Survey Boat Attitude Determination with GPS/IMU Systems

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Any vehicle such as a vessel or aeroplane has three attitude parameters. These are most commonly defined as pitch, roll and heading from true north. In hydrographic surveying, determination of these parameters is essential for the correction of multi-beam echo sounder measurements. In the study on which this paper is based, two of the three parameters, the pitch and roll angles, were measured with an inexpensive IMU (Inertial Measurement Unit) device. The third was calculated from GPS carrier phase measurements that were collected from two antennas on the boat. The GPS antennas depart from the vertical when they are subject to pitch and roll effects. In this case, the coordinates derived from GPS will be erroneous. Thus, if heading is to be calculated accurately, the horizontal coordinates have to be corrected for pitch and roll. This paper defines an algorithm for the purpose that enables pitch and roll angles to be measured with an accuracy of about 0·110 arcdeg and headings computed with an accuracy of about 0·010 arcdeg.

KEY WORDS

1. Hydrographic Surveying. 2. GPS. 3. Attitude Determination.

1. INTRODUCTION.

Hydrographic surveying boats move irregularly and three-dimensionally in the marine environment. The main reasons for the movement are waves and swell. Generally, the movement of a rigid object is described with translation and rotation of the centre of gravity of the body. The translation is represented by surge, sway and heave while the rotation is represented by pitch, roll and yaw (Work *et al*., 1998). Heave is described as the oscillatory rise and fall of a boat due to the entire hull being lifted by the force of the sea. Pitch is described as the oscillations of a ship around the transverse axis, due to the bow and stern being raised or lowered on passing through successive crests and troughs of waves, and roll is described as the oscillation of a ship around the longitudinal axis (IHO, 1994). The pitch, roll and heading angles are attitude parameters of a vehicle such as a vessel or aeroplane.

As a result of the motion of a boat that is subjected to the pitch and the roll (*P* & *R*) effects, a GPS antenna or a pole having a prism will not be vertical, and the coordinates derived from such equipment will contain some errors. In hydrographic surveying, the *P* & *R* motions affect not only position data, but also depth measurements because the sonar transducer departs from the vertical. The magnitude of the errors can reach several metres depending on the surveying environment. Hence, both coordinates and depth measurements should be corrected for the *P* & *R* effects when accurate and reliable data are needed such as for shallow water navigation, dredging, harbour manoeuvre charts etc. On the other hand, the value of

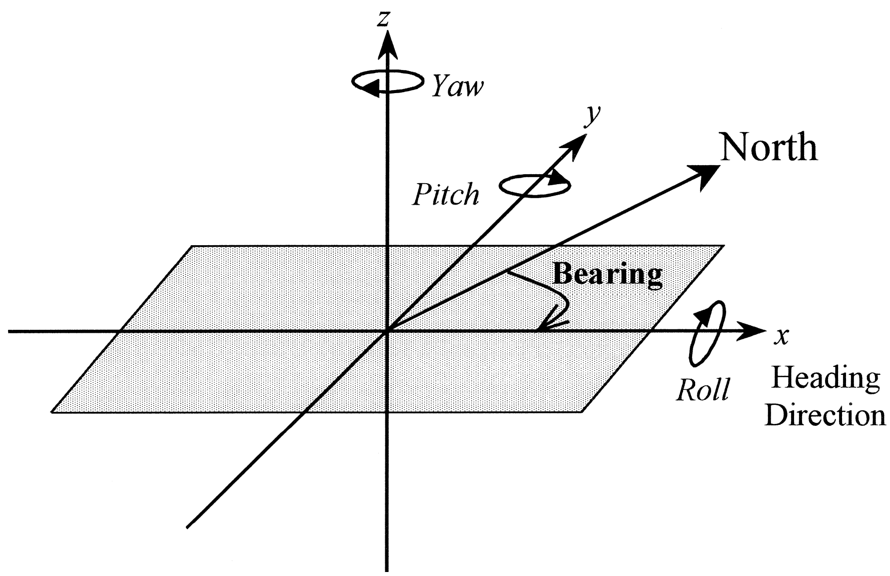

Figure 1. The Attitude Parameters of a Vehicle.

