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TREND IN CYCLE OR CYCLE IN TREND? NEW STRUCTURAL IDENTIFICATIONS FOR UNOBSERVED-COMPONENTS MODELS OF U.S. REAL GDP

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A well-documented property of the Beveridge–Nelson trend–cycle decomposition is the perfect negative correlation between trend and cycle innovations. We show how this may be consistent with a structural model where permanent innovations enter the cycle or transitory innovations enter the trend, and that identification restrictions are necessary to make this structural distinction. A reduced-form unrestricted version is compatible with either option, but cannot distinguish which is relevant. We discuss economic interpretations and implications using U.S. real GDP data.

Keywords: Trend-Cycle Decomposition, Data Revision, State-Space Form

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1. INTRODUCTION

Decomposing macroeconomic time series into trend (permanent) and cycle (transitory) components has a significant history. Macroeconomics is vitally interested in distinguishing between trends and cycles in series such as GDP and employment as the profession attempts to align theory, policy, and empirical estimation. Econometrics has responded with a basket of different methods including simple moving averages, fitted linear trends, and sophisticated linear filters such as the Hodrick–Prescott filter, the bandpass filters of Baxter and King (1999) and Christiano and Fitzgerald (2003) and the uncorrelated unobserved components models associated with structural time series analysis in Harvey (1989).[1] The Beveridge–Nelson (1981) decomposition, which specifically accounts for the unit root properties of many macroeconomic time series, has become a particularly useful tool, decomposing series into a deterministic trend, a random walk, and a cycle.[2] Morley et al. (2003; MNZ) were the first to investigate the equivalence between the unobserved components and Beveridge–Nelson approaches.

This paper considers identification of trend–cycle decompositions cast in a state-space form. We take the state-space version of the Beveridge–Nelson decomposition first provided by Morley (2002) and introduce identification insights drawn from the data revisions literature, in particular Jacobs and van Norden (2011; JvN). An important feature of this approach is that unlike the structural time series approach, where innovations to trends and cycles are typically assumed to be uncorrelated, such innovations are negatively correlated. Several recent papers argue that output data are better fit by models with negatively correlated innovations, including MNZ, Oh et al. (2008), Sinclair (2009), Jun et al. (2012), and Morley (2011), whereas Nelson (2008) also finds that models with negatively correlated innovations do as well at forecasting cyclic movements as models with uncorrelated innovations, or better.[3] Nonzero correlations between innovations in state-space models result in unequal weights on future and past values in the Kalman smoother [see Harvey and Koopman (2000)]. Proietti (2006) notes that negative correlations lead to higher weights on future observations in the Kalman smoother, resulting in relatively large revisions to filtered estimates.

In spite of the preceding evidence favoring negatively correlated innovations to trend and cycle, the economic interpretation of this correlation is the subject of considerable debate. The dominant view is that trend innovations lead to a requirement for cycles to "catch up," so that the deviation of the cycle from the shifted long-run path diminishes over time, resulting in a negative correlation. However, cycle innovations do not cause an analogous movement in trend. This view (long associated with Charles Nelson) implies that potential output is more

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volatile than observed output. This is consistent with the predominance of real shocks that directly affect potential output but not actual output.

In contrast, transitory innovations may be considered to influence the trend. In this case the literature interprets the results as supporting the effect of nominal shocks in determining long-term economic outcomes and as supporting a stronger role for macroeconomic policy, particularly in that monetary policy decisions or government expenditure or income changes may influence the equilibrium outcome path for an economy. Other options place emphasis on adjustments in the economy—for example, defense purchases, which may be stimulatory in the short run but detrimental in the long run [Clark (1987)], or increased long-term uncertainty created by short-run monetary policy actions [Weber (2011)]. Whether shocks to the trend influence cycle or shocks to the cycle influence trend has important implications for both policy and economic forecasting.

We show the difficulties in obtaining a structural form identification for the interactions between permanent and transitory innovations in the general state-space form of the Beveridge–Nelson decomposition as provided in MNZ, also noted by Proietti (2006) and Weber (2011), and how these may be resolved with assumptions adopted from JvN. By way of illustration, we apply these to U.S. GDP and show that the data support the interpretation of transitory innovations influencing trend rather than the alternative that permanent innovations influence cycle. This conclusion contributes to the ongoing debate about whether real shocks drive the economy, and nominal shocks are only temporary, or alternatively, nominal (cycle) shocks may indeed influence long-term economic outcomes.

