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Stimulated Raman scattering of ultra intense hollow Gaussian beam in relativistic plasma

PRERANA SHARMA
Physics Department, Ujjain Engineering College, Ujjain, India
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Abstract

Effect of relativistic nonlinearity on stimulated Raman scattering (SRS) of laser beam propagating carrying null intensity in center [hollow Gaussian beam (HGB)] is studied in collisionless plasma. The construction of the equations is done employing the fluid theory which is developed with partial differential equation and Maxwell's equations. The phenomenon of SRS is shown along with the excitation of seed plasma wave considering relativistic nonlinearity. The power of plasma wave is observed for higher order of HGB. The Raman back reflectivity is studied numerically for various orders of hollow Gaussian laser beam (HGLB) and the numerical analysis shows that these parameters play vital role on reflectivity characteristics. It is observed that the Raman back reflectivity is less for the higher order of HGLB.

Keywords: Hollow Gaussian beam; Relativistic nonlinearity; Stimulated Raman scattering

1. INTRODUCTION

The field of relativistic laser plasma interaction has branched out in two main directions first, motivated by laser fusion and fast ignition and second driven by particle acceleration (Hora *et al.*, 2013). In addition to that the studies of laser matter interactions have much relevance to basic plasma physics, advanced radiation sources, relativistic nonlinear optics and high density physics. All these applications need an efficient coupling between laser and plasma, which is governed by nonlinear processes during the interaction. The interaction of high power laser pulse with plasmas leads to various processes like, stimulated Raman scattering (SRS) (Kruer, 1974; Paknezhad & Dorranian 2011; Paknezhad, 2013*b*), stimulated Brillouin scattering (Kruer, 1974; Sharma *et al.*, 2009; Gao *et al.*, 2010; Paknezhad, 2013*a*), filamentation (Kruer, 1974), and harmonic generation (Milchberg *et al.*, 1995; Sharma & Sharma 2009*a*; 2009*b*; Gupta *et al.*, 2007), which affects the laser–plasma coupling. In the context of efficient laser–plasma coupling the scattering instabilities is of particular significance. These instabilities can modify the intensity distribution, affecting the uniformity of energy deposition. They reduce the laser–plasma coupling efficiency and produce energetic electrons. The SRS has largely been studied by many investigators. SRS of beat wave of two counter-propagating X-mode lasers in a magnetized plasma has been studied by Verma *et al.* (2014). Sharma *et al.* (2013) have studied the effect of laser beam filamentation on coexisting stimulated Raman and Brillouin scattering. Suppression of SRS due to localization of electron plasma wave (EPW) in laser beam filaments has been studied by Sharma and Sharma (2009*a*; 2009*b*). Purohit *et al.* (2012) have investigated the filamentation of laser beam and suppression of SRS due to localization of EPW. Recently, experimental work related to Raman plasma amplifiers has been performed by researchers (Turnbull *et al.*, 2012*a*; 2012*b*) in which SRS and SBS have been studied simultaneously.

Besides, several phenomena of the laser–plasma interaction have been studied considering various spatial profile of laser beam such as; Gaussian beam (Akhmanov *et al.*, 1968), super Gaussian beam (Grow *et al.*, 2006), dark hollow Gaussian beams (HGBs) (Sodha *et al.*, 2009*a*; 2009*b*). The self-focusing of HGBs or hollow Gaussian laser beams (HGLBs) (Sodha *et al.*, 2009*a*; 2009*b*; Gill *et al.*, 2010) and cross focusing of HGLBs (Gupta *et al.*, 2011*a*; 2011*b*) has been investigated theoretically in plasmas. Moreover, investigations related to HGLB have received much attention theoretically as well as experimentally due to its scope in wide area of science and technology. Numerous work has been done for the study of various laser plasma phenomena using dark HGB. Stimulated Raman backscattering of filamented HGLBs has been studied by Singh and Sharma (2013). Stimulated Brillouin backscattering of filamented HGLB

Address correspondence and reprint requests to: Prerana Sharma, Physics Department, Ujjain Engineering College, Ujjain, M. P. 456010, India. E-mail: preranaiitd@rediffmail.com

has been observed by Sharma & Singh (2013). Sodha *et al.* (2009*a*; 2009*b*) have studied the focusing of dark hollow Gaussian electromagnetic beams in a plasma. Focusing of a dark hollow Gaussian electromagnetic beam in a magnetoplasma has been investigated by Sodha *et al.* (2009*a*; 2009*b*). Relativistic and ponderomotive effects on evolution of dark hollow Gaussian electromagnetic beams in a plasma has been studied by Gill *et al.* (2010). Effect of relativistic self-focusing on plasma wave excitation by a hollow Gaussian beam has been investigated by Gupta *et al.* (2011*b*). Cross-focusing of two HGLBs in plasmas is observed by Gupta *et al.* (2011*a*). Singh *et al.* (2013) studied the THz generation by cosh-Gaussian lasers in a rippled density plasma. Saini and Gill (2006) have studied the self-focusing and self-phase modulation of an elliptic Gaussian laser beam in collisionless magnetoplasma.