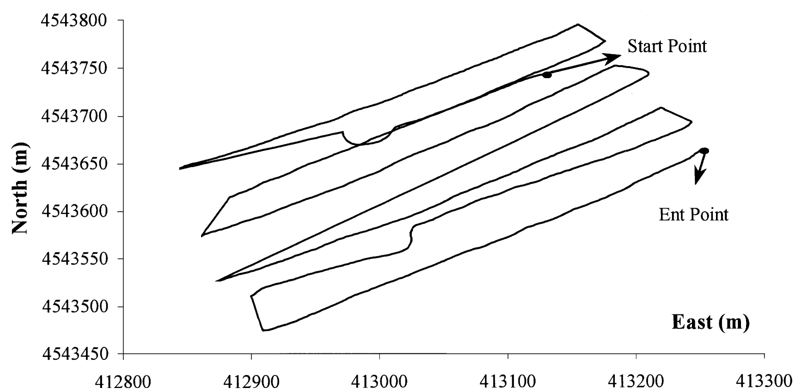

Figure 2. The Survey Profiles.

heading plays a very important role in some special hydrographic applications such as multi-beam or multi-transducer sounding systems. In fact, when the multi-beam echo sounder or multi-transducer are used, heading must be accurately known in addition to the *P* & *R* angles, to process the echo signals and to obtain a three-dimensional map of the sea bottom (Hare, 1995 & Luscombe, 1994).

In this study, the *P* & *R* angles are measured by a relatively low cost (< US$ 1000) IMU (Inertial Measurement Unit) device situated on the boat, and heading is calculated from a double antenna GPS system. The aim of this study was to determine the three attitude parameters of the boat and improve the accuracy of heading obtained from the GPS carrier phase measurements.

2. REVIEW OF ATTITUDE DETERMINATION.

Several methods and systems have been developed to determine the attitude parameters of a vehicle. GPS-based multi-antenna systems have been discussed extensively in the literature

(Martin-Neira *et al*., 1990; Wilson and Tonnemacher, 1992; Brown, 1992; Juang and Huang 1997; Juang and Huang, 1998). Enhanced methods that constitute the combination of GPS/INS (Inertial Navigation System) or GPS/GLONASS are also explained in the literature (Keong, 1999; McMillan *et al*., 1995). Some commercial GPS manufacturers have developed single packages that consist of multiple-GPS receivers for three-axis parameter estimation from a set of suitably placed antennas (Creamer *et al*., 1999).

Attitude parameters can be determined by GPS phase observations if at least three GPS antennas are suitably mounted to the rigid body of a boat. In this case, pitch and roll angles and heading can be calculated respectively by (Keong, 1999):

$$P = \arctan \frac{\Delta h_{1,3}}{\sqrt{(\Delta N_{1,3})^2 + (\Delta E_{1,3})^2}}, \tag{1}$$

$$R = \arctan \frac{\Delta h_{1,2}}{\sqrt{(\Delta N_{1,2})^2 + (\Delta E_{1,2})^2}}, \tag{2}$$

$$B = \arctan \frac{\Delta E_{1,3}}{\Delta N_{1,3}}, \tag{3}$$

where the suffixes 1, 2 and 3 represent the antennas numbers. In the derivation of equations (1), (2) and (3), it is assumed that antenna 1 and 2 are located so that the direction of 1, 2 is perpendicular to the heading direction of the vehicle and 1, 3 is also perpendicular to 1, 2. $\Delta N_{i,j}$, $\Delta E_{i,j}$ and $\Delta h_{i,j}$ are the north, east and height (up) coordinate differences that are obtained from the *ith* and *jth* antennas.

As described earlier, the P & R angles for this study were not determined by this method. Instead, they were measured with a low-cost IMU instrument located on the survey boat. The third parameter, heading, was derived from the coordinates obtained by the carrier phase measurements gathered from the double GPS receivers on the boat. One must keep in mind that these coordinates are derived from the tilted position of the antennas and must be corrected for P & R effects. An algorithm for this purpose is given in the following section.

