

## ARTICLES

# WEALTH INEQUALITY, OR $r - g$ , IN THE ECONOMIC GROWTH MODEL

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I investigate a simple continuous-time overlapping generations model with a neoclassical production function. I demonstrate that the degree of wealth inequality is positively related to the difference between the real interest rate  $r$  and the growth rate of income  $g$ , and if  $g$  falls, the  $r - g$  gap widens and inequality worsens. I also argue that a wealth tax reduces the wealth inequality. All these results are consistent with the famous predictions advanced by Thomas Piketty in *Capital in the Twenty-First Century* (2014).

**Keywords:** Overlapping Generations, Inequality, Wealth Tax

## 1. INTRODUCTION

In his influential book *Capital in the Twenty-First Century*, Thomas Piketty argues that when the saving rate  $s$  is constant, the capital-to-output ( $k/y$ ) ratio converges to  $s/g$  in the long run, where  $g$  is the per capita income growth rate. According to his claim, as the economic growth rate  $g$  goes to zero, the ratio  $k/y$  goes to infinity. This implies that capital becomes more and more important in the economy as the economic growth slows down. Piketty also predicts that the gap between the rate of return on capital  $r$  and the economic growth rate  $g$ , crucially affects the distribution of wealth. He argues that if  $r$  exceeds  $g$ , inherited wealth will grow faster than labor income and consequently wealth distribution will become highly concentrated. He also argues that if  $g$  is low, the gap  $r - g$  widens and the wealth inequality worsens. His claims on  $r - g$  are validated through the theory by Piketty and Zucman (2015); they construct a two-period overlapping generations (OLG) model in which the individuals are heterogeneous in their preferences toward wealth. They demonstrate that in a steady state, wealth inequality is an increasing function of the term  $\frac{1+r}{1+g}$ . In terms of economic policy, Piketty recommends a wealth tax to reduce wealth inequality.

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Recently, however, some authors have criticized these claims by using popular representative agent neoclassical growth models that are extensively studied in macroeconomics. With respect to his formula on capital-to-output ratio  $k/y = s/g$ , Krusell and Smith (2015) show that  $y$  should be net output instead of gross output, and similarly  $s$  must be net saving rate instead of the gross saving rate. Then, they find that Piketty's assumption on the constant net saving rate is not consistent with the data. Jones (2015, p. 47) also points out a problem of holding the net saving rate constant instead of gross saving rate, when the economic growth rate changes.

With respect to Piketty's prediction on  $r - g$ , Jones (2015) studies a dynamic general equilibrium model with heterogeneous agents and doubts the importance of  $r - g$  or a wealth tax in wealth inequality. He constructs a continuous-time OLG model with an AK production function, the details of which are in Jones (2014). The stationary wealth distribution is Pareto in his model. Jones represents the wealth inequality measure as a function of the population growth rate,  $r - g$ , and the wealth tax rate. However, he finds that in the general equilibrium where the interest rate is determined endogenously, the inequality measure depends only on the population growth rate. Thus, the gap  $r - g$  and wealth tax are *independent* of the wealth inequality. Moll (2014) considers a similar model and obtains the same conclusions.

In this paper, I argue that the conclusions of Jones and Moll are not robust by using a continuous time OLG model with a neoclassical production function. The set-up is very close to the models advanced by Jones (2015) as well as Blanchard (1985). I first obtain the wealth distribution which is the generalized Pareto distribution. Then, I show that the degree of wealth inequality is positively related to  $r - g$ , and, if the per capita income growth rate  $g$  slows down, the gap widens and the inequality worsens. Finally, I show that a wealth tax is useful in reducing wealth inequality. Therefore, my results support Piketty's claims. I also show that my main results continue to hold if the individuals buy annuities or if the technological progress is endogenously determined.

My model is closely related to Jones (2015) and the difference is only in the curvature of the production function. However, the difference changes the relationship between economic growth and inequality. In the general equilibrium,  $r$  equals the marginal product of capital. In Jones (2015), the production function is linear and then the steady-state growth rate  $g$  equals the marginal product (i.e.,  $r$ ) plus a constant term which depends only on the discount factor and the population growth rate. Thus, the gap  $r - g$  and also wealth inequality is independent of  $g$ . I use the concave production function and show that  $r - g$  is related to  $g$ . Wealth inequality depends on the economic growth only in my model.

