Macroeconomic Dynamics, **23**, 2019, 1921–1958. Printed in the United States of America.
doi:10.1017/S1365100517000505

DIRECTED STRUCTURAL CHANGE

ROBERT F. KANE
International University of Japan

This paper extends the existing theories of directed technical change by allowing the factors of production, skilled, and unskilled workers, to be employed in both the skill-intensive and unskilled-intensive sectors. Consequently, the direction of technical progress and the sectoral allocation of factors are jointly determined. The feedback between technical progress and the allocation of factors leads to new results concerning structural change and directed technical change. An increase in the endowment of a factor leads to a dynamic reallocation of factors toward the sector that uses the factor intensively. The reallocation of factors also affects the stability properties of directed technical change. When the parameter conditions necessary for strong bias are satisfied, the interior regime (nonspecialization) is at most locally stable. More importantly, if the relative endowment of skilled labor becomes too high (low), the economy necessarily specializes in the production of skilled (unskilled)-labor-intensive goods. Last, the relationship between the relative endowment of skilled labor and the steady-state relative wage rate is not necessarily monotonic.

Keywords: Endogenous Growth, Directed Technical Change, Structural Change

1. INTRODUCTION

The literature offers two explanations for structural change: the demand side, which relies on nonhomothetic preferences, and the supply side, which relies on biased sectoral growth. As households become richer, the demand for certain goods and services increases; consequently, employment rises in these sectors. Moreover, if technological improvements are inherently biased toward certain sectors, the relative sectoral productivity of labor changes and hence so does the sectoral employment. This paper analyzes a recently discovered phenomenon: the reallocation of labor from unskilled sectors toward skill-intensive sectors. This new type of structural change is documented by Buera and Kaboski (2012) and Rogerson et al. (2015), who coin the term skill-biased structural change. The present framework sheds light on skill-biased structural change through the lens of directed technical change.

In this paper, I provide a new explanation for structural change—*directed structural change*. Contrary to existing theories, structural change is ultimately driven

I thank Daron Acemoğlu, several anonymous referees, and Lei Ji for helpful insight. I am particularly indebted to John Seater and Pietro Peretto. Address correspondence to: Robert F. Kane, International University of Japan, 777 Kokusai-cho Minami-uonuma Niigata Japan 949-7277; e-mail: kane@iuj.ac.jp.

 1365-1005/17

by changes to the relative supply of factors of production. The building blocks of directed structural change are the "workhorse" model of directed technical change [Acemoglu (2002)] and the assumption of *imperfect* factor intensity (both sectors use both types of labor). By generalizing Acemoglu (2002) to include imperfect factor intensity, and hence allowing for the reallocation of factors, I obtain qualitatively new results concerning structural and directed technical change.

Despite providing the leading explanation for biased technical progress, the directed technical change literature has been silent concerning structural change. The reason directed technical change has not dealt with structural change is that, in its current form, it cannot. The framework, pioneered by Acemoglu (1998), Acemoglu and Zilibotti (2001), and Acemoglu (2002), is well known: entrepreneurs develop new machines targeting the skill-intensive and unskilled-intensive sectors. What has not received the attention it deserves is the assumption of *perfect* factor intensity in each sector: the skill-intensive sector only employs skilled labor and the unskilled-intensive sector only employs unskilled labor. It is worthwhile to reflect upon this assumption and its implications. However, unimportant unskilled labor is in the production of a given good or service, it is not completely unproductive; even CERN needs janitors. The assumption is more suspect when one considers unskilled-intensive goods.

To illustrate the importance of imperfect factor intensity, it is useful to compare the Heckscher–Ohlin and directed technical change frameworks. They share much in common. For example, both have two sectors that are distinguished by different factor intensities and both provide implications for factor rewards. However, a key component of the Heckscher–Ohlin framework is absent from directed technical change: the reallocation of factors. Consider the effect of an increase in the endowment of a factor in both models. The Rybczynski theorem states that an increase in the endowment of a factor, holding the prices constant, will lead to a more than proportional increase in the production of the sector that uses that factor intensively and a decrease in the production in the sector that uses the factor unintensively [Rybczynski (1955)]. The change in production is due to factor reallocations. In contrast, factor reallocations are absent in directed technical change. Instead, changes to endowments induce directed technical progress—which is absent in Heckscher–Ohlin. In the present framework, the direction of technical progress and the allocation of factors are jointly determined.

The first result of this new framework is "weak-induced structural change." An increase in the supply of a factor always leads to a dynamic reallocation of factors (both skilled and unskilled labor) away from the sector that uses the factor unintensively and toward the sector that uses the factor intensively. Weak-induced structural change is different from, though highly complementary to, the weak bias result demonstrated by Acemoglu (2002, 2007). Formally, weak bias states that an increase in the relative supply of a factor always induces technical progress that benefits that factor. The present model also exhibits weak bias; however, weak-induced structural change concerns the sectoral allocation of factors that is fixed, by assumption, in the directed technical change literature.

Weak-induced structural change can be viewed as a dynamic general equilibrium analog to the Rybczynski theorem. Unlike Rybczynski's framework, the prices and technology are endogenous in the present model. Consequently, an increase in the endowment of skilled labor does not necessarily lead to a rise in employment in the skill-intensive sector. The static response will, naturally, depend on the elasticity of substitution. Nonetheless, following the static response, weak-induced structural change implies that there will always be a dynamic reallocation of factors (or structural change) toward the sector that uses the factor intensively.

The second major result is that if the elasticity of substitution between the two sectors is above a certain threshold, the economy can exhibit "strong-induced structural change." I define strong-induced structural change as an endogenous regime switch from the interior solution (both sectors operating), to a corner (specialization in just one sector) in response to a change in the relative supply of skilled labor. When the elasticity of substitution is above the aforementioned threshold and the relative supply of skilled labor gets sufficiently high (low), the economy *necessarily* specializes in the production of the skilled (unskilled) labor-intensive good. This induced specialization is not a trivial outcome; technical progress, as well as its direction, is endogenous. It cannot occur when there is perfect factor intensity, as in the literature on induced technical progress [e.g., Acemoglu (2002)]. Strong-induced structural change has important implications for directed technical change, which will be discussed momentarily. Both types of structural change (weak and strong) are consistent with the definition used by Ngai and Pissarides (2007), who define structural change as "the state in which at least some of the labor shares are changing over time."

One of the many insights of directed technical change is that the magnitude of bias is affected by the "price effect" and "market size effect." Following an increase in the supply of skilled labor, the relative price of skill-intensive goods declines (the price effect), and the market size of the skill-intensive sector rises (the market size effect). The former encourages innovation in the sector that uses the scarce factor, and the latter encourages innovation in the sector with the more abundant factor employment. Acemoglu (2002) shows that when the elasticity of substitution is sufficiently high, the market size effect sufficiently dominates the price effect and the biased technical progress is so large that the steady-state relative wage rate is increasing in the relative supply of skilled labor. He terms this result strong bias. I show that when the sectoral allocations of factors are endogenous, the same forces that govern whether or not strong bias occurs *also* play a role in determining the stability of the dynamic system. In particular, when the parameter conditions allow for strong bias, the interior solution (nonspecialization) is at most locally stable and potentially globally unstable. The stability of the dynamic system has important ramifications for the model's predictions regarding the skill premium.

The third major result is that the relationship between the relative supply of skilled labor and the steady-state relative wage rate is not necessarily monotonic. The reason for the potential nonmonotonicity is strong-induced structural change. Provided the elasticity of substitution is high enough, strong bias will hold in the interior solution. However, strong-induced structural change implies that when the relative supply of skilled labor gets sufficiently high (low), the economy will specialize in the production of the skilled (unskilled)-labor-intensive good. When the economy is specialized, strong bias fails to hold. Consequently, if the parameter conditions necessary for strong bias are met, the relationship between the relative supply of skilled workers and the steady-state relative wage rate is not monotonic. In particular, it may vary across stages of development.

Goldin and Katz (2008) call the period from 1900 to 2000 "the human capital century." Directed structural change implies that the technical progress induced by the enormous rise in the supply of educated workers will lead to sectoral reallocations of skilled and unskilled workers toward the skill-intensive sector. The movement of labor (both skilled and unskilled) into the skill-intensive sector implies that the aggregate share of income going to unskilled labor will decrease over time. Therefore, in some sense, the present framework exhibits what Peretto and Seater (2013) refer to as factor elimination. It is useful to compare the two frameworks. Peretto and Seater (2013) develop a model in which firms engage in costly research and development (R&D) to increase the capital's share in a Cobb–Douglas production function; technical progress is directly factor eliminating.[1] In the present model, factor elimination only occurs due to general equilibrium forces—the reallocation of unskilled labor into a sector in which they are relatively less productive.

The structural change literature is far too vast for a detailed review.[2] The most closely related work is Dolores Guilló et al. (2011) followed by the working paper of Rogerson et al. (2015). The former explains structural change from agriculture to manufacturing using nonhomothetic preferences and endogenous sectoral biased growth. They find strong evidence that structural change is primarily driven by biased technical change. In particular, spillovers from manufacturing to agriculture are crucial for explaining the observed sectoral productivity growth. The latter explains skill-biased structural change using nonhomothetic preferences and exogenous-biased technical progress. In contrast to the existing literature, the present framework provides a *different* explanation for structural change: changes to the economy's relative supply of skilled labor and the corresponding induced technical progress.

The remainder of the paper is organized as follows. Section 2 sets up the model. Section 3 solves it. Section 4 analyzes the impact of demographic changes and introduces the concept of weak- and strong-induced structural change. Section 5 discusses the models implications for the skill premium. Finally, Section 6 concludes.

2. THE MODEL

The model presented here is similar to Acemoglu (2002) with the exception of the production technologies for skilled- and unskilled-intensive goods. In order to not obfuscate results, the only radical difference is that both sectors use both types of labor.

2.1. Households

Consider an economy consisting of a representative consumer with the following preferences:

$$\int_0^\infty \ln(C_t)\, e^{-\rho t} dt, \quad (1)$$

where $\rho > 0$ is the rate of time preference and C is consumption. I suppress time arguments to simplify notation whenever ambiguity does not arise. The budget constraint of the consumer is

$$C + I + R = Y, \quad (2)$$

where I denotes investment, and R the aggregate R&D expenditure. The present model assumes a "lab equipment" specification [Rivera-Batiz and Romer (1991), Acemoglu and Zilibotti (2001), Acemoglu, (2002)], in which consumption, investment, and R&D expenditure come out from aggregate production Y.

2.2. Final Good Production

The aggregate production is

$$Y = \left(Y_s^{\frac{\epsilon-1}{\epsilon}} + Y_u^{\frac{\epsilon-1}{\epsilon}}\right)^{\frac{\epsilon}{\epsilon-1}}, \quad (3)$$

where ϵ is the elasticity of substitution, Y_s denotes the production of the skilled-labor-intensive good, and Y_u denotes the production of the unskilled-labor-intensive good. Standard profit maximization implies

$$\frac{Y_s}{Y_u} = \left(\frac{P_s}{P_u}\right)^{-\epsilon}, \quad (4)$$

where P_s and P_u is the price of Y_s and Y_u, respectively.

Acemoglu and Autor (2011) provide a comprehensive summary of the CES production structure, assumed here, and its relation to skill-biased technical change. In particular, they point out that when $Y_s = A_s Z$ and $Y_u = A_u L$, the production technology (3) admits three interpretations:

1. There is only one good, and skilled (Z) and unskilled workers (L) are imperfect substitutes in its production.
2. The production function is equivalent to an economy where consumers have preferences defined over two goods; the skill-intensive good only uses high-skilled workers and the unskilled-intensive good only uses unskilled labor.
3. A mixture of one and two, where different sectors produce goods that are imperfect substitutes, and skilled and unskilled workers are employed in *both* sectors.

The third is, by all accounts, the correct interpretation of the world. However, it is difficult to interpret the model this way; moreover, even if one accepts this interpretation, the model is silent concerning the allocation of factors in each sector. It is therefore useful to formalize the model and assume that Y_s and Y_u are produced using both factors. To do so, I maintain the Cobb–Douglas specification used in Acemoglu (2002), but I assume that the marginal product of the unintensively used factor is positive. The production functions are thus

$$Y_s = \left(\int_0^{N_s} X_{si}^{1-\alpha-\beta} di\right) Z_s^{\alpha} L_s^{\beta}, \tag{5}$$

and

$$Y_u = \left(\int_0^{N_u} X_{ui}^{1-\alpha-\beta} di\right) L_u^{\alpha} Z_u^{\beta}, \tag{6}$$

where

$$\sum_j Z_j = Z \text{ and } \sum_j L_j = L,\ j = s, u. \tag{7}$$

The skill-intensive good is produced using skilled-complementary intermediates X_{si}, skilled labor Z_s, and unskilled labor L_s. The unskilled-labor-intensive good is produced using unskilled-complementary intermediates X_{ui}, skilled labor Z_u, and unskilled labor L_u. The parameter α governs the factor share of the intensively used labor and $\beta < \alpha$ is the factor share of the unintensively used labor, and of course $\alpha + \beta < 1$. The number of skill-complementary machines N_s and unskilled complementary machines N_u are endogenous state variables. Following Acemoglu (2002), the supply of skilled (Z) and unskilled (L) labor is exogenous. However, the sectoral allocations, Z_s, Z_u, L_s, and L_u, are endogenous.[3]

The sector-specific production functions use both skilled and unskilled labor. This assumption—and the Cobb–Douglas specification—is also made in the pioneering work of Romer (1990) and Rivera-Batiz and Romer (1991) in a one sector model.[4] Acemoglu (2002) drops the assumption that machines are used by both types of labor, but generalizes the model so that there are two sectors: the skilled labor intensive and unskilled labor intensive. In that model, each sector employs sector-specific machines: machines that can only be used by skilled labor (skill-intensive machines) and machines that can only be used by unskilled labor (unskilled-intensive machines). Furthermore, like Romer (1990) and Rivera-Batiz and Romer (1991), Acemoglu (2002) assumes that machines and labor are aggregated together according to a Cobb–Douglas production function. The present framework maintains the Cobb–Douglas assumption, but generalizes the model to capture that both sectors use both types of labor with different, *but not infinitely different*, factor intensities. The importance of imperfect factor intensity can be seen in Figure 1.