3. SEA TRIAL METHODOLOGY.

The trial measurements were performed at the Golden Horn (Halic) in April 2000. The Golden Horn is located in the southwest of the Istanbul Bosporus. It is approximately 8 km long and 200–700 metres wide. Its depth reaches to approximately 50 metres. The kinematic GPS carrier phase measurement method was used during the trials period. The initial integer ambiguity had to be resolved by some method before starting the measurement. In the marine environment, a vessel/boat can never be static even if it is anchored. Therefore, the On-The-Fly (OTF) carrier phase ambiguity resolution method was the most suitable one because it permits the initial integer ambiguity resolution even if the receiver is moving (Lachapelle *et al*., 1993). The raw data was collected at a 1 Hz rate at eight survey profiles with a fibre-glass boat measuring 6-metres in length (Figure 2). The study was carried out in a weak wave environment and windless weather condition.

The IMU used was a dual-axis sensor that measures the pitch and roll angles with an accuracy of 0·5% at 15 arcdeg and 0·25% at 7 arcdeg (AOSI, 2000). The collected pitch and roll angles varied between (−2·083 and +7·979) arcdeg and between (−9·577 and +13·966) arcdeg, respectively. The location of the antennas and the

IMU devices on the boat are shown in Figure 3. Note that, the antennas were placed so that their (x) coordinates were parallel to each other. This means, the antenna 1 and 2 baseline is perpendicular to the x axis (heading direction). On the other hand, the IMU device can be placed anywhere on the deck due to the pitch and roll angles being the same on every part of a rigid object.

4. THE PITCH AND ROLL CORRECTIONS. To calculate the P & R corrections, a coordinate system had to be defined. Furthermore, some definitions were also required. The boat coordinate system, which was used in the algorithm introduced in this study, is given in Figure 3. The Centre Of Gravity (COG) of the boat was defined as the origin of the coordinate system. It was determined by a technical collaboration with the Naval Architecture and Ocean Engineering Department of the ITU. For this purpose, the boat was lifted from the sea to the shore and mapped three-dimensionally by an electronic tachometer, with all the measurement equipment in place (i.e. GPS antennas, IMU device, etc.). All these measurements were realised by a Topcon GTS series instrument with distance and angle measurement accuracies of $\pm(2\text{mm}+2\text{ppm}*\text{Dist.})$ and 0·2*mgon* respectively.

The coordinate system was configured so that the heading of the boat was the x-axis. The arrows at the end of each axis indicate the positive directions of the relevant axis. Furthermore, the rotation around x and y axes were defined so that the pitch angle is positive when the bow of the boat goes up and the roll angle is positive when the starboard of boat goes up. The rotation around the z axis was defined so that the heading direction has the same sign with the bearing of the heading direction (see Figures 1 and 3).

The GPS coordinates could be corrected for the P & R effects for every measurement epoch, t_i, by

$$\begin{bmatrix} x'_{\text{GPS}} \\ y'_{\text{GPS}} \\ z'_{\text{GPS}} \end{bmatrix} = R_2(P)\,R_1(R) \begin{bmatrix} x_{\text{GPS}} \\ y_{\text{GPS}} \\ z_{\text{GPS}} \end{bmatrix} \tag{4}$$

$$\underline{\Phi} = \underline{R}_2(P)\,\underline{R}_1(R) = \begin{bmatrix} \text{Cos}P & 0 & -\text{Sin}P \\ 0 & 1 & 0 \\ \text{Sin}P & 0 & \text{Cos}P \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \text{Cos}R & \text{Sin}R \\ 0 & -\text{Sin}R & \text{Cos}R \end{bmatrix} \tag{5}$$

$$\underline{\Phi} = \begin{bmatrix} \text{Cos}P & \text{Sin}P\,\text{Sin}R & -\text{Sin}P\,\text{Cos}R \\ 0 & \text{Cos}R & \text{Sin}R \\ \text{Sin}P & -\text{Cos}P\,\text{Sin}R & \text{Cos}P\,\text{Cos}R \end{bmatrix} \tag{6}$$