My model is also close to Moll (2014), who studies the OLG model with neoclassical production function. In Moll (2014, p. 55), there are two types of individuals, capitalists and workers, and the capitalists have the entire capital. Here, I assume that the representative individuals are born in each period, just like Blanchard (1985). The difference seems to be small, but it crucially changes the relationship between the gap  $r - g$  and  $g$ . In Moll (2014),  $r - g$  is *independent* of  $g$  just as Jones (2015), whereas in my model,  $r - g$  depends negatively on  $g$ . Moll

(2014, p. 55) argues that “in general equilibrium, wealth distribution completely pinned down from demographics.” In my model, wealth distribution depends on both demographics and economic growth (i.e.,  $g$ ). Only my result is consistent with Piketty’s prediction that economic stagnation raises  $r - g$  and makes wealth distribution more unequal.

Although my conclusions are similar to those of Piketty and Zucman (2015), my model crucially differs from theirs on the source of agent heterogeneity. Piketty and Zucman (2015) assume that individuals differ in their preferences toward wealth inheritance, whereas in my model, individuals differ only in their life spans. I show that  $r - g$  affects wealth inequality in the popular Blanchard-type OLG models that are found in many macroeconomics textbooks including Barro and Sala-i-Martin (2004).

This paper is related to some recent literature on wealth inequality in dynamic general equilibrium models. Benhabib et al. (2016) explicitly characterize wealth distribution in an OLG model in which the agents receive idiosyncratic shocks on their investment and age. Nirei and Aoki (2015) investigate a neoclassical growth model in which individuals are subject to idiosyncratic investment shocks and borrowing constraints, and demonstrate that wealth is distributed according to Pareto. However, they do not focus on the relationship between  $r - g$  and inequality.

This paper is organized as follows. Section 2 describes the structure of the model. Section 3 investigates how the degree of inequality is related to  $r - g$ . Section 4 considers a case where the individuals by annuities. Section 5 studies a model with endogenous technological progress. Section 6 concludes.

## 2. THE MODEL

In this section, I provide an overview of my model.

### 2.1. Preferences

Time is continuous. In each period, a continuum of individuals is born. The number of newborns at date  $t$  is  $B_t = B_0 e^{nt}$ , where  $n > 0$  and  $B_0 > 0$ . Death follows the Poisson process with an arrival rate  $d$ . The population of agents born on date  $s$  (henceforth cohort  $s$ ) is  $L_{t,s} = d e^{-d(t-s)} B_s$  in period  $t$ . In accordance with Jones (2014), the total population  $L_t = \int_{-\infty}^t L_{t,s} ds$  evolves according to

$$\dot{L}_t = B_t - dL_t,$$

and in steady state,  $\frac{\dot{L}_t}{L_t} = n$  and  $L_t = \frac{B_t}{n+d}$ .

An agent supplies one unit of labor in the labor market and receives wage income in each period. Cohort  $s$  maximizes the following expected intertemporal utility:

$$U = \int_s^{\infty} e^{-(\rho+d)(t-s)} \ln C_{t,s} dt,$$

subject to the following budget constraint

$$\dot{Z}_{t,s} = (r_t - \tau)Z_{t,s} + W_t - C_{t,s}, \tag{1}$$

where  $\rho$  is the discount factor,  $C_{t,s}$  is the consumption level of cohort  $s$  in period  $t$ ,  $Z_{t,s}$  is the asset holdings of cohort  $s$  in period  $t$ ,  $r_t$  is the real interest rate in period  $t$ ,  $\tau$  is the linear capital income tax, and  $W_t$  is the wage income in period  $t$ . Here, I follow Jones (2015) and assume that the tax revenue is discarded. Let  $Z_t = \int_{-\infty}^t L_{t,s} Z_{t,s} ds$  denote the total wealth at time  $t$ . Individuals equally inherit the assets of the agents who die. Then, the initial asset level of cohort  $s$  is  $Z_{s,s} = \frac{dZ_s}{B_s} = \frac{d}{n+d} \frac{Z_s}{L_s}$ .

