

B.A.J. **8**, I, 183-194 (2002)

PAPERS FROM ACTUARIAL JOURNALS WORLDWIDE

Single copies of all the papers listed here can be obtained, subject to charge and copyright regulations, from the actuarial profession's libraries. Issues may be borrowed by members.
Tel: 0131 240 1311 or 01865 268206/208; libraries@actuaries.org.uk

ASTIN BULLETIN

Volume **31** (2), 2001

BACINELLO, A. R. *Fair pricing of life insurance participating policies with a minimum interest rate guaranteed.* 275-297. In this paper we analyse, in a contingent-claims framework, one of the most common life insurance policies sold in Italy during the last two decades. The policy, of the endowment type, is initially priced as a standard one, given a mortality table and a technical interest rate. Subsequently, at the end of each policy year, the insurance company grants a bonus, which is credited to the mathematical reserve and depends on the performance of a special investment portfolio. More precisely, this bonus is determined in such a way that the total interest rate credited to the insurance equals a given percentage (participating level) of the annual return on the reference portfolio and anyway does not fall below the technical rate (minimum interest rate guaranteed, henceforth). Moreover, if the contract is paid by periodical premiums, it is usually stated that the annual premium is adjusted at the same rate of the bonus, and thus the benefit is also adjusted in the same measure. In such policy the variables controlled by the insurance company (control-variables, henceforth) are the technical rate, the participation level and, in some sense, the riskiness of the reference portfolio measured by its volatility. However, as it is intuitive, not all sets of values for these variables give rise to a fair contract, ie to a contract priced consistently with the usual assumptions on financial markets and, in particular, with no-arbitrage. We derive then necessary and sufficient conditions under which each control-variable is determined by a fair pricing of the contract, given the remaining two.

BAMS, D. & WIELHOUWER, J. L. *Empirical issues in value-at-risk.* 299-315. For the purpose of value-at-risk (VaR) analysis, a model for the return distribution is important because it describes the potential behaviour of financial security in the future. What is important primarily, is the behaviour in the tail of the distribution since VaR analysis deals with extreme market situations. We analyse the extension of the normal distribution function to allow for fatter tails and for time-varying volatility. Equally important to the distribution function are the associated parameter values. We argue that parameter uncertainty leads to uncertainty in the reported VaR estimates. There is a trade-off between more complex tail-behaviour and this uncertainty. The 'best estimate'-VaR should be adjusted to take account of the uncertainty in the VaR. Finally, we consider the VaR forecast for a portfolio of securities. We propose a method to treat the modelling in a univariate, rather than a multivariate, framework. Such a choice allows us to reduce parameter uncertainty and to model directly the relevant variable.

PINQUET, J., GUILLÉN, M. & BOLANCÉ, C. *Allowance for the age of claims in bonus-malus systems.* 337-348. The purpose of the paper is to use the age of claims in the prediction of risks. A dynamic random effects model on longitudinal count data is presented, and estimated on the portfolio of a major Spanish insurance company. The estimated autocorrelation coefficients of stationary random effects are decreasing. A consequence is that the predictive ability of a claim decreases with the lag between the period of risk prediction and the period of occurrence. There is a wide gap between the long term properties of actuarial and real-world experience rating schemes. This gap can be partly filled if the age of claims is taken into account in the actuarial model.

REINHARD, J.-M. & SNOUSSI, M. D. *On the distribution of the surplus prior to ruin in a discrete semi-Markov risk model.* 255-273. In this paper we extend the work of Reinhard and Snoussi (2000) by developing a recursive system for finding the distribution of the surplus prior to ruin in a discrete semi-Markov risk model.

WAGNER, C. *A note on ruin in a two state Markov model.* 349-358. We are dealing with the ruin probability and the expected ruin time in a two state Markov model where the premium is the reciprocal of an integer and the initial surplus is a multiple of the premium.

WALHIN, J. F. & PARIS, J. *The practical replacement of a bonus-malus system.* 317-335. In this paper we will show how to set up a practical bonus-malus system with a finite number of classes. We will use the actual claim amount and claims frequency distributions in order to predict the future observed claims frequency when the new bonus-malus system will be in use. The future observed claims frequency is used to set up an optimal bonus-malus system as well as the transient and stationary distributions of the drivers in the new bonus-malus system. When the number of classes as well as the transition rules of the new bonus-malus system have been adopted, the premium levels are obtained by minimising a certain distance between the levels of the practical bonus-malus system and the corresponding optimal bonus-malus system. Some iterations are necessary in order to reach stabilization of the future observed claims frequency and the levels of the practical bonus-malus system.