$$x'_{\text{GPS}} = (\text{Cos}P)\,x_{\text{GPS}} + (\text{Sin}P\,\text{Sin}R)\,y_{\text{GPS}} - (\text{Sin}P\,\text{Cos}R)\,z_{\text{GPS}} \tag{7}$$

$$y'_{\text{GPS}} = (\text{Cos}R)\,y_{\text{GPS}} + (\text{Sin}R)\,z_{\text{GPS}} \tag{8}$$

$$z'_{\text{GPS}} = (\text{Sin}P)\,x_{\text{GPS}} - (\text{Cos}P\,\text{Sin}R)\,y_{\text{GPS}} + (\text{Cos}P\,\text{Cos}R)\,z_{\text{GPS}}, \tag{9}$$

where P is the pitch angle and R the roll angle, x_{GPS}, y_{GPS} and z_{GPS} are the antenna coordinates when $P=0$, $R=0$ and are defined in the local boat coordinate system. If the antennas are subjected to P & R effects, they depart from their vertical location. In this case, the coordinates affected by P & R effects are x'_{GPS}, y'_{GPS} and z'_{GPS}. The errors along the (x), (y) and (z) axes are $\delta\, x_i = x'_{\text{GPS}} - x_{\text{GPS}}$, $\delta\, y_i = y'_{\text{GPS}} - y_{\text{GPS}}$, $\delta\, z_i = z'_{\text{GPS}} - z_{\text{GPS}}$, respectively. The schematic depiction of the heading correction algorithm is given in the Figure 4.

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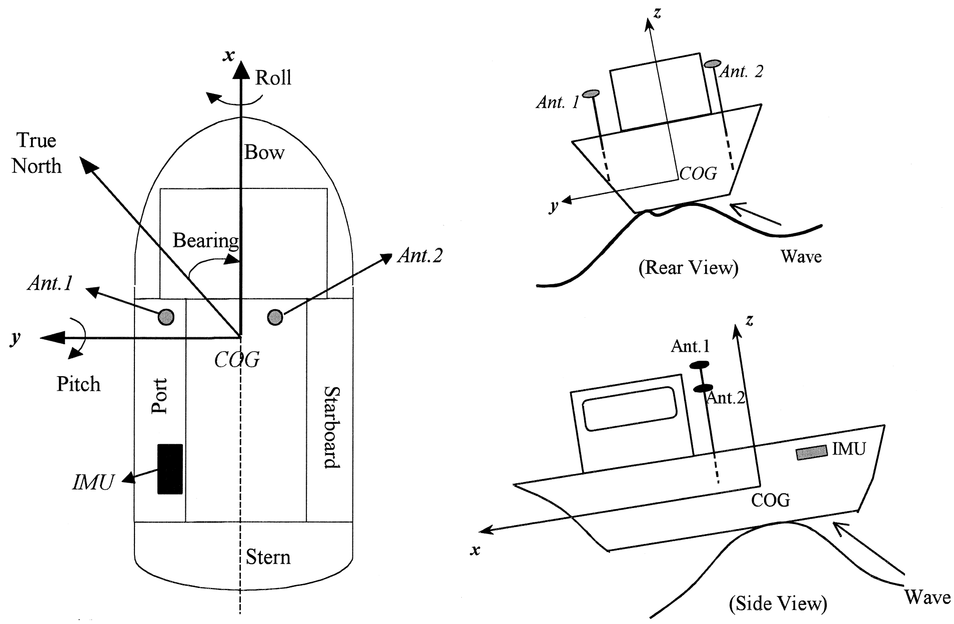

Figure 3. The GPS Antennas and IMU Configuration on the Boat.