When $\beta = 0$, the production possibilities frontier, depicted in Figure 1(a), is a rectangle. When $\beta > 0$, the familiar curvature, depicted in Figure 1(b), returns to the production possibilities frontier. Figures 1(a) and (b) illustrate two important

https://doi.org/10.1017/S1365100517000505 Published online by Cambridge University Press

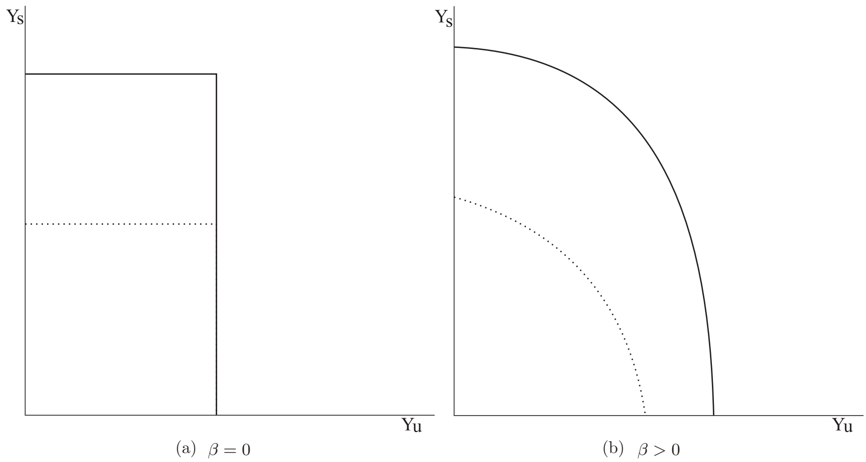


FIGURE 1. P.P.F following an increase in the supply of skilled labor.

differences between the two specifications. First, when $\beta = 0$, the economy must consume at the corner of the rectangle; when $\beta > 0$, the economy can consume at any point along the frontier. In other words, when $\beta = 0$, changes in the relative price of skill-intensive goods will have no effect on their production. When $\beta > 0$, the well-known tangency requirement, taught to all undergraduate economics students, implies that an increase in the price of skill-intensive goods will lead to an increase in their production. Second, imperfect factor intensity implies that an increase in the supply of a factor allows for an increase in the production of both goods. Figures 1(a) and (b) plot the production possibilities frontier following an increase in the supply of skilled workers (holding unskilled labor constant) under both formulations. When $\beta = 0$, an increase in skilled labor only allows for an increase in the production of the skill-intensive good; when $\beta > 0$, it is possible to produce more of both goods.

2.3. Demands and Factor Rewards

The profits in final output sector j are given by

$$\pi_{Yj} = P_j Y_j - w_Z Z_j - w_L L_j - \int_0^{N_j} p_{ji} X_{ji} di, \quad j = u, s \tag{8}$$

where w_Z and w_L is the wage rate for skilled and unskilled labor, respectively, P_j is the price of good Y_j, and p_{ji} is the price of intermediate good. The profit maximization yields the demand for machines

$$X_{si} = \left(\frac{P_s\,(1-\alpha-\beta)}{p_{si}}\right)^{\frac{1}{\alpha+\beta}} \left(\frac{Z_s}{L_s}\right)^{\frac{-\beta}{\alpha+\beta}} Z_s, \tag{9}$$

https://doi.org/10.1017/S1365100517000505 Published online by Cambridge University Press

and

$$X_{ui} = \left(\frac{P_u \left(1 - \alpha - \beta\right)}{p_{ui}} \right)^{\frac{1}{\alpha+\beta}} \left(\frac{Z_u}{L_u} \right)^{\frac{-\alpha}{\alpha+\beta}} Z_u. \quad \textbf{(10)}$$

Equations (9) and (10) imply that the demand for a machine is increasing in the price of the good it is used to produce, P_s and P_u, and decreasing in the price of the machine itself, p_s and p_u. In addition, the demand depends on the employment of factors in that sector, to which we now turn.

The first-order condition for the employment of skilled and unskilled labor yields the factor rewards

$$\begin{aligned} w_Z &= P_s \alpha \left(\int_0^{N_s} X_{si}^{1-\alpha-\beta} di \right) \left(\frac{Z_s}{L_s} \right)^{-\beta} Z_s^{\alpha+\beta-1} \\ &= P_u \beta \left(\int_0^{N_u} X_{ui}^{1-\alpha-\beta} di \right) \left(\frac{Z_u}{L_u} \right)^{-\alpha} Z_u^{\alpha+\beta-1}, \end{aligned} \quad \textbf{(11)}$$

and

$$\begin{aligned} w_L &= P_s \beta \left(\int_0^{N_s} X_{si}^{1-\alpha-\beta} di \right) \left(\frac{Z_s}{L_s} \right)^{1-\beta} Z_s^{\alpha+\beta-1} \\ &= P_u \alpha \left(\int_0^{N_u} X_{ui}^{1-\alpha-\beta} di \right) \left(\frac{Z_u}{L_u} \right)^{1-\alpha} Z_u^{\alpha+\beta-1}. \end{aligned} \quad \textbf{(12)}$$

The wage depends on the relative employment across the sectors. The following lemma yields the relationship between the ratio of skilled to unskilled labor employed in each sector.

LEMMA 1. *The ratio of skilled to unskilled labor employed in each sector can be written as*

$$\frac{Z_s}{L_s} = \left(\frac{\alpha}{\beta} \right)^2 \frac{Z_u}{L_u} = \left(\frac{\alpha^2 - \left(\alpha^2 - \beta^2\right) \upsilon}{\beta^2} \right) \frac{Z}{L}, \quad \textbf{(13)}$$

and the relative wage rate is

$$\omega \equiv \frac{w_Z}{w_L} = \left(\frac{\alpha\beta}{\alpha^2 - \left(\alpha^2 - \beta^2\right) \upsilon} \right) \left(\frac{Z}{L} \right)^{-1}, \quad \textbf{(14)}$$

where $\upsilon = Z_s / Z$ is the share of skilled labor devoted to the skill-intensive sector.

Proof. See the appendix.

Since $\alpha > \beta$, the first equality in (13) shows that Y_s is always skill intensive because the ratio of skilled to unskilled labor employed is higher than the ratio employed in in the production of Y_u. The second equality in (13) shows that the labor allocations (Z_s, Z_u, L_s, L_u) can be written as a function of the share of skilled labor devoted to the skill-intensive sector (υ). The sectoral allocations of labor are given by

$$Z_s = \upsilon Z \text{ and } Z_u = (1-\upsilon)\, Z, \tag{15}$$

and

$$L_s = \left(\frac{\beta^2 \upsilon}{\alpha^2 - \left(\alpha^2 - \beta^2\right)\upsilon}\right) L \text{ and } L_u = \left(\frac{\alpha^2\,(1-\upsilon)}{\alpha^2 - \left(\alpha^2 - \beta^2\right)\upsilon}\right) L. \tag{16}$$

To summarize, an increase in υ leads to a reallocation of skilled (by definition) and unskilled labor (due to general equilibrium forces) toward the skill-intensive sector.

It is worth emphasizing that equations (13) and (14) are not solutions yet; the sectoral allocation of skilled labor, υ, is endogenous. However, since the model is solved in terms of υ, insight can be gained by discussing the effects of changes in υ. If υ rises, holding the endowments fixed, the ratio of skilled to unskilled labor employed in both sectors falls. Intuitively when υ rises, the wage rate of unskilled labor would be higher in the skill-intensive sector than the unskilled-intensive sector; in order to maintain the equality of wage rates, unskilled labor follows the skilled labor. However, because it is relatively less productive there (its factor share is lower), proportionally less needs to move. This is precisely why the relative wage rate, equation (14), is increasing in υ.

2.4. Intermediate Goods

Following standard practice, the unit cost of producing a machine is assumed to equal one. The profit of the typical intermediate firm is thus

$$\pi_{ji} = X_{ji}\left(p_{ji} - 1\right), \quad j = u, s. \tag{17}$$

Firms choose their price to maximize their profits (17) subject to the demand for their machines, (9) and (10). The solution is well known; the optimal price is the marked up unit cost

$$p_s = p_u = (1-\alpha-\beta)^{-1}. \tag{18}$$

Substituting the price (18), the demands, (9) and (10), and the allocations of labor (13) into the profits (17) yields the relative profits, $\pi \equiv \pi_s/\pi_u$,

$$\pi = \left(\frac{P_s}{P_u}\right)^{\frac{1}{\alpha+\beta}} \left(\frac{\beta}{\alpha}\right)^{\frac{2\alpha}{\alpha+\beta}} \left(\frac{Z}{L}\right)^{\frac{\alpha-\beta}{\alpha+\beta}} \left(\frac{\alpha^2 - \left(\alpha^2-\beta^2\right)\upsilon}{\beta^2}\right)^{\frac{\alpha-\beta}{\alpha+\beta}} \left(\frac{\upsilon}{1-\upsilon}\right). \tag{19}$$

Equation (19) shows that the relative profits can be written as a function of three things: the price effect (P_s/P_u), the endowment effect (Z/L), and the endogenous allocation effect (written as a function of υ). It is worthwhile to discuss each in turn.

- The price effect: Holding the market size constant, an increase in the price of skill-intensive goods will increase the relative profitability of the skill-intensive sector.
- The endowment effect: Holding the prices and endogenous allocations constant, an increase in Z relative to L will increase the relative profitability of the skill-intensive sector.
- The endogenous allocation effect: Holding the endowment and prices constant, the movement of factors toward the skill-intensive sector has a positive effect on the relative profitability of the skill-intensive sector.

The first two effects, first discussed in Acemoglu (2002), are well known; the new feature of the model is the endogenous allocation of factors, υ. Recall that when υ increases, by definition, the sectoral allocation of skilled labor in the skill-intensive sector (Z_s) rises—by equation (16) the sectoral employment of unskilled labor in the skill-intensive sector (L_s) also rises. Therefore, the effects of a change in υ on equation (19) is not just the effects of reallocating skilled labor to the skill-intensive sector—it captures the movement of *both* factors. Similar to the endowment effect, an increase to the endogenous allocation of workers to the skill-intensive sector will increase the relative profits. The increase in profitability, however, is more nuanced; note that

$$\frac{\partial \ln(\pi)}{\partial \upsilon} = -\frac{(\alpha-\beta)^2}{\alpha^2 - \left(\alpha^2 - \beta^2\right)\upsilon} + \frac{1}{\upsilon(1-\upsilon)} > 0, \ \forall \upsilon. \qquad \textbf{(20)}$$

An increase in υ has two effects on the profits. The first effect, which is negative, is caused by the reduction in the ratio of skilled to unskilled workers employed. Since skilled labor is more important in the production of skill-intensive goods, a reduction of the skilled labor ratio employed reduces the relative profitability. The second, positive, effect is due to the larger market size induced by the reallocation of labor toward the skill-intensive sector. The second effect dominates the first for all parameter and state configurations.[5]

2.5. New Product Creation

Following Acemoglu (2002), entry costs are assumed to take the form

$$\dot{N}_s = \eta_s R_s \ \text{ and } \ \dot{N}_u = \eta_u R_u, \qquad \textbf{(21)}$$

where R_s is R&D spending for the skill-intensive good, and R_u is R&D spending for the unskilled-intensive good. As aforementioned, the present model assumes a "lab equipment" specification [Rivera- Batiz and Romer (1991), Acemoglu and Zilibotti (2001), Acemoglu (2002)].

Standard asset pricing conditions yield the well-known rate of return to new product creation

$$r_j = \eta_j \pi_j, \quad j = u, s. \tag{22}$$

Substituting (19) into (22) yields the relative rate of return

$$\frac{r_s}{r_u} = \frac{\eta_s}{\eta_u}\left(\frac{P_s}{P_u}\right)^{\frac{1}{\alpha+\beta}}\left(\frac{\beta}{\alpha}\right)^{\frac{2\alpha}{\alpha+\beta}}\left(\frac{Z}{L}\right)^{\frac{\alpha-\beta}{\alpha+\beta}}\left(\frac{\alpha^2-\left(\alpha^2-\beta^2\right)\upsilon}{\beta^2}\right)^{\frac{\alpha-\beta}{\alpha+\beta}}\left(\frac{\upsilon}{1-\upsilon}\right). \tag{23}$$

The relative rate of return is a function of the relative price, the endowment ratio, Z/L, and the endogenous allocations of factors (written as a function of υ). The next section shows that the relative price and endogenous allocations depend crucially on the endowment ratio and the relative technology.

3. GENERAL EQUILIBRIUM

The preceding section described the basic setup. We now turn to the interaction between the states of technology, the endowments, and the endogenous allocation of factors.

3.1. Exogenous Technology

In this section, the technology is exogenous. I derive the relationship between the sectoral allocations of labor and its determinants: the relative technology (N_s/N_u) and the endowment ratio (Z/L) .

Substituting the demand for machines, (9) and (10), into the production technologies, (5) and (6), and substituting in the relative price (4), I obtain

$$\frac{Y_s}{Y_u} = \left[\left(\frac{\beta}{\alpha}\right)^{\frac{2\alpha}{\alpha+\beta}}\left(\frac{Z}{L}\right)^{\frac{\alpha-\beta}{\alpha+\beta}}\left(\frac{N_s}{N_u}\right)\left(\frac{\alpha^2-\left(\alpha^2-\beta^2\right)\upsilon}{\beta^2}\right)^{\frac{\alpha-\beta}{\alpha+\beta}} \times\left(\frac{\upsilon}{1-\upsilon}\right)\right]^{\frac{\epsilon(\alpha+\beta)}{(\epsilon-1)(\alpha+\beta)+1}}. \tag{24}$$

Combining equation (11) with equation (4) yields

$$\left(\frac{Y_s}{Y_u}\right)^{\frac{\epsilon-1}{\epsilon}}\left(\frac{\alpha}{\beta}\right)\left(\frac{1-\upsilon}{\upsilon}\right) = 1. \tag{25}$$

Equations (24) and (25) together determine the allocation of skilled labor (υ) for a given technology and the endowment ratio. The next lemma solves for υ implicitly.