ASTIN papers are available for downloading, at no charge, from <http://www.casact.org/library/astin/index.htm> although there will be a delay of at least one year from date of publication.

Members of ASTIN and AFIR receive ASTIN Bulletin free of charge. Members of the Faculty of Actuaries and Institute of Actuaries can join ASTIN and AFIR by contacting Georgina Ivers (georgini@actuaries.org.uk) at the Institute of Actuaries.

Libraries, insurers, and individuals who are not members of the Faculty or Institute can subscribe to ASTIN Bulletin by contacting the publisher, Peeters, Journals Department, Bondgenotenlaan 153, B-3000 Leuven, Belgium. E-mail: peeters@peeters-leuven.be

AUSTRALIAN ACTUARIAL JOURNAL

Volume 7 (4), 2002

DAYANANDA, P. W. A. *Assessing the compensation for corporation executives.* 811-818.
Keywords: Executive stock options, executive award policies, Brownian motion, index stock, restriction or hurdles, valuation.

DUFRESNE, D. *A general class of risk models.* 755-791. We consider the actuarial risk model when the waiting times or claims have a Laplace transform which is a rational function. This generalises the classical model, where the waiting times are exponential, and gives more flexibility in the modelling of a risk business. Ruin is seen as a random walk crossing a barrier; the summands of the random walk are expressed as the difference of the waiting time and the claim. The class \mathcal{R}^f of distributions which have finite rational Laplace transforms includes the so-called phase-type distributions. For waiting times in \mathcal{R}^f , the Laplace transform of the ruin probability is obtained explicitly; if the claims are in \mathcal{R}^f , then the probability of ruin is a combination of exponentials times polynomials, which can be found in closed-form.

FITZHERBERT, R. *Volatility, beta and return: was there ever a meaningful relationship?* 681-754.
This paper deals with the relationship between volatility or, more correctly, β values and return within the equities asset class. The existence of such a relationship has been controversial in

actuarial circles, but the assumption that a positive linear relationship exists is now taught as part of the curriculum.

Most of the empirical studies of the relationship between β values and mean return have used arithmetic means of 'discrete' rates of return. By contrast, many of the apparently contradictory studies used continuous compounding, geometric means or their equivalent. A great deal of the controversy therefore hinges on the definition of mean return, which has not really been identified as an important issue.

If the appropriate definition of 'mean return' for long term investment or asset modelling is mean continuously compounded return or its equivalent, then much of the empirical support for a positive relationship between β values and return may need to be re-evaluated.

INSTITUTE OF ACTUARIES OF AUSTRALIA. *Proposed review of the Life Act 1995: Submission, October 2001: Submission, November 2001.* 795-803, 805-808. In October 2001 the IAAust was invited to submit comments to the Australian Prudential Regulation Authority on the proposed review of the Life Act 1995. In November 2001, the IAAust submitted further comments to the Australian Prudential Regulation Authority on the proposed review of the Life Act 1995, following the submission in October 2001.

Reproduced with the permission of The Institute of Actuaries of Australia.

Subscription details available from: IAA, Level 7 Challis House, 4 Martin Place, Sydney, NSW 2000, Australia. E-mail: insact@actuaries.asn.au

GENEVA PAPERS ON RISK AND INSURANCE THEORY

Volume 26 (2), 2001

DIONNE, G. & INGABIRE, M.-G. *Diffidence theorem, state-dependent preferences, and DARA.* 139-154. C. Gollier (The Economics of risk and time, MIT Press 2001) has developed a standard technique based on the diffidence theorem. This theorem provides a very simple instrument to solve relatively sophisticated problems when preferences are state-independent. The object of this article is to show that the theorem is also very useful to derive significant results with state-dependent preferences. Using the reference set notion and an extension of the diffidence theorem, we establish formally necessary and sufficient conditions on the reference set, in order to obtain prudence and decreasing absolute risk aversion. Examples of DARA utility functions compatible with non-linear reference sets are presented in the appendix.

ERMOLIEV, Y. M. & FLÂM, S. D. *Finding Pareto optimal insurance contracts.* 155-167. This paper deals with on-line computation — or step-wise learning — of Pareto optimal insurance contracts. Our approach tolerates that the loss distribution might be unknown, intractable, or not well-specified. Thus we accommodate fairly inexperienced parties. Losses are here simulated or observed, one at a time, and they cause iterated revisions of the premium. The mechanical and global nature of probability calculus thereby yields to more tentative, myopic procedures, possibly closer to how humans operate or reason in face of risk. Sequential revisions may also reduce the expense of insurers' time and money in seeking sufficient statistics. Emphasized ... is the remarkable simplicity and stability of the resulting adaptive procedures. Special attention goes to catastrophic risks, and to subsidized or competitive insurance.

