A Variational Approach to Understanding White Dwarf Evolution

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I. Background

The observed falloff in the white dwarf luminosity function at $\log(L/L_{\odot}) \approx -4.5$ (Liebert, Dahn, and Monet 1988) is most easily explained as a result of the finite age of the Galactic disk. Using a simple perturbation approach as our method and a white dwarf evolution code as our tool, we have mapped the sensitivity of the ages of the model sequences in this low-luminosity regime to the uncertainties in the input physics and model parameters. We present here a preliminary overview of what we've learned.

II. The Code

We have updated the White Dwarf Evolution Code of Lamb and Van Horn (1975) to include both carbon and oxygen in the core. We interpolate to the mixture composition using our pure-C and pure-O tables and the additive-volume technique (Fontaine, Graboske, and Van Horn 1977). The envelope subroutines calculate stratified H/He/C envelopes of essentially arbitrary layer masses within the range 0 to $\sim 10^{-2} M_{\star}$, and treat the composition transition zones as discontinuities. Because the equation of state tables referenced by the envelope routines do not include crystallization, a given sequence ends when the crystallization front reaches the core/envelope boundary.

III. Two Representative Sequences

In Figure 1 we plot $\log(L/L_{\odot})$ *vs.* $\log(Age)$ and in Figure 2 we plot $\log(T_c)$ *vs.* $\log(Age)$ for two representative model sequences. The first simulates a $0.6 M_{\odot}$ WD with a 50/50 C/O mixture in the core and an outer helium layer with mass $10^{-4} M_{\star}$ (sequence CO60400[**]). The second simulates a $0.7 M_{\odot}$ WD with the C/O convective overshooting profile found by Mazzitelli and D'Antona (1986, hereafter MD) as shown in their Figure 4, and a helium layer with mass $10^{-2} M_{\star}$ (sequence MD70200). Roughly speaking, the MD composition profile has $X_{12} \sim .25$ from the center to $q (\equiv M_r/M_{\star}) = 0.5$, and $X_{12} \approx 0.4 \cdot q + 0.3$ for $0.5 < q < (1 - M_{env})$. Tables 1 and 2 contain the summary listings for the two sequences.

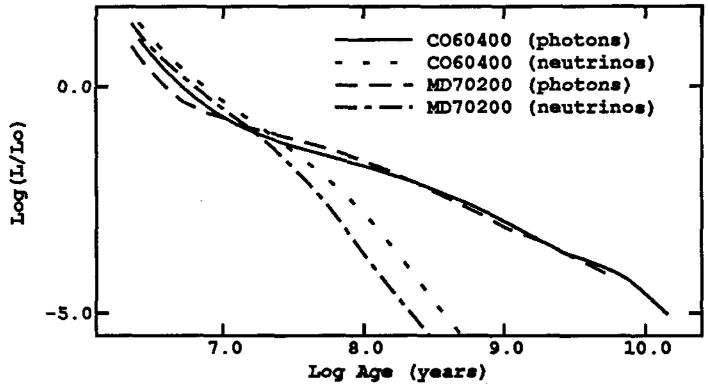

Figure 1: $\mathrm{Log}(L/L_{\odot})$ *vs.* $\mathrm{Log}(Age)$. The MD70200 sequence is younger than the CO60400 sequence at the lowest luminosities. As expected, the plateau caused by the onset of crystallization (near $\log(L/L_{\odot}) = -3.6$) is larger in the sequence which crystallizes at a lower luminosity. Note that above $\log(L/L_{\odot}) \sim -1.0$, neutrino cooling dominates photon energy losses.

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[**] For all sequences, "CO" implies a 50/50 mix throughout the interior, "C" implies pure carbon, "O" implies pure oxygen, and "MD" implies the profile shown in MD's Figure 4.

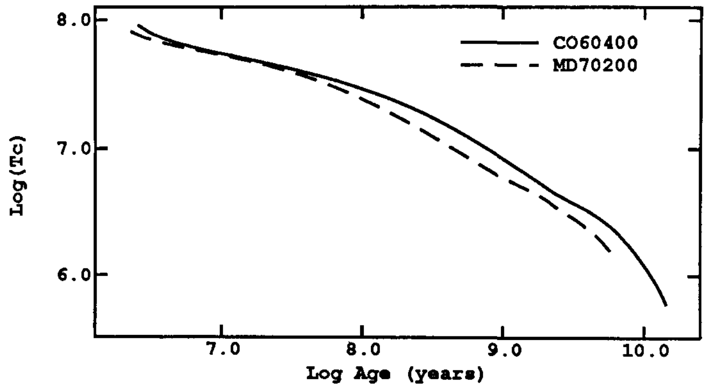

Figure 2: Log(T_C) *vs.* Log(*Age*). The MD70200 sequence has a lower core temperature than the CO60400 sequence as a result of the neutrino cooling in the core. Note also the sharp drop in the core temperature as Debye cooling sets in at the lowest luminosities.