Further, it is very well known that the nonlinearity in the refractive index governs the propagation of laser beam in the plasma through distinct mechanisms like thermal, relativistic, and ponderomotive etc. When the laser propagates in the plasma the generated nonlinearity depends upon laser pulse duration and electron and ion plasma period. In the situation when the laser beam follows the time regime $\tau < \tau_{pe}$ relativistic nonlinearity is created. Here τ is laser pulse duration and τ_{pe} is the electron plasma period. This nonlinearity is setup almost immediately. The relativistic dependence of electronic mass on the quiver speed of electrons in the field of the beams causes a modification in the plasma frequency ω_p. It is the change of the plasma frequency which is responsible for the attraction or repulsion of the beams. The theoretical work related to the relativistic nonlinearity was first started by Max *et al.* (1974) and Sun *et al.* (1987). The experimental observation on relativistic self-focusing of a laser beam was reported by Borisov *et al.* (1992). Growth of laser ripple and its influence on plasma wave have been investigated by Purohit *et al.* (2004) taking into account the relativistic nonlinearity. In their work the radially symmetrical ripple is considered to be superimposed on an intense laser beam in collisionless unmagnetized plasma. Effect of relativistic mutual interaction of two laser beams on the growth of laser ripple in plasma is studied by Purohit *et al.* (2005). They have shown that one laser beam affects the dynamics of the second beam and a mutual nonlinear interaction has shown to be take place. Relativistic Landau damping of electron plasma waves has been studied by Bers *et al.* (2009). The formulation of the collisionless EPW damping rate in a relativistic thermal equilibrium plasma is presented, and evaluated for such waves in both forward and backward SRS. Excitation of an upper hybrid wave by a high power laser beam in plasma has been studied by Purohit *et al.* (2008). The amplitude of the upper hybrid wave is shown to be nonlinearly coupled with the relativistic laser beam propagating perpendicular to the static magnetic field and having its electric vector polarized along the direction of the static magnetic field. Relativistic self-focusing and its effect on stimulated Raman and stimulated Brillion scattering has been studied by Mahmoud & Sharma (2001). The effect of finite laser beam size and that of scattered beam and relativistic self-focusing of the pump laser beam on Raman and Brillion reflectivity have also been studied, including pump depletion. Filamentation of a relativistic short pulse laser in a plasma is studied by Kumar *et al.* (2006). They have studied the filamentation instability under the combined effect of both relativistic and ponderomotive nonlinearity. SRS in laser–plasma interaction has been studied by Sharma and Gupta (2006) considering both relativistic and ponderomotive nonlinearities simultaneously. The effect of filamentation on SRS back reflectivity has been shown in their work. All the above mentioned works deal the laser beam propagation with relativistic nonlinearity and treated plasma wave excitation, upper hybrid wave excitation, Raman and Brillion scattering, etc. but none of these works treated Raman scattering of HGB considering relativistic nonlinearity.

In the present work, we study the effect of relativistic nonlinearity on the SRS of dark cylindrical HGBs, in which the irradiance along the axis is zero, and the maximum is away from the axis. The motivation of this study is the work done by Singh and Sharma (2013). The central focus here is to observe the effect of relativistic nonlinearity on the stimulated back Raman scattering process from HGB. In the present work, we have used a paraxial like approach; similar to one set by Akhmanov *et al.* (1968) and further established by Sodha *et al.* (1976) has been used in the present analysis. The relativistic change in the mass of electron causes a redistribution of the plasma density. The plasma channel, thus produced, guides the HGLB. However, the EPW excitation is studied and SRS has been explored in the presence of relativistic nonlinearity. Raman back reflectivity for the different orders of self-focused HGLBs in collisionless plasma, considering relativistic nonlinearity has been numerically calculated.

The paper is organized as follows: Section 2 is devoted to the basic equations for general propagation of HGLB with relativistic nonlinearity and excitation of EPW has been presented. In Section 3, the equations governing the dynamics of SRS process has been given. The discussions of numerical results are presented in Section 4. A brief conclusion of investigation is presented in Section 5.

2. EXCITATION OF PLASMA WAVE IN PRESENCE OF HGLB

To study Raman scattering of a HGLB propagating in collisionless plasma, first the excitation process of EPW through the same is to be studied, since the amount of scattered light is proportional to the plasma wave's amplitude. Therefore, in this section first the propagation of high power HGLB through the plasma is being studied and hence the excitation process of plasma wave is presented considering relativistic nonlinearity. For this purpose, a high power HGLB of frequency ω_0 and the wave vector k_0 is considered to be propagating in hot, collisionless, and homogeneous plasma in the

z-direction. The irradiance distribution ($z = 0$) of the incident beam is

$$\mathbf{E_0} \cdot \mathbf{E_0^*}\big|_{z=0} = E_{00}^2 (r^2/2r_0^2)^{2m} \exp(-r^2/r_0^2) \quad (1)$$

where the direction of propagation has been assumed to be along the z-axis and r refers to radial coordinate of the cylindrical coordinate system, r_0 is the spot size of the beam, respectively, E_0 is a real constant characterizing the amplitude of the HGLB, respectively, E_{00} refers to the complex amplitude of the beam which denotes the electric field maximum at $r_{\max} = r_0\sqrt{2m}$, corresponding to $z = 0$. The positive integer number m decides order of HGLB which characterizes the shape of the HGLB and position of its irradiance maximum. Equation (1) represents fundamental Gaussian beam of width r_0 for $m = 0$. We can write the following equation for the field inside the plasma

$$\nabla^2 \mathbf{E_0} + \frac{\omega_0^2}{c^2}\left(1 - \frac{\omega_p^2}{\omega_0^2}\right)\mathbf{E_0} = 0 \quad (2)$$

where ω_p is the plasma frequency. The wave equation governing the electric vector of the HGLB in plasma can be written as assuming the variation of electric field in terms of vector potential as

$$\mathbf{E_0} = \mathbf{A}_0(x, y, z)\, e^{-ik_0 z}. \quad (3)$$

For the considered HGLB (in a steady state) the vector potential $\mathbf{A}_0$ satisfies the wave equation as

$$2ik_0 \frac{\partial \mathbf{A}_0}{\partial z} + i\mathbf{A}_0 \frac{\partial k_0}{\partial z} = \left(\frac{\partial^2 \mathbf{A}_0}{\partial r^2} + \frac{1}{r}\frac{\partial \mathbf{A}_0}{\partial r}\right) + \frac{\omega_0^2}{k_0 c^2}(\varepsilon - \varepsilon_0) \quad (4)$$

where $k_0(z) = (\omega_0/c)\sqrt{\varepsilon_0(z)}$ is the wave number of the system, c is the speed of light in vacuum, and ε_0 is the linear part of the plasma dielectric constant given by

$$\varepsilon_0 = 1 - \frac{\omega_{pe}^2}{\omega_0^2} \quad (5)$$

where, $\omega_{pe} = (4\pi n N_0 e^2/m_c)^{1/2}$ is the plasma frequency. Here N_0 is the electron density in absence of laser beam; e and m_c are the charge and relativistic mass of the electron, respectively. Substituting the relativistic mass ($m_c = \gamma m_0$) where m_0 is the electron rest mass, and $\omega_p = (4\pi N_0 e^2/m_0)^{1/2}$ we have,

$$\varepsilon = 1 - \frac{\omega_p^2}{\gamma\,\omega_0^2} \quad (6)$$

where the relativistic factor γ is given by,

$$\gamma = \left[1 + \frac{e^2}{c^2 m_0^2 \omega_0^2}\mathbf{E_0} \cdot \mathbf{E_0^*}\right]^{1/2} \quad (7)$$