GURIEV, S. *On microfoundations of the dual theory of choice.* 117-137. We show that Yaari's dual theory of choice under risk may be derived as an indirect utility when a risk-neutral agent faces financial imperfections. We consider an agent that maximizes expected discounted cash

flows under a bid-ask spread in the credit market. It turns out that the agent evaluates lotteries as if she were maximizing Yaari's dual utility function. We also generalize the dual theory of choice for unbounded lotteries.

LUCIANO, E. & KAST, R. *A value at risk approach to background risk*. 91-115. This paper studies the effects of an uninsurable background risk (BR) on the demand for insurance (proportional and with deductible). We study both the case of BR uncorrelated with the insurable one and the perfectly correlated one, in a Gaussian world. In order to perform our study, we exploit the new risk measure known as Value at Risk (VaR) and consider insurance contracts which are Mean-VaR efficient. We obtain results which depend on the parameters (moments) of both risks and on the magnitude of loadings charged by the insurance company, instead of depending on the risk attitudes of the insured, such as risk aversion and prudence. We demonstrate that, if loadings are not too high, the demand for insurance increases with positively correlated BR; it decreases with BR negatively correlated if the latter is less risky than the insurable one (in this case it can even go to zero, if loadings are too high); it goes to zero with BR which is negatively correlated and more risky than the insurable one.

Reproduced with the permission of Kluwer Academic Publishers.

Subscription details available from: Kluwer Academic Publishers, PO Box 322, 3300 AH Dordrecht, The Netherlands

INSURANCE: MATHEMATICS & ECONOMICS

Volume 30 (1), 2002

BOUTSIKAS, M. V. & KOUTRAS, M. V. *Modeling claim exceedances over thresholds*. 67-83. In this paper, we consider a simple risk model and study the occurrences of clusters of threshold exceedances by the individual claims. The statistic used to study the model is the discrete multiple scan statistic. A compound Poisson approximation is established and certain asymptotic results are obtained for both the risk model and a similar in nature financial problem. Finally, we review two typical examples from areas of applied science where the outcomes of this paper may have beneficial impact.

CENTENO, M. DE L. *Measuring the effects of reinsurance by the adjustment coefficient in the Sparre Anderson model*. 37-49. We study the insurer's adjustment coefficient as a function of retention levels for combinations of quota-share with excess of loss reinsurance in the Sparre Anderson (1957) model. We show that the insurer's adjustment coefficient is a unimodal function of the retention levels when the quota-share reinsurance premium is calculated on original terms and when the excess of loss premium is calculated according to the expected value principle.

DENUIT, M., LEFÈVRE, C. & UTEV, S. *Measuring the impact of dependence between claims occurrences*. 1-19. The purpose of this paper is to provide a quantitative measure of the impact of a possible dependence between the occurrences of claims in an individual risk model. Firstly, probabilistic distances, of stop-loss or total variation types, specific to arithmetic random variables are introduced and studied, especially in connection with related probabilistic orderings. Then, these results are applied to derive effective bounds for the distance between the total number of claims in the original model and under a standard independence assumption.

HÜRLIMANN, W. *On the accumulated aggregate surplus of a life portfolio*. 27-35. The present paper considers a simple stochastic model for a multi-period (non-profit) life portfolio with

two sources of uncertainty, the returns on investments and the claims. Within this model the bonus of a portfolio is determined numerically using the concept of stable reserve associated to a financial gain. The accumulated aggregate surplus of the portfolio after a finite number of insurance periods is expressed as the difference between the accumulated value on investments and the accumulated actuarial reserves and aggregate claims. It is shown that replacing the dependence between the aggregate claims of successive periods by an assumption of independence increases the riskiness in stop-loss order of the accumulated aggregate surplus. This results in higher stable reserves under the independence assumption. It is shown that the stable reserve can be determined numerically by solving an implicit expected value equation, which involves Black-Scholes formula for the stop-loss premiums on the accumulated return on investments and De Pril's two-stage recursive formulas for the distribution of the accumulated aggregate claims. The obtained results are illustrated numerically at a portfolio of endowment insurance policies.

