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Redundant nonholonomic mechanical systems: characterization and control

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SUMMARY

This paper introduces the notion of kinematic redundancy in nonholonomic mechanical systems, identifies some of the interesting properties which result because of the presence of the redundancy, and initiates a study of the control and application of these systems. It is shown that kinematic redundancy in nonholonomic system can be exploited both to simplify the problem of controlling these systems and to enhance their performance capabilities. Moreover, it is demonstrated that these results can be obtained even in the presence of considerable uncertainty regarding the system model. The proposed ideas are illustrated through the study of three example systems: a space robot, a mobile manipulator, and a tractor-trailer system with two steering inputs (fire truck).

KEYWORDS: Kinematic redundancy; Nonholonomic systems; Adaptive control.

1. INTRODUCTION

There is great theoretical and practical interest in controlling mechanical systems in the presence of constraints on the realizable system motions. Systems for which the motion constraints are nonholonomic (nonintegrable) have been of particular interest recently. Much of the attention devoted to nonholonomic systems is a consequence of the importance of such systems in applications. For example, nonholonomic constraints arise in systems with rolling contact, such as wheeled (and other) mobile robots and multifingered robotic hands, and in systems for which the dynamics admits a symmetry, such as space systems with angular momentum conservation. Interest in studying nonholonomic systems is also motivated by the fact that for these systems traditional control methods are insufficient and new approaches must be developed.

Most of the work reported to date on controlling nonholonomic mechanical systems has focused on the *kinematic control* problem, in which it is assumed that the system velocities are the control inputs and that the system dynamics can be adequately represented using the system kinematic model. While this work has produced useful results, there are important reasons for formulating the nonholonomic system control problem at the *dynamic control* level, where the control inputs are those produced by the system actuators and the system model contains the mechanical system dynamics. For example, since this is the level at which control actually takes place in practice, designing controllers at this level can lead to significant improvements in performance and implementability and can help in the early identification and resolution of difficulties. Recognizing the importance of considering the nonholonomic system control problem at the dynamic control level, several researchers have considered this problem in recent years.[1–9] Progress has been made in understanding the fundamental characteristics of these systems and several useful dynamic controllers have been presented. Additionally, work has been initiated to address important practical issues, such as the effects of model uncertainty.

While research on the dynamic control of nonholonomic mechanical systems has produced valuable results, much work remains to be done. For instance, the transient performance of many nonholonomic system controllers is inadequate for practical applications, and effective, comprehensive solutions to such problems as collision avoidance, environmental interaction, sensor-based planning and control, and user interfacing have not yet been developed. Additionally, in many cases the control algorithms proposed for nonholonomic systems are very complicated and computationally expensive, which has limited the applicability of nonholonomic systems. In this paper we initiate an exploration of the extent to which various structural properties present in nonholonomic mechanical systems can be exploited to resolve some of these difficulties. Toward this end, we consider the notion of kinematic redundancy in nonholonomic systems. While the utility of kinematic redundancy in robotic manipulators is well known,[10] this property has not been studied in a systematic way for nonholonomic systems. We begin our investigation by introducing a definition of kinematic redundancy which seems both natural and useful when applied to nonholonomic mechanical systems. We then identify some of the properties which result because of the presence of the redundancy. Next we consider the problem of controlling these systems, and show that kinematic redundancy can be used both to simplify the control problem and to enhance system performance. Additionally, we demonstrate that these results can be obtained even in the presence of uncertainty regarding the system model.

To introduce the basic ideas in a concrete way and provide additional motivation for much of what follows, we close this section by considering three examples of nonholonomic systems which are clearly "redundant" in some sense. We will return to these examples as we develop an understanding of this notion of redundancy.