Further, the complex amplitude $\mathbf{A}_0(r, z)$ may be expressed as, $\mathbf{A}_0(r, z) = \mathbf{A}_0(r, z)\exp[-ik_0(z)S_0(r, z)]$ where $S_0(r, z)$ is the eikonal associated with the HGLB. Now putting $\mathbf{A}_0$ in Eq. (4) and segregating the real and imaginary parts from the resulting equation, we get the following set of equations as

$$2\frac{\partial S_0}{\partial z} + \left(\frac{\partial S_0}{\partial z}\right)^2 = \frac{\omega_0^2 \varepsilon}{c^2 k_0^2} + \frac{1}{k_0^2 \mathbf{A}_0}\left(\frac{\partial^2 \mathbf{A}_0}{\partial r^2} + \frac{1}{r}\frac{\partial \mathbf{A}_0}{\partial r}\right) \quad (8)$$

$$\frac{\partial A_0^2}{\partial z} + \frac{\partial S_0}{\partial r}\frac{\partial \mathbf{A}_0^2}{\partial r} + \mathbf{A}_0^2\left(\frac{\partial^2 S_0}{\partial r^2} + \frac{1}{r}\frac{\partial S_0}{\partial r}\right) = 0 \quad (9)$$

Moreover, in Gaussian beams, we expand all the parameters around central point where the intensity is maximum that is, $r = 0$. However, in case of HGLB, the irradiance is not maximum at the center, thus it is better to define a point along the radial distance of the beam at which all the power of the beam is supposed to be concentrated, let say $r = r_0 f_0(z)\sqrt{2m}$ which indeed justified in the paraxial like approximation, where $f_0(z)$ is the beam width parameter for the HGLB. We expand all the parameters around this point and define this point η as

$$\eta = \left[(r/r_0 f_0) - \sqrt{2m}\right] \quad (10)$$

Here, the condition $\eta \ll \sqrt{2m}$ is appropriate like the case of paraxial theory. The conversion indicates as

$$\frac{\partial}{\partial z} = \frac{\partial}{\partial z} - \frac{\left(\sqrt{2m} + \eta\right)}{f_0}\frac{df_0}{d\eta}\frac{\partial}{\partial \eta} \quad (11)$$

$$\frac{\partial}{\partial r} = \frac{1}{r_0 f_0}\frac{\partial}{\partial \eta} \quad (12)$$

The appropriate parameters like eikonal, ε, and irradiance may be expanded around the maximum of the HGLB. So,

$$\varepsilon(\eta, z) = \varepsilon_0(z) + \phi(E.E^*) \quad (13)$$

where $\varepsilon_0(z)$ is given by Eq. (5) and $\phi(E.E^*)$ is given as

$$\phi(E.E^*) = \frac{\omega_p^2}{\omega_0^2}\left\{1 - \frac{1}{\left[1 + \left(e^2/c^2 m_0^2 \omega_0^2\right) E_0.E_0^*\right]}\right\} \quad (14)$$

Moreover, expanding the effective dielectric function using Taylor series around the irradiance maximum $\eta = 0$ of the HGLB as

$$\varepsilon(\eta, z) = \varepsilon_f(\eta = 0) + \eta^2 \varepsilon_2(\eta = 0) \quad (15)$$

where ε_2 is defined as $\partial\varepsilon/\partial\eta^2$. The expressions for these coefficients ε_f and ε_2 have been derived later. Substitution for $\varepsilon(\eta, z)$ from Eq. (15) in real and imaginary part equations that is, Eqs. (8) and (9) leads to

$$\frac{1}{r_0^2 f_0^2}\left(\frac{\partial S_0}{\partial \eta}\right)^2 + \frac{2\partial S_0}{\partial z} = \frac{1}{k_0^2 \mathbf{A}_0 r_0^2 f_0^2}\left[\frac{1}{(\sqrt{2m}+\eta)}\frac{\partial \mathbf{A}_0}{\partial \eta} + \frac{\partial^2 \mathbf{A}_0}{\partial \eta^2}\right] + \eta^2 \frac{\omega_0^2(\varepsilon_2)}{c^2 k_0^2} \quad (16)$$

$$\frac{\partial \mathbf{A}_0^2}{\partial z} + \frac{\mathbf{A}_0^2}{r_0^2 f_0^2}\left[\frac{\partial^2 S_0}{\partial \eta^2} + \frac{1}{(\sqrt{2m}+\eta)}\frac{\partial S_0}{\partial \eta}\right] + \frac{1}{r_0^2 f_0^2}\frac{\partial \mathbf{A}_0^2}{\partial \eta}\frac{\partial S_0}{\partial \eta} = 0 \quad (17)$$

One can express solution of Eq. (16), with the paraxial approximation $\eta \ll \sqrt{2m}$ as

$$\mathbf{A}_0^2 = \frac{E_0^2}{2^{2m} f_0^2}\left(\sqrt{2m}+\eta\right)^{4m} \exp\left(-\left(\sqrt{2m}+\eta\right)^2\right) \quad (18)$$

with

$$S_0(\eta, z) = \frac{\left(\sqrt{2m}+\eta\right)^2}{2}\beta(z) + \varphi(z)$$

where

$$\beta(z) = r_0^2 f_0 \frac{d f_0}{d z} \quad (19)$$

Here $\varphi(z)$ is an arbitrary function of z. As we assumed that nearly all the power of the beam is concentrated in the region around $\eta = 0$, there is certainly some power of the beam beyond this limitation, which is accounted for in an approximate manner by Eq. (18).

Further, the present study considers a plasma, characterized by relativistic nonlinearity caused by the relativistic change in the mass of electron. To derive the value of the coefficients ε_f and ε_2 given in Eq. (15), it is convenient to expand the solution for $\mathbf{A}_0^2$ as a polynomial in η^2; for further algebraic analysis,

$$A_0^2 = d_0 + d_2\eta^2 \quad (20)$$

where d_0 and d_2 are expressed as

$$d_0^2 = \frac{E_0^2}{f_0^2} m^{2m} \exp(-2m) \quad (21)$$

$$d_2^2 = \frac{2E_0^2}{f_0^2} m^{2m} \exp(-2m) \quad (22)$$

Following the paraxial like approximation one can expand the dielectric function in axial and radial parts around the maximum of the HGLB. Thus, from the set of Eqs. (6, 7) and (20–22), one finds

$$\varepsilon_f(\eta = 0) = 1 - \frac{\omega_p^2}{\omega_0^2}\frac{1}{(1+d_0)^{1/2}} \quad (23)$$

$$\varepsilon_2(\eta = 0) = -\frac{\omega_p^2}{2\omega_0^2}\frac{d_2}{(1+d_0)^{3/2}} \quad (24)$$

