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Technical Note

Clarifying the definition of redundancy as used in robotics

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SUMMARY

Several descriptions of redundancy are presented in the literature, often from widely different perspectives. Therefore, a discussion of these various definitions and the salient points would be appropriate. In particular, any definition and redundancy needs to cover the following issues; the difference between multiple solutions and an infinite number of solutions; degenerate solutions to inverse kinematics; task redundancy; and the distinction between non-redundant, redundant and highly redundant manipulators.

KEYWORDS: Redundancy; Robotics; Survey.

1. INTRODUCTION

The major problem with definitions of redundancy is that it is a term used for quite disparate, but related, situations. This note looks at a number of widely used definitions with a view to identifying the key features and proposing some workable definitions.

Starting at the highest level, redundancy concerning robotic manipulators can be categorised as sensor redundancy and mechanical redundancy,[1] **Sensor redundancy** occurs when there are more sensors than theoretically necessary, usually when high reliability is required. Although sensory redundancy is important, it is not considered in this paper. **Mechanical redundancy** can be further divided into **kinematic** and **actuation redundancy**. The term **redundancy** used in this paper means **kinematic redundancy** unless otherwise stated.

2. REDUNDANCY IN THE LITERATURE

Redundancy is described in McKerrow's "Introduction to Robotics"[2] as follows;

> '*When a manipulator can reach a specified position with more than one configuration of the linkages, the manipulator is said to be* **redundant**.'

According to this definition, redundancy means more than one solution to the inverse kinematic transform. The example given in Introduction to Robotics is the two link planar manipulator shown in Figure (1). Because the joint variable θ_2 can be either positive or negative, there are two possible arm configurations which are called **elbow down** and **elbow up**. One of these solutions is selected depending on external constraints, for instance sometimes it may not be physically possible to reach the required location using one of the solutions.

Within the same book redundancy is also described as the state of having more degrees of 'mobility' than the task requires (**task redundancy**) and manipulators with more than six degrees of mobility are said to be **infinitely redundant**.

There are two definitions of redundancy given at the beginning of the Chapter 4 in "Robot Control".[3] The first one is;

> '*. . . it is common to say that a robot is* ***redundant*** *when it has more than six joints.*'

The second one is;

> '*From a general point of view, any robotic system in which the way of achieving a given task is not unique may be called* ***redundant***.'

The first statement is plainly misleading as it stands since there are many robot configurations that have less than 6 degrees of freedom that are redundant, particularly planar designs.

Using the second definition above, the concept of redundancy is related to the definition of the task in "Robot Control" and is not considered as an intrinsic feature of the structure of the robot. To illustrate this second definition an example given by Samson uses the ubiquitous two link planar robot. Suppose that a task is specified in terms of only the x direction (Figure 2). When $x_r < l_1 + l_2$, there are an infinite number of solutions. When $x_r = l_1 + l_2$, there is only one solution, joint variable values are zero and the manipulator is no longer redundant. When $x_r > l_1 + l_2$, the task cannot be achieved. This issue of task redundancy is obviously important and is returned to later.

A finite number of solutions are called multiple solutions by Samson[3] and it is expressed that a robotic system is **truly redundant** when there is an infinite set of solutions in the joint space for a given end-effector configuration. Moreover, Samson draws attention to the fact that an infinite number of solutions should not be confused with a finite number of solutions. Together with Samson, Craig[4] also named finite number of solutions as **multiple solutions**. Koivo[5] also described redundancy in the same way as the first of Samson's descriptions.

Since they are related to redundancy, degenerate

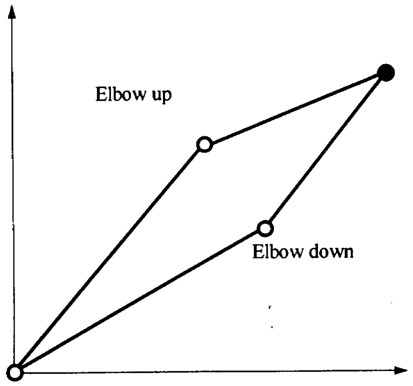

Fig. 1. Two solution for two link planar manipulator ($n = 2, m = 2$).

configurations should also be considered. One result of degeneracy is a situation in which there are an infinite number of configurations of the manipulator that achieve the desired end-effector configuration. However, in such a configuration, one or more degrees of end-effector freedom is lost. Koivo provides the following description for degeneracy;[5]

> *When the inverse solution to the kinematics equation is not unique, the inverse solution to the kinematic equations is said to be* ***degenerate****.*

The example of the same two link planar manipulator as the ones in[2,3] is given and the two possible solutions to the inverse kinematics (Figure 1) are treated as **degenerate configurations** by Koivo.

