

K. TENT and M. ZIEGLER. *A Course in Model Theory*. Lecture Notes in Logic, vol. 40. Cambridge University Press, United Kingdom, 2012, x + 248 pp.

Long ago there were only a handful of sources from which to learn model theory: Robinson's "Complete theories", a few sections of Shoenfield, Sack's "Saturated model theory" and finally, in the early 70's Chang and Keisler produced their book which stood for a generation. Times have certainly changed. In the past 15 years or so there has been an explosion of books on model theory: introductory books, specialized books on particular applications and general themes. In the introductory category, one could choose as a reference for a standard graduate course in model theory from among the books by Marker, Poizat, Hodges, or Marcja and Toffalori. To this list, there is a new entrant: "A course in model theory" by Tent and Ziegler.

The absolute basics of introductory model theory - languages, structures, compactness and the like - are handled very efficiently in the first four chapters of the book. As evidence of how rapid the introduction is, the definition of complete type (referred to as a type: be forewarned) is introduced on page 22. The notion of a multi-sorted structure is introduced at the start, as it should be, and is freely used throughout the book. The authors make a conscious decision to put some necessary background information in three very useful appendices. They vary in depth; I would imagine that someone completely new to any of the topics - basic set theory, a variety of facts about fields and related topics and partition theorems - would need to consult other reference materials. Chapter 3 contains an excellent section devoted to a wide range of examples. It is very nice to see the theory of modules covered at such an early stage even if one has to take Neumann's lemma on faith until Chapter 6. The examples in this section are very challenging and will probably require the reader to consult the appendices and other material. Alternatively, these examples could act as very good projects for students in an introductory graduate course. Chapter 4 discusses the omitting types theorem,  $\omega$ -categorical theories, prime models, and Fraïssé constructions. A full discussion of the topology on the type space is delayed until later in the book which momentarily makes the definition of isolated seem a bit odd. Random graphs are introduced in the exercises to Section 3.3 and maybe a pointer to Section 4.4 on the amalgamation method would have been welcome.

The cat is out of the bag when one gets to Chapter 5. The authors have chosen  $\aleph_1$ -categorical theories and Morley's categoricity theorem as the main motivator for the book. Given how central these topics have been over the past 50 years, this is not surprising. What is novel is how quickly the categoricity theorem is attacked. The authors leave a variety of topics until later chapters as they make a full out assault on the categoricity theorem starting from the first section of Chapter 5. More general discussions of indiscernibles are put off until Chapter 7; stability in general is discussed later in Chapter 8. The entirety of Morley's theorem is dispatched in 25 pages with all the essentials given. This leaves one essentially half way through the book with a lot of pieces to be picked up but with a sense that something quite important is going on.

Chapter 6 goes back and revisits  $\omega$ -stable theories through the lens of Morley rank. The logic topology on the type space now plays a key role and the devoted reader should probably return to earlier sections in the book and put the pieces together. As an application of the use of Morley rank, countable models of an  $\aleph_1$ -categorical theory are classified by their dimension. Chapter 7 seems initially to be out of place as it is devoted to simple theories; those theories that have a well-behaved notion of independence given by dividing. This perception is misplaced since the goal of the authors is to give a very slick introduction to independence in stable theories by developing forking in the original manner of Shelah via indiscernibles. The key results are due to Kim and Pillay but the presentation owes a lot to Casanovas. The introduction to forking is then accomplished in as quick a manner as possible and Chapter 8 gets down to the business of looking at stable theories.

The book concludes with two specialized topics: Chapter 9 deals with the existence and uniqueness of prime models in stable theories and Chapter 10 looks at generalized Fraïssé constructions and Hrushovski's examples of non-modular  $\aleph_1$ -categorical theories which do not interpret a field. These are both challenging topics. Readers of Chapter 9 will see the definition of the average type of an indiscernible sequence which wasn't discussed earlier.

Chapter 10 introduces some of the most fundamental fine structure tools in a stability theorist's toolbox - internality and analysability. If one gets through the construction of the Hrushovski examples, one is rewarded with a nice use of the group configuration to see that a group is not interpretable in the resulting structure, let alone a field.

This book could serve as an introduction to model theory for students who have seen almost no logic but these would have to be very good students. Alternatively, the book could be used as a first free-standing book in model theory after students have seen a basic course in logic. This would make the first few chapters a quick refresher and allow the students to concentrate on the latter chapters. The book contains some excellent exercises and there is a fourth appendix which hints at solutions. It must be noted that many of the exercises have a forward looking aspect: topics are often first encountered in exercises and then handled more thoroughly in later sections. There are a few things to quibble about: a discussion of the monster model is handled very thoroughly. This is a good thing since this is often a source of confusion for new students. However, given the manner in which the rest of the book was written, it might have been better to include this discussion as another appendix. Imaginary elements are only introduced in the chapter on stable theories. The authors go to great lengths to note that the construction of the imaginary universe is a general construction but its placement perpetuates the feeling that imaginaries have something to do with stability theory and are not a fundamental object of general model theory. Overall the book is an excellent addition to the growing library of books from which one can learn model theory.

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