The dimensionless beam width parameter f_0 can be obtained using the boundary conditions at $z = 0$ as $f_0 = 1$ and $df_0/dz = 0$ (Akhmanov *et al.*, 1968) as

$$\frac{\partial^2 f_0}{\partial \xi^2} = \frac{4}{f_0^3} - \frac{R_{d0}^2 \omega_p^2 d_2}{2\omega_0^2 k_0^2 \varepsilon_0 f_0 (1+d_0)^{3/2}} \quad (25)$$

where $R_{d0} = k_0 r_0^2$, $\xi = z/R_{d0}$. The above Eq. (25) gives the variation of beam width parameter with the normalized distance and Eq. (18) provides the intensity profile of the laser beam in the plasma along with the radial direction when relativistic nonlinearity is operative. We perform numerical computation of Eqs. (18) with (25). The equation has been solved for an initial plain wave front and the boundary conditions for HGB, and the results are presented in Section 4.

Further, the propagation of the laser beam through the plasma leads the variation of density of the plasma channel. The relativistic mass change of electron is responsible for the change in the density of the plasma and hence changes in the refractive index of the medium. As a result the laser beam self focuses in the plasma. Here, it is important to mention that the seed EPW exists in the plasma nonlinearly interacts with the propagating laser beam and as a result the plasma wave gets excited. The excitation process of EPW in the presence of relativistic nonlinearity can be described with the equations presented as follows.

(a) Momentum equation

$$m_c\left[\frac{\partial V}{\partial t} + (V\cdot\nabla)V\right] = -eE - \frac{e}{c}V\times B - 2\Gamma_e m_c V - \frac{3K_0 T_e}{N}\nabla N \quad (26)$$

where the Landau damping factor is

$$2\Gamma_e \simeq \sqrt{\frac{\pi}{8}}\frac{\omega_p}{k^3\lambda_d^3}\exp\left(-\frac{3}{2} - \frac{1}{2k^2\lambda_d^2}\right) \quad (27)$$

where $\lambda_d = (k_\beta T_0/4\pi N_0 e^2)^{1/2}$ is the Debye length V is the velocity of electron fluid, N is the instantaneous electron density, E the electric and B the magnetic field vectors, and k is the wave number of the electrostatic wave.

(b) Equation of continuity

$$\frac{\partial N}{\partial t} + \nabla.(NV) = 0, \quad (28)$$

(c) Poisson's equation for electric field

$$\nabla\cdot E = -4\pi e N. \quad (29)$$

Using the customary techniques (Akhmanov *et al.*, 1968) and factorizing the density into equilibrium and perturbed part as $N = N_{0e} + n_{e0}$ the equation describing the electron density variation as

$$\frac{\partial^2 n_{e0}}{\partial t^2} - v_{th}^2 \nabla^2 n_{e0} + 2\Gamma_e \frac{\partial n_{e0}}{\partial t} + \frac{\omega_p^2}{\gamma} n_{e0} = 0 \quad (30)$$

The solution of the form $n_{e0} = n_{e00}(r, z)\exp(i\omega t - ikz)$ satisfies the dispersion relation

$$\omega^2 = \frac{\omega_p^2}{\gamma} + k^2 v_{th}^2 \quad (31)$$

Further, assuming $n_{e00}(r, z)$ in the form of $\exp(-ikS)$ in Eq. (8) and splitting the real and imaginary quantities, we get the real part equation as

$$2\left(\frac{\partial S}{\partial z}\right) + \left(\frac{\partial S}{\partial r}\right)^2 = \frac{1}{k^2 n_{e00}}\left(\frac{\partial^2 n_{e00}}{\partial r^2} + \frac{1}{r}\frac{\partial n_{e00}}{\partial r}\right) + \frac{\omega_p^2}{k^2 v_{th}^2}\left(1 - \frac{1}{\gamma}\right) \quad (32)$$

and imaginary part equation as

$$\frac{\partial n_{e00}^2}{\partial z} + \frac{\partial S}{\partial r}\frac{\partial n_{e00}^2}{\partial r} + \frac{n_{e00}^2}{k^2}\left(\frac{\partial^2 S}{\partial r^2} + \frac{1}{r}\frac{\partial S}{\partial r}\right) + 2\Gamma_e \frac{\omega\, n_{e00}^2}{k v_{th}^2} = 0 \quad (33)$$

The term S describes the eikonal for plasma wave. Here, to deal the HGLB the transformation of (r, z) coordinate into (η, z) coordinate is required. So, using the Eq. (10) we made the transformation. However, using paraxial like approximation Eqs. (32) and (33) can be solved and for that we assume the initial radial variation of density perturbation and the solution as

$$n_{e00}^2 = \frac{N_{e00}^2}{2^{2m} f_e^2}\left(\sqrt{2m} + \eta\right)^{4m}\left(\frac{r_0 f_0}{a f_e}\right)^{4m} \exp\left[-\left(\sqrt{2m} + \eta\right)^2\right] \times \exp[-k_i(z)] \quad (34)$$

$$S(\eta, z) = \frac{\left(\sqrt{2m} + \eta\right)^2}{2} r_0^2 f_0^2 \beta(z) + \phi(z), \quad \beta(z) = \frac{1}{f_e}\frac{df_e}{dz} \quad (35)$$

where f_e is a dimensionless parameter of the electron plasma wave, a is the initial beam width of plasma wave, $k_i = 2\Gamma_e \omega / k V_{th}^2$ is the damping factor and N_{e00}^2 is the magnitude of the excited plasma wave. Further, we have used the following boundary conditions (for an initially plane wave front): At $df_e/dz = 0$ and $f_e = 1$ and $S = 0$ $z = 0$. Substituting, Eqs. (34) and (35) in Eq. (32) and equating the coefficients of η^2 on both sides, we get the equation for f_e as

$$\frac{\partial^2 f_e}{\partial \xi^2} = \frac{R_{d0}^2 f_e}{f_0^2}\left\{\frac{1}{k^2 r_0^4 f_0^2}\left[3 + \left(\frac{r_0 f_0}{a f_e}\right)^4\right]\right\} - \frac{R_{d0}^2 f_e}{2\omega_0^2 f_0^2}\frac{\omega_p^4}{k^2 V_{th}^2}\frac{d_2}{(1 + d_0)^{3/2}} \quad (36)$$

where $R_d = ka^2$, Further, the amplitude of the density perturbation at finite z is to be obtained, for that purpose, Eq. (34) has to be solved numerically with Eq. (36) and results are given in Section 4.