PROMISLOW, S. D. & YOUNG, V. R. *Measurement of relative inequity and Yaari's dual theory of risk*. 95-109. We apply Yaari's dual theory of risk, a type of non-expected utility theory, to the problem of measuring the inequity of a premium relative to the (unknown) net premium. This work parallels that of Promislow and Young [SAJ, 2000]. In the latter work, however, we apply expected utility to the problem of measuring relative inequity. This paper generalizes Yaari's risk measure, itself a generalization of a well-known measure of inequity called the Gini coefficient.

SCHMIDT, K. D. *A note on the overdispersed Poisson family*. 21-25. In the Poisson model for loss reserving it is assumed that the incremental claims are independent and Poisson distributed with expectations being the product of two factors, depending on the occurrence year and the development year, respectively. It is well-known that maximum-likelihood estimation in the Poisson model yields the chain-ladder estimators of the expected ultimate aggregate claims. Recently, this result has been extended to overdispersed Poisson models. In the present paper, it is shown that every overdispersed Poisson model can be transformed into the Poisson model by rescaling all incremental claims by a common factor.

TSAI, C. C.-L. & WILLMOT, G. E. *A generalized defective renewal equation for the surplus process perturbed by diffusion*. 51-66. In this paper, we consider the surplus process of the classical continuous time risk model containing an independent diffusion (Wiener) process. We generalize the defective renewal equation for the expected discounted function of a penalty at the time of ruin in Garber and Landry [IME 22 (1998)]. Then an asymptotic formula for the expected discounted penalty function is proposed. In addition, the associated claim size distribution is studied, and reliability-based class implications for the distribution are given.

WANG, N., PANG, W. K. & HUANG, W. K. *A discussion on Buhlmann's criterion for asset valuation*. 85-93. In this paper, we consider the criterion of Buhlmann [NAAJ 1 (1997)] for asset valuation. The limiting behavior of the trend of the valuation compared with the real development is studied for both i.i.d. growth rates and Markov dependent growth rates. The average sojourn time within Buhlmann's band is assessed for independent growth rates by the technique of Wald's sequential analysis. A continuous asset model discussed in [IME 24 (1999)] is also studied with this criterion. From this continuous model, we will see some interesting links of this topic with ruin theory.

Reproduced with the permission of Elsevier Science.

Subscription details available from: Elsevier Science, PO Box 311, 1000 AE Amsterdam, The Netherlands. E-mail: nlinfo-f@elsevier.nl

- DOHERTY, N. A. & SCHLESINGER, H. *Insurance contracts and securitization*. 45-62. High correlations between risks can increase required insurer capital and/or reduce the availability of insurance. For such insurance lines, securitization is rapidly emerging as an alternative form of risk transfer. The ultimate success of securitization products depends on the ability of securitization to facilitate and/or be facilitated by insurance contracts. The authors consider how insured losses might be decomposed into separate components, one of which is a type of 'systemic risk' that is highly correlated among insureds. Such a correlated component might conceivably be hedged directly by individuals but is more likely to be hedged by the insurer. The authors examine how insurance contracts may be designed to allow the insured a mechanism to retain all or part of the systemic component. Examples are provided that illustrate this methodology in several types of insurance markets subject to systemic risk.
- DOHERTY, N. A. & RICHTER, A. *Moral hazard, basis risk, and gap insurance*. 9-24. The article addresses the trade-off between moral hazard and a basis risk. A decision maker, eg a primary insurer, is considered who can purchase an index hedge and a (re)insurance contract that covers the gap between actual losses and the index-linked payout, or part of this gap. The results suggest that combining insurance with an index hedge may extend the possibility set and by that means lead to efficiency gains. Naturally the results depend heavily on the transaction costs associated with both instruments. In particular, the authors show that if the index product is without transaction costs, at least some index-linked coverage is always purchased, so long as there is positive correlation between the index and the actual losses. So under these circumstances, there is in any case a benefit from the availability of index products. Furthermore, it is shown that the index hedge would always be supplemented by a positive amount of gap insurance.
- GROSEN, A. & JØRGENSEN, P. L. *Life insurance liabilities at market value: an analysis of insolvency risk, bonus policy, and regulatory intervention rules in a barrier option framework*. 63-91. This article takes a contingent claim approach to the market valuation of equity and liabilities in life insurance companies. A model is presented that explicitly takes into account the following: (i) the holders of life insurance contracts (LICs) have the first claim on the company's assets, whereas equity holders have limited liability; (ii) interest rate guarantees are common elements of LICs; and (iii) LICs according to the so-called contribution principle are entitled to receive a fair share of any investment surplus. Furthermore, a regulatory mechanism in the form of an intervention rule is built into the model. This mechanism is shown to significantly reduce the insolvency risk of the issued contracts, and it implies that the various claims on the company's assets become more exotic and obtain barrier option properties. Closed valuation formulas are nevertheless derived. Finally, some representative numerical examples illustrate how the model can be used to establish the set of initially fair contracts and to determine the market values of contracts after their inception.
- LEE, J.-P. & YU, M.-T. *Pricing default-risky CAT bonds with a moral hazard and basis risk*. 25-44. This article develops a contingent claim model to price a default-risky, catastrophe-linked bond. This model incorporates stochastic interest rates and more generic loss processes and allows for practical considerations of moral hazard, basis risk, and default risk. The authors compute default-free and default-risky CAT bond prices by using the Monte Carlo method. The results show that both moral hazard and basis risk drive down the bond prices substantially; these effects should not be ignored in pricing the CAT bonds. The authors also show how the bond prices are related to catastrophe occurrence intensity, loss volatility, trigger level, the issuing firm's capital position, debt structure, and interest rate uncertainty.