Example 1: Space robot. Consider the simple model of a "free-flying" space robot shown in Figure 1. The system is modeled as a rigid "space platform" with inertia J pinned to the ground at its center of mass, and a three link planar manipulator with link lengths $l_1=l_2=l_3=l$ and link masses $m_1=m_2=m_3=m$, assumed for simplicity to be concentrated at the distal ends of the three links. The manipulator has three actuators, one at each joint, while the platform's pinned connection to the ground is unactuated. Note that pinning the platform in this way permits it to rotate freely but prevents translation. Thus the nonholonomic constraints arising from angular momentum conservation is retained, while the holonomic constraints arising from linear momentum conservation in a truly "free-flying" system are replaced with holonomic pinned constraints; observe that this simplifies the subsequent analysis but removes none of the essential structure of the system. Background information on free-flying space robots is given in reference 11.

Let ϕ denote the angle of the platform, θ_1, θ_2, θ_3, be joint coordinates for the manipulator, and x, y represent the position of the tip of the manipulator relative to some fixed reference frame. For many applications with space robots, the task requirements involve placing the manipulator end-effector at a user-specified location while simultaneously controlling the orientation of the space platform. For the simple planar model considered here, this implies that for such tasks the configuration space of interest is three dimensional (with coordinates ϕ, x, y) and thus is a subset of the full four dimensional configuration space. This suggests the possibility that the "extra" degree-of-freedom (DOF) could be used to achieve some additional benefit, such as improved performance or reduced controller complexity.

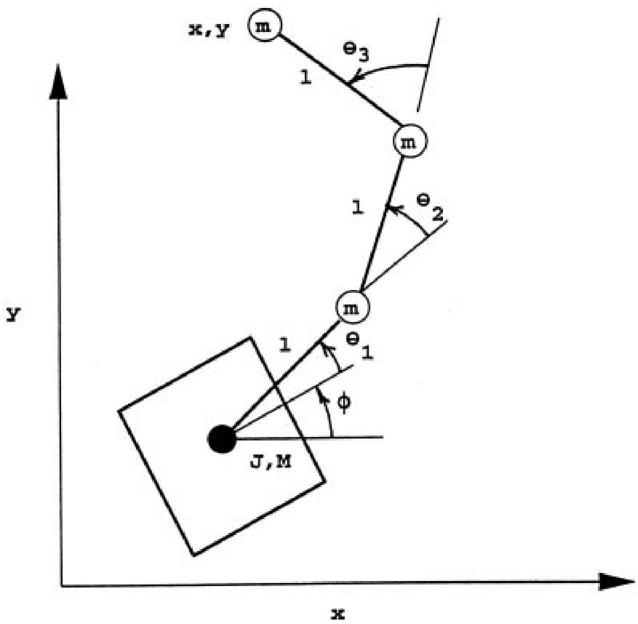

Fig. 1. Illustration of space robot.

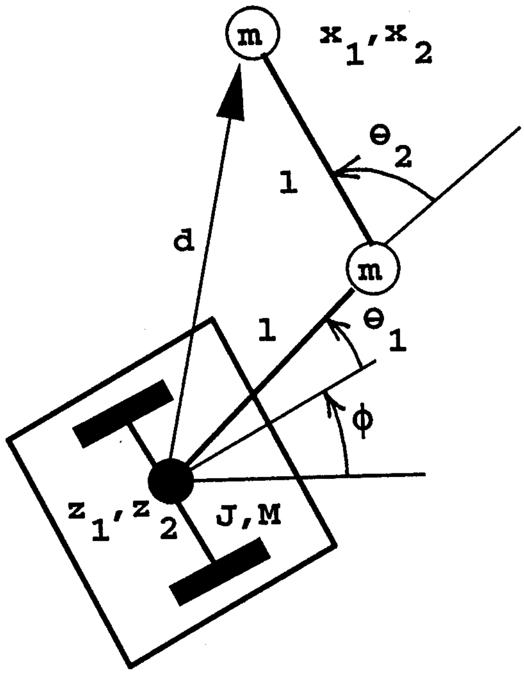

Fig. 2. Illustration of mobile manipulator.