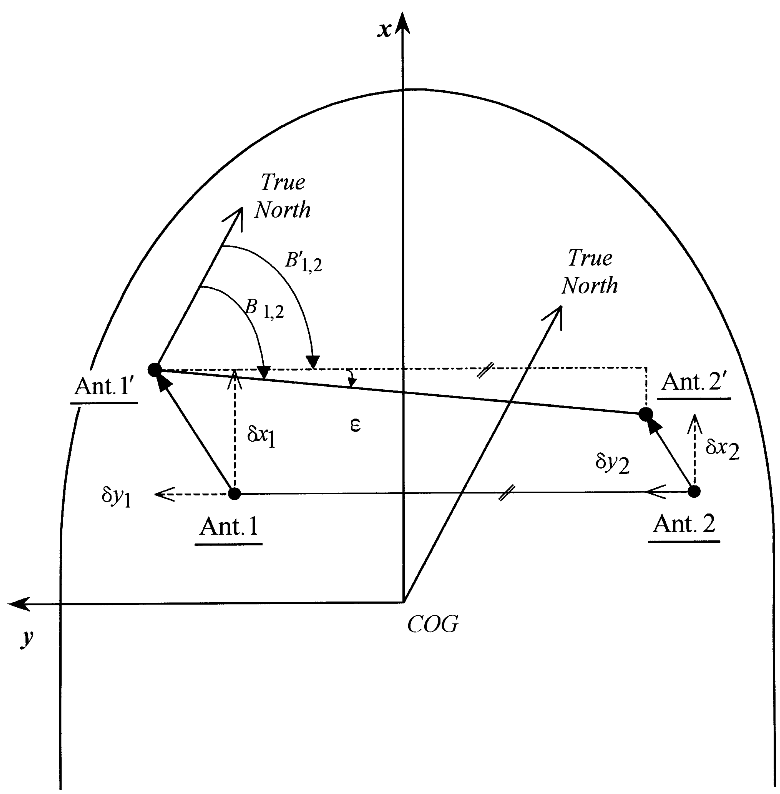

Figure 4. Schematic Depiction of the Heading Correction.

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According to the Figure 4, the corrected heading is calculated by:

$$B'_{1-2} = B_{1-2} - \epsilon \tag{10}$$

where B_{1-2} is the heading calculated from reported antennas coordinates (not corrected for the P & R angles) and can be obtained by:

$$B'_{1-2} = Arctn\frac{y^{\text{ant.2}} - y^{\text{ant.1}}}{x^{\text{ant.2}} - x^{\text{ant.1}}} \tag{11}$$

On the other hand, ϵ_i can be calculated from Figure 4 by:

$$\begin{aligned} \tan(\epsilon_i) &= \frac{\delta x_i^{\text{ant.1}} - \delta x_i^{\text{ant.2}}}{\bar{L}_{12} - (\delta y_i^{\text{ant.2}} - \delta y_i^{\text{ant.1}})} \\ &= \frac{(\text{Cos}P\, x^{\text{ant.1}} + \text{Sin}P\,\text{Sin}R\, y^{\text{ant.1}} - \text{Sin}P\,\text{Cos}R\, z^{\text{ant.1}}) - x^{\text{ant.1}}}{\bar{L}_{12} + \{(\text{Cos}R\, y^{\text{ant.1}} + \text{Sin}R\, z^{\text{ant.1}}) - y^{\text{ant.1}}\} - \{(\text{Cos}R\, y^{\text{ant.2}} + \text{Sin}R\, z^{\text{ant.2}}) - y^{\text{ant.2}}\}} \\ &\quad - \frac{(\text{Cos}P\, x^{\text{ant.2}} + \text{Sin}P\,\text{Sin}R\, y^{\text{ant.2}} - \text{Sin}P\,\text{Cos}R\, z^{\text{ant.2}}) - x^{\text{ant.2}}}{\bar{L}_{12} + \{(\text{Cos}R\, y^{\text{ant.1}} + \text{Sin}R\, z^{\text{ant.1}}) - y^{\text{ant.1}}\} - \{(\text{Cos}R\, y^{\text{ant.2}} + \text{Sin}R\, z^{\text{ant.2}}) - y^{\text{ant.2}}\}}, \end{aligned} \tag{12}$$

where $\bar{L}_{12}$ is the distance between the antennas 1 and 2 and measured, like the GPS antennas coordinates, when the boat was lifted on the shore for the determination of the COG.