The remainder of this paper is structured as follows. First, in Section 2, we introduce the modeling framework to illustrate different assumptions that may be used in trend–cycle decompositions and consider different interpretations associated with these assumptions. Section 3 provides the empirical application to U.S. real GDP data and discusses the evidence for whether transitory innovations enter trend or permanent innovations enter cycle. Section 4 concludes.

2. A SIMPLE MODEL FOR DECOMPOSITIONS WITH MULTIPLE INTERPRETATIONS

Consider the decomposition

$$y_t = \widetilde{y}_t + e_t,$$

where y_t is observable, $\widetilde{y}_t$ is a latent variable, and $e_t \equiv y_t - \widetilde{y}_t$. We will assume that $\widetilde{y}_t$ is a random walk—which is equivalent to $\Delta\widetilde{y}_t$ being i.i.d.—but we will make no identifying assumptions about e_t for the moment. Although macroeconomists can easily think of $\tilde{y}_t$ as trend (permanent component) and e_t as cycle (transitory component), this decomposition is also entirely compatible with the JvN approach of $\tilde{y}_t$ as the "truth" and e_t as measurement errors, as will be shown later.

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We can write one such very simple model in state-space form as

$$\text{Measurement Equation} \quad y_t = \begin{bmatrix} 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} \widetilde{y}_t \\ e_t \end{bmatrix}, \tag{1}$$

$$\text{Transition Equation} \quad \begin{bmatrix} \widetilde{y}_t \\ e_t \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} \widetilde{y}_{t-1} \\ e_{t-1} \end{bmatrix} + \begin{bmatrix} \sigma_\eta & 0 \\ 0 & \sigma_\nu \end{bmatrix} \cdot \begin{bmatrix} \eta_t \\ \nu_t \end{bmatrix}, \tag{2}$$

where $[\eta_t \ \nu_t]' \sim$ i.i.d. $N(\mathbf{0}, \boldsymbol{I}_2)$. Note that because y_t is just $\widetilde{y}_t$ plus i.i.d. noise, $\text{var}(\Delta y_t) > \text{var}(\Delta \widetilde{y}_t)$, $\forall \sigma_\nu > 0$.

The model implies that $y_t \sim \text{IMA}(1, 1)$, which might not be realistic. In particular, if y_t is thought to contain cycles, we can nest this possibility by allowing e_t to follow an AR(2) process, as in MNZ.[4] Now the measurement equation becomes

$$y_t = \begin{bmatrix} 1 & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} \widetilde{y}_t \\ e_t \\ e_{t-1} \end{bmatrix}, \tag{3}$$

with transition equation

$$\begin{bmatrix} \widetilde{y}_t \\ e_t \\ e_{t-1} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \phi_1 & \phi_2 \\ 0 & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} \widetilde{y}_{t-1} \\ e_{t-1} \\ e_{t-2} \end{bmatrix} + \begin{bmatrix} \sigma_\eta & 0 \\ 0 & \sigma_\nu \\ 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} \eta_t \\ \nu_t \end{bmatrix}. \tag{4}$$

In the trend–cycle decomposition literature the final term in (4) is usually expressed as

$$\begin{bmatrix} \tilde{\eta}_t \\ \tilde{\nu}_t \\ 0 \end{bmatrix} = \begin{bmatrix} \sigma_\eta & 0 \\ 0 & \sigma_\nu \\ 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} \eta_t \\ \nu_t \end{bmatrix},$$

where $\tilde{\eta}_t$ is the "trend" or permanent innovation and $\tilde{\nu}_t$ is the "cycle" or transitory innovation.

Although this is consistent with the prototypical unobserved-components model of the business cycle with *orthogonal* innovations, i.e., the seminal model of Watson (1986), orthogonality is not essential. We could instead assume that the innovations are perfectly correlated, which results in a restricted single source of error (SSE) decomposition as in Anderson et al. (2006),[5] with transition equation

$$\begin{bmatrix} \widetilde{y}_t \\ e_t \\ e_{t-1} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \phi_1 & \phi_2 \\ 0 & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} \widetilde{y}_{t-1} \\ e_{t-1} \\ e_{t-2} \end{bmatrix} + \begin{bmatrix} \sigma_\eta \\ \sigma_\nu \\ 0 \end{bmatrix} \cdot [\eta_t]. \tag{5}$$

We could also consider the polar opposite case, where the innovations are perfectly negatively correlated:

$$\begin{bmatrix} \widetilde{y}_t \\ e_t \\ e_{t-1} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \phi_1 & \phi_2 \\ 0 & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} \widetilde{y}_{t-1} \\ e_{t-1} \\ e_{t-2} \end{bmatrix} + \begin{bmatrix} \sigma_\eta \\ -\sigma_\nu \\ 0 \end{bmatrix} \cdot [\eta_t]. \tag{6}$$