The definition of redundancy in "Advanced Robotics"[1] is especially important since it is written about redundancy and optimisation and approaches the subject from a mathematical perspective.

> '*In a system with* ***kinematic redundancy****, we are able to change the internal structure of configuration of the mechanisms without changing the position and orientation of the end-effector or of the object.*'

Redundancy is also described in a number of papers[6–19] which discuss several different issues related to redundant robots. The definitions are very similar to each other and do not add extra knowledge to the definition of redundancy under discussion here, e.g.

> '*Redundant robots are mechanisms with more degrees of freedom (DOF) than required for realization of a prescribed task in a task space.*'

3. DISCUSSION ON DEFINITIONS

When a standard English dictionary is consulted, the basic meaning of redundancy is seen to be in a state of no longer being needed because of the fact that the same function is being fulfilled to excess.[20] In reality, this definition does not exactly reflect the meaning which is implied in robotics. When extra axes of motion are employed, it is obvious that the design does envisage a purpose and is not adding complexity for complexity's sake. However redundancy can be seen as implying extra degrees of freedom which are **not** needed for tasks in well-organised environments. For instance using a six axis machine for welding or for stereotactic surgery is not kinematically required since five axes will place a uniform cross section tool at the appropriate position with the required orientation to complete the task – no final rotation about the tool axis is required. This is more in accordance with basic definition in the dictionary.

More than one solution to the inverse kinematics is not a clear expression. A distinction is made between a finite number of solutions and an infinite number of solutions. The question is whether there is a real difference between these two expressions. A finite number of solutions are treated sometimes as redundancy, sometimes as multiple solutions. Technically, choosing a solution out of an infinite number of solutions can be completely different from choosing one out of a finite number of solutions. Besides, a robot with a finite number of solutions does not have the same degree of flexibility as one with an infinite number of solutions. More importantly, if robots with a finite number of solutions are treated as redundant, the distinction between non-redundant robot and redundant robots blurs, because, as known, the simplest two link planar manipulator does not have a unique solution to the inverse kinematics.

Redundancy is also deemed to be a task dependent concept as seen from the example that Samson[3] gives. This is clearly important since most robots and automated machines are defined around task requirements. Hence the implementation of the SCARA type robots for a specific range of pick and place tasks.

The four link planar manipulator shown in Figure 3 has a kind of redundancy which has an infinite number of

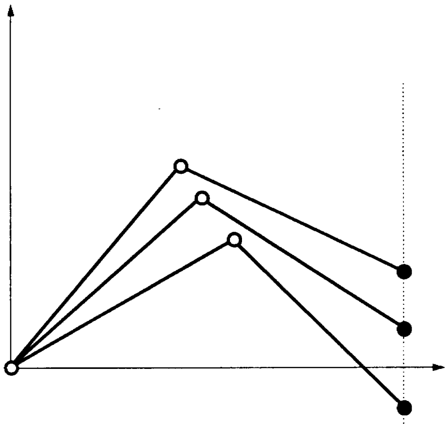

Fig. 2. Three Solution out of an Infinite Number of Solutions for Two Link Planar Manipulator ($n = 2, m = 2, r = 1$).

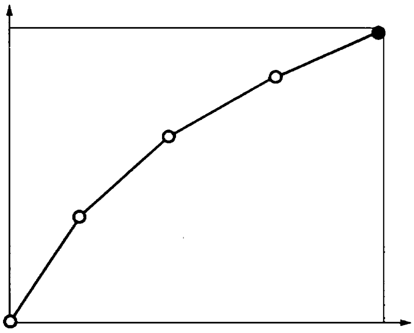

Fig. 3. Four link manipulator ($n = 4, m = 3, r = 1, 2$ or 3).

solutions to the inverse kinematics, independent of the task. This type of device, when not in a degenerate configuration, clearly fulfils Nakamura's definition since even the simplest tasks cannot be completed without selecting a solution out of an infinite number of solutions to control the internal configuration of links to reach the desired location. Control over the internal configuration of the manipulator is the key point to using redundancy.