Example 2: Mobile manipulator. Consider the simple mobile manipulator system obtained by mounting a two link planar arm on a two wheel mobile platform (see Figure 2). We remark that background material on mobile manipulators and their applications can be found in reference 12. Let z_1, z_2, ϕ denote the position and orientation coordinates for the (axle of the) mobile platform and M, J represent the platform inertial parameters. The two link planar manipulator has joint coordinates θ_1, θ_2, end-effector coordinates x_1, x_2, link lengths $l_1=l_2=l$, and link masses $m_1=m_2=m$, assumed for simplicity to be concentrated at the distal ends of the two links. The platform and the manipulator each have two actuators, so that the overall system possesses four actuators for its five DOF.

For many applications with mobile manipulators, the task requires that the manipulator end-effector be positioned at a given location and that the mobile platform be positioned in such a way that the arm is in a "good" configuration (e.g., far from arm singularities). For the simple planar model described above, it is clear that the arm can be maintained in a good configuration for a wide range of platform configurations. For example, to place the arm far from singularities imposes only one constraint on the three DOF of the platform. Thus, again, there is the possibility that the "extra" DOF could be used to achieve some additional benefit.

Example 3: Fire truck. Consider a tractor-trailer system with independent steering inputs at the front and rear axles (see Figure 3); this vehicle geometry is typical of a fire truck, and provides enhanced maneuverability for the system as a result of the extra steering input at the rear axle. Background material on fire truck systems is provided in reference 13. For this system x, y, θ_1 are the position and orientation coordinates for the rear axle of the tractor, θ_2 is the angle of the trailer, ϕ_1, ϕ_2 represent the front and rear

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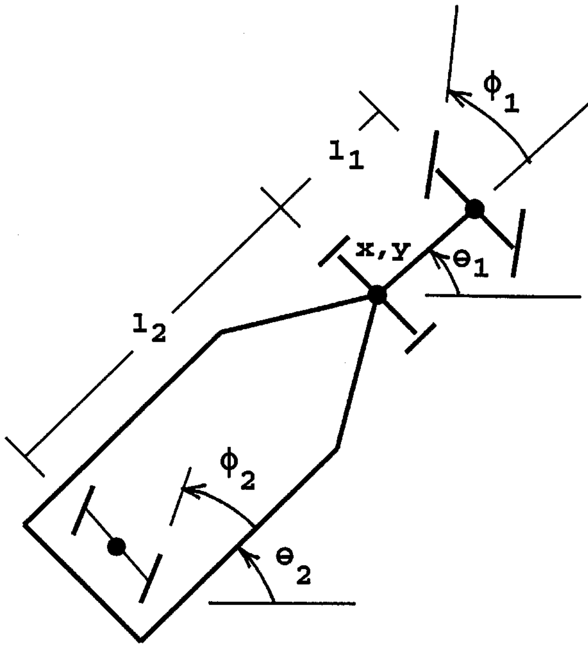

Fig. 3. Illustration of fire truck.

steering angles, respectively, I_{b_1}, I_{b_2}, I_{s_1}, I_{s_2}, m_1, m_2 are the system inertial parameters, and $l_1 = l_2 = l$ are the wheel base lengths. The fire truck is assumed to have two steering inputs and one drive input, so that the overall system possesses three inputs for its six DOF. Observe that the fire truck can be maneuvered without using the rear axle steering input (so that ϕ_2 is a constant); for example, setting $\phi_2 = 0$ yields the usual tractor-trailer system. Thus it can be seen that this system possesses a form of redundancy associated with its kinematics, and it is of interest to utilize this redundancy to improve the performance of the system.

2. REDUNDANT NONHOLONOMIC SYSTEM CHARACTERIZATION

The focus of this paper is the introduction of a notion of kinematic redundancy for nonholonomic mechanical systems and the initiation of a study of the control and application of these systems. We wish to work with nonholonomic systems arising from both explicit kinematic constraints and from symmetries of the system dynamics. Thus in this section we first develop models for these two classes of systems which are useful for characterizing redundancy in nonholonomic systems, and then establish a natural and useful definition of kinematic redundancy in terms of these models. Once this definition is in place we shall turn to the problem of controlling the motion of these systems.