PEKÖZ, E. A. *Samuelson's fallacy of large numbers and optional stopping*. 1-7. Accepting a sequence of independent positive mean bets that are individually unacceptable is what Samuelson called a fallacy of large numbers. Recently, utility functions were characterized where this occurs rationally, and examples were given of utility functions where any finite number of good bets should never be accepted. Here the author shows how things change if you are allowed the option to quit early. Subject to some mild conditions, you should essentially always accept a sufficiently long finite sequence of good bets. Interestingly, the strategy of quitting when you get ahead does not perform well, but quitting when you get behind does. This sheds some light on more possible behavioral reasons for Samuelson's fallacy, as well as strategies for handling a series of sequentially observed good investments.

Reproduced with the permission of the American Risk and Insurance Association.

Subscription details available from: the American Institute for CPCU, 720 Providence Road, Malvern, PA 19355, USA. E-mail: aria@cpcuia.org

NORTH AMERICAN ACTUARIAL JOURNAL

Volume 5 (2), 2001

BROWN, R. L., DAMM, R. & SHARARA, I. *A macro-economic indicator of age at retirement*.

1-10. This paper explores the relationship between the wealth transfer index (WTI), a statistic defined by Brown and Bilodeau (1997), and retirement age, which is the age at which the workers in an economy cease to be economically productive. Appropriately expressed as a ratio of consumption demand to labour productivity, WTI is a barometer for the demand for wealth placed on the workers of an economy. This paper explains why a relationship between this statistic and retirement age must exist. Using Canadian historical median retirement age data compiled by Statistics Canada and calculated values of the WTI for the same period, three linear regression models are fitted. The conclusion from these models indicates that there is a strong positive correlation between the WTI and average retirement age. This paper also briefly looks at the well-documented shift expected to occur in Canada because of the baby boom–baby bust tidal wave. The aged dependency ratio is expected to increase dramatically, reaching 45% in 2036. A practical application of the WTI model suggests that the baby boom cohort may experience a rise in the normal retirement age in the period 2017–34. They will, in effect, be forced to retire at ages that will allow an 'acceptable' transfer of wealth from the worker to dependent Canadians. Using one of the fitted linear regression models, and projected values of the WTI, the paper concludes by projecting the median retirement age to 2041 for Canadian workers.

BROWNE, M. J., CARSON, J. M. & HOYT, R. E. *Dynamic financial models of life insurers*.

11-26. The Society of Actuaries seeks to provide actuaries of life insurance companies with a systematic approach for estimating the adverse effects of economic developments that could impede insurer performance. Toward that end, this study combines market and economic factors with insurer-specific data to form dynamic financial models of life insurers. Empirical analysis is based on annual data from 1985 through 1995 for 1,593 life insurers. By identifying important exogenous and insurer-specific factors related to life insurer performance, this study provides a basis for actuaries to build dynamic financial models for individual insurers. The study also identifies and describes several web sites that provide access to relevant economic and financial data.

CARRIÈRE, J. F. & HILL, C. F. *Analysis of incremental returns of Canadian mutual funds*. 27-40.

