This book demands the reader's attention and practically forces our interaction—students who would benefit from some wider reading would find it hard not to be drawn in. Some of my sixth form students can often tend to be rather passive and I am sure they would gain from working with the author through some of these questions.

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An introduction to functional analysis by James C. Robinson, pp 248, £29.99 (paper), ISBN 978-0-52172-839-3, Cambridge University Press (2020)

This excellent new introductory text on functional analysis manages to accomplish two objectives that are both important but also, to some extent, at odds with one another. On the one hand, this book is genuinely accessible, even perhaps to undergraduates (and certainly to first-year graduate students) on my side of the Atlantic. On the other hand, it covers a substantial amount of material, including some topics (e.g., Sturm-Liouville problems and unbounded operators) that go beyond what is generally taught in an introductory course.

The book is divided into four Parts, each one consisting of several chapters. Part I (*Preliminaries*) contains two chapters covering, respectively, background material in linear algebra (including a proof, using Zorn's Lemma, of the existence of a Hamel basis for an arbitrary vector space) and metric spaces. Because the author assumes the reader already has some familiarity with this material, the exposition here is efficient rather than leisurely, with some proofs omitted (but quite a few included).

Part II covers normed linear spaces, with a focus on those that are complete. The term "Banach space" is introduced at this point, but extensive discussion of these spaces is reserved for future chapters. Many examples are given, but discussion of the Lebesgue spaces L^p are, initially anyway, replaced by discussion of the sequence spaces l^p; the L^p spaces are, however, defined in the last chapter of this part as the completion of a space of continuous functions with the L^p norm. (A prior knowledge of Lebesgue theory is not assumed by the author. An Appendix discusses, without proof, Lebesgue measure and the Lebesgue integral, and it is then proved that this construction of L^p agrees with the one given in chapter 7). This last chapter, and the appendix section, evince a fairly sharp spike in level of difficulty from what has come before, but the author does take pains to make it as clear and detailed as possible. Other topics covered in this part of the book include the Contraction Mapping principle (with applications to differential equations discussed as exercises), the Arzela-Ascoli theorem, the Weierstrass Approximation theorem, and the Stone-Weierstrass theorem.

The remaining two parts of the book cover Hilbert and Banach spaces, in that order. (Hilbert spaces are done first because the author felt their additional structure made them easier to study.) The chapters on Hilbert spaces cover, in addition to the usual introductory material (orthonormal bases, projections, the Riesz representation theorem, self-adjoint operators), such additional topics as compact operators on Hilbert spaces and their spectra, and Sturm-Liouville problems.

Part IV of the book covers material that is standard for a first course, including the 'big theorems' (Hahn-Banach, Uniform Boundedness, Open Mapping, Closed Graph) and some of their applications. The Hahn-Banach theorem is initially given an analytic statement but a geometric version (separating hyperplanes) is presented later, as is the Krein-Millman theorem. Part IV also contains discussions of more sophisticated topics, including weak and weak* convergence, a spectral theorem for compact operators on a Banach space, and (returning to Hilbert spaces) a spectral theorem for unbounded self-adjoint operators.

In addition to the core text and exercise solutions, there are three appendices, covering, respectively, the equivalence of Zorn's Lemma and the Axiom of Choice, Lebesgue integration, and the Banach-Alaoglu theorem (preceded by a brisk introduction to topological spaces, up to and including a proof of the Tychonoff theorem). No exercises appear in these Appendices.

Of course, being a relatively short book (less than 300 pages of actual text, not counting the aforementioned appendices and exercise solutions) and one that is intended for a beginning audience, the omission of some topics is inevitable. Unitary operators are mentioned only once, in an exercise, and normal operators are not discussed at all. Banach algebras are not discussed, nor are topological vector spaces in that level of generality (though some results often stated in the context of topological vector spaces, like the Krein-Millman theorem, are here stated for Banach spaces). Distributions and Sobolev spaces are also not discussed.

Throughout, the exposition is of very high quality—clear, well-motivated, and enjoyable to read. There are lots of examples, as well as a good number of exercises, solutions to which appear in the back of the book. This is a boon to students reading this book independently, and to faculty members teaching out of another text who would like a good source of problems, but is perhaps something of an annoyance for people actually using this book as a text who like to assign graded homework assignments.

Bottom line: this is a valuable book. It is an accessible yet serious look at the subject, and anybody who has worked through it will be rewarded with a good understanding of functional analysis, and should be in a position to read more advanced books with profit.

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A modern introduction to differential equations (3rd edn.) by Henry J. Ricardo, pp. 539, £115 (hard), ISBN 978-0-12823-417-4, Academic Press/Elsevir (2020)

This is an established textbook aimed at a wide range of undergraduate courses, from mathematics to the social sciences. It covers basic methods of finding explicit algebraic solutions, and qualitative analysis of differential equations (DEs) which do not have such elementary solutions, the emphasis being on interpreting the solutions rather than on rigour. The chapters include first-order DEs (with slope fields, a.k.a. iron-filings diagrams), numerical methods (improved Euler, Runge-Kutta), second- and higher-order equations, the Laplace transform, systems of linear DEs (the longest chapter, with much discussion of stability), and systems of non-linear DEs. The background mathematical knowledge required is no more than "two semesters of calculus"; what linear algebra is needed is mentioned in the course of the book, and as an appendix. Students with no more equipment than this may have to take on trust results such as $r^2\dot{\theta} = x\dot{y} - y\dot{x}$ (page 403), but this is a minor issue. Further appendices cover complex numbers and series solutions. There are many worked examples, clearly illustrated, and substantial exercises with answers and hints to odd-numbered questions. An Instructor's Resource Manual and a student Solutions Manual are also available online.