Alternatively, following MNZ, we can encompass (4), (5), and (6) in the form

$$\begin{bmatrix} \widetilde{y}_t \\ e_t \\ e_{t-1} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \phi_1 & \phi_2 \\ 0 & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} \widetilde{y}_{t-1} \\ e_{t-1} \\ e_{t-2} \end{bmatrix} + \begin{bmatrix} \sigma_\eta & r_{12} \\ r_{21} & \sigma_\nu \\ 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} \eta_t \\ \nu_t \end{bmatrix}, \qquad (7)$$

where r_{12} and r_{21} are nonzero. In the following, we will refer to the model with (6) as the original BN model and that with (7) as the MNZ model.[6]

The critical component for estimating such models in state-space format is the variance–covariance matrix of the innovations, denoted by $\mathbf{Q}$. In our most general case, given in (7), the relevant form is given as

$$\mathrm{E}\left(\begin{bmatrix} \sigma_\eta & r_{12} \\ r_{21} & \sigma_\nu \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \eta_t \\ \nu_t \end{bmatrix} \begin{bmatrix} \eta_t \\ \nu_t \end{bmatrix}' \begin{bmatrix} \sigma_\eta & r_{12} \\ r_{21} & \sigma_\nu \\ 0 & 0 \end{bmatrix}'\right) = \begin{bmatrix} \sigma_\eta^2 + r_{12}^2 & \sigma_\eta r_{21} + \sigma_\nu r_{12} & 0 \\ \sigma_\eta r_{21} + \sigma_\nu r_{12} & \sigma_\nu^2 + r_{21}^2 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \qquad (8)$$

that is, $\mathrm{E}\left(\mathbf{R}\varepsilon\varepsilon'\mathbf{R}'\right) = \mathbf{Q}$.

Estimation of (7) allows us to exactly identify the three elements in $\mathbf{Q}$. However, the four elements in $\mathbf{R}$ are not identified. We may instead entertain a number of restrictions on $\mathbf{R}$ consistent with the economic argument. For example, if only permanent (or real) economic innovations have long-term effects, then transitory (or nominal) innovations will not have a sustained influence. This implies that $r_{12} = 0$ and $\mathbf{Q}$ simplifies to

$$\mathrm{E}\left(\begin{bmatrix} \sigma_\eta & 0 \\ r_{21} & \sigma_\nu \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \eta_t \\ \nu_t \end{bmatrix} \begin{bmatrix} \eta_t \\ \nu_t \end{bmatrix}' \begin{bmatrix} \sigma_\eta & 0 \\ r_{21} & \sigma_\nu \\ 0 & 0 \end{bmatrix}'\right) = \begin{bmatrix} \sigma_\eta^2 & \sigma_\eta r_{21} & 0 \\ \sigma_\eta r_{21} & r_{21}^2 + \sigma_\nu^2 & 0 \\ 0 & 0 & 0 \end{bmatrix}. \qquad (9)$$

We also entertain the opposite case, where permanent innovations do not influence the cycle but transitory innovations affect the trend, which implies that $r_{21} = 0$ and $\mathbf{Q}$ simplifies to

$$\mathrm{E}\left(\begin{bmatrix} \sigma_\eta & r_{12} \\ 0 & \sigma_\nu \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \eta_t \\ \nu_t \end{bmatrix} \begin{bmatrix} \eta_t \\ \nu_t \end{bmatrix}' \begin{bmatrix} \sigma_\eta & r_{12} \\ 0 & \sigma_\nu \\ 0 & 0 \end{bmatrix}'\right) = \begin{bmatrix} \sigma_\eta^2 + r_{12}^2 & \sigma_\nu r_{12} & 0 \\ \sigma_\nu r_{12} & \sigma_\nu^2 & 0 \\ 0 & 0 & 0 \end{bmatrix}. \qquad (10)$$

These two models are observationally equivalent to the unrestricted unobserved-components model of MNZ, and are hence labeled unrestricted trend-in-cycle (UT2C) and unrestricted cycle-in-trend (UC2T) models, respectively. This is consistent with the fact that, whereas MNZ (p. 241) write, "If we accept the implication that innovations to trend are strongly negatively correlated with innovations to the cycle, then the case for the importance of real shocks in the macro economy is strengthened," Proietti (2006) shows that this need not always be the case.