Consider first the class of nonholonomic mechanical systems arising from the presence of explicit constraints on the system kinematics; these systems can be modeled as[1]

$$M(\mathbf{x})\mathbf{T} = H^*(\mathbf{x})\ddot{\mathbf{x}} + V^*_{cc}(\mathbf{x}, \dot{\mathbf{x}})\dot{\mathbf{x}} + \mathbf{G}^*(\mathbf{x}) + A^T(\mathbf{x})\lambda \qquad (1a)$$

$$A(\mathbf{x})\dot{\mathbf{x}} = \mathbf{0} \qquad (1b)$$

where $\mathbf{x} \in \Re^n$ is the vector of system generalized coordinates, $\mathbf{T} \in \Re^p$ is the vector of actuator inputs, $M: \Re^n \rightarrow \Re^{n \times p}$ is bounded and of full rank, $H^*: \Re^n \rightarrow \Re^{n \times n}$ is the system inertia matrix, $V^*_{cc}: \Re^n \times \Re^n \rightarrow \Re^{n \times n}$ quantifies Coriolis and centripetal acceleration effects, $\mathbf{G}^*: \Re^n \rightarrow \Re^n$ arises from the system potential energy, $A: \Re^n \rightarrow \Re^{m \times n}$ is a bounded full rank matrix quantifying the nonholonomic constraints, $\lambda \in \Re^m$ is the vector of constraint multipliers, and all functions are assumed to be smooth. It is well known that the mechanical system dynamics (1) possesses considerable structure. For example, for any set of generalized coordinates $\mathbf{x}$, the matrix H^* is symmetric and positive definite, the matrix V^*_{cc} depends linearly on $\dot{\mathbf{x}}$, and the matrices H^* and V^*_{cc} are related according to $\dot{H}^* = V^*_{cc} + V^{*T}_{cc}$.

It is useful for our subsequent development to rewrite the nonholonomic system model by employing a reduction procedure to decrease the dimension of the dynamics (1). Toward this end, observe that the assumption that A is full rank implies that the codistribution spanned by the rows of A has dimension m. The annihilator of this codistribution is then an $r = n - m$ dimensional smooth distribution $\Delta = \text{span}[\mathbf{r}_1(\mathbf{x}), \mathbf{r}_2(\mathbf{x}), \ldots, \mathbf{r}_r(\mathbf{x})]$, where the $\mathbf{r}_i$ are smooth vector fields on the configuration space which satisfy $A\mathbf{r}_i = 0$ $\forall \mathbf{x}$. Defining $R = [\mathbf{r}_1, \mathbf{r}_2, \ldots, \mathbf{r}_r] \in \Re^{n \times r}$ permits this relationship to be expressed more concisely as $AR = 0$. As an example, let the matrix A be partitioned as $A = [A_1 \; A_2]$, with $A_1 \in \Re^{m \times m}$ and $A_2 \in \Re^{m \times r}$ and where A_1 is nonsingular (this is always possible, possibly with a reordering of the configuration coordinates). Then R can be constructed as follows:

$$R = \begin{bmatrix} -A_1^{-1}A_2 \\ I_r \end{bmatrix} \qquad (2)$$

where I_r is the $r \times r$ identity matrix. Defining a partition of $\mathbf{x}$ corresponding to the partition specified for A, so that $\mathbf{x} = [\mathbf{x}_1^T \; \mathbf{x}_2^T]^T$ with $\mathbf{x}_1 \in \Re^m$ and $\mathbf{x}_2 \in \Re^r$, permits the system velocities to be determined by $\dot{\mathbf{x}}_2$ via $\dot{\mathbf{x}} = R(\mathbf{x})\dot{\mathbf{x}}_2$. This parameterization then allows (1) to be reformulated as

$$\dot{\mathbf{x}}_1 = A^*(\mathbf{x})\dot{\mathbf{x}}_2 \qquad (3a)$$

$$\mathbf{F} = H(\mathbf{x})\ddot{\mathbf{x}}_2 + V_{cc}(\mathbf{x}, \dot{\mathbf{x}}_2)\dot{\mathbf{x}}_2 + \mathbf{G}(\mathbf{x}) \qquad (3b)$$