3. SRS

SRS governs the amount of laser energy that can be propagated over long distances through plasma without being lost to scattering and electron heating. SRS is the process in which light from an incident pump pulse is scattered by the electron density perturbations of a plasma wave. If the wave has frequency $\omega_p^2 = 4\pi n_0 e^2/m$, with electron charge e, mass m and number density n_0 and the incident light's frequency is ω_0, then light will be scattered from noise to frequencies $\omega_1 = \omega_0 - \omega_p$ and $\omega_1 = \omega_0 + \omega_p$, which are called the Stokes and anti-Stokes lines. The beating between the ω_0 light and that scattered to ω_1 resonantly drive a plasma wave, which creates a feedback loop, as the amount of scattered energy is proportional to the amplitude of excited plasma wave. To study the scattering of HGLB, first we split the total electric field of the pump wave into two parts scattered and remaining, and further only the scattered part of the laser beam is treated to observe the beam width parameter and reflectivity of the same. Thus, the high frequency pump wave is propagating through the plasma is considered to have total electric field E_T as

$$E_T = E_0 \exp(i\omega_0 t) + E_s \exp(i\omega_s t) \quad (37)$$

where E_0 is the electric field of the pump laser beam and E_S, electric field of the scattered wave. The electric field E_S originates owing to scattering of the pump beam off the plasma wave. Thus, the wave equation

$$\nabla^2 E_T - \nabla\left(\nabla \cdot E_T\right) = \frac{1}{c^2}\frac{\partial^2 E_T}{\partial t^2} + \frac{4\pi}{c^2}\frac{\partial J_T}{\partial t} \quad (38)$$

where J_T is the total current density vector corresponding to E_T. Comparing the zeroth order terms in above equation, one gets Eq. (2) and equating the terms at scattered frequency. Further, separating the terms at different frequencies, one obtains

$$\nabla^2 E_s + \frac{\omega_s^2}{c^2}\left(1 - \frac{\omega_p^2}{\omega_s^2}\frac{1}{\gamma}\right)E_s = \frac{1}{2}\frac{\omega_p^2}{c^2}\frac{\omega_s}{\omega_0}\frac{n^*}{N_0}E_0 \quad (39)$$

Further substituting E_s as

$$E_s = E_{s0}(r, z)\exp(ik_{s0}t) + E_{s1}(r, z)\exp(-ik_{s1}t) \quad (40)$$

where

$$k_{s0}^2 = \frac{\omega_s^2}{c^2}\left(1 - \frac{\omega_p^2}{\omega_s^2}\right)^{1/2} = \frac{\omega_s^2}{c^2}\sqrt{\varepsilon_{s0}} \quad (41)$$

and k_{s1} and ω_s follows

$$\omega_s = \omega_0 - \omega, \quad k_{s1} = k_0 - k$$

Substituting Eq. (40) in (39), one gets

$$-k_{s0}^2 E_{s0} + 2ik_{s0}\frac{\partial E_{s0}}{\partial z} + \left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r}\right)E_{s0} + \frac{\omega_s^2}{c^2}\left[\varepsilon_{s0} + \frac{\omega_p^2}{\omega_s^2}\left(1 - \frac{1}{\gamma}\right)\right]E_{s0} = 0 \tag{42}$$

$$-k_{s1}^2 E_{s1} - 2ik_{s1}\frac{\partial E_{s1}}{\partial z} + \left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r}\right)E_{s1} + \frac{\omega_s^2}{c^2}\left\{\varepsilon_{s0} + \frac{\omega_p^2}{\omega_s^2}\left(1 - \frac{1}{\gamma}\right)\right\} \tag{43}$$

$$E_{s1} = \frac{\omega_p^2 \omega_s n^* E_0}{2c^2\omega_0 N_0} e^{(-ik_0 S_0)}$$

Moreover, using $E_{s0} = E_{s00}(r, z)\, e^{+ikS_c}$ in Eq. (42) and separating the real and imaginary parts, we get the two equations as

$$\frac{2\partial S_c}{\partial z} + \left(\frac{\partial S_c}{\partial r}\right)^2 = \frac{1}{k_{s0}^2 E_{s00}}\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r}\right)E_{s00} + \frac{\omega_p^2}{\varepsilon_{s0}\omega_s^2}\left(1 - \frac{1}{\gamma}\right) \tag{44}$$

$$\frac{\partial E_{s00}^2}{\partial z} + E_{s00}^2\left(\frac{\partial^2 S_c}{\partial r^2} + \frac{1}{r}\frac{\partial S_c}{\partial r}\right) + \frac{\partial S_c}{\partial r}\frac{\partial E_{s00}^2}{\partial r} = 0 \tag{45}$$

The solution of these equations can be written as

$$E_{s00}^2 = \frac{B_1^2}{2^{2m} f_s^2}\left(\sqrt{2m} + \eta\right)^{4m}\left(\frac{r_0 f_0}{b_0 f_s}\right)^{4m} \exp\left[-(\sqrt{2m} + \eta)^2\right] \tag{46}$$

$$S_c(\eta, z) = \frac{\left(\sqrt{2n} + \eta\right)^2}{2} r_0^2 f_0^2 \beta(z) + \phi(z), \quad \beta(z) = \frac{1}{f_s}\frac{df_s}{dz} \tag{47}$$

where B_1 is the constant determined by the boundary conditions, f_s is a dimensionless parameter of the backscattered beam, S_c is the eikonal of the scattered beam and b_0 is the initial beam width of scattered beam. In the presence of relativistic nonlinearity the dielectric constant at scattered frequency

$$\varepsilon_s(\eta, z) = \varepsilon_{f_s}(\eta = 0) + \eta^2 \varepsilon_{2s}(\eta = 0) \tag{48}$$

Next, using the paraxial like approximation the dielectric function can be expanded around the maximum of the HGLB. Thus, we get

$$\varepsilon_{f_s}(\eta = 0) = 1 - \frac{\omega_p^2}{\omega_s^2}\frac{1}{(1 + d_0)^{1/2}} \tag{49}$$

$$\varepsilon_2(\eta = 0) = -\frac{\omega_p^2}{2\omega_s^2}\frac{d_2}{(1 + d_0)^{3/2}} \tag{50}$$