An alternative interpretation of our original model is as a measurement error model where e_t is the measurement error in observing our object of interest $\widetilde{y}_t$.

Typical measurement error models assume that $E\,(\widetilde{y}_t \cdot e_t) = 0$, so that what we observe is the "truth" plus a random "noise" term e_t. However, we might prefer to think of measurement error as "news" rather than "noise," so that $E\,(y_t \cdot e_t) = 0$. This would be more consistent with the idea of an "efficient" statistical agency, as suggested by Sargent (1989), for example. In that case, the transition equations become

$$\begin{bmatrix} \widetilde{y}_t \\ e_t \\ e_{t-1} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \phi_1 & \phi_2 \\ 0 & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} \widetilde{y}_{t-1} \\ e_{t-1} \\ e_{t-2} \end{bmatrix} + \begin{bmatrix} \sigma_\eta & -\sigma_\nu \\ 0 & \sigma_\nu \\ 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} \eta_t \\ \nu_t \end{bmatrix}, \qquad \textbf{(11)}$$

where now η_t is the "truth" shock and ν_t is the "news" shock, and we have allowed the measurement errors to be correlated over time. Note that for any observation y_t, the news shock in the "truth" $\tilde{y}_t$ is exactly offset by the shock in the measurement error e_t, so that only the portion of the shock due to η_t is initially observable. This model is a special case of the UC2T model introduced earlier, with the additional restriction $r_{12} = -\sigma_\nu$, so that

$$\mathbf{Q} = \begin{bmatrix} \sigma_\eta^2 + \sigma_\nu^2 & -\sigma_\nu^2 & 0 \\ -\sigma_\nu^2 & \sigma_\nu^2 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

We refer to this as the restricted cycle-in-trend (RC2T) model. Alternatively, we could impose the restriction $r_{21} = -\sigma_\eta$ on the UT2C model, to obtain the restricted trend-in-cycle (RT2C) model, with

$$\mathbf{Q} = \begin{bmatrix} \sigma_\eta^2 & -\sigma_\eta^2 & 0 \\ -\sigma_\eta^2 & \sigma_\eta^2 + \sigma_\nu^2 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

In this case, in any observation y_t, the shock to the "truth" $\tilde{y}_t$ is exactly offset by the shock in the measurement error e_t, so that only the portion of the shock due to the transitory shock ν_t is initially observable. Both the RC2T and RT2C models imply that $\mathrm{var}(\Delta y_t) < \mathrm{var}(\Delta \tilde{y}_t)$ for all $\sigma_\nu > 0$.

The different assumptions and interpretations just described capture the essential differences between a number of important and much more general state-space models. The difference between $E\,(\widetilde{y}_t \cdot e_t) = 0$ and $E\,(y_t \cdot e_t) = 0$ captures the essential difference between structural time series models (which use the former assumption) and the Beveridge–Nelson decomposition (which typically imposes the latter in estimation). The Beveridge–Nelson trend–cycle decomposition interprets the results as a stochastic trend and a cycle, whereas the JvN approach interprets them as a "true value" contaminated by measurement error. All of these models also have multivariate extensions that may play important roles in the identification of the model; for example, see Sinclair (2009) and Morley (2011).

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TABLE 1. Observationally equivalent trend–cycle decomposition models

	MNZ	UT2C	UC2T
Trend process			
Drift μ	0.78	0.78	0.78
Variance of shocks Q_{11}	1.53	1.53*	1.53*
Cycle process			
ϕ_1	1.25	1.25	1.25
ϕ_2	−0.65	−0.65	−0.65
Variance of shocks Q_{22}	0.68	0.68*	0.68*
Correlation	−0.92	−0.92*	−0.92*
Matrix **R**			
σ_η		1.24	0.48
r_{12}			−1.14
r_{21}		−0.76	
σ_ν		0.32	0.82
Log likelihood	−334.18	−334.18	−334.18

Notes: UT2C refers to the unrestricted trend-cycle model. UC2T refers to the unrestricted cycle-in-trend model. An asterisk indicates that a parameter is not estimated, but calculated from the elements of the **R** matrix.