This paper presents some stochastic models of mutual fund returns to explain the risks associated with the net incremental return over a benchmark due to active investment

- management practices. This model can describe the stochastic behaviour of the returns on established funds, but, more importantly, it also models the uncertainty or risk associated with new funds or funds with no track record. Using Canadian data, the paper also shows how to estimate the parameters of the model and check the model assumptions.
- HARDY, M. R. *A regime-switching model of long-term stock returns*. 41-53. In this paper I first define the regime-switching lognormal model. Monthly data from the Standard and Poor's 500 and the Toronto Stock Exchange 300 indices are used to fit the model parameters, using maximum likelihood estimation. The fit of the regime-switching model to the data is compared with other common econometric models, including the generalised autoregressive conditionally heteroskedastic model. The distribution function of the regime-switching model is derived. Prices of European options using the regime-switching model are derived and implied volatilities explored. Finally, an example of the application of the model to maturity guarantees under equity-linked insurance is presented. Equations for quantile and conditional tail expectation (tail-VaR) risk measures are derived, and a numerical example compares the regime-switching lognormal model results with those using traditional lognormal stock return model.
- MACDONALD, A. S. & PRITCHARD, D. J. *Genetics, Alzheimer's disease, and long-term care insurance*. 54-78. This paper applies a model of Alzheimer's disease (AD) developed by Macdonald and Pritchard (2000) to the question of the potential for adverse selection in long-term care (LTC) insurance introduced by the existence of DNA tests for variants of the ApoE gene, the $\epsilon 4$ allele of which is known to predispose one to earlier onset of AD. It computes the expected present values (EPVs) of model LTC benefits with respect to AD for each of five ApoE genotypes, weighted average EPVs with and without adverse selection, and sample underwriting ratings. The paper concludes that adverse selection could increase costs significantly in a small LTC insurance market only if current population genetic risk is not much smaller than that observed in case-based studies, and if carriers of the $\epsilon 4$ allele are very much more likely to buy LTC insurance. Finally, the paper considers the cost of a combined retirement package, providing both pension and LTC insurance, and shows that it can reduce adverse selection.
- MØLLER, T. *Hedging equity-linked life insurance contracts*. 79-95. This paper examines a portfolio of equity-linked life insurance contracts and determines risk-minimizing hedging strategies within a discrete-time setup. As a principal example, I consider the Cox-Ross-Rubinstein model and an equity-linked pure endowment contract under which the policyholder receives $\max(S_T, K)$ at time T if he or she is then alive, where S_T is the value of a stock index at the term T of the contract and K is a guarantee stipulated by the contract. In contrast to most of the existing literature, I view the contracts as contingent claims in an incomplete model and discuss the problem of choosing an optimality criterion for hedging strategies. The subsequent analysis leads to a comparison of the risk (measured by the variance of the insurer's loss) inherent in equity-linked contracts in the two situations where the insurer applies the risk-minimizing strategy and the insurer does not hedge. The paper includes numerical results that can be used to quantify the effect of hedging and describe how this effect varies with the size of the insurance portfolio and assumptions concerning the mortality.
- SCOLLNIK, D. P. M. *Actuarial modeling with MCMC and BUGS*. 96-125. In this paper, the author reviews some aspects of Bayesian data analysis and discusses how a variety of actuarial models can be implemented and analysed in accordance with the Bayesian paradigm using Markov chain Monte Carlo techniques via the BUGS (Bayesian inference using Gibbs sampling) suite of software packages. The emphasis is placed on actuarial loss models, but other applications are references, and directions are given for obtaining documentation for additional worked examples on the world wide web.

VINSONHALER, C., RAVISHANKER, N., VADIVELLOO, J. & RASOANAIVO, G. *Multivariate analysis of pension plan mortality data*. 126-138. This paper uses the logistic regression model to examine private pension plan data for 1989-95 collected by the Retirement Plans Experience Committee of the Society of Actuaries. When only one explanatory variable, such as annuity class size, is used in modelling mortality rates, the model provides a reasonable fit to the data. Multiple explanatory variables give less satisfactory results.

NORTH AMERICAN ACTUARIAL JOURNAL

Volume 5 (3), 2001

BOYLE, P. P., KOLKIEWICZ, A. W. & TAN, K. S. *Valuation of the reset options embedded in some equity-linked insurance*. 1-18. This paper proposes a method for valuing American options using a Monte Carlo simulation approach. Our approach can be used to price the reset feature found in some equity-linked insurance contracts. We model this feature as a multiple shout option and give examples based on certain equity-linked insurance products that are very popular in Canada. These contracts are known as segregated fund contracts and the valuation of the embedded options in these contracts has posed serious challenges for actuaries. One of the advantages of the Monte Carlo approach in this connection is that it can be extended to handle different investment assumptions as well as multiple assets. We show how to modify the stochastic mesh model of Broadie and Glasserman (1997b) to incorporate quasi-Monte Carlo in the simulation and thus improve the efficiency. We benchmark the efficiency gains in our method using standard American options and multiple shout options.