LEMMA 2. *The "short-run" share of skilled labor employed in the skill-intensive sector is defined by the implicit function*

$$\upsilon(z,n) \equiv \underset{\upsilon}{\text{argsolve}} \left\{ \left(\frac{\alpha^2 - (\alpha^2 - \beta^2)\,\upsilon}{\beta^2} \right)^{\beta-\alpha} \left(\frac{\upsilon}{1-\upsilon} \right)^{\frac{1}{\epsilon-1}} = \left(\frac{\beta}{\alpha} \right)^{\frac{(\epsilon-1)(\alpha-\beta)-1}{\epsilon-1}} z^{\alpha-\beta} n^{\alpha+\beta} \right\}, \tag{26}$$

where

$$z \equiv \frac{Z}{L},$$

and

$$n \equiv \frac{N_s}{N_u}.$$

Proof. Substitute equation (24) into (25).

Unfortunately, no closed-form solution exists for $\upsilon(z,n)$.[6] However, it is straightforward to show that the left-hand side of (26) is monotonically increasing in υ when $\epsilon > 1$, and monotonically decreasing when $\epsilon < 1$; since the left-hand side can be made arbitrarily large or small, equation (26) always admits a unique solution.[7] I define $\upsilon(z,n)$ as the short-run share because the relative technology, at a given moment in time, is exogenous. In the long run—which is discussed in the next subsection—the relative technology, n, is endogenous.

In Section 4, I analyze the effects of changes in the endowment ratio, z. Before doing so, it is worthwhile to discuss the relationship between the technology and the allocation of factors. Taking the log of equation (26) and differentiating it yields the implicit relationship between υ and n.

COROLLARY 1. *When* $\epsilon > 1$,

$$\frac{d\upsilon(z,n)}{d\ln(n)} = \frac{(\alpha+\beta)\,\upsilon(z,n)\,[1-\upsilon(z,n)]}{\frac{(\alpha-\beta)(\alpha^2-\beta^2)\upsilon(z,n)[1-\upsilon(z,n)]}{\alpha^2-(\alpha^2-\beta^2)\upsilon(z,n)} + \frac{1}{\epsilon-1}} > 0; \tag{27}$$

when $\epsilon < 1$, *the sign is reversed.*

Proof. See the appendix.

Equation (27) shows that as n rises, υ will rise (fall) when the goods are substitutes (complements). The elasticity of substitution drives the direction of the reallocation of factors because it determines the relative strength of the price effect. Note that the relative price is

$$\frac{P_s}{P_u} = \left[\left(\frac{\beta}{\alpha} \right)^{\frac{2\alpha}{\alpha+\beta}} z^{\frac{\alpha-\beta}{\alpha+\beta}} n \left(\frac{\alpha^2 - (\alpha^2-\beta^2)\,\upsilon}{\beta^2} \right)^{\frac{\alpha-\beta}{\alpha+\beta}} \left(\frac{\upsilon}{1-\upsilon} \right) \right]^{\frac{-(\alpha+\beta)}{(\epsilon-1)(\alpha+\beta)+1}}. \tag{28}$$

When n rises there are two competing forces that govern the reallocation of factors: the productivity of (skilled and unskilled) labor rises in the skill-intensive sector and the relative price of skill-intensive goods declines. If the price effect is weak, $\epsilon > 1$, the rise in productivity dominates the reduction in the price; consequently, labor is reallocated to the skill-intensive sector. If the price effect is strong, $\epsilon < 1$, the price reduction dominates the rise in productivity and labor is reallocated to the unskilled-labor-intensive sector.

3.2. Endogenous Technology

Having solved for the sectoral allocations (υ) as an implicit function of its determinants, the technology (n) and the endowment ratio (z), we now turn to the model with an endogenous technology. Rates of return are equal across sectors in the steady state, which leads to my first proposition.

PROPOSITION 1. *In the interior steady state, the long-run allocation of skilled labor is given by*

$$\upsilon_{ss}(z) = \underset{\upsilon}{\text{argsolve}} \left\{ \left(\frac{\upsilon}{1-\upsilon}\right)^{\frac{(\epsilon-1)(\alpha+\beta)-1}{(\epsilon-1)(\alpha+\beta)}} \left(\frac{\alpha^2 - \left(\alpha^2 - \beta^2\right)\upsilon}{\beta^2}\right)^{\frac{\alpha-\beta}{\alpha+\beta}} = \frac{\eta_u}{\eta_s}\left(\frac{\beta}{\alpha}\right)^{\frac{1-(\epsilon-1)2\alpha}{(\alpha+\beta)(\epsilon-1)}} z^{\frac{\beta-\alpha}{\alpha+\beta}} \right\}, \tag{29}$$

and the steady-state relative technology is

$$n_{ss}(z) \equiv \left(\frac{N_s}{N_u}\right)_{ss} = \frac{\eta_s}{\eta_u}\left(\frac{\beta}{\alpha}\right)\left(\frac{\upsilon_{ss}(z)}{1-\upsilon_{ss}(z)}\right). \tag{30}$$

Proof. See the appendix.

Note that, unlike the short-run allocation of skilled labor which is given by equation (26), the long-run allocation of skilled labor $[\upsilon_{ss}(z)]$ can be written as a function of the endowment ratio. Once again, the distinction between the short-run and long-run is that in the long-run the relative technology, $n_{ss}(z)$, is endogenous.

The proof of the existence of a solution to (29) is an elementary application of the intermediate value theorem. The uniqueness of the equilibrium is more subtle, and deserves a thorough discussion. There are three cases to analyze: the case of complements ($\epsilon < 1$), "weak" substitutes $[(\alpha+\beta)^{-1} + 1 > \epsilon > 1]$, and "strong" substitutes $[\epsilon > (\alpha+\beta)^{-1} + 1]$. In the next subsection, I show when $(\alpha+\beta)^{-1}+1 > \epsilon$, there exists one root to equation (29) and the system is globally stable; when $\epsilon > (\alpha+\beta)^{-1} + 1$, there either exists one root (which is unstable), or three (one locally stable and two unstable).

3.3. Stability of the Steady State

Using equations (24), (4), and (23), the relative rate of return can be written as

$$\frac{r_s}{r_u} = K\left[\left(z\frac{\alpha^2-\left(\alpha^2-\beta^2\right)\upsilon(z,n)}{\beta^2}\right)^{\frac{\alpha-\beta}{\alpha+\beta}}\left(\frac{\upsilon(z,n)}{1-\upsilon(z,n)}\right)\right]^{\frac{(\epsilon-1)(\alpha+\beta)}{(\epsilon-1)(\alpha+\beta)+1}} \times n^{\frac{-1}{(\epsilon-1)(\alpha+\beta)+1}}, \qquad (31)$$

where K is a positive constant,

$$K = \frac{\eta_s}{\eta_u}\left(\frac{\beta}{\alpha}\right)^{\frac{2\alpha(\epsilon-1)}{(\epsilon-1)(\alpha+\beta)+1}}.$$

The transitional dynamics in this class of models is well known: when $r_s > r_u$, all R&D is directed to the skill-intensive sector—hence n grows. The condition for stability is thus that the relative rate of return is declining in n. An increase in n, in isolation, reduces the relative rate of return—the mechanics work through the price effect. This is what drives stability in Acemoglu (2002). However, in the present work, stability is no longer guaranteed through this mechanism. As equation (27) shows, as n changes so does υ. The relationship between the relative rate of return and υ deserves attention.

As υ rises, the relative price of skill-intensive goods falls and employment in the skill-intensive sector rises. When the goods are substitutes ($\epsilon > 1$), the price effect is relatively weak; consequently, an increase in υ leads to an increase in the relative rate of return. When the goods are complements ($\epsilon < 1$), the price effect dominates; the relative rate of return is declining in υ. Interestingly, for both cases (substitutes and complements), the feedback mechanism, the changes in υ induced by changes in n, work against stability. The condition for stability is that the combined effects—the direct and negative price effect, and the indirect positive feedback—are negative.[8]

Totally differentiating equation (31) with respect to n [and using equation (27)] yields the requirement for stability:

$$\frac{d\ln(r_s/r_u)}{d\ln(n)} < 0 \iff \frac{(\alpha+\beta)}{\frac{(\alpha-\beta)\left(\alpha^2-\beta^2\right)\upsilon(z,n)[1-\upsilon(z,n)]}{\alpha^2-\left(\alpha^2-\beta^2\right)\upsilon(z,n)}+\frac{1}{\epsilon-1}} \times\left[\frac{(\epsilon-1)(\alpha+\beta)-1}{(\alpha+\beta)(\epsilon-1)}-\frac{(\alpha-\beta)^2\upsilon(z,n)[1-\upsilon(z,n)]}{\alpha^2-\left(\alpha^2-\beta^2\right)\upsilon(z,n)}\right] < 0. \qquad (32)$$

Equation (32) yields the condition for global stability.

PROPOSITION 2. *The system is globally stable provided*

$$(\alpha+\beta)^{-1}+1 > \epsilon. \qquad (33)$$

https://doi.org/10.1017/S1365100517000505 Published online by Cambridge University Press

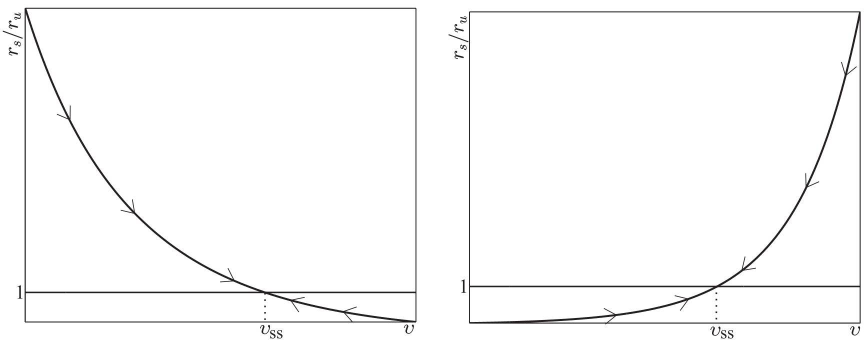
Proof. When the goods are weak substitutes, $(\alpha+\beta)^{-1}+1>\epsilon>1$, the first term of (32) is positive, and the second term is negative. When the goods are complements ($\epsilon<1$), the first term of equation (32) is negative and the second term is positive. ■

Intuitively when the goods are not strong enough substitutes, the feedback induced by the reallocation is not strong enough to overcome the direct and negative price effect of increasing n. To understand the stability of the system, it is useful to reduce the dimensionality of the problem.[9] Given equation (26), the relative rate of return, equation (31), can be rewritten as

$$\frac{r_s}{r_u}=\frac{\eta_s}{\eta_u}\left(\frac{\beta}{\alpha}\right)^{\frac{2\alpha(\epsilon-1)-1}{(\alpha+\beta)(\epsilon-1)}}\left(\frac{\upsilon(z,n)}{1-\upsilon(z,n)}\right)^{\frac{(\alpha+\beta)(\epsilon-1)-1}{(\alpha+\beta)(\epsilon-1)}} \left(\frac{\alpha^2-\left(\alpha^2-\beta^2\right)\upsilon(z,n)}{\beta^2}\right)^{\frac{\alpha-\beta}{\alpha+\beta}} z^{\frac{\alpha-\beta}{\alpha+\beta}}. \tag{34}$$

The globally stable system [as a function of $\upsilon(z,n)$] is depicted in Figure 2 for the cases of weak substitutes and complements.

In both cases, if the initial conditions $\upsilon(z_0,n_0)$ are such that the relative rate of return is bigger than one, the economy will develop machines targeting the skill-intensive sector. The difference between substitutes and complements is the effect that this biased technical progress has on employment. We know from (27) that an increase in n leads to an increase in υ when the goods are substitutes, and a decrease in υ when the goods are complements. Figure 2(a) depicts the dynamics when the good are substitutes. When n rises, labor is reallocated toward the growing sector. Figure 2(b) depicts the dynamics when the goods are complements. When n rises, labor is reallocated toward the stagnating sector. In both cases, the equilibrium is unique, and the system is globally stable.



(a) Weal sibstotites: $1<\epsilon<(1+\alpha+\beta)/(\alpha+\beta)$ (b) Complements: $\epsilon<1$

FIGURE 2. Weak substitutes and complements: $\epsilon<(\alpha+\beta)^{-1}+1$.

https://doi.org/10.1017/S1365100517000505 Published online by Cambridge University Press

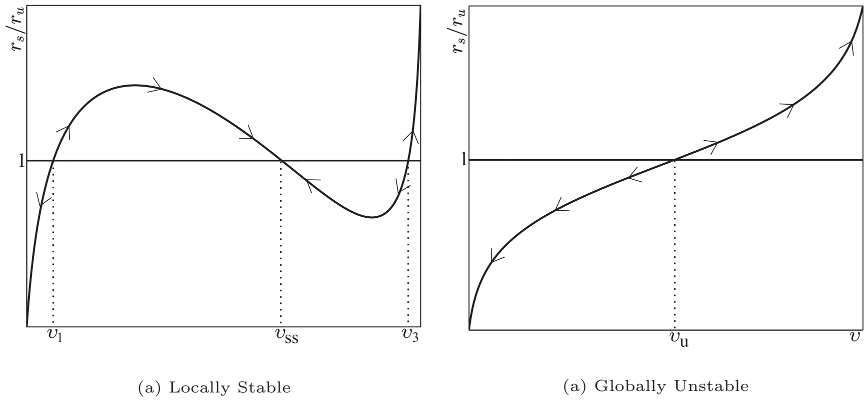


FIGURE 3. Strong substitutes: $\epsilon > (\alpha + \beta)^{-1} + 1$.