3. ESTIMATES

To examine these findings, we estimate various specifications of the unobserved-component models that were discussed in the previous section, using U.S. real GDP data from 1947Q1 to 2012Q3.[7] Table 1 compares the implied estimates of the **R** matrix across the three observationally equivalent models discussed previously (MNZ), the unrestricted trend-into-cycle model (UT2C) associated with the **Q** matrix in (9), and the unrestricted cycles-into-trend model (UC2T) with the **Q** matrix in (10). Estimates for the drift term in the trend process, the autoregressive parameters for the cycle process, and the log likelihood value are the same for all three models. The estimates for the elements of **R** show that the MNZ specification is compatible with very different structural models of cycle and trends. On one hand, the UT2C model has a relatively small coefficient (0.32) on the transitory component ν, whereas the impact of the permanent innovation η is more than twice as large (−0.76). This is consistent with the view that business cycles are dominated by the impact of permanent, real shocks. On the other hand, the UC2T model has the opposite result, with innovations to the trend dominated by transitory, nominal shocks (−1.14) rather than permanent real shocks (0.48).[8]

Table 2 compares the estimated parameters of the MNZ model with five nested models, each of which imposes a different restriction. In addition to the RT2C and RC2T models, the table shows the original BN model, the SSE model, and the Watson (1986) model; t-ratios (using standard errors estimated from the outer

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TABLE 2. Restricted trend–cycle decomposition models

		MNZ		Watson (1986)		RT2C		RC2T		BN		SSE	
						Trend process							
	μ	**0.78**	[10.11]	**0.79**	[19.47]	**0.79**	[20.67]	**0.78**	[10.20]	**0.77**	[10.31]	**0.79**	[22.99]
	Q_{11}	**1.53**	[3.37]	0.24	[1.79]	**0.05**	[3.06]	**1.51**	[2.63]	**1.47**	[3.97]	0.07	[0.96]
						Cycle process							
	ϕ_1	**1.25**	[5.59]	**1.50**	[15.35]	**1.37**	[20.30]	**1.19**	[6.60]	**1.34**	[20.44]	**1.50**	[15.95]
	ϕ_2	**−0.65**	[−2.66]	**−0.51**	[−5.03]	**−0.38**	[−5.46]	**−0.50**	[−3.82]	**−0.74**	[−8.42]	**−0.51**	[−5.30]
	Q_{22}	0.68	[1.05]	**0.54**	[3.31]	**0.87**	[8.71]	1.08	[1.86]	0.48	[1.40]	**0.41**	[2.27]
Covariance	Q_{21}	−0.94	[−1.83]	—	.	**−0.05**	[−3.06]	−1.08	[−1.86]	**−0.84**	[−2.10]	**0.17**	[3.06]
Log likelihood		−344.18		−347.32		−348.01		−345.23		−344.49		−347.26	
LR stat				**6.28**		**7.66**		2.11		0.63		**6.16**	
P-value				0.012		0.006		0.146		0.428		0.013	

Notes: RT2C refers to the restricted trend-cycle model. RC2T refers to the restricted cycle-trend model. The table shows parameter estimates with t-ratios (based on standard errors estimated from the outer product of the score matrix), the value of the log likelihood function, and the likelihood ratio (LR) test statistics comparing each restricted model against the MNZ model. Parameter estimates that are significantly different from zero at the 5% level are in **bold**. P-values indicated for the LR statistics are based on the $\chi^2(1)$ distribution.

product of the score matrix) are reported in brackets next to each parameter estimate, and parameters significantly different from zero based on a two-sided standard normal distribution are indicated in boldface.[9] Likelihood-ratio (LR) statistics test the restrictions imposed by each model on the MNZ model.

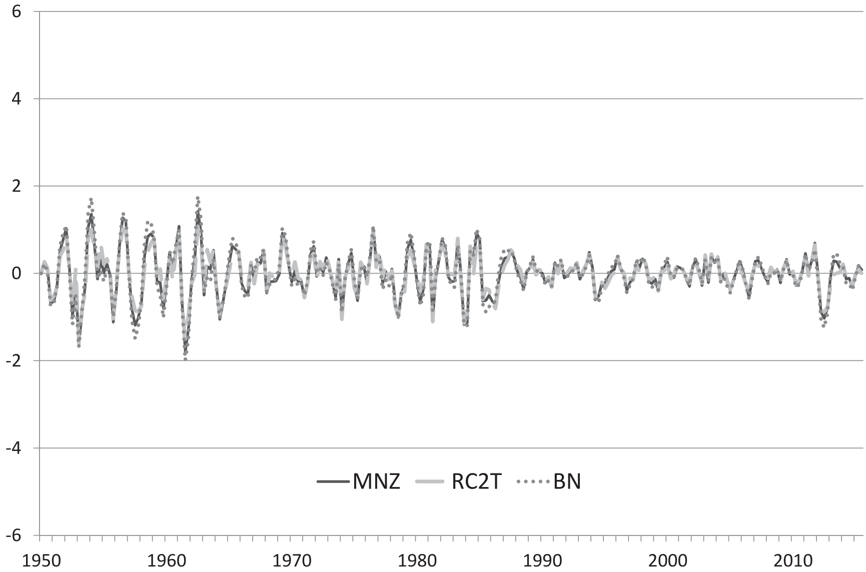
The parameter estimates for the MNZ model are similar to those reported by MNZ (2003). In addition to the familiar "hump-shaped" AR(2) coefficients, we find the variance of permanent innovations to be just over double the variance of transitory innovations, and the covariance of the innovations is strongly negative. Although the covariance of -0.94 is not (quite) significantly different from zero at the 5% level, based on its t-ratio, the LR test comparing the Watson and MNZ models allows a rejection of the same hypothesis at almost the 1% level and is typically considered to be more reliable in finite samples. Note also that the variance of innovations to the cycle is very imprecisely estimated; this reflects in part a high correlation between the estimated variance and the estimated covariance of the innovations.