TABLE 1

CO60400: Sequence Summary

$\log(L/L_\odot)$	*Age*	$\log(T_c)$	$\log(P_c)$	$\log(\rho_c)$	$\log(R_\star)$	$\log(L_\nu/L_\odot)$	$M_x/M_\star$	T_{eff}
−0.6	$9.097e+06$	7.747	23.231	6.533	8.969	−0.22	0.	35496
−0.8	$1.173e+07$	7.722	23.238	6.537	8.963	−0.48	0.	31803
−1.0	$1.596e+07$	7.690	23.244	6.541	8.958	−0.77	0.	28525
−1.2	$2.350e+07$	7.650	23.249	6.545	8.954	−1.14	0.	25534
−1.4	$3.807e+07$	7.594	23.254	6.549	8.950	−1.63	0.	22902
−1.6	$6.584e+07$	7.523	23.260	6.553	8.946	−2.25	0.	20518
−1.8	$1.106e+08$	7.445	23.264	6.556	8.942	−2.96	0.	18335
−2.0	$1.750e+08$	7.364	23.268	6.559	8.939	−3.71	0.	16413
−2.2	$2.645e+08$	7.278	23.271	6.562	8.936	−4.43	0.	14663
−2.4	$3.822e+08$	7.191	23.274	6.563	8.933	−5.06	0.	13137
−2.6	$5.426e+08$	7.099	23.276	6.565	8.931	−5.65	0.	11715
−2.8	$7.432e+08$	7.009	23.278	6.566	8.929	−6.19	0.	10474
−3.0	$1.001e+09$	6.921	23.280	6.567	8.928	−6.71	0.	9345
−3.2	$1.337e+09$	6.832	23.281	6.568	8.927	−7.22	0.	8330
−3.4	$1.781e+09$	6.742	23.282	6.569	8.926	−9.96	0.	7437
−3.6	$2.385e+09$	6.646	23.283	6.569	8.925	< -10.00	0.	6634
−3.8	$3.511e+09$	6.549	23.284	6.570	8.924	< -10.00	0.27	5922
−4.0	$5.165e+09$	6.434	23.284	6.571	8.924	< -10.00	0.63	5280
−4.2	$7.017e+09$	6.298	23.285	6.571	8.923	< -10.00	0.87	4704
−4.4	$8.677e+09$	6.177	23.285	6.571	8.923	< -10.00	0.96	4204
−4.6	$1.026e+10$	6.065	23.286	6.572	8.923	< -10.00	0.99	3745

TABLE 2

MD70200: Sequence Summary

$\log(L/L_\odot)$	*Age*	$\log(T_c)$	$\log(P_c)$	$\log(\rho_c)$	$\log(R_\star)$	$\log(L_\nu/L_\odot)$	$M_x/M_\star$	T_{eff}
−0.6	$7.926e+06$	7.747	23.573	6.763	8.924	−0.24	0.	37389
−0.8	$1.237e+07$	7.704	23.586	6.772	8.918	−0.71	0.	33560
−1.0	$1.925e+07$	7.656	23.596	6.779	8.910	−1.17	0.	30171
−1.2	$3.437e+07$	7.578	23.601	6.783	8.906	−1.90	0.	27016
−1.4	$5.845e+07$	7.491	23.606	6.786	8.904	−2.72	0.	24176
−1.6	$9.121e+07$	7.404	23.610	6.789	8.900	−3.53	0.	21581
−1.8	$1.327e+08$	7.318	23.612	6.790	8.897	−4.20	0.	19288
−2.0	$1.868e+08$	7.233	23.615	6.792	8.896	−4.77	0.	17235
−2.2	$2.566e+08$	7.149	23.617	6.794	8.894	−5.28	0.	15417
−2.4	$3.486e+08$	7.065	23.619	6.795	8.892	−5.77	0.	13757
−2.6	$4.684e+08$	6.982	23.620	6.796	8.891	−6.26	0.	12260
−2.8	$6.310e+08$	6.898	23.621	6.797	8.890	−6.75	0.	10945
−3.0	$8.426e+08$	6.816	23.622	6.797	8.888	−7.22	0.	9779
−3.2	$1.148e+09$	6.743	23.622	6.797	8.891	< -10.00	0.	8696
−3.4	$1.730e+09$	6.646	23.623	6.798	8.890	< -10.00	0.06	7754
−3.6	$2.362e+09$	6.548	23.624	6.799	8.887	< -10.00	0.38	6927
−3.8	$3.250e+09$	6.450	23.623	6.798	8.893	< -10.00	0.66	6133
−4.0	$4.381e+09$	6.336	23.624	6.799	8.888	< -10.00	0.86	5505
−4.2	$5.773e+09$	6.192	23.623	6.798	8.895	< -10.00	0.96	4859