At the end of the calculation using (12), the heading corrections were between $(-0{\cdot}600) \sim (+1{\cdot}033)$ arcdeg intervals. The corrections, belonging to the every measurement epoch, are shown in the Figure 5. It is clearly seen from Figure 4 that,

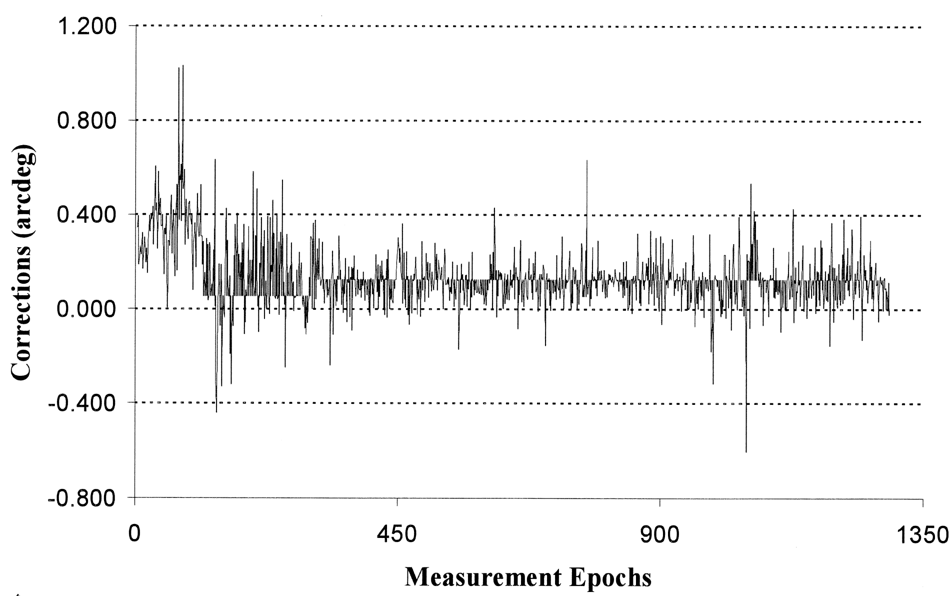

Figure 5. The Bearing Corrections.

if the two antennas are situated on the surveying boat symmetrically with respect to the COG in horizontal plane and their phase centres are the same distance from the

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COG in vertical plane, the two antennas will be influenced by the pitch and the roll effects at the same magnitude. In this case, correction is not necessary. But these conditions are not provided easily because of the logistical or physical features of the boat. In fact, the establishment of symmetry is not easy – furthermore, not necessary if the heading angles are properly corrected for pitch and roll effects.

On the other hand, if triple antennas are used for the determination of the attitude parameters, first the pitch and roll angles are determined by the use of GPS carrier phase measurements obtained from the triple antennas. Then the heading can be calculated from the corrected coordinates.

5. ERROR ESTIMATION OF CORRECTION VALUE.

If the error propagation low is applied to the equation (12) with respect to the independent variables ($x^{ant.1}$, $x^{ant.2}$, $y^{ant.1}$, $y^{ant.2}$, $z^{ant.1}$, $z^{ant.2}$, P, R, $\bar{L}_{12}$), the accuracy of the correction value can be estimated. Let the letters A and B represent the numerator and denominator of the equation (12), respectively.

$$f(\epsilon) = f(x_1, x_2, y_1, y_2, z_1, z_2, P, R, L_{12}) \tag{13}$$

$$df = \frac{\delta f}{\delta x_1} dx_1 + \frac{\delta f}{\delta x_2} dx_2 + \frac{\delta f}{\delta y_1} dy_1 + \frac{\delta f}{\delta y_2} dy_2 + \frac{\delta f}{\delta z_1} dz_1 + \frac{\delta f}{\delta z_2} dz_2 + \frac{\delta f}{\delta P} dP + \frac{\delta f}{\delta R} dR + \frac{\delta f}{\delta L_{12}} dL_{12} \tag{14}$$