Two models fit the data almost as well as the MNZ model: the original BN model and the RC2T model. LR statistics are unable to reject either model at even the 10% significance level. An examination of (11) reveals that this model incorporates both permanent innovations and transitory innovations in determining the growth of U.S. GDP, a result consistent with both Sinclair (2010) and the findings of Weber (2011). Weber (2011) shows support for a switch between a cycle-into-trend specification and a trend-into-cycle specification over the postwar sample, but does not allow the possibility of influences in both directions.

In addition to having estimated cyclic dynamics similar to that of the MNZ model, both Sinclair and Weber also estimate permanent innovations to be much more variable than transitory innovations. The ability of the RC2T model to fit the variance properties of the data, particularly that Q_{11} exceeds Q_{22}, as present in the MNZ results, may be a contributing factor. Although the variance of the transitory innovations remained imprecise, higher estimated variances were associated with more negative covariances.[10]

Figure 1 compares the smoothed and filtered estimates of the cycle for the MNZ, BN, and RC2T models and shows that they are extremely similar. All three models produce smoothed estimates of the cycle that are much more variable than filtered estimates, implying that although cycles are initially estimated to be quite small, these estimates subsequently undergo substantial revision. As the figure shows, although filtered estimates of the cycle only rarely exceed 1% of GDP, smoothed estimates are occasionally four times as large. For example, during the most recent recession, filtered estimates from all three models initially indicated a large recession, with output roughly 1% below trend in early 2009. Recent smoothed estimates, however, revise that figure to near-zero and instead put 2008 output at 4% above trend (the highest cyclic peak in the post-1947 period).[11]

In contrast to these three similar models, the other three models (Watson, RT2C, and SSE) do not fit the data as well and produce distinctly different results. All three produce cycles with very highly persistent AR(2) dynamics. (The sum of



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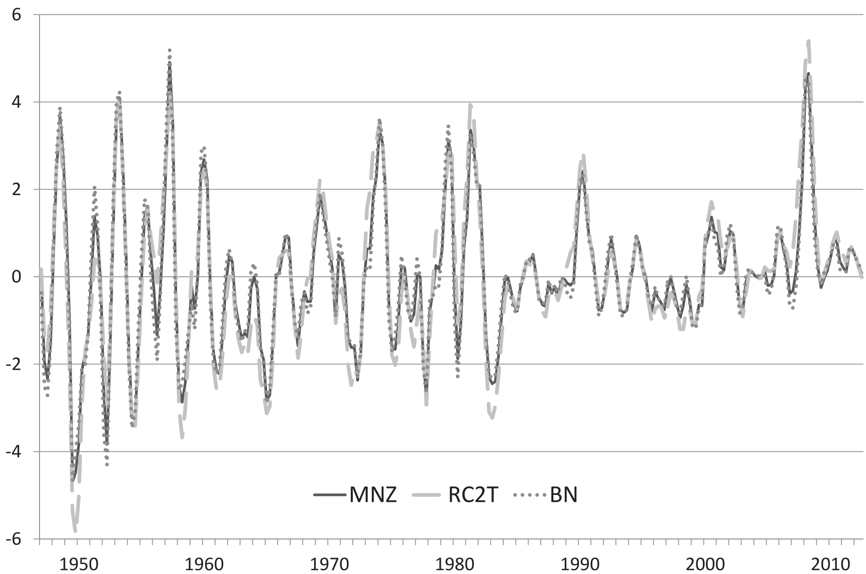

Smoothed

FIGURE 1. Smoothed and filtered estimates of the cycle for the MNZ and the RC2T models (100 × ln real GDP).