BROWN, R. L. & LIU, J. *The shift to defined contribution pension plans: why did it not happen in Canada?* 65-77. There has been a strong shift away from defined benefit (DB) pension plans toward defined contribution (DC) pension plans in the United States over the last 20 years. A variety of reasons for this shift have been proposed. In another paper in this issue, Krzysztof Ostaszewski presents a new hypothesis to explain the shift to DC plans in the United States. He argues that the decline in importance of DB plans is due to a shift in the way relative returns to macroeconomic factors of production, that is, capital and labour, are being rewarded in the national economy. This paper attempts to test the Ostaszewski hypothesis using Canadian data. In Canada there has been only a slight decrease in DB plan coverage. It is shown that the Ostaszewski theory does not fit the Canadian experience well. Instead, it is argued that pension regulation and tax legislation play a crucial role in pension design and reform. It is also argued that the difference in pension regulation and taxation in Canada versus the United States has directly influenced plan sponsors in considering their pension objectives, costs, and risks. Differences in the proportion of the workforce that is unionised may also be important. The paper concludes that pension regulation and taxation are more important variables than are macroeconomic reward systems in the use of DB versus DC pension plans.

CARRIÈRE, J. F. *A Gaussian process of yield rates calibrated with strips*. 19-30. This paper presents a Gaussian multivariate factor model of the term structure of interest rates. It shows that there exists a martingale valuation law of the factors so that the price function of a zero-coupon bond is an exponential spline. The model's linear and Gaussian structure yields a simple model where estimation and calibration are relatively easy to do. Using yield data on stripped bonds, the spline model gives a very good approximation of the yield curve at all times. Moreover, the crucial Gaussian assumption is reasonable when modelling the dynamics for short periods like one year.

IMAI, J. & BOYLE, P. P. *Dynamic fund protection*. 31-51. Dynamic fund protection provides an investor with a floor level of protection during the investment period. This feature generalises the concept of a put option, which provides only a floor value at a particular time. The dynamic

- protection feature ensures that the fund value is upgraded if it ever falls below a certain threshold level. Gerber and Pafumi (2000) have recently derived a closed-form expression for the price of this protection when the basic portfolio follows geometric Brownian motion. In this paper we examine the pricing of this feature under the constant elasticity of variance process. Two approaches are used to obtain numerical results. First, we show how to extend the basic Monte Carlo approach to handle the particular features of dynamic protection. When a discrete-time simulation approach is used to value a derivative that is subject to continuous monitoring, there is a bias. We show how to remove this bias. Second, a partial differential equation approach is used to price dynamic protection. We demonstrate that the price of the dynamic protection is sensitive to the investment assumptions. We also discuss a discrete time modification of the dynamic protection feature that is suitable for practical implementation. The paper deals just with pricing and does not consider the important question of reserving for these contracts.
- OSTASZEWSKI, K. M. *Macroeconomic aspects of private retirement programs*. 52-64. The decline in importance of private defined benefit plans in relation to defined contribution plans in the United States is a major issue of interest to pension actuaries. This decline has been attributed to numerous factors: costs of government regulation, societal and cultural changes, changed employer attitudes, and employees' lack of understanding of defined benefit plans. This paper analyses possible macroeconomic factors contributing to the crisis of defined benefit plans and proposes an alternative hypothesis for the cause of the crisis: the decline of the relative attractiveness of defined benefit plans in relation to defined contribution plans when these are viewed as investments, that is, as securities in capital markets.
- SU, T. K., TONG, H. & YANG, H. *Bayesian risk measures for derivatives via random Esscher transform*. 78-91. This paper proposes a model for measuring risks for derivatives that is easy to implement and satisfies a set of four coherent properties introduced in Artzner et al (1999). We construct our model within the context of Gerber-Shiu's option-pricing framework. A new concept, namely Bayesian Esscher scenarios, which extends the concept of generalised scenarios, is introduced via a random Esscher transform. Our risk measure involves the use of the risk-neutral Bayesian Esscher scenario for pricing and a family of real-world Bayesian Esscher scenarios for risk measurement. Closed-form expressions for our risk measure can be obtained in some special cases.
- YANG, H. & ZHANG, L. *The joint distribution of surplus immediately before ruin and the deficit at ruin under interest force*. 92-103. In this paper we consider a compound Poisson risk model with a constant interest force. We investigate the joint distribution of the surplus immediately before and after ruin. By adapting the techniques in Sundt and Teugels (1995), we obtain integral equations satisfied by the joint distribution function and a Lundberg-type inequality. In the case of zero initial reserve and the case of exponential claim sizes, we obtain explicit expressions for the joint distribution function.
- YAO, Y. *State price density, Esscher transforms, and pricing options on stocks, bonds, and foreign exchange rates*. 104-117. The state price density is modelled as an exponential function of the underlying state variables, and the Esscher transform is used to specify the forward-risk-adjusted measure. With the aid of state price densities, Esscher transforms, and characteristic functions, this paper provides a consistent framework for pricing options on stocks, interest rates, and foreign exchange rates. The framework discussed is quite general and is related to many popular models.

