-1 and no type-2b seasonal commodities. Thus, the dimensionality of the commodity space is constant over each season so that $N_m = N$ for $m = 1, \dots, M$ and we can aggregate commodities over seasons.

Define the year- t quantity for commodity n as the sum over the season- m quantities:

$$Q_n^t \equiv \sum_{m=1}^M q_n^{tm}; \quad n = 1, \dots, N; \quad t = 0, 1, \dots, T. \quad (24)$$

Using the inflation-adjusted normalized prices p_n^{tm*} , an annual normalized value for commodity n in year t is defined as

$$V_n^{t*} \equiv \sum_{m=1}^M p_n^{tm*} q_n^{tm}; \quad n = 1, \dots, N; \quad t = 0, 1, \dots, T. \quad (25)$$

The normalized unit value for good n is defined as

$$P_n^{t*} \equiv V_n^{t*} / Q_n^t; \quad n = 1, \dots, N; \quad t = 0, 1, \dots, T. \quad (26)$$

Define the year- t vector of normalized unit values as $\mathbf{P}^{t*} \equiv [P_1^{t*}, \dots, P_N^{t*}]$ and the year- t vector of total quantities consumed as $\mathbf{Q}^t \equiv [Q_1^t, \dots, Q_N^t]$ for $t = 0, 1, \dots, T$.

The annual price and quantity vectors \mathbf{P}^{t*} and \mathbf{Q}^t can be used in calculating annual quantity indexes. We want to justify the use of such an index. We assume

that intertemporal utility function satisfies the assumptions (5). One assumption that appears to be necessary for total annual year- t quantities $Q^t = \sum_{m=1}^M q^m$ to solve (15) is

$$f(x^1, x^2, \dots, x^M) = g\left(\sum_{m=1}^M x^m\right), \tag{27}$$

where g is an increasing, concave, and linearly homogeneous function of N variables.¹³ However, to ensure that the quantity vectors $[q^1, \dots, q^M]$ are solutions to (15) when f is defined by (27), we also require equality of the normalized price vectors; i.e., we require¹⁴

$$p^{t1*} = p^{t2*} = \dots = p^{tM*}; \quad t = 0, 1, \dots, T. \tag{28}$$

To see why this is so, rewrite (15) when f is defined by (27) as follows:

$$\max_{x^1, \dots, x^M} \left\{ g\left(\sum_{m=1}^M x^m\right) : \sum_{m=1}^M p^{tm*} \cdot x^m = \sum_{m=1}^M p^{tm*} \cdot q^{tm} \right\} = g\left(\sum_{m=1}^M q^{tm}\right), \tag{29}$$

$t = 0, 1, \dots, T.$

If (28) were not true for some t , then in (29), we would find that *all* of the seasonal purchases in year t for any commodity for which unequal prices prevailed would have to be concentrated in the seasons with the lowest prices, which would contradict the observed data.

Assuming that (27) and (28) are satisfied, we can apply exact index number theory and derive the following annual index number equalities:

$$g(Q^t)/g(Q^s) = Q^*(P^{s*}, P^{t*}, Q^s, Q^t); \quad 0 \leq s, t \leq T \tag{30}$$

for any index number formula Q^* that is exact for the annual aggregator function g . Thus, we have provided an economic justification for the use of annual normalized unit values P^{t*} and total annual quantities Q^t in an index number formula.

Suppose that Q^* in (30) and Q in (17) are both Fisher ideal quantity indexes. Under what conditions will the annual unit value approach (which leads to (30) with $Q^* = Q_F^*$) give us the same *numerical* answer as the less restrictive Mudgett-Stone approach (which leads to (17) with $Q = Q_F$)?

Using definitions (24–26), it is easy to see that

$$p^{t*} \cdot q^t = \sum_{m=1}^M p^{tm*} \cdot q^{tm} = P^{t*} \cdot Q^t; \quad t = 0, 1, \dots, T. \tag{31}$$

Hence a Fisher ideal index used in (17) will equal a Fisher ideal index used in (30); i.e.,

$$Q_F^*(P^{s*}, P^{t*}, Q^s, Q^t) = Q_F(p^{s*}, p^{t*}, q^s, q^t); \quad 0 \leq s, t \leq T, \tag{32}$$

if and only if

$$P^{s*} \cdot Q^t = p^{s*} \cdot q^t \quad \text{for } 0 \leq s, t \leq T. \tag{33}$$

A simple set of conditions that will ensure the equalities in (33) are the following Leontief-type aggregation conditions:

$$q^{tm} = \alpha_t \beta_m \bar{q}; \quad t = 0, 1, \dots, T; \quad m = 1, \dots, M, \tag{34}$$

where $\alpha_t > 0$ is a year- t growth factor, $\beta_m > 0$ is a shift factor for season m , and $\bar{q} \equiv [\bar{q}_1, \dots, \bar{q}_N]$ is a fixed-quantity vector. If the β_m form an increasing sequence, they may be interpreted as monthly growth factors. If the β_m fluctuate with mean 1, they can be interpreted as pure seasonal fluctuation factors with *all* commodities subject to the *same* pattern of fluctuations.