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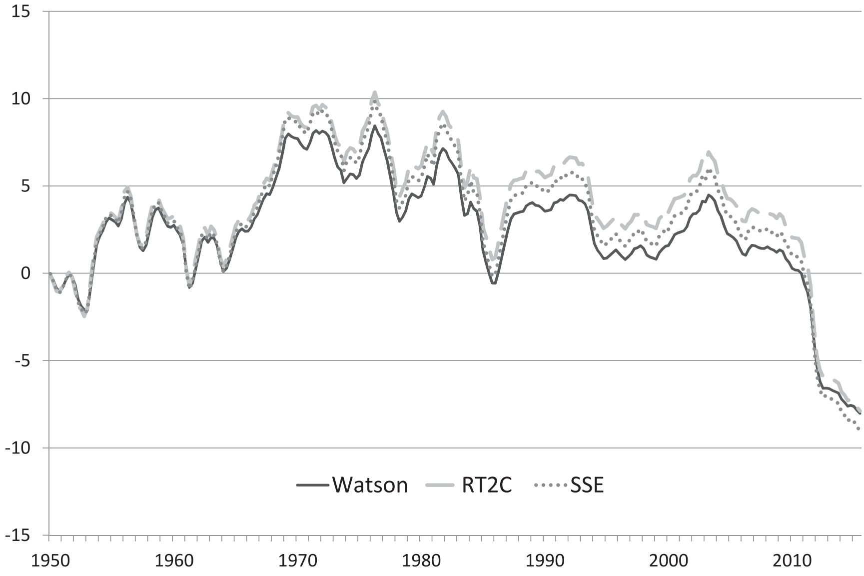

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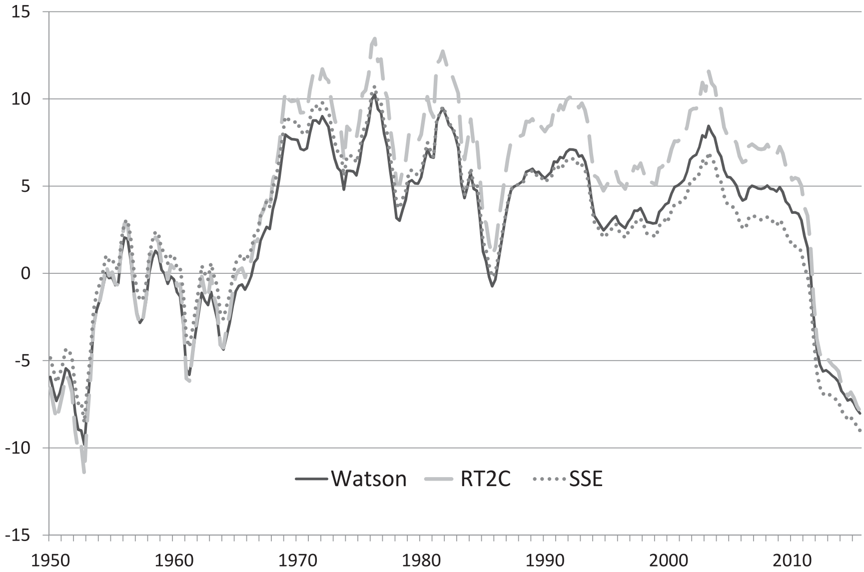

Smoothed

FIGURE 2. Smoothed and filtered estimates of the cycle for the Watson and the RT2C models (100 × ln real GDP).

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the AR coefficients is 0.99 for all three models.) Each estimates the variance of innovations to the cycle to be at least twice that of the innovations to the trend; in the case of the RT2C model, they are more than 10 times larger. However, when compared with the MNZ model, each of these models is strongly rejected by the data.[12]

Figure 2 compares the smoothed and filtered estimates of the cycle for the Watson, the RT2C, and the SSE models. Estimated cycles are large and highly persistent; filtered estimates were consistently positive for twenty years, starting in the early 1960s. Filtered estimates of the most recent recession are without precedent, implying nonstop decline relative to trend since 2006, culminating in a cycle 8% below trend by 2012Q3.[13] The largest revisions of the filtered estimates occur at the start of the sample, with estimates for the 1940s and early 1950s revised downward by 5% of GDP or more. Although the estimated standard errors for both the smoothed and filtered estimates from these models are large, smoothed estimates are occasionally significantly different from zero at the 5% level, as are filtered estimates from the SSE model. Neither the Watson nor the RT2C model finds that current estimates of the cycle are significantly different from zero, which is perhaps surprising, given their size.