We now turn to the case of strong substitutes; when $\epsilon > (\alpha + \beta)^{-1} + 1$, the possibility of multiple regimes emerge. In particular, the following proposition holds.

PROPOSITION 3. *When $\epsilon > (\alpha + \beta)^{-1} + 1$, there either exists three steady states, the middle one being locally stable, or one globally unstable steady state. The condition for local stability is*

$$\frac{(\alpha + \beta)(\epsilon - 1) - 1}{(\alpha + \beta)(\epsilon - 1)} < \frac{(\alpha - \beta)^2 \upsilon_{ss}(z)[1 - \upsilon_{ss}(z)]}{\alpha^2 - \left(\alpha^2 - \beta^2\right)\upsilon_{ss}(z)}. \tag{35}$$

Proof. See the appendix.

Proposition 3 highlights two important features of the model. First, when the two goods are strong substitutes, the system is *at most* locally stable. The condition for local stability is that the rate of return is declining in the neighborhood of the steady state. Second, the system has the potential to be globally unstable. The intuition behind both of these features can be seen from the condition for local stability: when υ is made arbitrarily small, or large, the right-hand side is necessarily smaller than the left-hand side. In other words, if the initial allocation $\upsilon(z_0, n_0)$ is small enough, the rate of return to unskilled directed technical change is increasing in the level of technology, N_u; if $\upsilon(z_0, n_0)$ is large enough, the rate of return to skill-intensive directed technical change is increasing in N_s.

If α and β are close in value, the system is more likely to be globally unstable; consequently, the range of the interior solution becomes wider when β is small relative to α. The intuition is that "gap" between α and β determines how difficult it is to reallocate factors across sectors. If the factor intensities across sectors are similar, it is easy to reallocate factors and the system is globally unstable. If the factor intensities differ greatly, it is more difficult to reallocate factors across sectors. Figure 3 plots the relative rate of return, (34), for both possibilities.

Figure 3(a) shows that if $\upsilon(z_0, n_0)$ (the endogenous allocation of skilled labor to the skill-intensive industry as a function of the initial conditions) is smaller than the first root (υ_1), the economy will specialize in the production of unskilled goods—υ goes to zero. If $\upsilon(z_0, n_0)$ lies between the first (υ_1) and third root (υ_3), the economy will produce both goods—υ goes to υ_{ss}. If $\upsilon(z_0, n_0)$ is larger than the third root, the economy will specialize in the production of skill intensive goods—υ goes to one.

Figure 3(b) depicts the system when α and β are close together. If $\upsilon(z_0, n_0)$ is smaller than υ_u, the economy will specialize in the production of unskilled goods. If $\upsilon(z_0, n_0)$ is larger than υ_u, the economy will specialize in the production of skill-intensive goods.

4. STRUCTURAL CHANGE

This section analyzes the economy's *dynamic* response to a change in the endowment ratio, z. Following an increase in the relative supply of skilled labor: there is a "short-run" reallocation of factors which holds the technology fixed, and a "long-run" reallocation that features an endogenous technology.

The short-run reallocation is obtained by taking the log of (26) and implicitly differentiating it; doing so yields the following corollary.

COROLLARY 2. *The short-run reallocation of skilled labor following an increase in z is*

$$\vartheta_{\mathrm{SR}} \equiv \frac{d\upsilon\,(z,n)}{dln\,(z)} = \frac{(\alpha-\beta)\,\upsilon\,(z,n)\,[1-\upsilon\,(z,n)]}{\frac{(\alpha-\beta)\left(\alpha^2-\beta^2\right)\upsilon(z,n)[1-\upsilon(z,n)]}{\alpha^2-\left(\alpha^2-\beta^2\right)\upsilon(z,n)}+\frac{1}{\epsilon-1}}. \qquad \textbf{(36)}$$

When $\epsilon > 1$, ϑ_{SR} is positive; when $\epsilon < 1$, ϑ_{SR} is negative.

Proof. See the appendix.

Recall from equation (26) that $\upsilon\,(z,n)$ is determined by z and n; consequently, the derivative is a function of z and n. The direction of the reallocation, like equation (27), is determined by the elasticity of substitution. In Acemoglu (2002), an increase in the relative endowment of skilled labor, z, trivially leads to an increase in employment in the skill intensive sector. In the present model whether or not employment rises is determined by the elasticity of substitution between the sectors. It natural follows that the Rybczynski theorem [Rybczynski (1955)] does not generalize to a general equilibrium setting unless the two sectors are substitutes.[10] Holding the technology and L fixed, an increase in Z will lead to a reallocation of factors (skilled and unskilled labor) to the Z-intensive sector only if $\epsilon > 1$. If $\epsilon < 1$, factors are reallocated toward the sector that uses the factor unintensively—the precise *opposite* prediction of Rybczynski.

The intuition behind equation (36) can be gained from the maintenance of equal marginal revenue products of skilled labor in both sectors. Holding the allocations constant, an increase in z will increase production in the skill-intensive sector;

https://doi.org/10.1017/S1365100517000505 Published online by Cambridge University Press

therefore the relative price, P_s/P_u, declines. Simultaneously, the relative marginal product of labor in the skill-intensive sector increases. When the goods are substitutes, the price effect is relatively weak and the increase in the relative marginal product dominates; consequently, υ increases. When the goods are complements, the price effect dominates and υ decreases.

To obtain the long-run reallocation of factors—which incorporates an endogenous technology—we must once again turn to the implicit function theorem. Taking the log of equation (29) and totally differentiating it yields the following corollary.

COROLLARY 3. *The long-run reallocation of skilled labor following an increase in z is*

$$\vartheta_{\text{LR}} \equiv \frac{d\upsilon_{ss}(z)}{dln(z)} = \frac{(\alpha-\beta)\,\upsilon_{ss}(z)\,[1-\upsilon_{ss}(z)]}{\frac{(\alpha-\beta)\left(\alpha^2-\beta^2\right)\upsilon_{ss}(z)[1-\upsilon_{ss}(z)]}{\alpha^2-\left(\alpha^2-\beta^2\right)\upsilon_{ss}(z)} - \left(\frac{(\epsilon-1)(\alpha+\beta)-1}{\epsilon-1}\right)}. \qquad \textbf{(37)}$$

When $\epsilon > 1$, ϑ_{LR} is positive; when $\epsilon < 1$, ϑ_{LR} is negative.

Proof. See the appendix.

Corollary 3 implies that—including the endogenous direction of technical progress—an increase in the relative supply of skilled labor (z) will lead to a reallocation of labor toward (away from) the skill-intensive sector when the two goods are substitutes (complements). In the next subsection, I compare the short-run and long-run reallocation of labor and introduce weak- and strong-induced structural change.

4.1. Weak-Induced Structural Change

We now turn to the first major result of the model, *weak-induced structural change*: following an increase in the supply of a factor, there is always a dynamic reallocation of factors toward the sector that uses the factor intensively. The next proposition characterizes weak-induced structural change formally.

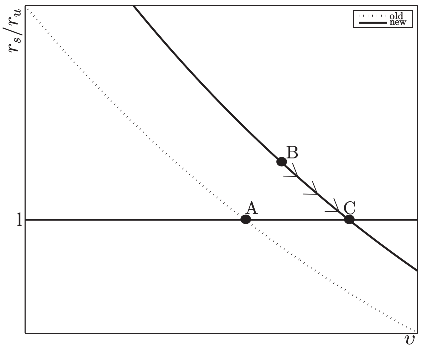
PROPOSITION 4. *Suppose the relative supply of skilled workers increases from z_1 to z_2, then*

$$\upsilon_{ss}(z_2) > \upsilon(z_2, n_{ss}(z_1)), \qquad \textbf{(38)}$$

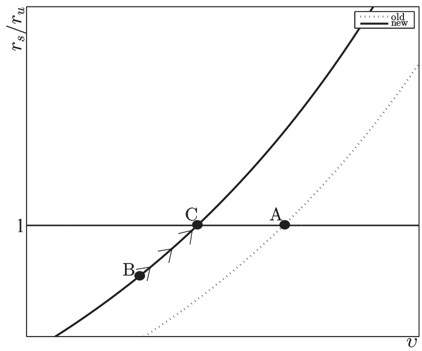
where $\upsilon_{ss}(z_2)$ is the new steady-state allocation of skilled labor, and $\upsilon(z_2, n_{ss}(z_1))$ is the short-run allocation, which holds the technology constant at the original steady-state value.

Proof. See the appendix.

Proposition 4 characterizes the dynamic reallocation of factors. The distinguishing feature between the two allocations is the technology. The short-run allocation, $\upsilon(z_2, n_{ss}(z_1))$, holds the relative technology constant at the original steady-state value, $n_{ss}(z_1)$. In the long -run, when the technology is endogenous, the allocation



(a) Substitutes



(b) Complements

FIGURE 4. Weak-induced structural change.

is given by $\upsilon_{ss}(z_2)$. Following an increase in the relative endowment of skilled labor, and corresponding short-run reallocation of factors, there is always structural change biased toward the skill-intensive sector—what I refer to as weak-induced structural change.

Figure 4 illustrates the static and dynamic reallocation of skilled labor following an increase in the relative supply of skilled workers from z_1 to z_2. If the two sectors are substitutes, as depicted in Figure 4(a), the short-run reallocation is the increase in υ from point A to point B. At point B, the relative rate of return for skill-intensive machines is more profitable; consequently, n grows. Corollary 1 implies that as n increases, because the goods are substitutes, so too does υ—weak-induced structural change is positive reallocation of labor from point B to the new steady state, C. If the two sectors are complements, as depicted in Figure 4(b), the short-run reallocation is the reduction in υ from point A to point B. At point B, the creation of unskilled-labor-intensive machines is higher; consequently, n decreases. Corollary 1 implies that, because the goods are complements, υ rises. Weak-induced structural change is, again, the movement from B to the new steady state C. Although the long-run change (from A to C) is negative, the dynamic reallocation, from B to C, is always positive.[11]

Figure 4 illustrates why it is remarkable that weak-induced structural change holds regardless of the elasticity of substitution. Consider the case when the goods are substitutes. The short-run increase in employment (from A to B) is not as large as the long-run response (from A to C). The case of complements is the opposite. The short-run decrease in employment (from A to B) is too large. The biased technical progress leads to a reallocation of labor *back* to the skill-intensive sector. Intuitively, when the goods are substitutes (complements), the increase in z leads to a short-run increase (decrease) in υ because the price effect is relatively weak (strong). Correspondingly, the relative rate of return also increases (decreases) when the goods are substitutes (complements). However, since the relationship

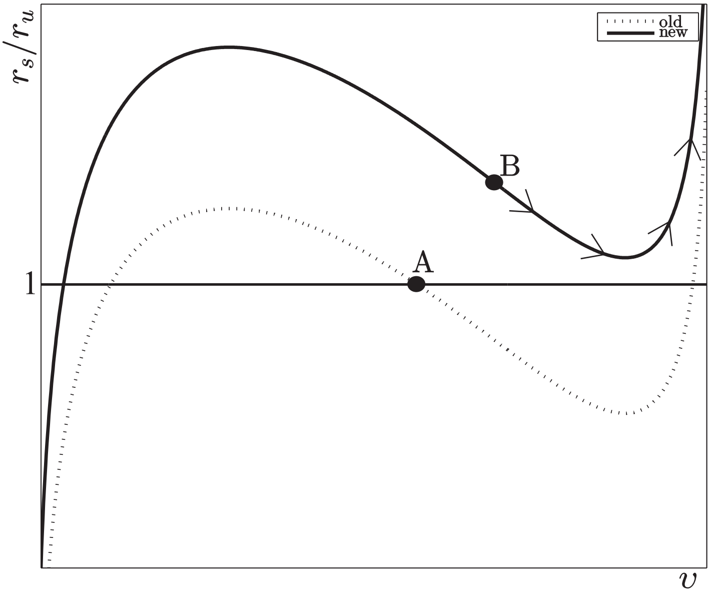


FIGURE 5. Strong-induced structural change.

between n and υ works through the same price effect, in both cases the dynamic reallocation is positive.

4.2. Strong-Induced Structural Change

I define strong-induced structural change as an endogenous regime switch from the interior regime to specialization induced by a change in the endowment ratio, z. Figure 5 demonstrates strong-induced structural change.

Following an increase in the endowment of skilled labor, the relative rate of return shifts up and υ rises. Since the rate of return to creating skill-intensive machines is higher, the economy reallocates labor toward the skill-intensive sector. As depicted, the relative rate of return never drops back down to one; therefore following the jump from A to B, υ keeps growing until the economy becomes completely specialized. Similarly, but not depicted, a decrease in the endowment of skilled labor can lead to specialization in the production of unskilled-labor-intensive goods.[12] The next proposition characterizes strong-induced structural change formally.

PROPOSITION 5. *When* $\epsilon > (\alpha + \beta)^{-1} + 1$ *and* $\beta > 0$, *if a locally stable regime exists, it only exists for*

$$z \in [\bar{z}_u, \bar{z}_s].$$

https://doi.org/10.1017/S1365100517000505 Published online by Cambridge University Press

If z falls below $\bar{z}_u$, the economy will converge to the unskilled-labor-intensive technology. If z rises above $\bar{z}_s$, the economy will converge to the skill-intensive technology.

Proof. See the appendix.