$$\begin{aligned}
\frac{1}{\mathrm{Cos}^2 \epsilon} d\epsilon = & \left[\frac{1}{B}(-1 + \mathrm{Cos}P)\, dx^{ant.1}\right] \\
& + \left[\frac{1}{B}(1 - \mathrm{Cos}P)\, dx^{ant.2}\right] \\
& + \left[\frac{1}{B^2}\{(\mathrm{Sin}P\,\mathrm{Sin}R)*B - A*(-1 + \mathrm{Cos}R)\}\, dy^{ant.1}\right] \\
& + \left[\frac{1}{B^2}\{(-\mathrm{Sin}P\,\mathrm{Sin}R)*B - A*(1 - \mathrm{Cos}R)\}\, dy^{ant.2}\right] \\
& + \left[\frac{1}{B}\left(-\mathrm{Sin}P\,\mathrm{Cos}R - \frac{A}{B}\mathrm{Sin}R\right) dz^{ant.1}\right] \\
& + \left[\frac{1}{B}\left(\mathrm{Sin}P\,\mathrm{Cos}R + \frac{A}{B}\mathrm{Sin}R\right) dz^{ant.2}\right] \\
& + \left[\frac{1}{B}(-\mathrm{Sin}P(x^{ant.1} - x^{ant.2}) + \mathrm{Cos}P\,\mathrm{Sin}R(y^{ant.1} - y^{ant.2}) \right. \\
& \left. + \mathrm{Cos}P\,\mathrm{Cos}R(z^{ant.2} - z^{ant.1}))\, dP\right] \\
& + \left[\frac{1}{B^2}\{(\mathrm{Cos}R\,\mathrm{Sin}P(y^{ant.1} - y^{ant.2}) - \mathrm{Sin}P\,\mathrm{Sin}R(z^{ant.2} - z^{ant.1}))*B \right. \\
& \left. - A*(\mathrm{Sin}R(y^{ant.2} - y^{ant.1}) + \mathrm{Cos}R(z^{ant.1} - z^{ant.2}))\}dR\right] \\
& - \frac{A}{B^2} dL_{12}.
\end{aligned} \tag{15}$$

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So, the standard deviation equation is expressed by:

$$
\begin{aligned}
\sigma_\epsilon^2 = {} & (\mathrm{Cos}^2\epsilon)^2\left[\frac{1}{B}(-1+\mathrm{Cos}P)\,\rho\,\sigma_{x_1}\right]^2 \\
& +(\mathrm{Cos}^2\epsilon)^2\left[\frac{1}{B}(1-\mathrm{Cos}P)\,\rho\,\sigma_{x_2}\right]^2 \\
& +(\mathrm{Cos}^2\epsilon)^2\left[\frac{1}{B^2}\{(\mathrm{Sin}P\,\mathrm{Sin}R)*B-A*(-1+\mathrm{Cos}R)\}\,\rho\,\sigma_{y_1}\right]^2 \\
& +(\mathrm{Cos}^2\epsilon)^2\left[\frac{1}{B^2}\{(-\mathrm{Sin}P\,\mathrm{Sin}R)*B-A*(1-\mathrm{Cos}R)\}\,\rho\,\sigma_{y_2}\right]^2 \\
& +(\mathrm{Cos}^2\epsilon)^2\left[\frac{1}{B}\left(-\mathrm{Sin}P\,\mathrm{Cos}R-\frac{A}{B}\mathrm{Sin}R\right)\rho\,\sigma_{z_1}\right]^2 \\
& +(\mathrm{Cos}^2\epsilon)^2\left[\frac{1}{B}\left(\mathrm{Sin}P\,\mathrm{Cos}R+\frac{A}{B}\mathrm{Sin}R\right)\rho\,\sigma_{z_2}\right]^2 \\
& +(\mathrm{Cos}^2\epsilon)^2\left[\frac{1}{B}(-\mathrm{Sin}P(x^{\mathrm{ant.1}}-x^{\mathrm{ant.2}})+\mathrm{Cos}P\,\mathrm{Sin}R(y^{\mathrm{ant.1}}-y^{\mathrm{ant.2}})\right. \\
& \left.+\mathrm{Cos}P\,\mathrm{Cos}R(z^{\mathrm{ant.2}}-z^{\mathrm{ant.1}}))\,\sigma_P\right]^2 \\
& +(\mathrm{Cos}^2\epsilon)^2\left[\frac{1}{B^2}\{(\mathrm{Cos}R\,\mathrm{Sin}P(y^{\mathrm{ant.1}}-y^{\mathrm{ant.2}})-\mathrm{Sin}P\,\mathrm{Sin}R(z^{\mathrm{ant.2}}-z^{\mathrm{ant.1}}))*B\right. \\
& \left.-A*(\mathrm{Sin}R(y^{\mathrm{ant.2}}-y^{\mathrm{ant.1}})+\mathrm{Cos}R(z^{\mathrm{ant.1}}-z^{\mathrm{ant.2}}))\}\sigma_R\right]^2 \\
& -(\mathrm{Cos}^2\epsilon)^2\left(-\frac{A}{B^2}\rho\,\sigma_{L_{12}}\right)^2.
\end{aligned}
\tag{16}
$$