Following Piketty and Zucman (2015), let  $\bar{r}_t = r_t - \tau$  denote the rate of return (net-of-tax) on capital. The human wealth is defined as  $\Omega_t = \int_t^\infty e^{-\bar{R}_{x,t}} W_x dx$ , where  $\bar{R}_{x,t} = \int_t^x \bar{r}_z dz$  is the compound interest rate (net-of-tax). It evolves according to

$$\dot{\Omega}_t = \bar{r}_t \Omega_t - W_t.$$

As the utility function is logarithmic, the consumption of cohort  $s$  at time  $t$  is given by

$$C_{t,s} = (\rho + d)(Z_{t,s} + \Omega_t). \tag{2}$$

In the competitive equilibrium, total wealth is equal to total capital, which means that  $Z_t = K_t$ . Therefore, the aggregate consumption at time  $t$ ,  $C_t = \int_{-\infty}^t L_{t,s} C_{t,s} ds$  is

$$C_t = (\rho + d) \int_{-\infty}^t L_{t,s} (Z_{t,s} + \Omega_t) ds = (\rho + d)(K_t + L_t \Omega_t). \tag{3}$$

**2.2. Production**

There are many identical firms. The production function  $F$  has constant returns to scale and is given by  $F(K_t, A_t L_t)$ , where  $K_t$  is the capital,  $A_t$  is the technology level, and  $L_t$  is the labor supply. The growth rate of  $A_t$ ,  $\frac{\dot{A}_t}{A_t}$  is exogenous and equals  $g$ . Let  $k_t = \frac{K_t}{A_t L_t}$  denote the capital per efficiency unit of labor at time  $t$ . The production function per efficiency unit of labor  $f(k) = F(k, 1)$  satisfies  $f(0) = 0$ ,  $f' > 0$ ,  $f'' < 0$ , and  $f'(0) = +\infty$ .

Factor markets are perfectly competitive, and the equilibrium wage rate in period  $t$  is  $W_t = A_t[f(k_t) - k_t f'(k_t)]$  and the capital rental rate in period  $t$  is  $r_t = f'(k_t)$ . As the total tax revenue  $\tau K_t$  is thrown away as in Jones (2015), the resource constraint is

$$\dot{K}_t = F(K_t, A_t L_t) - C_t - \tau K_t. \tag{4}$$

Equation (4) is reexpressed as

$$\dot{k}_t = f(k_t) - c_t - (g + n + \tau)k_t, \tag{5}$$

where  $c_t = \frac{C_t}{A_t L_t}$  denotes the consumption per efficiency unit of labor.

Let  $\omega_t = \frac{\Omega_t}{A_t}$  and  $w_t = \frac{W_t}{A_t}$ . From equation (3), I have  $c_t = (\rho + d)(k_t + \omega_t)$ . As  $\omega_t$  evolves according to  $\dot{\omega}_t = (\bar{r}_t - g)\omega_t - w_t$ , I have

$$\dot{c}_t = (\bar{r}_t - g - \rho - d)c_t - (\rho + d)nk_t. \quad (6)$$

The path of  $(c_t, k_t)$  is determined by equations (5) and (6).

### 2.3. Balanced Growth Path

I now focus on the balanced growth path (BGP), where  $c_t = c$ ,  $k_t = k$ ,  $w_t = w$ , and  $r_t = r$  are all constant, and the growth rate of output per capita,  $A_t f(k)$  is  $g$ . Equations (5) and (6) imply that the stationary allocation  $(c, k)$  is determined by

$$c = \frac{n(\rho + d)k}{\bar{r} - g - \rho - d}, \quad (7)$$

$$c = f(k) - (g + \tau + n)k, \quad (8)$$

where  $\bar{r} = f'(k) - \tau$ . When  $n$  and  $d$  are zero, my model coincides with the Ramsey model, and in the steady state,  $\bar{r} - g$  coincides with the discount factor, as Krusell and Smith (2015) have pointed out.