The bounds, $\bar{z}_u$ and $\bar{z}_s$, are calculated in the appendix. Before discussing the intuition of Proposition 5, it is worth noting that a locally stable regime does not necessarily exist *for any* endowment; Figure 3(b) shows that a locally stable interior solution does not exist when $\alpha \approx \beta$. Proposition 5 implies that if a locally stable solution exists, it only exists for a finite range of relative endowments. If the relative endowment of skilled labor increases to a value that is greater than $\bar{z}_s$, strong-induced structural change occurs and the economy specializes in the production of skill-intensive goods. Symmetrically, if the relative endowment of skilled falls below $\bar{z}_u$, the economy will specialize in the production of unskilled-labor-intensive goods. Figure 6 demonstrates the relative endowments that admit a locally stable regime.

As discussed at length in Section 3.3, the stability of the system requires that the relative rate of return (in the neighborhood of the steady state) is declining in n. It is worthwhile to discuss the mechanics in more detail. The general equilibrium effect of an increase in n can be broken down into two forces, or three when disaggregated:

- the direct price effect (negative);
- the feedback effect
  - the ratio effect (negative)
  - the market size effect (positive).

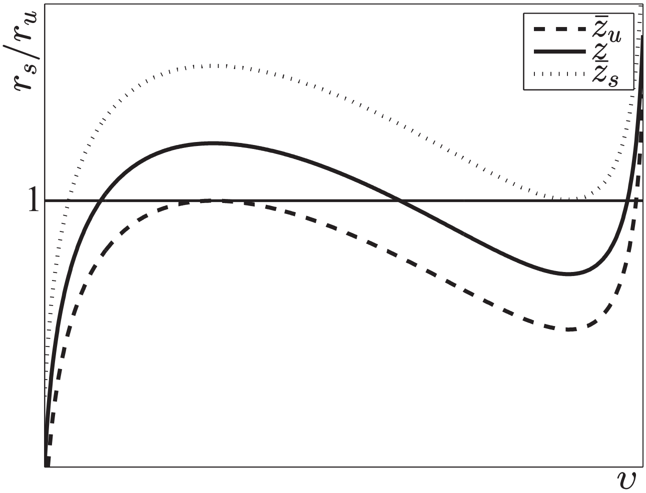


FIGURE 6. Boundaries for strong-induced structural change.

https://doi.org/10.1017/S1365100517000505 Published online by Cambridge University Press

When n rises, holding the allocation of factors constant, the price of skill-intensive goods declines and hence the relative profitability of creating skill-intensive machines declines. However, as equation (27) shows, when the goods are substitutes (as we are assuming here), factors are reallocated to the growing sector. The reallocation of factors induced by the biased technical progress, the feedback effect, has its own effect on technical progress. The feedback effect can be disaggregated into two forces: the reduction in the ratio of skilled to unskilled labor employed in both sectors (the ratio effect), and the increased employment of factors in the skill-intensive sector (the market size effect).

The ratio effect is similar to the mechanics driving the Stopler–Samuelson theorem.[13] Since the employment ratio of skilled to unskilled labor differs in both sectors, the reallocation of factors requires that the sectoral input utilization must change. To maintain the endowment constraints, the ratio of skilled labor to unskilled labor employed in both sectors falls. Since the skill-intensive sector, by definition, uses relatively more skilled labor, the reduction in the skilled to unskilled labor ratio reduces the relative profits. Though the reallocation of factors is absent from Acemoglu (2002), the effect of increased employment of both factors in the skill-intensive sector is similar to the well-known market size effect. When the skill-intensive sector employs more factors of production, the creation of machines used in that sector becomes more profitable.[14]

When the goods are strong substitutes [$\epsilon > (\alpha + \beta)^{-1} + 1$], the market size effect can dominate the ratio and price effects. Intuitively, when the goods are strong enough substitutes, the price effect is weak. Why then does an interior solution exist at all? For some parameter and state configurations, the price effect, *when combined with the ratio effect*, dominates the market size effect. The reason strong-induced structural change necessarily occurs when z gets too large (or small) is that the reduction in the sectoral employment ratio is by its nature, bounded—while the market size effect, the relative employment of factors, is not.[15] Consequently, when z rises (falls) enough, the market size effect dominates the ratio and price effects and the relative rate of return is increasing in n.

5. IMPLICATIONS FOR THE SKILL PREMIUM

Having solved for the results on structural change, we now turn the theory's implications for the skill premium. At any given moment in time, the relative wage rate can be written as

$$\omega\left(z, \upsilon\left(z, n\right)\right) = \left(\frac{\alpha\beta}{\alpha^2 - \left(\alpha^2 - \beta^2\right)\upsilon\left(z, n\right)}\right) z^{-1}. \qquad \textbf{(39)}$$

Equation (39) shows that the relative wage rate can be written as a function of three things: the relative technology (n), the endowments (z), and the endogenous allocation of factors (which is itself determined by n and z). Before turning to

https://doi.org/10.1017/S1365100517000505 Published online by Cambridge University Press

weak and strong bias [Acemoglu (2002)], I discuss the implications of exogenous changes in the relative technology, n.

5.1. Exogenous Skill-Biased Technical Change

Taking the log of equation (39) and differentiating it with respect to n [and using equation (27)] yields the relationship between the relative technology and the relative wage rate

$$\frac{d\ln(\omega(z,\upsilon(z,n)))}{d\ln(n)} = \left(\frac{\alpha+\beta}{\alpha-\beta+\left(\frac{\alpha^2-\left(\alpha^2-\beta^2\right)\upsilon(z,n)}{\left(\alpha^2-\beta^2\right)\upsilon(z,n)[1-\upsilon(z,n)]}\right)\frac{1}{\epsilon-1}}\right). \quad \mathbf{(40)}$$

Equation (40) illustrates an interesting result: the effects of identical skill-biased technical progress *will not have identical effects* on economies with different relative supplies of skilled labor. This is in stark contrast to the prediction of what Acemoglu and Autor (2011) call the canonical model.[16] The model collapses to the canonical model when $\beta = 0$. In this special case

$$\frac{d\ln(\omega)}{d\ln(n)} = \frac{\alpha(\epsilon-1)}{\alpha(\epsilon-1)+1}. \quad \mathbf{(41)}$$

Why is (40) different from (41)? When the allocation of factors is endogenous, the effects of skill-biased technical change on the relative wage rate depends on the sectoral allocations of labor. Intuitively, when $\upsilon(z,n)$ is small, a given improvement in the skill-intensive technology does not have a large effect on the relative wage rate because employment in that sector is low. On the other hand, as $\upsilon(z,n)$ approaches one, the economy is specialized in the production of skill-intensive goods and the technological improvement becomes hicks neutral. The presence of imperfect factor intensity ($\beta > 0$) fundamentally alters the time series and cross-section predictions of the model.

The canonical model ($\beta = 0$) implies that constant-biased exogenous growth will have a constant impact on the relative wage rate. When $\beta > 0$, the same biased exogenous growth will affect economies—which have different endowments and hence different allocations—in different ways. In the remainder of this section, I focus on the case of substitutes ($\epsilon > 1$), which implies that technical progress in the skill-intensive sector increases the relative wage rate of skilled labor.[17] Contrary to the canonical model, constant-biased exogenous growth does not yield a constant growth rate in the relative wage. Holding z constant, starting from a low value of n, as n increases the relative wage rate increases a small amount. At an intermediate range of n, the relative wage rate will increase faster. Finally as the economy approaches specialization, the technical progress becomes hicks neutral and the growth rate in the relative wage rate declines. Therefore, even holding z fixed and the growth of n constant, the growth rate of the relative wage rate is not constant.

Equation (40) highlights an interesting issue. There are two forces that govern the relative wage rate: changes in the supply of skills and skill-biased technical change.[18] The former reduces inequality, whereas the later increases it. Suppose that technological transfer is perfect, then the observed skill-biased technical change in rich countries should lead to skill-biased technical change in laggard countries. The canonical model predicts that the skill-biased technical change will have identical effects on the relative wage rates in both types of countries: the laggards and those on the frontier. However, there exists huge variations in the growth rates of the relative wage rates. If the canonical model is to be believed, there are two explanations for the observed differences. First, the differences are due to varying growth rates in the supply of skilled labor across countries. Second, technological transfer is not playing a large role. The present model provides a third explanation: identical technical change will have different effects on different economies.

5.2. Strong Bias

Unsurprisingly, the present model exhibits weak bias.[19] Recall that weak-bias implies: following an increase in the relative endowment of skilled labor, the transitional dynamics are such that the relative wage rate of skilled labor increases. The next proposition characterizes the relationship between the steady-state relative wage rate and the endowment of skilled labor.

PROPOSITION 6. *The long-run relationship between the supply of skilled labor and the relative wage rate is*

$$\frac{dln(\omega_{ss})}{dln\,(z)} = \frac{(\epsilon-1)\,(\alpha+\beta)-1}{\frac{(\epsilon-1)(\alpha+\beta)(\alpha-\beta)^2\upsilon_{ss}(z)[1-\upsilon_{ss}(z)]}{\alpha^2-\left(\alpha^2-\beta^2\right)\upsilon_{ss}(z)} - ((\epsilon-1)\,(\alpha+\beta)-1)}, \qquad \textbf{(42)}$$

where

$$\frac{dln(\omega_{ss})}{dln\,(z)} > 0 \implies \epsilon > (\alpha+\beta)^{-1}+1. \qquad \textbf{(43)}$$

Proof. See the appendix.

Equation (42) illustrates many points that are worthwhile to discuss. The effects of a change in z on the steady-state relative wage rate depends on the endogenous allocation of factors, $\upsilon_{ss}\,(z)$. An obvious application of this is cross-country comparisons. The denominator will be smaller when $\upsilon_{ss}\,(z)$ is very small, or very large. Hence, changes to the supply of educated workers will have a larger impact on relative wage rates for countries at polarizing ends of the relative supply; this feature is especially interesting when the parameters allow for strong bias.

Recall that strong bias refers to a positive relationship between the steady-state relative wage rate and the relative supply of a factor [Acemoglu (2002)]. Equation (43) shows that strong bias only holds if the elasticity of substitution is sufficiently high. What is remarkable is that the necessary conditions for strong bias are inconsistent with global stability (35). Interestingly, in the neighborhood of the

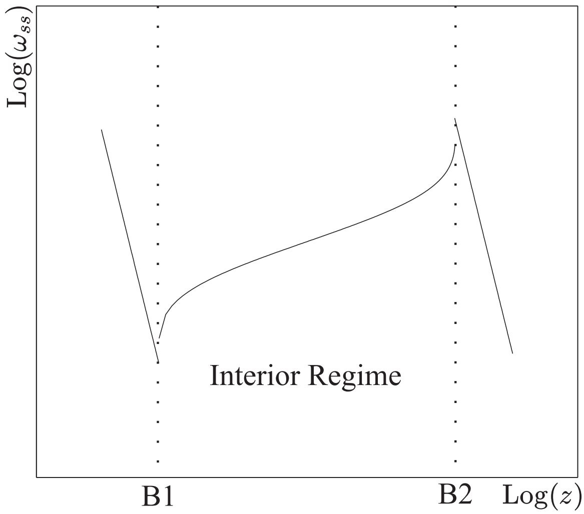


FIGURE 7. Locally stable steady-state relative wage rate.

locally stable equilibrium, strong bias necessarily holds because the condition for local stability (35) ensures that (42) is in fact positive. Figure 7 depicts the locally stable steady-state relative wage rate as a function of the relative supply of skilled labor.

The steady-state relative wage rate is rotated-Z shaped in the relative supply of skilled labor. To understand Figure 7, it is useful to begin in the interior (both sectors operating) regime. Inside the interior, a small increase in the supply of skills induces technical change that is sufficiently biased toward skilled labor so that the relative wage increases. However, strong-induced structural change implies that if z gets too large, the interior regime becomes globally unstable. Consequently, when z reaches a critical threshold [Log(z) $>$ $B2$], the economy specializes in the production of skill-intensive goods. There is a discontinuity at this point because the economy transitions from the interior solution to specialization and the steady-state relative wage rate jumps up.[20] Likewise, beginning in the interior regime, and reducing the relative supply of skills, the relative wage rate declines until [Log (z) $<$ $B1$], at this point strong-induced structural change implies that the economy necessarily specializes in the production of unskilled-labor-intensive goods. This implication is important for empirical tests of directed technical change. If an economy's relative supply of skills is outside the bounds, strong bias will fail to hold.

It is hard to know if the regime switching cases discussed above are of empirical interest; if one is true, the first seems the most likely. Consider Sub-Saharan Africa, the relative supply of skilled workers is very low. Because of this, our theory implies that the economies in this region have historically specialized in the production of unskilled-intensive goods. Therefore, small increases in the supply of skilled workers do not induce a transition to the interior solution. However, if there is a sufficiently large rise in the relative supply of skilled labor that pushes the economy into the interior regime, increases in the supply of skilled labor can induce directed technical change. If strong bias occurs, as Acemoglu and Autor (2011) deems empirically plausible, the transition to a "modern" economy will elicit an enormous increase in inequality.

6. CONCLUSION

This paper has provided a simple, yet fruitful, generalization of Acemoglu (2002) that yields a set of new predictions. The first prediction of "weak-induced structural change" implies that the technical progress induced by an increase in the endowment of a factor always yields a dynamic reallocation of factors toward the sector that uses the factor intensively, even when the two goods are complements. The second prediction of "strong-induced structural change" implies that the directed technical change induced by a change in the relative endowment of a factor can fundamentally transform the economy. It is possible that the economy can completely specialize and converge to a skill (unskilled) intensive production technology. Finally, the model yields fundamentally new predictions concerning strong bias. Namely, the relationship between the relative supply of skilled labor and the steady-state relative wage rate is not necessarily monotonic.