Reproduced with the permission of the Society of Actuaries.

Subscription details available from: Society of Actuaries, 475 N. Martingale Road, Schaumburg, ILL 60173 USA, www.soa.org

2002

AASE, K. K. *Perspectives of risk sharing*. 73-128. In this paper we present an overview of the standard risk sharing model of insurance. We discuss and characterize a competitive equilibrium, Pareto optimality, and representative agent pricing, including its implications for insurance premiums. We only touch upon the existence problem of a competitive equilibrium, primarily by presenting several examples. Risk tolerance and aggregation is the subject of one section. Risk adjustment of the probability measure is one topic, as well as the insurance version of the capital asset pricing model.

The competitive paradigm may be a little demanding in practice, so we alternatively present a game theoretic view of risk sharing, where solutions end up in the core. Properly interpreted, this may give rise to a range of prices of each risk, often visualized in practice by an ask price and a bid price. The nice aspect of this is that these price ranges can be explained by 'first principles', not relying on transaction costs or other frictions.

We also include a short discussion of moral hazard in risk sharing between an insurer and a prospective insurance buyer.

We end the paper by indicating the implications of our results for a pure stock market. In particular we find it advantageous to discuss the concepts of incomplete markets in this general setting, where it is possible to use results for closed, convex subspaces of an L^2 -space to discuss optimal risk allocation problems in incomplete financial markets.

CENTENO, M. DE L., ANDRADE E. S. & JOÃO, M. *Optimal bonus scales under path-dependent bonus rules*. 129-136. Bonus malus systems have been studied by several authors under the framework of Markov chains. Optimal scales have been deduced by Norberg (1976), Borgan, Hoem & Norberg (1981) and Gilde & Sundt (1989). In these articles the authors assumed that the bonus system forms a first order Markov chain. In the present paper we deduce the optimal scales, using the same criteria as in the cited papers, for bonus systems that are not first order Markovian processes, but that can be regarded as Markovian by increasing the number of states of the system.

DENUIT, M., GENEST, C. & MARCEAU, É. *Criteria for the stochastic ordering of random sums, with actuarial applications*. 3-16. It is shown that vectors $(S_{M_1}, \dots, S_{M_n})$ and $(S'_{M'_1}, \dots, S'_{M'_n})$ of random sums of positive random variables are stochastically ordered by upper orthant dependence, lower orthant dependence, concordance or by the supermodular ordering whenever their corresponding random numbers of terms (M_1, \dots, M_n) and (M'_1, \dots, M'_n) are themselves ordered in this fashion. Actuarial applications of these results are given to different dependence structures for the collective risk model with several classes of business.

DENUIT, M. *S-convex extrema, Taylor type expansions and stochastic approximations*. 45-67. The present work studies s -convex orders using a remarkable probabilistic generalization of Taylor's theorem obtained by Massey & Whitt (1993) and further discussed by Lin (1994). We propose two methods for approximating a given risk with known first moments by means of s -convex extremal distributions. The goodness of those approximations is explored using stop-loss distances. Several applications show the interest of this approach in actuarial sciences.

GOMEZ-DENIZ, E., HERNANDEZ-BASTIDA, A. & VAZQUEZ, F. J. *Bounds for ratios of posterior expectations: applications in the collective risk model*. 37-44. This paper considers the collective risk model for the insurance claims process. We will adopt a Bayesian point of view, where uncertainty concerning the specification of the prior distribution is a common question. The robust Bayesian approach uses a class of prior distributions which model uncertainty about the prior, instead of a single distribution. Relatively little research has dealt with robustness with respect to ratios of posterior expectations as occurs with the Esscher and Variance

premium principles. Appropriate techniques are developed in this paper to solve this problem using the ε -contamination class in the collective risk model.

JURI, A. *Supermodular order and Lundberg exponents*. 17-36. A risk process where the claims are sums of dependent random variables is considered. Using the supermodular order the influence, the dependence has on the infinite- and finite-time Lundberg exponent is investigated and monotonicity results are obtained.

Reproduced with the permission of Taylor and Francis, www.tandf.no/saj