Here, I focus on the Cobb–Douglas production function  $f(k) = k^\alpha$ , where  $\alpha \in (0, 1)$ . In this case,  $r = \alpha k^{\alpha-1}$ . Let  $c_1(k)$  and  $c_2(k)$  denote the right-hand side of equations (7) and (8), respectively. The two functions are expressed as

$$c_1(k) = n(\rho + d) \frac{k}{\alpha k^{\alpha-1} - g - \tau - \rho - d}, \quad (9)$$

$$c_2(k) = k^\alpha - (g + \tau + n)k. \quad (10)$$

The function  $c_1(k)$  is increasing, convex, and satisfies  $c_1(0) = 0$  and  $\lim_{k \rightarrow k^m} c_1(k) = +\infty$  with  $k^m = (\frac{\alpha}{g+\rho+\tau+d})^{1/(1-\alpha)}$ . Similarly, the function  $c_2(k)$  is concave and satisfies  $c_2(0) = 0$  and  $c_2'(0) = +\infty$ . Thus, the curves  $c = c_1(k)$  and  $c = c_2(k)$  have a unique intersection.

I have the following proposition on  $\bar{r} - g$ .

**PROPOSITION 1.** *Along the BGP,  $\bar{r} - g$  solves the following quadratic equation on  $x$ :*

$$[x + (1 - \alpha)(g + \tau) - \alpha n](x - \rho - d) - \alpha n(\rho + d) = 0. \quad (11)$$

*The gap  $\bar{r} - g$  is a strictly decreasing function of  $g$  and  $\tau$ .*

**Proof.** Equations (9) and (10) imply that

$$(r/\alpha - g - \tau - n)(\bar{r} - g - \rho) = n(\rho + d). \quad (12)$$

Therefore,  $\bar{r} - g$  solves equation (11). In equation (11), if  $g$  and  $x (= \bar{r} - g)$  increase, the left-hand side increases. This is impossible, and  $\frac{dx}{dg} < 0$ . I can argue the same point on  $\tau$ . ■

Proposition 1 shows that when the economic growth rate slows down, the gap  $r - g$  widens, and that the wealth tax reduces the gap. This is consistent with Piketty’s prediction.

### 3. WEALTH DISTRIBUTION AND $r - g$

Here, investigate the wealth distribution along the BGP. The investigation of the equilibrium path out of the steady state is not easy, because along the transition paths, the interest rate is not constant and wealth distribution is not analytically tractable. Therefore, I focus on the BGP. From equations (1) and (2), the assets of cohort  $s$  evolve according to

$$\dot{Z}_{t,s} = (\bar{r}_t - \rho - d)Z_{t,s} + W_t - (\rho + d)\Omega_t.$$

If  $z_{t,s} = Z_{t,s}/A_t$ , then  $z_{s,s} = \frac{d}{n+d}k_s$  and

$$\dot{z}_{t,s} = (\bar{r} - g - \rho - d)z_{t,s} + w_t - (\rho + d)\omega_t.$$

Along the BGP, where  $k$ ,  $w$ , and  $\omega$  are constant and  $(\bar{r} - g)\omega = w$ , I have

$$z_{t,s} = e^{(\bar{r}-g-\rho-d)(t-s)} \left( \omega + \frac{d}{n+d}k \right) - \omega.$$

At time  $t$ , the relative population of cohort  $s$  is  $\frac{L_{t,s}}{L_t} = (d+n)e^{-(d+n)(t-s)}$ . Therefore,

$$\Pr(Z_{t,s} \geq x) = \left[ \frac{\omega + x/A_t}{\omega + dk/(n+d)} \right]^{-\frac{d+n}{\bar{r}-g-\rho-d}}. \tag{13}$$

The individual wealth is distributed according to the generalized Pareto distribution, and the Pareto inequality measure  $\eta$  which is the inverse of the exponent in equation (13) is

$$\eta = \frac{\bar{r} - g - \rho - d}{d + n}. \tag{14}$$

This equation is the same as equation (13) in Jones (2014). If  $\bar{r} - g$  widens, so inequality definitely worsens. As shown in Proposition 1, the reduction of  $g$  increases  $\bar{r} - g$ , and subsequently raises  $\eta$ . Similarly, the increase of  $\tau$  reduces the gap, which in turn reduces  $\eta$ . Thus, I have the following proposition.