Directed structural change provides a new explanation for structural change—in particular skill-biased structural change. In contrast to existing work, structural change is ultimately driven by changes in the relative endowments of the factors of production and the corresponding induced technical progress. But, importantly, this new framework also provides a new tool that can be applied to issues that traditional directed technical change is unsuitable for. I briefly discuss three potential applications. First, consider the introduction of international trade into directed technical change [Acemoglu (1998, 2002)]; imperfect factor intensity allows trade to influence the allocation of factors (the driving force behind all Heckscher–Ohlin results). Second is the application of directed technical change to labor misallocations. Hsieh and Klenow (2009) show that there exists substantial misallocations of factors of production in China and India. If these countries improved labor market flexibility, what implications would this have for directed technical change? The removal of the artificial barriers would induce labor reallocations which would, by itself, induce directed technical change. Furthermore, the induced technical progress would lead to even more reallocations. Finally, the introduction of labor market frictions might prove useful. Weak-induced structural change implies that increases in the supply of skilled workers always yield dynamics in which factors are reallocated toward the skill-intensive sector. On the transition path unemployment might be biased.

Though the model presented here is admittedly simple, fundamentally new results emerge. The new results concerning strong bias are of particular interest given the importance of understanding the distributional implications of directed technical change. As long as the factor share of the unintensively used resource is positive ($\beta > 0$), strong bias *cannot* hold everywhere. Strong bias can at most hold for a finite range of relative supplies. Therefore, empirical tests of directed technical change must be done very carefully. Even if two economies differ only by their relative supply of skill, strong bias might hold in one country but fail in the other.

NOTES

1. Zuleta and Young (2013) develop a model of structural change that relies on factor elimination; the manufacturing sector engages in investment to increase the capital's share (in the Cobb–Douglas production function). As the manufacturing technology becomes "AK," labor is reallocated to the service sector.

2. See Herrendorf et al. (2014) for a comprehensive review of the structural change literature.

3. It would be nice to incorporate endogenous education, as in Galor and Moav (2000), but this would make the model intractable.

4. Acemoglu and Guerrieri (2008) explain capital deepening using a similar structure, but with exogenous-biased technical progress.

5. The proof that the second effect always dominates the first is needed several times throughout the paper; it is proved in the appendix under subsection "Lemma 2."

6. In the appendix, I discuss a special case, $\alpha = \beta$, that yields a closed-form solution.

7. See the appendix for the proof and discussion. Following Acemoglu (2002), I do not analyze the case in which $\epsilon = 1$, (all technical progress is hicks neutral).

8. Of course one can obtain the conditions for stability by calculating the eigenvalues for the system. Nonetheless, because of the model's close relation to Acemoglu (2002), I present the stability analysis in similar fashion.

9. Of course, normally one would like to write the relative rate of return in terms of the state variable, n. However, the system does not admit a closed form solution for υ as a function of n.

10. Recall that the Rybczynski theorem holds prices fixed.

11. In the appendix, "Relation to price and market size effects," I discuss the model through the lens of price and market size effects.

12. For brevity, I omit the symmetric case of a fall in the supply of skills. Moreover, most countries have seen increases in the supply of skilled labor.

13. Similar in that the employment ratios are endogenous. In Stolper and Samuelson (1941), exogenous price movements drive the reallocation of factors; they do not discuss changes in the technology.

14. Of course as υ increases, the relative price falls. It is straightforward to show that when $\epsilon > 1$, the combined effect is positive. In the appendix, I discuss price and market size effects in more detail.

15. The ratio effect is bounded because as $\upsilon(z, n)$ goes to one or zero, Z_s/L_s and Z_u/L_u approaches Z/L. The market size effect is not bounded because the relative employment: $Z_s/Z_u = \upsilon(z, n) / (1 - \upsilon(z, n))$ and $L_s/L_u = \beta^2 \upsilon(z, n) / \alpha^2 (1 - \upsilon(z, n))$, goes to infinity (zero) as $\upsilon(z, n)$ goes to one (zero).

16. Recall that the canonical model is the CES production structure, assumed here, with perfect factor intensity.

17. It is straightforward to show that equations (40) and (41) are positive if and only if $\epsilon > 1$.

18. There is a large literature that demonstrates the importance of skill-biased technical change; see, for example, Acemoglu and Autor (2011) and McAdam and Willman (2017).

19. Acemoglu (2007) shows that weak bias is quite general.

20. Recall Figure 7 plots the *steady-state* relative wage rate. At a given moment in time, the relationship between z and ω is continuous.

REFERENCES

Acemoglu, D. (1998) Why do new technologies complement skills? Directed technical change and wage inequality. *Quarterly Journal of Economics* 113(4), 1055–1089.

Acemoglu, D. (2002) Directed technical change. *Review of Economic Studies* 69(4), 781–809.

Acemoglu, D. (2007) Equilibrium bias of technology. *Econometrica* 75(5), 1371–1409.

Acemoglu, D. and D. H. Autor (2011) Skills, tasks and technologies: Implications for employment and earnings. In O. Ashenfelter and D. E. Card (eds.), *Handbook of Labor Economics*, vol. 4, chap. 12, pp. 1043–1171. Amsterdam: Elsevier.

Acemoglu, D. and V. Guerrieri (2008) Capital deepening and nonbalanced economic growth. *Journal of Political Economy* 116(3), 467–498.

Acemoglu, D. and F. Zilibotti (2001) Productivity differences. *Quarterly Journal of Economics* 116(2), 563–606.

Buera, F. J. and J. P. Kaboski (2012) The rise of the service economy. *American Economic Review* 102(6), 2540–69.

Dolores Guilló, M., C. Papageorgiou, and F. Perez-Sebastian (2011) A unified theory of structural change. *Journal of Economic Dynamics and Control* 35(9), 1393–1404.

Galor, O. and O. Moav (2000) Ability-biased technological transition, wage inequality, and economic growth. *Quarterly Journal of Economics* 115(2), 469–497.

Goldin, C. D. and L. F. Katz (2008) *The Race between Education and Technology.* Cambridge, MA: Belknap Press of Harvard University Press.

Herrendorf, B., R. Rogerson, and K. Valentinyi (2014) Growth and structural transformation. In P. Aghion and S. N. Durlauf (eds.), *Handbook of Economic Growth*, vol. 2B, chap. 6, pp. 855–941. Amsterdam and New York: North Holland.

Hsieh, C.-T. and P. J. Klenow (2009) Misallocation and manufacturing TFP in China and India. *Quarterly Journal of Economics* 124(4), 1403–1448.

McAdam, P. and A. Willman (2017) Unraveling the skill premium. *Macroeconomic Dynamics* 1–30. doi:10.1017/S1365100516000547.

Ngai, L. R. and C. A. Pissarides (2007) Structural change in a multisector model of growth. *American Economic Review* 97(1), 429–443.

Peretto, P. F. and J. J. Seater (2013) Factor-eliminating technical change. *Journal of Monetary Economics* 60(4), 459–473.

Rivera-Batiz, L. A. and P. M. Romer (1991) Economic integration and endogenous growth. *Quarterly Journal of Economics* 106(2), 531–55.

Rogerson, R., J. Kaboski, and F. Buera (2015) Skill-Biased Structural Change and the Skill Premium. Meeting papers 895, Society for Economic Dynamics.

Romer, P. M. (1990) Endogenous technological change. *Journal of Political Economy* 98(5), S71–S102.

Rybczynski, T. M. (1955) Factor endowment and relative commodity prices. *Economica* 22(88), 336–341.

Stolper, W. F. and P. A. Samuelson (1941) Protection and real wages. *Review of Economic Studies* 9(1), 58–73.

Zuleta, H. and A. T. Young (2013) Labor shares in a model of induced innovation. *Structural Change and Economic Dynamics* 24(C), 112–122.

APPENDIX

A.1. PROOF OF LEMMA 1

Combining (12) and (11) yields

$$\frac{P_u\alpha\left(\int_0^{N_u} X_{ui}^{1-\alpha-\beta}di\right)\left(\frac{L_u}{Z_u}\right)^{\alpha-1} Z_u^{\alpha+\beta-1}}{P_u\beta\left(\int_0^{N_u} X_{ui}^{1-\alpha-\beta}di\right)\left(\frac{L_u}{Z_u}\right)^{\alpha} Z_u^{\alpha+\beta-1}}=\frac{P_s\beta\left(\int_0^{N_s} X_{si}^{1-\alpha-\beta}di\right)\left(\frac{L_s}{Z_s}\right)^{\beta-1} Z_s^{\alpha+\beta-1}}{P_s\alpha\left(\int_0^{N_s} X_{si}^{1-\alpha-\beta}di\right)\left(\frac{L_s}{Z_s}\right)^{\beta} Z_s^{\alpha+\beta-1}},$$

trivial algebra yields (13)

$$\frac{Z_u}{L_u}=\left(\frac{\beta}{\alpha}\right)^2\frac{Z_s}{L_s}.$$

Substituting in the labor constraints: $L_u=L-L_s$ and $Z=Z_s+Z_u$, and defining $\upsilon Z=Z_s$ yields

$$L_s=\frac{\left(\frac{\beta}{\alpha}\right)^2\upsilon L}{1-\upsilon+\left(\frac{\beta}{\alpha}\right)^2\upsilon}.$$

Therefore,

$$\frac{Z_s}{L_s}=\frac{\upsilon Z}{\frac{\left(\frac{\beta}{\alpha}\right)^2\upsilon L}{1-\upsilon+\left(\frac{\beta}{\alpha}\right)^2\upsilon}}=\left(\frac{\alpha^2-\left(\alpha^2-\beta^2\right)\upsilon}{\beta^2}\right)\frac{Z}{L}.$$

We now turn to the derivation of the relative wage rate. Equation (11) and (12) imply

$$\frac{w_Z}{w_L}=\frac{P_u\beta\left(\int_0^{N_u} X_{ui}^{1-\alpha-\beta}di\right)\left(\frac{Z_u}{L_u}\right)^{-\alpha} Z_u^{\alpha+\beta-1}}{P_u\alpha\left(\int_0^{N_u} X_{ui}^{1-\alpha-\beta}di\right)\left(\frac{Z_u}{L_u}\right)^{1-\alpha} Z_u^{\alpha+\beta-1}}=\frac{\beta}{\alpha}\left(\frac{Z_u}{L_u}\right)^{-1}.$$

Substituting in equation (13) yields

$$\omega=\frac{w_Z}{w_L}=\left(\frac{\beta\alpha}{\alpha^2-\left(\alpha^2-\beta^2\right)\upsilon}\right)\left(\frac{Z}{L}\right)^{-1}. \qquad \blacksquare$$

A.2. PROOF OF LEMMA 2

Substituting equation (24) into equation (25) yields (26) (repeated here for convenience):

$$\upsilon(z,n)\equiv\underset{\upsilon}{\text{argsolve}}\left\{\left(\frac{\alpha^2-\left(\alpha^2-\beta^2\right)\upsilon}{\beta^2}\right)^{\beta-\alpha}\left(\frac{\upsilon}{1-\upsilon}\right)^{\frac{1}{\epsilon-1}}=\left(\frac{\beta}{\alpha}\right)^{\frac{(\epsilon-1)(\alpha-\beta)-1}{\epsilon-1}} z^{\alpha-\beta}n^{\alpha+\beta}\right\}.$$

The left-hand side is clearly monotonically increasing when $\epsilon>1$. When $\epsilon<1$, the situation is more complex. The first term is monotonically increasing and the second term

https://doi.org/10.1017/S1365100517000505 Published online by Cambridge University Press

is monotonically decreasing. Taking the log of the left-hand side and differentiating with respect to υ yields

$$\frac{d\ln\left(\left(\frac{\alpha^2-(\alpha^2-\beta^2)\upsilon}{\beta^2}\right)^{\beta-\alpha}\left(\frac{\upsilon}{1-\upsilon}\right)^{1/(\epsilon-1)}\right)}{d\upsilon}$$
$$=\frac{1}{\upsilon(1-\upsilon)}\left[(\alpha-\beta)\left(\alpha^2-\beta^2\right)\frac{\upsilon(1-\upsilon)}{\alpha^2-\left(\alpha^2-\beta^2\right)\upsilon}-\frac{1}{1-\epsilon}\right].$$

When $\epsilon < 1$, the first term is positive and the second is negative. To prove that the left-hand side is always declining in υ when $\epsilon < 1$ it suffices to show

$$\frac{(\alpha-\beta)\left(\alpha^2-\beta^2\right)\upsilon(1-\upsilon)}{\alpha^2-\left(\alpha^2-\beta^2\right)\upsilon}<\frac{1}{1-\epsilon}\ \forall\ 0<\epsilon<1, \upsilon<1.$$

First, note that

$$\frac{(\alpha-\beta)^2}{(\alpha+\beta)}=\underset{\upsilon}{\operatorname{argmax}}\left\{\frac{(\alpha-\beta)\left(\alpha^2-\beta^2\right)\upsilon(1-\upsilon)}{\alpha^2-\left(\alpha^2-\beta^2\right)\upsilon}\right\},$$

and

$$1=\underset{\epsilon}{\operatorname{argmin}}\left\{\frac{1}{1-\epsilon}\right\}.$$

Since

$$\frac{1}{1-\epsilon}\geq 1>\frac{(\alpha-\beta)^2}{(\alpha+\beta)}\geq\frac{(\alpha-\beta)\left(\alpha^2-\beta^2\right)\upsilon(1-\upsilon)}{\alpha^2-\left(\alpha^2-\beta^2\right)\upsilon},\ \forall 0<\epsilon, \upsilon<1, \qquad \textbf{(A.1)}$$

the left-hand side is monotonically decreasing in υ when $\epsilon < 1$. ■

A.3. PROOF OF COROLLARY 1, 2, AND 3

This section is divided into two parts, the derivation of Corollary 1 and 2, and the derivation of Corollary 3. ■