The boundary conditions $df_s/dz = 0, f_s = 1$ and $S_c = 0$ at $z = 0$ is used for initially plane wave front. Using Eq. (44) and comparing the coefficients of η^2 one gets

$$\frac{\partial^2 f_s}{\partial \xi^2} = \frac{R_{d0}^2 f_s}{f_0^2}\left[\frac{1}{k_{s0}^2 r_0^4 f_0^2}\left\{3 + \left(\frac{r_0 f_0}{b_0 f_e}\right)^4\right\}\right] - \frac{R_{d0}^2 f_s}{2\omega_s^2 f_0^2}\frac{\omega_p^4}{k_{s0}^2 c^2}\frac{d_2}{(1 + d_0)^{3/2}} \tag{51}$$

where $R_{ds}^2 = k_{s0}^2\, b_0^2$ is the diffraction length of the scattered radiation. It is clear from Eq. (34) that the plasma wave is damped as it propagates along the z-axis. Consequently the scattered wave amplitude should also decrease with increasing z. Thus, the appropriate boundary condition would be

$$E_S = E_{S0}(r, z)e^{+ik_{s0}z} + E_{S1}(r, z)e^{-ik_{s1}z} = 0 \text{ at } Z = Z_c \tag{52}$$

If L is the interaction length then length Z_c is chosen ($Z_c = L - z$) sufficiently large such that n_{e00} is nearly zero. Substituting Z_c in Eq. (40), one gets

$$\frac{1}{b_0^2 f_s(Z_c)} = \frac{1}{r_0^2 f_0^2(Z_c)} + \frac{1}{a^2 f_e^2(Z_c)} \tag{53}$$

and

$$B_1 \approx -\frac{1}{2}\frac{\omega_p^2}{c^2}\frac{\omega_s}{\omega_0}\frac{f_s^2}{f_e^2}\left(\frac{b_0 f_s}{a f_e}\right)^{4m}\left(\frac{N_{e00}^2}{N_0}\right) E_{00}\exp(i\omega t - kz - S)\exp \times \frac{\left[-\left(\sqrt{2m} + \eta\right)^2 - 2k_i z\right]\exp[-(ik_0 S_0 + ik_{s1} Z_c)]}{\left\{k_{s1}^2 - k_{s0}^2 - (\omega_p^2/c^2)[1 - (1/\gamma)]\right\}\exp\left\{-\left(\sqrt{2m} + \eta\right)^2\left[\left(r_0^2 f_0^2\right)/\left(b_0^2 f_s^2\right)\right]\right\}\exp(ik_{s0} S_c + ik_{s0} Z_c)} \tag{54}$$

The back reflectivity $R = |E_S|^2/|E_{00}|^2$ is calculated by using the above set of equations

$$R = \frac{\left\{\left(B_1/2^{4m} f_s^2\right)\left(\sqrt{2m} + \eta\right)^{4m}\left(r_0 f_0/b_0 f_s\right)^{4m} \exp\left[-\left(\sqrt{2m} + \eta\right)^2\left(r_0^2 f_0^2/b_0^2 f_s^2\right)\right] \exp[-(ik_{s0} S_0 + ik_{s0} Z_c)] + (1/2)(\omega_p^2/c^2)(\omega_s f_s^2/\omega_0 f_e^2)(b_0 f_s/a f_e)^{2m} \left\{[E_{00}\exp(i\omega t - kz - S)]/[(k_{s1}^2 - k_{s0}^2) - (\omega_p^2/c^2)(1 - 1/\gamma)]\right\}\exp[-(ik_0 S_c + ik_{s1} Z_c)](n^*/n_0)\right\}^2}{(E_0^2/2^{2m} f_0^2)\left(\sqrt{2m} + \eta\right)^{4m}\exp\left[-\left(\sqrt{2m} + \eta\right)^2\right]} \tag{55}$$

The above equation expresses the reflectivity of back scattered beam. The SRS reflectivity can be calculated numerically using the relevant above set of equations. It is clear from the above equation that the reflectivity R depends upon the beam width parameter of incident pump wave, EPW, and scattered wave.

4. NUMERICAL RESULTS AND DISCUSSION

In the present work, the phenomena of SRS of HGLB have been studied considering relativistic nonlinearity in collisionless plasma. In the process the weak EPW propagating in the z-direction is observed to be nonlinearly coupled with the laser beam. The nonlinear coupling between these two waves leads to excitation of the EPW. This pump beam scattered by the EPW and back reflectivity has been calculated. In this process, first we see the propagation of HGLB and the focusing of beams depends upon dielectric function corresponding to the relativistic nonlinearity. The dielectric function is modified due to the presence of high power HGLB in the plasma. In the analysis all the relevant parameters have been expanded around the axis of maximum irradiance of the HGLB in paraxial like approach. Equations (18) and (19) show the intensity profile of HGLB in plasma along the radial direction in the presence of relativistic nonlinearity, while Eq. (25) determines the focusing/defocusing of the HGBs, along the distance of propagation in the plasma. The numerical calculation have been done with using following typical laser beam parameters: The HGLB power flux (10^{22} W cm^{-2}), $r_0 = 15$ μm, $\omega_0 = 1.776 \times 10^{14}$ rad s^{-1}, and plasma density $n = 0.35$ $n_{cr.}$, the electron thermal speed $v_{th} = 0.05c$. The boundary condition used here for an initial plane wave front of the laser beams, $f_0 = 1$, $df_0/d\xi = 0$, and $S_0 = 0$ at $\xi = 0$.

Further, Eq. (25) explains the beam dynamics in plasma with relativistic nonlinearity taken into account and decides the propagation behavior of HGLB inside the plasma. Equation (25) is nonlinear second order differential equations governing the normalized beam width parameter f_0 in the plasma. The first term on the right-hand side of Eq. (25) represents spatial dispersion and is responsible for diffractional divergence. On the other hand, second term is nonlinear in nature leading to self-focusing of the beam. Analytical solutions to these equations are not possible. Thus, we go for the numerical computational techniques to study beam dynamics. The variation of initial intensity of the HGLB for the order $m = 2$, has been shown in Figure 1a at $\xi = 0$. It is obvious from the figure that in paraxial regime the intensity of laser beam is maximum at $\eta = 0$. Similarly the laser profile has also been shown to the first focal point for $\xi = 0$ and the results is shown in the form of Figure 1b. Figure 2a illustrates the focusing of HGLB for the purely Gaussian mode ($m = 0$) of HGLB when relativistic nonlinearity is operative in the system. Figure 2a clearly displays the oscillatory self-focusing in the presence of relativistic nonlinearity, while Figure 2b gives the effect of relativistic nonlinearity on the propagation of HGLB for $m = 1$ mode. Figure 2c depicts the dynamics of laser beam for $m = 2$ mode and Figure 2d give the same for $m = 3$ mode of propagation of HGLB.