Degenerate configurations are described in different ways as well. Degeneracy occurs at certain positions, or degenerate configurations, where there is a change in the solution space. For a normally non-redundant device a degenerate configuration may lead to an infinite number of solutions to the inverse kinematics. Therefore, the definition of degeneracy made by Koivo[5] does not fully express the situation. However considering a degeneracy in a redundant device it would be possible to reach a non-redundant configuration. One point is clear – degeneracy decreases mobility.

Two examples illustrate the effect of degenerate configurations on non-redundant mechanisms. Degeneracy can be observed when the joint angle between the first and second links of a two link manipulator with equal link lengths is 180° from fully extended, Figure 4. Clearly the position of the end-effector is independent of the first joint angle. The second example occurs in wrist mechanisms which have joints where two axes become collinear as a result of a specific value of a third joint which separates those two joints, e.g. PUMA 560 wrist.

Taking an example that illustrates the effect of degeneracy on redundant devices: a planar four link device effectively becomes a three link device when the most distal link is 180° from fully extended, Figure 5.

To summarise, there are three distinct concepts used to describe the kinematic status of the device itself: multiple solutions; redundancy; and degenerate configurations, which are all effected by the external feature of the mobility required to complete a specific task.

4. CLEARER DEFINITIONS OF REDUNDANCY

As yet not mentioned there is in fact a clear and simple method of defining all the terms used above, relating the terms to the dimensions of the spaces defining the device and the task.

The device can be described as having 'n' axes of motion. Similarly the space defined by the achievable motion of the end-effector will have a dimension 'm'. The task space will have dimension 'r'.

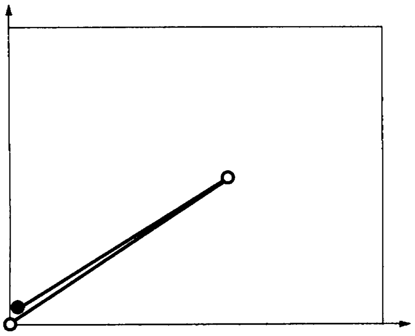

Fig. 4. Degenerate two link planar manipulator ($n = 2, m = 1$).

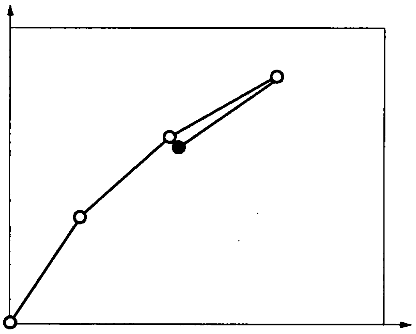

Fig. 5. Degeneration of a four link planar manipulator into a non-redundant three link planar manipulator ($n = 3, m = 3$).

Case 1:

$$n = m$$

This is the standard non-redundant robot.

Case 1a:

$$n > m$$

When there is a reduction of the dimension m in specific configurations. This device is now in a degenerate configuration.

Case 2:

$$n > m$$

When 'n' is designed to be greater than 'm' then the device is redundant. In such situations the self shape of a device can be varied without changing the end-effector configuration, since the joints do not produce independent motion in end-effector space. This is therefore the key situation which is examined when considering navigation through cluttered workspaces and collision avoidance generally.

Case 3:

$$m > r$$

When the task space 'r' is completely within the end-effector space, and the dimension of the end-effector space, irrespective of the dimension of the joint space, is greater than the task space, then this describes task redundancy.

Case 4:

As a distinct case in any of these situations there can be examples where for a particular configuration there exists a mirror configuration. This gives rise to multiple solutions where there is a finite and well defined set of solutions for the end-effector configuration.

These mathematical expressions can now be converted into definitions:

Definition 1: *If the number of solutions to the inverse kinematics of a manipulator is not unique but finite, the manipulator is said to have* ***multiple solutions****.*

Definition 2: *If the dimension of joint space is greater than the dimension of end-effector space then the device is* ***kinematically redundant****.*

Definition 3: *If the task space is completely contained by, and has a lower dimensionality than the end-effector space, the manipulator is said to be **task redundant**.*

Two adjectives are often coupled with the idea of a redundant manipulator. **Highly** or **hyper redundant manipulator**[21–23] are often used in conjunction with the term snake robots. The intent is clear, such devices either being planar or fully spatial should have a joint space dimension that is much greater than the dimension of the end-effector space, i.e. $n \gg m$. Since the maximum 'm' can reach is 6 any value of 'n' greater than 10 fulfils this criterion.[24] Often the implication of such designs is the need to consider non-Jacobian based inverse kinematics techniques for controlling the self shape of the device.

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