**PROPOSITION 2.** *A slowdown in the per capita income growth rate raises the inequality measure  $\eta$ , and wealth tax reduces  $\eta$ .*

Figure 1 shows a negative relationship between the wealth inequality and  $g$  when  $\alpha = 0.3$ ,  $\rho = 0.05$ ,  $n = 0.03$ , and  $d = 0.05$ . The parameters for  $\alpha$ ,  $\rho$ , and

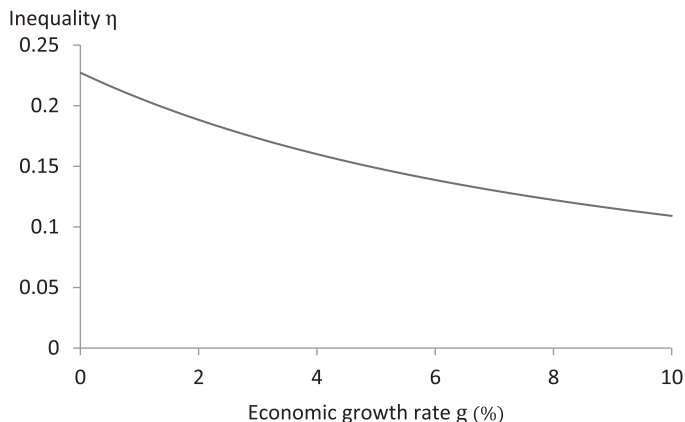


FIGURE 1. Inequality and growth.

$d$  are from Mankiw and Weinzierl (2006). As the economic growth rate declines from 10% to zero, the Pareto inequality measure almost doubles from 0.12 to 0.22.

My result differs from Jones (2015), who demonstrated that the inequality index is independent of  $r - g$  and  $\tau$ . The production function he uses is an  $AK$  type, with the wage income and human wealth both equal to zero. In this case, equation (3) becomes  $C = (\rho + d)K$ , and equation (4) is expressed as  $\dot{K} = (A - \rho - d - \tau)K$ . Thus, the per capita income growth rate is  $A - \rho - d - \tau - n \equiv \hat{g}$ . The interest rate is  $r = A$ , and the inequality measure is  $\eta = \frac{\bar{r} - \hat{g} - \rho - d}{d + n} = \frac{n}{d + n}$ , which is unrelated to  $r - g$  and  $\tau$ . As this paper illustrates, Jones' conclusion crucially depends on the linearity of the production function.

#### 4. MODEL WITH ANNUITIES

In the preceding section, I assumed that the wealth of the people is redistributed to the newborns when they die. Here, I consider a model where the individuals purchase annuities as in Blanchard (1985), and show that my result on the relevance of  $\bar{r} - g$  on inequality continues to hold. I follow Blanchard (1985) and assume that annuity markets are competitive and the insurance company pays to the individual with financial wealth  $Z$  by  $dZ$  units. The budget constraint of the agent born at date  $s$  is slightly different from the previous one and the rate of interest on asset is now  $\bar{r} + d$ :

$$\dot{Z}_{t,s} = (\bar{r}_t + d)Z_{t,s} + W_t - C_{t,s}. \tag{15}$$

The initial asset level  $Z_{s,s}$  is zero. The human wealth evolves according to  $\dot{\Omega}_t = (\bar{r}_t + d)\Omega_t - W_t$  and then the term  $\omega_t = \Omega_t/A_t$  satisfies  $\dot{\omega}_t = (\bar{r}_t + d - g)\omega_t - w_t$ . The individual consumption function (2), the aggregate consumption (3), and the resource constraint (5) are the same as before. Thus, I have

$$\dot{c}_t = (\bar{r}_t - g - \rho)c_t - (\rho + d)(n + d)k_t. \tag{16}$$

The path of  $(c_t, k_t)$  is determined by equations (5) and (16). Substitution of equation (2) into equation (15) yields  $\dot{Z}_{t,s} = (\bar{r}_t - \rho)Z_{t,s} + W_t - (\rho + d)\Omega_t$ . Thus, the term  $z_{t,s} = Z_{t,s}/A_t$  evolves according to

$$\dot{z}_{t,s} = (\bar{r}_t - g - \rho)z_{t,s} + w_t - (\rho + d)\omega_t.$$

Along the BGP,  $\dot{\omega} = 0$  and then  $(\bar{r} + d - g)\omega = w$ . Therefore,  $z_{t,s} = (e^{(\bar{r}-g-\rho)(t-s)} - 1)\omega$ , and I get

$$\Pr(Z_{t,s} \geq x) = \left(1 + \frac{x}{\omega e^{g t}}\right)^{-\frac{d}{\bar{r}-g-\rho}}.$$

The Pareto inequality measure is  $\eta^* = \frac{\bar{r}-g-\rho}{d}$ , which is very close to the previous one  $\eta = \frac{\bar{r}-g-\rho-d}{d+n}$ . I have the following proposition.