To observe the consequences of the propagation of laser beam and numerical appreciation of them, the excitation of EPW has been studied. It is clearly seen that the EPW excited due to the coupling between seed plasma wave and pump beam. Since the propagation of high power HGLB leads to

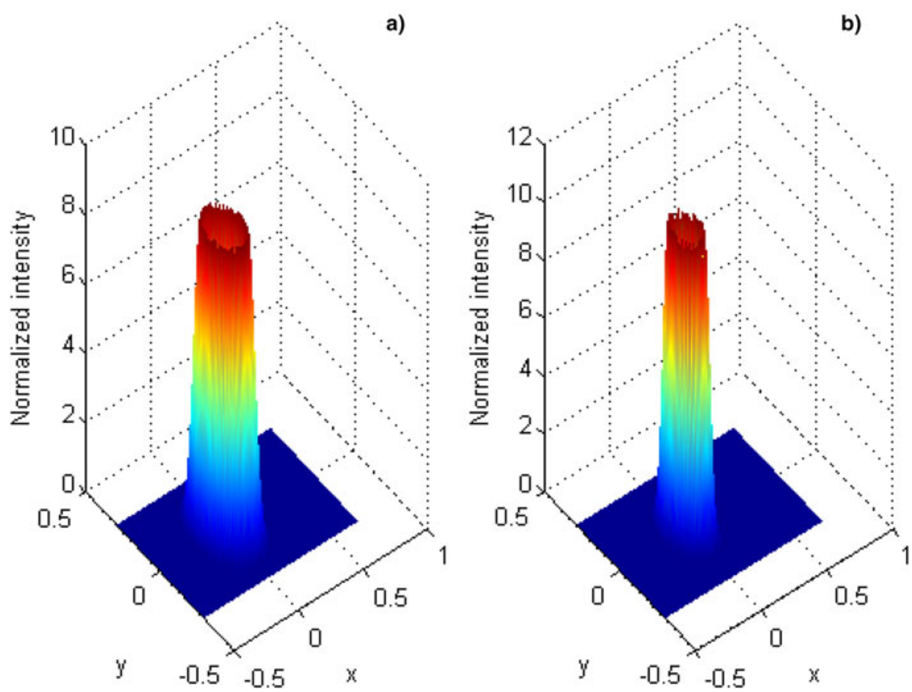

Fig. 1. Normalized intensity of HGLB for laser power $\alpha E_{00}^2 = 1.6$ when only relativistic nonlinearity is operative; (**a**) at $\xi = 0$, (**b**) at the first focal point.

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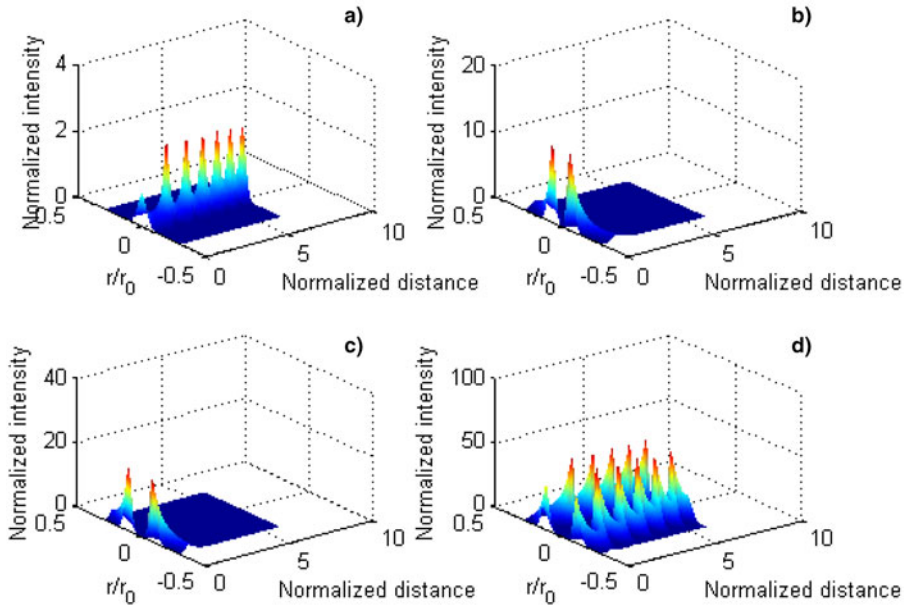

Fig. 2. Normalized intensity of HGLB with normalized distance ($\xi = z/R_{d0}$) for various order of propagation for laser power $\alpha E_{00}^2 = 1.6$ when only relativistic nonlinearity is operative; (**a**) $m = 0$, (**b**) $m = 1$, (**c**) $m = 2$, and (**d**) $m = 3$.

the modification of electron density in the plasma. In the present formulation the density profile of the EPW is governed by Eq. (34) with Eq. (35). The variation of initial intensity of the HGLB for the order $m = 2$ has been shown in Figure 3a at $\xi = 0$. It is obvious from the figure that in paraxial regime the intensity of laser beam is maximum at $\eta = 0$. Figure 3b expresses the EPW at first focal point. It is clear from Eq. (36) that the dynamics of EPW is influenced by the coupling term. The beam width parameter of EPW controls the intensity profile of the excited EPW. Therefore, Eq. (36) has been numerically calculated to observe the dynamics of the EPW for various orders of HGLB for the same chosen parameters as previously. The beam width parameter f_e of the EPW as a function of dimensionless distance of propagation has been computed with relativistic nonlinearity and the results has been presented in the form of Figure 4. Figure 4b–4d depicts the excited EPW for $m = 1$, 2, 3 modes of HGLB, respectively.