We now verify that assumptions (34) imply the equalities (33). Using the definition of an inner product, we have for $0 \leq s, t \leq T$;

$$\begin{aligned} P^{s*} \cdot Q^t &= \sum_{n=1}^N P_n^{s*} Q_n^t \\ &= \sum_{n=1}^N \left[\sum_{m=1}^M P_n^{sm*} q_n^{sm} / \sum_{j=1}^M q_n^{sj} \right] \left[\sum_{i=1}^M q_n^{ti} \right] \quad \text{using definitions (24–26)} \\ &= \sum_{n=1}^N \left[\sum_{m=1}^M P_n^{sm*} \alpha_s \beta_m \bar{q}_n / \sum_{j=1}^M \alpha_s \beta_j \bar{q}_n \right] \left[\sum_{i=1}^M \alpha_t \beta_i \bar{q}_n \right] \quad \text{using (34)} \\ &= \sum_{n=1}^N \sum_{m=1}^M P_n^{sm*} \alpha_t \beta_m \bar{q}_n \\ &= \sum_{n=1}^N \sum_{m=1}^M P_n^{sm*} q_n^{tm} \quad \text{using (34)} \\ &= p^{s*} \cdot q^t, \end{aligned}$$

where the last equality follows from the definitions of the annual vectors p^{s*} and q^t .

Thus, assumptions (34) do indeed imply the equality of the Fisher indexes in (32) but they are not consistent with the simultaneous existence of both seasonal and nonseasonal commodities or with the existence of nonconstant monthly growth rates.

Another set of conditions that will ensure that the equalities in (33) hold are the following Hicks aggregation conditions:

$$p^{tm*} = \gamma_t \bar{p}; \quad t = 0, 1, \dots, T; \quad m = 1, \dots, M, \tag{35}$$

where $\gamma_t > 0$ is a year- t price-level factor and $\bar{p} \equiv [\bar{p}_1, \dots, \bar{p}_N]$ is a constant price vector.

We now verify that assumptions (35) imply the equalities (33). Using definitions (24–26) again, we have, for $0 \leq s, t \leq T$,

$$\begin{aligned}
 \mathbf{P}^{s*} \cdot \mathbf{Q}^t &= \sum_{n=1}^N \left[\sum_{m=1}^M p_n^{sm*} q_n^{sm} / \sum_{j=1}^M q_n^{sj} \right] \left[\sum_{i=1}^M q_n^{ti} \right] \\
 &= \sum_{n=1}^N \left[\sum_{m=1}^M \gamma_s \bar{p}_n q_n^{sm} / \sum_{j=1}^M q_n^{sj} \right] \left[\sum_{i=1}^M q_n^{ti} \right] \quad \text{using (35)} \\
 &= \sum_{n=1}^N [\gamma_s \bar{p}_n] \left[\sum_{i=1}^M q_n^{ti} \right] \\
 &= \sum_{n=1}^N \sum_{m=1}^M \gamma_s \bar{p}_n q_n^{tm} \\
 &= \sum_{n=1}^N \sum_{m=1}^M p_n^{sm*} q_n^{tm} \quad \text{using (35)} \\
 &= \mathbf{P}^{s*} \cdot \mathbf{Q}^t.
 \end{aligned}$$

Thus conditions (35) imply the equalities in (33) and (32). Note that conditions (35) are just a different way of writing our earlier conditions (28). These conditions are very restrictive: They require absolute equality of all discounted seasonal price vectors \mathbf{p}^{tm*} within each year t . In particular, these conditions rule out seasonal fluctuations in prices.

The above analysis indicates that the existence of seasonal commodities generally will cause the annual unit value index numbers to differ (perhaps substantially) from the Mudgett-Stone annual indexes studied in the preceding two sections. Because the assumptions on the underlying annual aggregator function needed to derive exact indexes are much less restrictive for the Mudgett-Stone indexes, we recommend the use of these indexes over the use of annual unit value indexes.