**PROPOSITION 3.** *The slowdown in the economic growth rate worsens wealth inequality if the individuals buy annuities.*

**Proof.** In the steady state,  $c = f(k) - (g + n + \tau)k = (n + d)(\rho + d)\frac{k}{\bar{r}-g-\rho}$ . As  $f(k) = k^\alpha$ ,  $f(k)/k = r/\alpha$  and the gap  $x = \bar{r} - g$  solves

$$[x + (1 - \alpha)(g + \tau) - \alpha n](x - \rho) = \alpha(n + d)(\rho + d). \tag{17}$$

The left-hand side is an increasing function of  $g$  and  $x$ , whereas the right-hand side is independent of  $g$  and  $x$ . Thus,  $dx/dg < 0$ . The inequality measure  $\eta^* = (x - \rho)/d$  is an increasing function of  $x$  and then  $d\eta^*/dg < 0$ . ■

### 5. ENDOGENOUS TECHNOLOGICAL PROGRESS

In the preceding section, the growth rate  $g$  was exogenous. In this section, I endogenize the process of technological progress by assuming that the technology level depends positively on the level of the aggregate capital. Specifically, I assume that the level of technology  $A_t$  is expressed as

$$A_t = (\bar{K}_t)^\theta,$$

where  $\bar{K}_t$  is the economy-wide stock of capital, and  $\theta \in (0, 1)$  is the capital intensity in the technological progress. A similar assumption is found in García-Peñalosa and Turnovsky (2006). Firms take  $\bar{K}_t$  as exogenous. Along the equilibrium path,  $\bar{K}_t = K_t$  and then  $g_t = \frac{\dot{A}_t}{A_t} = \theta \frac{\dot{K}_t}{K_t}$  and  $k_t = \frac{K_t^{1-\theta}}{L_t}$ . As before,  $\frac{\dot{L}_t}{L_t} = n$  and therefore the variables  $k_t$ ,  $c_t$ , and  $g_t$  evolve according to

$$\dot{k}_t = f(k_t) - c_t - (g_t + n + \tau)k_t, \tag{18}$$

$$\dot{c}_t = [f'(k_t) - g_t - \tau - \rho - d]c_t - (\rho + d)nk_t, \tag{19}$$

$$g_t = \frac{\theta}{1 - \theta} \left( \frac{\dot{k}_t}{k_t} + n \right). \tag{20}$$



Equations (18) and (19) are almost the same as equations (5) and (6), and the only difference is that here the growth rate of technology  $g_t$  varies with time. Along the BGP,  $k_t$  and  $c_t$  are constant, and the balanced growth rate  $g$  equals

$$g = \frac{\theta n}{1 - \theta}.$$

Economic growth rate  $g$  is positively related to the capital intensity parameter  $\theta$ . An increase in  $\theta$  raises  $g$  without affecting any other parameters. I have the following proposition.

**PROPOSITION 4.** *A reduction in the capital intensity parameter  $\theta$  reduces economic growth rate  $g$ , raises the gap  $r - g$  and worsens wealth inequality.*

My main result between economic growth and inequality is unchanged even when I endogenize the technological progress.

## 6. CONCLUSION

In this paper, I investigate a continuous-time OLG model with capital accumulation in which agents are subject to the constant death probability, and demonstrate that the gap  $r - g$  and wealth tax are closely related to wealth inequality. All of these results are consistent with the famous predictions advanced by Thomas Piketty in *Capital in the Twenty-First Century* (2014). In my model, the individual does not have a bequest motive for simplification. Benhabib and Bisin (2007) study the bequest in a continuous-time OLG model and find that the stationary wealth distribution is Pareto. However, their model does not have capital. As a future study, I would like to incorporate a bequest motive into my model with endogenous capital accumulation. My model also ignores the heterogeneity in the rate of return. I would like to consider the idiosyncratic risk on the rate of return on capital, just as Benhabib et al. (2011) do in their finite-horizons OLG model.

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