A.3.1. Proof of Corollary 1 and 2

First, take the log of (26), then totally differentiate the left-hand side and the right-hand side:

$$d\ln\left(\left(\frac{\alpha^2-\left(\alpha^2-\beta^2\right)\upsilon(z,n)}{\beta^2}\right)^{\beta-\alpha}\left(\frac{\upsilon(z,n)}{1-\upsilon(z,n)}\right)^{1/(\epsilon-1)}\right)$$
$$=(\alpha-\beta)\,d\ln(z)+(\alpha+\beta)\,d\ln(n),$$

https://doi.org/10.1017/S1365100517000505 Published online by Cambridge University Press

which yields

$$\left[\frac{(\alpha-\beta)\left(\alpha^2-\beta^2\right)}{\alpha^2-\left(\alpha^2-\beta^2\right)\upsilon(z,n)}+\frac{(\epsilon-1)^{-1}}{\upsilon(z,n)\left[1-\upsilon(z,n)\right]}\right]d\upsilon(z,n)$$
$$=(\alpha-\beta)\,d\ln(z)+(\alpha+\beta)\,d\ln(n)\,.$$

When $dz=0$, trivial algebra yields

$$\frac{d\upsilon(z,n)}{d\ln(n)}=\frac{(\alpha+\beta)\,\upsilon(z,n)\left[1-\upsilon(z,n)\right]}{\frac{(\alpha-\beta)\left(\alpha^2-\beta^2\right)\upsilon(z,n)\left[1-\upsilon(z,n)\right]}{\alpha^2-\left(\alpha^2-\beta^2\right)\upsilon(z,n)}+\frac{1}{\epsilon-1}}.$$

Holding the technology constant, the same algebra yields

$$\vartheta_{SR}\equiv\frac{d\upsilon}{d\ln(z)}=\frac{(\alpha-\beta)\,\upsilon(z,n)\left[1-\upsilon(z,n)\right]}{\frac{(\alpha-\beta)\left(\alpha^2-\beta^2\right)\upsilon(z,n)\left[1-\upsilon(z,n)\right]}{\alpha^2-\left(\alpha^2-\beta^2\right)\upsilon(z,n)}+\frac{1}{\epsilon-1}}.$$

We now turn to proving monotonicity. When $\epsilon>1$, the denominator is clearly positive of (36) and (27) is positive. When $\epsilon<1$, the situation is more complex. It suffices to prove that

$$\frac{(\alpha-\beta)\left(\alpha^2-\beta^2\right)\upsilon(z,n)\left[1-\upsilon(z,n)\right]}{\alpha^2-\left(\alpha^2-\beta^2\right)\upsilon(z,n)}<\frac{1}{1-\epsilon}\ \forall\,0<\epsilon,\upsilon<1.$$

The proof of this is identical to Lemma 2 [see (A.1)] and is therefore omitted. ■

A.3.2. Proof of Corollary 3

Taking the log of equation (29) and totally differentiating it yields

$$\left[\frac{(\epsilon-1)(\alpha+\beta)-1}{(\epsilon-1)(\alpha+\beta)}\left(\frac{1}{\upsilon_{ss}(z)\left[1-\upsilon_{ss}(z)\right]}\right)\right.$$
$$\left.-\left(\frac{\alpha-\beta}{\alpha+\beta}\right)\frac{\left(\alpha^2-\beta^2\right)}{\alpha^2-\left(\alpha^2-\beta^2\right)\upsilon_{ss}(z)}\right]d\upsilon_{ss}(z)=\frac{\beta-\alpha}{\alpha+\beta}d\ln(z)\,.$$

Trivial algebra yields equation (37) (repeated here for convenience):

$$\frac{d\upsilon_{ss}}{d\ln(z)}=\frac{(\alpha-\beta)\,\upsilon_{ss}(z)\left[1-\upsilon_{ss}(z)\right]}{\frac{(\alpha-\beta)\left(\alpha^2-\beta^2\right)\upsilon_{ss}(z)\left[1-\upsilon_{ss}(z)\right]}{\alpha^2-\left(\alpha^2-\beta^2\right)\upsilon_{ss}(z)}-\left(\frac{(\epsilon-1)(\alpha+\beta)-1}{\epsilon-1}\right)}.$$

The sign depends on the denominator, in particular whether or not

$$\frac{(\alpha-\beta)\left(\alpha^2-\beta^2\right)\upsilon_{ss}(z)\left[1-\upsilon_{ss}(z)\right]}{\alpha^2-\left(\alpha^2-\beta^2\right)\upsilon_{ss}(z)} > \left(\frac{(\epsilon-1)(\alpha+\beta)-1}{(\epsilon-1)}\right). \qquad \textbf{(A.2)}$$

The case of substitutes is straightforward; the condition for stability (35) ensures that (A.2) holds; when the goods are weak substitutes, the right-hand side of (A.2) is negative therefore it is trivially satisfied. The case of complements is more complicated. Lemma 2 demonstrated proved that

$$\frac{(\alpha-\beta)\left(\alpha^2-\beta^2\right)\upsilon(1-\upsilon)}{\alpha^2-\left(\alpha^2-\beta^2\right)\upsilon} < \frac{1}{1-\epsilon}\ \forall\, 0<\epsilon<1, \upsilon<1;$$

hence (A.2) does not hold and ϑ_{LR} is negative. ■

A.4. PROOF OF PROPOSITION 1

Combining equation (24) with equation (4) implies that

$$\frac{P_s}{P_u} = \left[\left(\frac{\beta}{\alpha}\right)^{\frac{2\alpha}{\alpha+\beta}} n \left(z\frac{\alpha^2-\left(\alpha^2-\beta^2\right)\upsilon}{\beta^2}\right)^{\frac{\alpha-\beta}{\alpha+\beta}} \left(\frac{\upsilon}{1-\upsilon}\right)\right]^{\frac{-(\alpha+\beta)}{(\epsilon-1)(\alpha+\beta)+1}}.$$

Substituting the foregoing equation into equation (23) yields

$$\frac{r_s}{r_u} = \frac{\eta_s}{\eta_u}\left[\left(\frac{\beta}{\alpha}\right)^{\frac{2\alpha}{\alpha+\beta}} \left(z\frac{\alpha^2-\left(\alpha^2-\beta^2\right)\upsilon}{\beta^2}\right)^{\frac{\alpha-\beta}{\alpha+\beta}}\right.$$
$$\left.\times\left(\frac{\upsilon}{1-\upsilon}\right)\right]^{\frac{(\epsilon-1)(\alpha+\beta)}{(\epsilon-1)(\alpha+\beta)+1}} n^{\frac{-1}{(\epsilon-1)(\alpha+\beta)+1}}.$$

From equation (26), we know

$$n = \left(z\frac{\alpha^2-\left(\alpha^2-\beta^2\right)\upsilon}{\beta^2}\right)^{\frac{\beta-\alpha}{(\alpha+\beta)}} \left(\frac{\upsilon}{1-\upsilon}\right)^{\frac{1}{(\epsilon-1)(\alpha+\beta)}} \left(\frac{\beta}{\alpha}\right)^{\frac{1-(\epsilon-1)(\alpha-\beta)}{(\epsilon-1)(\alpha+\beta)}}.$$

Combining the previous two equations yields equation (29).

Equation (29) implies

$$\left(\frac{\upsilon_{ss}(z)}{1-\upsilon_{ss}(z)}\right)^{\frac{(\epsilon-1)(\alpha+\beta)-1}{(\epsilon-1)(\alpha+\beta)}} \frac{\eta_s}{\eta_u}\left(\frac{\beta}{\alpha}\right)^{\frac{(\epsilon-1)2\alpha-1}{(\alpha+\beta)(\epsilon-1)}}$$
$$= \left(z\frac{\alpha^2-\left(\alpha^2-\beta^2\right)\upsilon_{ss}(z)}{\beta^2}\right)^{\frac{\beta-\alpha}{\alpha+\beta}}.$$

Combining the two previous equations yields the steady-state technology, (30). ■

https://doi.org/10.1017/S1365100517000505 Published online by Cambridge University Press

A.5. PROOF OF PROPOSITION 3

Differentiating equation (34) with respect to υ yields

$$\frac{d\ln(r)}{d\ln(\upsilon)} = K\left[\left(\frac{(\alpha+\beta)(\epsilon-1)-1}{\epsilon-1}\right) - \frac{(\alpha-\beta)\left(\alpha^2-\beta^2\right)\upsilon\left[1-\upsilon\right]}{\alpha^2-\left(\alpha^2-\beta^2\right)\upsilon}\right], \qquad \textbf{(A.3)}$$

where K is a positive constant. Therefore, if

$$\left(\frac{(\alpha+\beta)(\epsilon-1)-1}{\epsilon-1}\right) > \underset{\upsilon}{\text{argmax}}\left\{\frac{(\alpha-\beta)\left(\alpha^2-\beta^2\right)\upsilon(1-\upsilon)}{\alpha^2-\left(\alpha^2-\beta^2\right)\upsilon}\right\}, \qquad \textbf{(A.4)}$$

the derivative is positive throughout. Hence, if (A.4) is satisfied, there exists one root which is unstable.

Second, note that $d\ln(r_s/r_u)/d\ln(\upsilon) = 0$ has a maximum of two roots from $\upsilon \in 0, 1$. Consequently, the derivative changes signs at most twice (recall that it can be positive throughout). Therefore, there exists a maximum of three *possible* roots to equation (29). It is straightforward to show that if there is more than one root to (29), there must be three. To see this, it is worthwhile to repeat equation (29):

$$\left(\frac{\upsilon}{1-\upsilon}\right)^{\frac{(\epsilon-1)(\alpha+\beta)-1}{(\epsilon-1)(\alpha+\beta)}}\left(\frac{\alpha^2-\left(\alpha^2-\beta^2\right)\upsilon}{\beta^2}\right)^{\frac{\alpha-\beta}{\alpha+\beta}}$$
$$= \frac{\eta_u}{\eta_s}\left(\frac{\beta}{\alpha}\right)^{\frac{1-(\epsilon-1)2\alpha}{(\alpha+\beta)(\epsilon-1)}} z^{\frac{\beta-\alpha}{\alpha+\beta}}.$$

The left-hand side goes to zero as υ goes to zero, and goes to infinity as υ goes to one. Recall that the derivative is initially positive (for low values of υ). Consequently, the first root is unstable. If there exists a second root, the derivative must have changed signs, and that root is locally stable. Finally, because the left-hand side of equation (29) goes to infinity as υ goes to one, if there is a second root, there must be a third root—which is unstable. ■

A.6. PROOF OF PROPOSITION 4

To prove proposition 4, it suffices to show that for all ϵ,

$$\vartheta_{\text{LR}} > \vartheta_{\text{SR}}. \qquad \textbf{(A.5)}$$

Note that

$$\frac{\vartheta_{\text{LR}}}{\vartheta_{\text{SR}}} = \frac{M}{M-(\alpha+\beta)},$$

where

$$M = \frac{(\alpha-\beta)\left(\alpha^2-\beta^2\right)\upsilon_{ss}(z)\left[1-\upsilon_{ss}(z)\right]}{\alpha^2-\left(\alpha^2-\beta^2\right)\upsilon_{ss}(z)} + \frac{1}{\epsilon-1}.$$

There are three cases to analyze: complements, weak substitutes $[(\epsilon-1)(\alpha+\beta) < 1,$ $\epsilon > 1]$, and strong substitutes $[(\epsilon-1)(\alpha+\beta) > 1]$.

When the goods are strong substitutes, the condition for local stability implies that $M > \alpha+\beta$, therefore, $\vartheta_{\text{LR}} > \vartheta_{\text{SR}}$.

When the goods are weak substitutes, trivial algebra yields that again $M > \alpha + \beta$, so $\vartheta_{\mathrm{LR}} > \vartheta_{\mathrm{SR}}$.

When the goods are complements, M is negative therefore $\vartheta_{\mathrm{LR}} > \vartheta_{\mathrm{SR}}$. Though ϑ_{LR} is negative, it is less negative than ϑ_{SR}. ■

A.7. PROOF OF PROPOSITION 5

There are two roots to

$$\frac{d\ln\left(r_s/r_u\right)}{d\upsilon\left(z,n\right)}=0.$$

They are given implicitly by the quadratic

$$0=-\left(\alpha^2-\beta^2\right)\upsilon\left(z,n\right)^2+\left(\alpha^2-\beta^2\right)\left(1+\Omega\right)\upsilon\left(z,n\right)-\Omega\alpha^2,$$

where

$$\Omega\equiv\left(\frac{\left(\alpha+\beta\right)\left(\epsilon-1\right)-1}{\left(\alpha+\beta\right)\left(\epsilon-1\right)}\right)\left(\frac{\alpha-\beta}{\alpha+\beta}\right)^{-1}.$$

The roots are

$$\upsilon_1=\frac{\left(1+\Omega\right)}{2}-\frac{\sqrt{\left(\alpha^2-\beta^2\right)^2\left(1+\Omega\right)^2-4\left(\alpha^2-\beta^2\right)\Omega\alpha^2}}{2\left(\alpha^2-\beta^2\right)};$$

$$\upsilon_2=\frac{\left(1+\Omega\right)}{2}+\frac{\sqrt{\left(\alpha^2-\beta^2\right)^2\left(1+\Omega\right)^2-4\left(\alpha^2-\beta^2\right)\Omega\alpha^2}}{2\left(\alpha^2-\beta^2\right)}.$$

The cutoff for strong-induced structural change that leads to specialization in the unskilled-intensive sector is that the local maximum of the relative rate of return is equal to one. Therefore,

$$\frac{r_s}{r_u}=\frac{\eta_s}{\eta_u}\left(\frac{\beta}{\alpha}\right)^{\frac{2\alpha(\epsilon-1)-1}{(\alpha+\beta)(\epsilon-1)}}\left(\frac{\upsilon_1}{1-\upsilon_1}\right)^{\frac{(\alpha+\beta)(\epsilon-1)-1}{(\alpha+\beta)(\epsilon-1)}}\left(\frac{\alpha^2-\left(\alpha^2-\beta^2\right)\upsilon_1}{\beta^2}\right)^{\frac{\alpha-\beta}{\alpha+\beta}}\left(z_B^u\right)^{\frac{\alpha-\beta}{\alpha+\beta}}=1$$

implies

$$z_B^u=\left(\frac{\eta_u}{\eta_s}\right)^{\left(\frac{\alpha+\beta}{\alpha-\beta}\right)}\left(\frac{\alpha}{\beta}\right)^{\frac{2\alpha(\epsilon-1)-1}{(\alpha-\beta)(\epsilon-1)}}\left(\frac{\upsilon_1}{1-\upsilon_1}\right)^{\frac{1-(\alpha+\beta)(\epsilon-1)}{(\alpha-\beta)(\epsilon-1)}}\left(\frac{\alpha^2-\left(\alpha^2-\beta^2\right)\upsilon_1}{\beta^2}\right)^{-1}.$$

Symmetrically the cutoff for strong induced structural change in the skill-intensive sector is

$$z_B^s=\left(\frac{\eta_u}{\eta_s}\right)^{\left(\frac{\alpha+\beta}{\alpha-\beta}\right)}\left(\frac{\alpha}{\beta}\right)^{\frac{2\alpha(\epsilon-1)-1}{(\alpha-\beta)(\epsilon-1)}}\left(\frac{\upsilon_2}{1-\upsilon_2}\right)^{\frac{1-(\alpha+\beta)(\epsilon-1)}{(\alpha-\beta)(\epsilon-1)}}$$
$$\times\left(\frac{\alpha^2-\left(\alpha^2-\beta^2\right)\upsilon_2}{\beta^2}\right)^{-1}.$$

■

https://doi.org/10.1017/S1365100517000505 Published online by Cambridge University Press

A.8. PROOF OF PROPOSITION 6

Note that

$$\frac{d\ln(\omega_{ss})}{d\ln(z)} = \frac{((\epsilon-1)(\alpha+\beta)-1)}{\frac{(\epsilon-1)(\alpha+\beta)(\alpha-\beta)^2 \upsilon_{ss}(1-\upsilon_{ss})}{\alpha^2-(\alpha^2-\beta^2)\upsilon_{ss}} - ((\epsilon-1)(\alpha+\beta)-1)}$$

There are three cases to analyze,

1. $\epsilon > (1+\alpha+\beta)/(\alpha+\beta)$;
2. $1 < \epsilon < (1+\alpha+\beta)/(\alpha+\beta)$;
3. $\epsilon < 1$.