4. CONCLUSION

Trend–cycle decompositions are deeply important to macroeconomics and econometrics, and the implementation and identification assumptions used in cycle extraction influence the estimated outcomes. This paper draws insights from the identification conditions used in the state-space formulation of the structure of data revisions in JvN to motivate an identification scheme in the Beveridge–Nelson equivalent state-space formulation of trend–cycle decomposition, ensuring negative correlation between trend and cycle innovations. Most authors are agreed that innovations to GDP are predominantly permanent and negatively correlated. Indeed, recently Sinclair (2009) has found the same for unemployment, and noted the importance of this commonality between GDP and unemployment for Okun's law.

We show that using a state-space formulation for trend–cycle time series, such as GDP, will not ensure a structural interpretation of whether transitory innovations enter trend or permanent innovations enter cycle. Instead, we implement restricted models that do admit such an interpretation. When they are applied to U.S. GDP data, we find that the results for 1947Q1 to 2012Q3 are more consistent with a model where transitory innovations enter trend, rather than where permanent innovations enter cycle. There is some support for this result in the existing literature in the two-regime model for industrial production of Weber (2011) and the importance of incorporating both types of innovations to explain U.S. recessions in Sinclair (2010). Furthermore, all of the models consistent with the data (BN, MNZ, and RC2T) imply that smoothed cyclical fluctuations are many times larger than filtered cycles, reflecting that filtered estimates are not reliable indicators of business cycles.

The paper shows how the parallels between the trend–cycle decompositions literature and the data revisions literature may be used to aid in identification when there is assumed to be a negative correlation between the two types of innovations—the common presumption in both literatures. Using these parallels, we explore the relationships between permanent and transitory innovations, which may provide the driving influence in an economy. In this way, the paper seeks to align economic theory and econometric technique in the spirit of, for example, Murray and Nelson (2004) and Lee and Nelson (2007).

NOTES

1. See Jacobs (1998), Mills (2003), and Harvey (2006) for further information.
2. However, Nelson (2008) notes that it was left on the shelf for nearly a decade.
3. Perron and Wada (2009) take a different view, and emphasize the role of breaks.
4. Ma and Wohar (2013) take an alternative route by adding AR dynamics to the trend.
5. Anderson et al. (2006) consider a more general model in which the cycle follows an ARMA(2,1) process. This more general model is observationally equivalent to the MNZ model of random walk plus AR(2) cycle, as discussed in Morley (2011).
6. MNZ and Morley (2011) provide insightful and nuanced discussions of the relationships between the Beveridge–Nelson decomposition and the BN and MNZ models described here.
7. MNZ used the same data series ending in 1998Q2. We follow them in fitting the model to 100 times the natural logarithm of the series. We also reestimated all our models on the 1947–1998 sample used by MNZ and obtained results very similar to those reported here. All estimates were produced using the `CMLmt` package in GAUSS to maximize the likelihood function. Two constraints were imposed in estimation: The AR(2) coefficients were constrained to ensure a stationary cycle and models with multiple sources of innovations were constrained to have a positive definite innovation covariance matrix. These constraints were never binding at the maximum likelihood estimates.
8. The UC2T model estimates imply that positive transitory shocks permanently “lower” trend output.
9. As in all state-space models, caution should be exercised in testing the null hypothesis that one or more variances are zero, as standard asymptotic inference theory is not generally applicable when parameters are constrained to lie on the boundary of the parameter set under the null. See Morley et al. (2013) for a recent discussion of the problem.
10. The covariance for the RC2T model implies a correlation of -0.85 between the two innovations.
11. The extensive revision of filtered estimates is reflected in their estimated standard errors. Filtered estimates of the cycle for these three models are not presented with their standard errors to conserve space; however, they are never close to being statistically different from zero at conventional levels of significance. These results are available from the authors on request.
12. This may reflect the weakness of these models in explaining the time-varying trend growth rate of output over the past sixty years. Faced with faster growth in the earlier part of the sample, they use a nearly nonstationary cycle to capture an upward trend that plateaus in the early 1970s, coincident with the growth slowdown. This is related to the critiques of the inability of unobserved component models to capture structural breaks; see Perron and Wada (2009).
13. Smoothed estimates imply a deeper recession in 1950. However, caution should be exercised in making this comparison, as recent estimates of the cycle may yet undergo substantial revision.

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