Note that, it is assumed there is no correlation between the independent variables.

The standard deviation of the pitch and the roll angles (σ_P, σ_R) are estimated as 0·110 arcdeg; the antenna coordinates and the baseline (σ_x, σ_y, σ_z and $\sigma_{L_{12}}$) are estimated as 0·005 metres in (16). At the end of the calculation, the average of the standard deviations is found as 0·010 arcdeg.

In addition to this, the correction value (ϵ_i) derived from (12) has to be tested for significance. For this purpose, the *Student* test was applied. In the first stage, a test value should be determined by the calculated correction and its standard deviation for every measurement epoch by

$$t_{\epsilon_i} = \frac{|\epsilon_i|}{\sigma_{\epsilon_i}}\, i = 1, 2, \ldots, n \tag{17}$$

where n is the number of the measurement epochs.

The test value is then compared with the critical value of t obtained as 1·65 from the *Student* distribution table with the degrees of freedom $f = \propto$ and a level of significance of 5% ($\alpha = 0{\cdot}05$). Note that, significance level is usually taken as 5% or 1% (Davis, 1986). In case the value obtained from (17) is smaller than the critical value, namely, $t_{\epsilon_i} < t_{\alpha,f}$, then:

$$H_0 : \mathrm{E}(\epsilon_i) = 0, \tag{18}$$

the null hypothesis is accepted. In other words, this correction is not significant. When the test was applied to the whole data group, it was found that the null hypothesis was accepted for only a few corrections. That is, most of the corrections were significant and should be applied to the relevant headings.

6. CONCLUSIONS AND SUGGESTIONS. The three attitude parameters of the surveying boat were successfully determined by a low-cost IMU device and multi-antenna GPS carrier phase measurements. The algorithm presented in this paper successfully achieves correction of the heading obtained by the GPS carrier phase measurement. It can be said that, if not only heading but also other data such as coordinates are needed to a higher accuracy, the pitch and the roll angles have to be considered and their effects have to be removed from the measurements.

If the corrections for heading are examined in Figure 5, it can be seen that they reach to 1 arcdeg, even though the study was carried out in a weak wave environment and windless day with the small boat. The early trial studies have shown that, if a bigger boat is used and surveying is performed in a strong wave environment, the corrections might reach much higher magnitude. This experience shows that the corrections are significant and should not be neglected if high precision hydrographic data is needed.

The most important advantage of this method is its low cost. The angle measurement device is not expensive and measured the angles with an accuracy of 0·110 arcdeg. In this way, two GPS receivers (except the one on the shore) together with the IMU device are sufficient for the determination of the three attitude parameters. Furthermore, a third gyro is no longer necessary because heading can be derived from the two GPS antennas with an accuracy of 0·010 arcdeg. Although the GPS is already used for many aspects of hydrographic surveying, this study showed that only one more GPS receiver and low-cost IMU device are sufficient for attitude parameters determination.

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