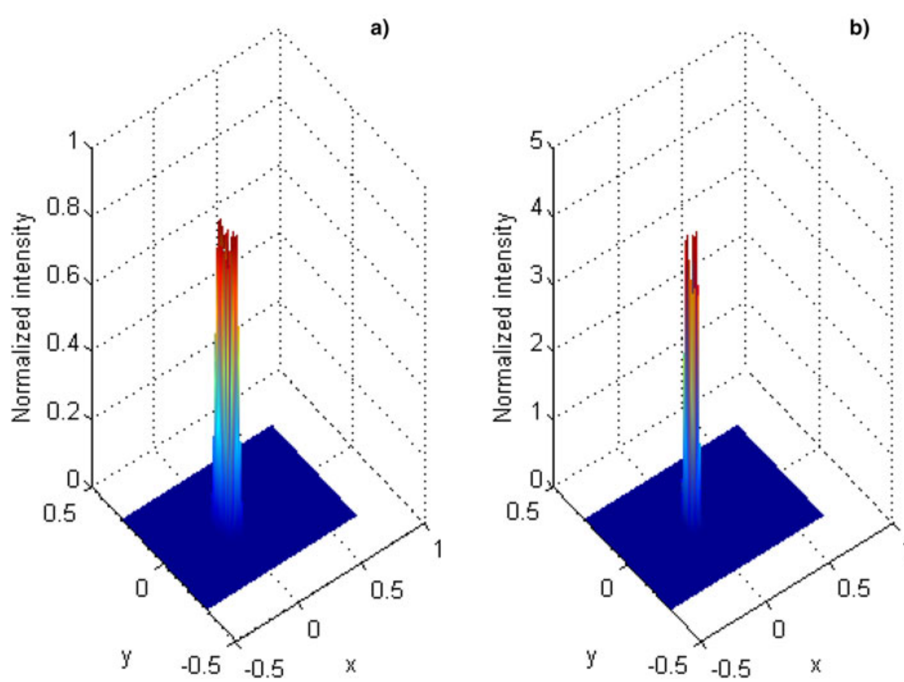

Fig. 3. Normalized intensity of EPW when relativistic nonlinearity is operative; (**a**) at $\xi = 0$, (**b**) at the first focal point.

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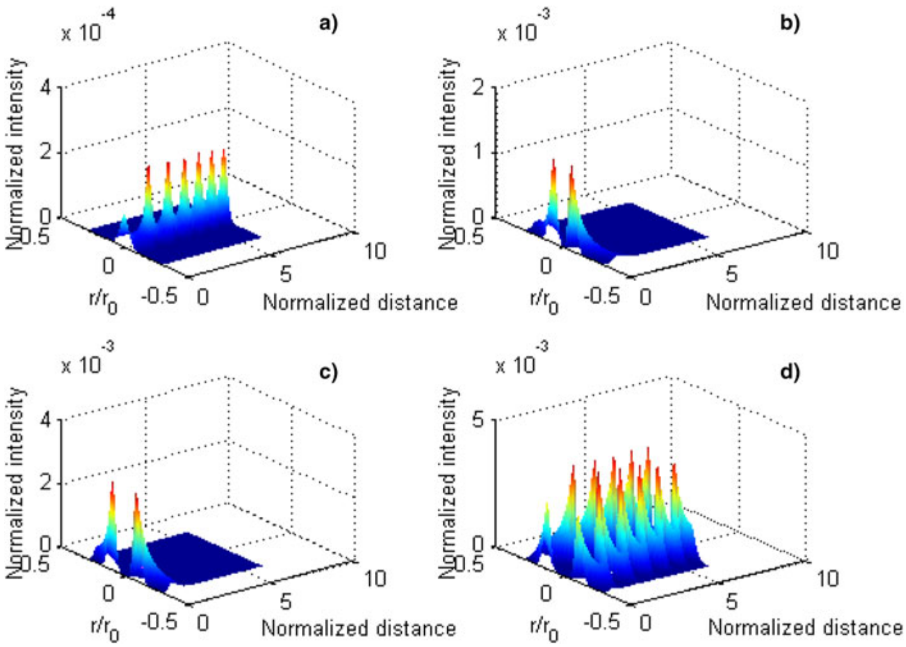

Fig. 4. Normalized intensity of EPW with normalized distance ($\xi = z/R_{d0}$) for various order of propagation for laser power $\alpha E_{00}^2 = 1.6$ when relativistic nonlinearity is operative; (**a**) $m = 0$, (**b**) $m = 1$, (**c**) $m = 2$, and (**d**) $m = 3$.

In addition, Eq. (51) shows that the beam width parameter of scattered HGLB depends upon the beam width parameter of EPW and incident HGLB. The beam width parameter f_s of the scattered beam as a function of dimensionless distance of propagation have been shown with relativistic nonlinearity by Eq. (51). Moreover, Eq. (55) gives the reflectivity against the distance of propagation. We have numerically solved Eq. (55) and the results are given in the form of Figure 5. Figure 5 clearly depicts the change in the back reflectivity with normalized distance for different modes of propagation of HGLB. The back reflectivity is displayed with normalized distance ξ for different order of HGLB. It is found that the reflectivity is higher at the points where the HGLB focuses. It is also shown that as we increase the order of the HGLBs the reflectivity decreases because the focusing of HGLBs decreases with increasing order of the beam.

In conclusion, the SRS of HGLB has been presented in the present work. It is shown that all the modes of propagation of HGLB contribute to the reflectivity of the propagating pump HGLB. In the scattering process of HGLB when the relativistic nonlinearity is taken into consideration the intensity profile is modified due to the presence of higher modes of propagation.

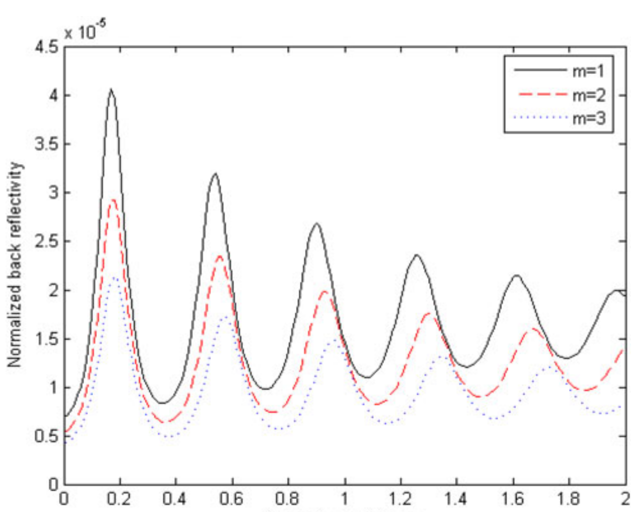

Fig. 5. Variation in normalized back scattered reflectivity of HGLB with normalized distance for various orders of propagation for $\alpha E_{00}^2 = 1.6$ when relativistic nonlinearity is operative.

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