5. CONCLUSION

We have discussed the problem that Statistical Agencies face when constructing price and quantity aggregates under conditions of high inflation when there are seasonal commodities. Without seasonal commodities, the index number problem is still straightforward (but expensive): the Statistical Agency must collect subannual price and quantity (or value) information more frequently in order to

make the subannual periods of time short enough that variations in prices within the periods can be neglected.¹⁵ However, when there are seasonal commodities, this solution to the high-inflation index number problem is not valid: We cannot make meaningful bilateral index number comparisons (from the viewpoint of the economic approach) between consecutive months or quarters if the dimensionality of the commodity space varies from period to period.

When there are seasonal commodities and high inflation, Statistical Agencies first *must* undertake a preliminary deflation of the quarterly or monthly prices, using either seasonal nominal interest rates or an index of nonseasonal commodity prices as the deflators. Once these deflated seasonal prices or normalized prices of the form (16) have been constructed, then annual Mudgett-Stone price and quantity indexes of the form (17) and (18) in Section 3 can be constructed. Alternatively, the approach outlined in Section 4 could be used, which involves constructing annual quantity series [recall (24)] and annual normalized unit values [recall (26)] and then using these annual quantities and prices in an index number formula. However, the Mudgett-Stone approach is preferable to the annual normalized unit value approach because the former approach requires much weaker assumptions on preferences. Moreover, the latter approach is not consistent with the existence of seasonal commodities.

NOTES

1. This classification corresponds to Balk's (1980a, p. 7; 1980b, p. 110; 1980c, p. 68) narrow- and wide-sense seasonal commodities.

2. This classification is due to Mitchell (1927, p. 236). See Mitchell (1927, p. 237) and Granger (1978, p. 33) for examples of seasonal fluctuations due to custom.

3. A bilateral index number formula uses the price and quantity information that pertains to only two periods or two countries. A multilateral formula uses information that pertains to many periods.

4. Using the nonparametric tests for maximizing behavior due to Afriat (1967) and Diewert (1973, p. 424), we can test whether a given set of price and quantity data is consistent with the maximization of a homothetic or linearly homogeneous utility function; see Diewert (1981, pp. 198–199). If a combination of seasonal and nonseasonal data pass this test, then the seasonal commodities are of type 2a.

5. See Flux (1921, pp. 184–185), Bean and Stine (1924), Crump (1924, p. 185), Mudgett (1955), Stone (1956), Rothwell (1958), Zarnowitz (1961), Turvey (1979), and Balk (1980a,b,c, 1981).

6. See Diewert (1980, pp. 506–508; 1983c) on the economic approach to seasonal indexes. This paper focuses on the theory of the seasonal consumer price index. An analogous theory exists for the seasonal producer price index with separability assumptions on the producer's intertemporal production function or factor requirements function. See Fisher and Shell (1972) and Diewert (1980, 1983b).

7. The present paper generalizes Diewert's (1983c) earlier economic approach.

8. Our assumptions are admittedly unrealistic. The consumer is assumed to know future spot prices p^t , know his or her future income streams, be able to freely borrow and lend between years at the same rates, and have unchanging tastes over years. Under these assumptions, the consumer at the beginning of year 0 chooses a sequence of annual consumption plans, q^t , $t = 0, 1, \dots, T$, and sticks to them.

9. The function f is defined over the annual commodity space of dimension $\sum_{m=1}^M N_m \equiv N^*$; i.e., each physical commodity in each season is treated as a separate economic commodity from the perspective of the annual utility function f . The concavity assumption on f can be replaced by the weaker condition of continuity; see Diewert (1974, p. 111).

10. See Diewert (1976, p. 116; 1981, pp. 180–193; 1983a, p. 184; 1993, pp. 45–50).

11. Because it is difficult to distinguish type-2a from type-2b seasonal commodities, it may be more practical to stick with nonseasonal ones. Of course, there may be difficulties in distinguishing nonseasonal from seasonal commodities as well.

12. See Dalén (1992, p. 135), Diewert (1995, pp. 20–24), and Balk (1995).

13. Assumption (27) is restrictive: It says that the consumer is indifferent to the annual consumption of each commodity taking place in a single season or spread across seasons.

14. These conditions are Hicksian aggregation conditions which guarantee the existence of annual aggregates [See Hicks (1946, p. 312)]. In fact, if conditions (28) hold, we do not have to make the restrictive assumption (27) to determine that $[q^{t1}, q^{t2}, \dots, q^{tM}]$ solves (15). To determine the annual aggregator function g^* under conditions (28), let $c(p^{t1*}, \dots, p^{tM*})$ be the unit cost function dual to $f(x^1, \dots, x^M)$. Define the N variable unit cost function $c^*(p^t) \equiv c(p_1^t, \dots, p_N^t)$. Then, $g^*(\sum_{m=1}^M x^m)$ is dual to c^* .

15. Diewert (1995, p. 22) advocated this solution to the index number problem under high inflation but he neglected the seasonal commodities problem.

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