When (1) holds, the condition for local stability ensure that the denominator is positive; therefore, strong bias holds in the interior solution. When (2) holds, the numerator is negative, and the denominator is positive, therefore strong bias fails to hold.

When $\epsilon < 1$, the numerator is negative. To show that strong bias fails to hold, we must show that the denominator is positive. Straightforward algebra implies that the denominator is positive provided

$$\frac{1+(1-\epsilon)(\alpha+\beta)}{(1-\epsilon)(\alpha+\beta)} > \frac{(\alpha-\beta)^2 \upsilon_{ss}(1-\upsilon_{ss})}{\alpha^2-(\alpha^2-\beta^2)\upsilon}.$$

Noting that

$$1 > \left(\frac{\alpha-\beta}{\alpha+\beta}\right)^2 = \underset{\upsilon_{ss}}{\operatorname{argmax}} \left\{\frac{(\alpha-\beta)^2 \upsilon_{ss}(1-\upsilon_{ss})}{\alpha^2-(\alpha^2-\beta^2)\upsilon_{ss}}\right\},$$

the denominator is negative and hence strong bias fails to hold. ■

A.9. SPECIAL CASE ($\alpha = \beta$)

Before delving into the general case it might be worthwhile to discuss the special case of identical skill intensity across sectors ($\alpha = \beta$). Because this special case admits a closed-form solution, it helps illustrate the underlying mechanics. In the special case of $\alpha = \beta$, substituting equation (24) into equation (25) yields

$$\underbrace{\left(n^{\frac{\epsilon 2\alpha}{(\epsilon-1)2\alpha+1}}\left(\frac{\upsilon}{1-\upsilon}\right)^{\frac{\epsilon 2\alpha}{(\epsilon-1)2\alpha+1}}\right)^{\frac{-1}{\epsilon}}}_{P_s/P_u}\underbrace{\left(n^{\frac{\epsilon 2\alpha}{(\epsilon-1)2\alpha+1}}\left(\frac{\upsilon}{1-\upsilon}\right)^{\frac{\epsilon 2\alpha}{(\epsilon-1)2\alpha+1}}\right)\left(\frac{1-\upsilon}{\upsilon}\right)}_{\text{MPZ}_s/\text{MPZ}_u} = 1, \quad \textbf{(A.6)}$$

which has the solution

$$\upsilon = \frac{n^{2\alpha(\epsilon-1)}}{1+n^{2\alpha(\epsilon-1)}}. \quad \textbf{(A.7)}$$

Equation (A.7) pins down υ explicitly, and hence—by equations (15) and (16)—the sectoral allocation of both factors in terms of the technology.

When n grows, employment in the "s" sector will rise (decline) if the two goods are substitutes (complements). The intuition can be seen from equation (A.6)—which captures the equal marginal revenue products of skilled labor in both sectors. Holding the allocations constant, an increase in n has two effects: the price effect, and the effect on the relative

https://doi.org/10.1017/S1365100517000505 Published online by Cambridge University Press

marginal products. When n increases, the ratio of production, Y_s/Y_u, rises and consequently P_s/P_u falls. The fall in the relative price reduces the relative marginal revenue product. However, because labor is more productive in the s sector, the relative marginal product rises. Whether or not the relative marginal revenue product rises depends on the elasticity of substitution between the two sectors. When the goods are substitutes, $\epsilon > 1$, the price effect is weak; consequently, labor is reallocated toward the s sector. When the goods are complements, $\epsilon < 1$, the price effect dominates and marginal revenue product falls; consequently, labor is reallocated away from the s sector.

A.10. RELATION TO PRICE AND MARKET SIZE EFFECTS (WISC)

To understand weak and strong structural change, it is worthwhile to analyze the model through the lens of price and market size effects. To do so, recall that equation (19) shows that the relative profit, and hence relative rate of return, can be written as a function of the relative price (P_s/P_u), the endowment (z), and the endogenous allocation of factors. Taking the log of equation (19) and differentiating it with respect to z (and holding n constant) yields

$$\frac{d\ln(\pi)}{d\ln(z)} = \underbrace{\left(\frac{1}{\alpha+\beta}\right)\frac{d\ln(P_s/P_u)}{d\ln(z)}}_{\text{price effect}} + \underbrace{\underbrace{\frac{\alpha-\beta}{\alpha+\beta}}_{\text{endowment effect}} + \underbrace{\frac{d\ln\left(\left(\frac{\alpha^2-(\alpha^2-\beta^2)\upsilon(z,n)}{\beta^2}\right)^{\frac{\alpha-\beta}{\alpha+\beta}}\left(\frac{\upsilon(z,n)}{1-\upsilon(z,n)}\right)\right)}{d\ln(z)}}_{\text{endogenous allocation effect}}}_{\text{increased production}},$$

where

$$\frac{d\ln(P_s/P_u)}{d\ln(z)} = \left(\frac{-(\alpha+\beta)}{(\epsilon-1)(\alpha+\beta)+1}\right) \times \left[\frac{\alpha-\beta}{\alpha+\beta} + \frac{d\ln\left(\left(\frac{\alpha^2-(\alpha^2-\beta^2)\upsilon(z,n)}{\beta^2}\right)^{\frac{\alpha-\beta}{\alpha+\beta}}\left(\frac{\upsilon(z,n)}{1-\upsilon(z,n)}\right)\right)}{d\ln(z)}\right].$$

There are two main forces that affects relative profitability: changes in prices and changes in production. In the present setup, contrary to Acemoglu (2002), the changes in production are not driven solely by "market size" effects. Instead, there are several factors at play. The first is the "endowment" effect. Since the skill-intensive sector uses relatively more skilled labor, increasing z, holding the sectoral allocations constant, will increase production in the skill-intensive sector. The second is the "endogenous allocation effect." The change in z causes labor to be reallocated across the sectors. This itself can be broken down into two separate elements (I will return to that in the appendix section: "Relation to price and market size effects (SISC).")

As is quite clear from this representation, the effect of increased production and the price effect works in opposite directions. The effect of an increase in z on the relative profits depends on which dominates. Combining the two foregoing equations yields

$$\frac{d\ln(\pi)}{d\ln(z)} = \left(\frac{(\epsilon-1)(\alpha+\beta)}{(\epsilon-1)(\alpha+\beta)+1}\right) \times \left[\frac{\alpha-\beta}{\alpha+\beta} + \frac{d\ln\left(\left(\frac{\alpha^2-(\alpha^2-\beta^2)\upsilon(z,n)}{\beta^2}\right)^{\frac{\alpha-\beta}{\alpha+\beta}}\left(\frac{\upsilon(z,n)}{1-\upsilon(z,n)}\right)\right)}{d\ln(z)}\right]. \quad \textbf{(A.8)}$$

The first term of equation (A.8) is positive (negative) when the goods are substitutes (complements), while the second term is unambiguously positive. Consequently, an increase in z will increase (decrease) relative profits when the goods are substitutes (complements). Intuitively, when the goods are substitutes, the price effect is relatively weak and the increased production dominates. When the goods are complements, the price effect will dominate. It is worthwhile to discuss both cases separately.

When the goods are substitutes ($\epsilon > 1$), both terms in the inside of the brackets are positive. Because the goods are substitutes, the increase in market size increases the relative profit; hence n will increase on the transition path. Corollary 1 equation (27) established: when the goods are substitutes, an increase in n will lead a rise in employment in the skill-intensive sector. Directed structural change, the structural change induced by directed technical change, magnifies the positive short-run reallocation.

When the goods are complements ($\epsilon < 1$), the second term inside of the brackets is negative. Contrary to the case of substitutes, the reallocation of labor works against the endowment effect. Nonetheless, it is straightforward to show that the change in production is positive. Because the price effect dominates the increased production (the first term is negative), the relative profit declines. Consequently, on the transition path n will fall. However, Corollary 1 equation (27) established that when the goods are complements: as n declines, labor is reallocated *back to* the skill-intensive sector. Directed structural change, *weakens* the negative short-run reallocation.

A.11. RELATION TO PRICE AND MARKET SIZE EFFECTS (SISC)

Just as an increase in z has price and "market size" effects, changes in n induce similar responses. It is worthwhile to discuss them in more detail. Since strong-induced technical change only occurs when $\epsilon > (\alpha+\beta)^{-1}+1$, I focus on the case of substitutes. The effect of an increase in n on the relative profits can be broken down as follows:

$$\frac{d\ln(\pi)}{d\ln(n)} = \underbrace{\frac{-1}{(\epsilon-1)(\alpha+\beta)+1}}_{\text{direct price effect}} + \underbrace{\frac{(\epsilon-1)(\alpha+\beta)}{(\epsilon-1)(\alpha+\beta)+1}\left(\underbrace{\frac{d\ln\left(\frac{\alpha^2-(\alpha^2-\beta^2)\upsilon(z,n)}{\beta^2}\right)^{\frac{\alpha-\beta}{\alpha+\beta}}}{dn}}_{\text{ratio effect}} + \underbrace{\frac{d\ln\left(\frac{\upsilon(z,n)}{1-\upsilon(z,n)}\right)}{dn}}_{\text{market size effect}}\right)}_{\text{feedback effect}}.$$

The first term captures the price effect of an increase in n. When n rises, holding the allocations constant, the price of skill intensive goods declines. However, the increase in n leads to an increase in υ (a feedback effect). It is straightforward to show that, on net, the feedback effect is positive. But the feedback effect can be disaggregated to two elements: the ratio effect and the market size effect.

The ratio effect is negative. Intuitively, as factors are reallocated to the skill-intensive sector, just as in the Stopler–Samuelson framework, the skilled labor to unskilled labor ratio must fall. This force reduces production in the skill-intensive sector. However, the market size effect, the increased use of both factors in the skill-intensive sectors, leads to an increase in the production of the skill-intensive sector. The relative strength of the ratio effect and the market size effect is not constant. To see this, note that

$$\underbrace{\frac{d\ln\left(\frac{\alpha^2-\left(\alpha^2-\beta^2\right)\upsilon(z,n)}{\beta^2}\right)^{\frac{\alpha-\beta}{\alpha+\beta}}}{dn}}_{\text{ratio effect}} = \frac{-1}{\frac{(\alpha-\beta)\left(\alpha^2-\beta^2\right)\upsilon(z,n)[1-\upsilon(z,n)]}{\alpha^2-\left(\alpha^2-\beta^2\right)\upsilon(z,n)}+\frac{1}{\epsilon-1}}\left[\frac{(\alpha-\beta)\left(\alpha^2-\beta^2\right)\upsilon(z,n)\left[1-\upsilon(z,n)\right]}{\alpha^2-\left(\alpha^2-\beta^2\right)\upsilon(z,n)}\right].$$

As υ approaches one (zero) the ratio effect disappears. In contrast the market size effect,

$$\underbrace{\frac{d\ln\left(\frac{\upsilon(z,n)}{1-\upsilon(z,n)}\right)}{dn}}_{\text{market size effect}} = \frac{(\alpha+\beta)}{\frac{(\alpha-\beta)\left(\alpha^2-\beta^2\right)\upsilon(z,n)[1-\upsilon(z,n)]}{\alpha^2-\left(\alpha^2-\beta^2\right)\upsilon(z,n)}+\frac{1}{\epsilon-1}},$$

approaches a constant as υ approaches one (zero). The total effect of an increase in n, as υ goes to one (zero), is thus

$$-\underbrace{\frac{1}{(\epsilon-1)(\alpha+\beta)+1}}_{\text{direct price effect}}+\underbrace{\frac{(\epsilon-1)^2(\alpha+\beta)^2}{(\epsilon-1)(\alpha+\beta)+1}}_{\text{feedback effect}}=(\epsilon-1)(\alpha+\beta)-1.$$

The reason SISC occurs is when z gets sufficiently large (small), the stabilizing force (the ratio effect) disappears.

https://doi.org/10.1017/S1365100517000505 Published online by Cambridge University Press