IV. Variation of Selected Parameters

For the remainder of this paper we focus on the ages of the low-luminosity white dwarfs. Although Liebert, Dahn and Monet (1988) give $\log(L/L_\odot) = -4.5$ as the nominal luminosity of the falloff, they make the point that there is some uncertainty in this result. With this in mind, we will report the differential effects (in Gyr) that the variation of selected model input quantities has on the ages at the luminosities $\log(L/L_\odot) = -4.4$ and -4.6, as compared with some standard model. We denote these age differences as $\Delta\tau_{-4.4}$ and $\Delta\tau_{-4.6}$.

- The phase diagram of the dense C–O plasma has only recently been computed using a density functional approach by Barrat, Hansen, and Mochkovitch (1988, hereafter BHM). They found that the two nuclear species are miscible in the solid phase, but that the solid phase should be slightly more oxygen-rich than the fluid phase. They further suggest that convection will redistribute the carbon and oxygen, allowing the buildup of an oxygen-rich core. BHM estimate that the gravitational potential energy released by this redistribution will extend the WD lifetime by roughly +0.5 Gyr at the luminosity of the falloff. At present, our code does not include convection in the core, and so we cannot directly incorporate the BHM results; however, we were interested in the sensitivity of the WD ages to the uncertainty in the freezing temperature. We considered the two cases, $T_{xtal} = T_{xtal}(C)$, and $T_{xtal} = T_{xtal}(O)$. Between the sequences CO60300 and CO60400 these two different prescriptions gave $\Delta\tau_{-4.4} \approx 0.30$ Gyr and $\Delta\tau_{-4.6} \approx 0.45$ Gyr, where the $T_{xtal}(C)$ model was the older in each case. These results suggest that we will not be too far wrong if we use $T_{xtal} = X_{12} \cdot T_{xtal}(C) + X_{16} \cdot T_{xtal}(C)$, and this is what we use for the remainder of the C–O core models.
- As mentioned above, we have considered two different C/O profiles in our C–O models, a 50/50 mix and the MD profile. Comparing the two, we find $\Delta\tau(\text{CO70400} \rightarrow \text{MD70400}) \approx +0.4$ Gyr.
- We have varied the helium layer mass in several DB sequences, and find that $\Delta\tau$ is remarkably linear as a function of $\log(M_{He}/M_\star)$ at a given luminosity in our $0.6M_\odot$ sequences. The best fit to the results at $\log(L/L_\odot) = -4.4$ over the range $10^{-2} > M_{He}/M_\star > 10^{-5}$ is

$$\tau_{-4.4} = 5.92 - 0.86 \cdot \log(M_{He}/M_\star) \text{ Gyr},$$

and the best fit at $\log(L/L_\odot) = -4.6$ over the range $10^{-3} > M_{He}/M_\star > 10^{-5}$ is

$$\tau_{-4.6} = 6.01 - 1.29 \cdot \log(M_{He}/M_\star) \text{ Gyr}.$$

The conductive opacities for carbon are higher than those for helium, and so the thinner the helium layer, the longer the evolutionary timescale. Pelletier *et al.* (1986) used accretion-diffusion theory to understand the abundances of C found in cool DB white dwarfs, and found that their models which had $10^{-3.5} > M_{He}/M_\star > 10^{-4}$ most closely matched the observations. This implies an uncertainty in $\tau_{-4.5}$ of order ±0.5 Gyr.

- Finally, we need to worry about uncertainties in the the radiative opacities. As an extreme test of the sensitivity of the WD ages to the radiative opacities we arbitrarily divided the $Z = 10^{-3}$ radiative opacities by a factor of 10. The resulting effect on the ages is small: $\Delta\tau_{-4.4} = -0.5$ Gyr and $\Delta\tau_{-4.6} = -0.6$ Gyr.

V. Summary and Conclusions

We have used a variational approach to map out the effects that uncertainties in the theoretical model parameters have upon the derived ages near the observed cutoff in the white dwarf luminosity function. We find that although there are a number of parameters whose uncertainties imply uncertainties in $\Delta\tau_{-4.5}$ of order ±0.5 Gyr, none of the parameters we explored causes a shift in the age so large as to bring into question the value of the technique. On the contrary, because our internal theoretical uncertainties are fairly small and getting smaller with time, we feel that our results underscore the power of using the observed white dwarf luminosity function for studying the history of star formation in our galaxy.

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