where $A^* = -A_1^{-1}A_2$, $\mathbf{F} = R^T M \mathbf{T}$, $H = R^T H^* R$, $V_{cc} = R^T(H^*\dot{R} + V^*_{cc}R)$, and $\mathbf{G} = R^T \mathbf{G}^*$. In what follows, it is assumed that $p \geqslant r$ and $R^T M$ is full rank, so that any desired $\mathbf{F}$ can be realized through proper specification of $\mathbf{T}$ and the system (3*b*) is fully actuated. Additionally, we suppose that the involutive closure of Δ (defined as the smallest involutive distribution containing Δ) has constant rank n on the configuration space, so that the constraints are nonintegrable and the system (1) is (completely) nonholonomic.[14]

Note that (3) consists of a “reduced” dynamic model (3*b*), which defines the evolution of the “reducing outputs” $\mathbf{x}_2$, together with a purely kinematic relationship (3*a*). Therefore the representation (3) provides a simpler description of the nonholonomic mechanical system than that given in (1). Moreover, as shown in the next lemma, the dynamics (3*b*) retains much of the mechanical system structure of the original system (1).

Lemma 1: The dynamic model terms H, $\mathbf{G}$ are bounded functions of $\mathbf{x}$ whose time derivatives $\dot{H}$, $\dot{\mathbf{G}}$ are also bounded

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in $\mathbf{x}$ and depend linearly on $\dot{\mathbf{x}}_2$, the matrix H is symmetric and positive definite, and the matrices H and V_{cc} are related according to $\dot{H}=V_{cc}+V_{cc}^T$.

Proof: The proof is given in reference 9. ■

We now turn our attention to those nonholonomic mechanical systems which arise because of the presence of a symmetry in the system dynamics. More specifically, consider the class of mechanical systems for which the system Lagrangian is G-invariant for some Lie group G,[15] and suppose for concreteness that $G=SO(2)$. By decomposing the configuration space into irreducible representations of $SO(2)$, it is always possible to choose (local) configuration coordinates $\mathbf{x}=[\mathbf{x}_1^T\ \mathbf{x}_2^T]^T$ for this system in such a way that the elements of $\mathbf{x}_2\in\Re^r$ are coordinates for $\Re^n/G$ and the elements of $\mathbf{x}_1\in\Re^m$ are the coordinates which transform nontrivially under G. Selecting coordinates in this way permits the G-invariant system Lagrangian to be written in the form $L(\mathbf{x},\dot{\mathbf{x}})=\dot{\mathbf{x}}^T H^*(\mathbf{x}_2)\dot{\mathbf{x}}/2-U(\mathbf{x}_2)$ for some potential U and inertia matrix

$$H^*=\begin{bmatrix} J_1(\mathbf{x}_2) & Q(\mathbf{x}_2)\\ Q^{\mathrm{T}}(\mathbf{x}_2) & J_2(\mathbf{x}_2)\end{bmatrix} \tag{4}$$

with submatrices J_1, J_2, Q which are independent of $\mathbf{x}_1$.

The fact that L is independent of $\mathbf{x}_1$ means that the Euler-Lagrange equations corresponding to the $\mathbf{x}_1$ coordinates have the character of a velocity constraint:

$$\frac{\partial L}{\partial \dot{\mathbf{x}}_1}=J_1(\mathbf{x}_2)\dot{\mathbf{x}}_1+Q(\mathbf{x}_2)\dot{\mathbf{x}}_2=\mathbf{0} \tag{5}$$

where it is assumed that the system is initially at rest. Observe that (5) can be used to parameterize the system velocities via $\dot{\mathbf{x}}=R(\mathbf{x}_2)\dot{\mathbf{x}}_2$, with R defined as

$$R=\begin{bmatrix} -J_1^{-1}Q\\ I_r\end{bmatrix} \tag{6}$$

We again assume that the smallest involutive distribution containing the span of the columns of R has constant rank n, in which case the constraints (5) are nonintegrable and the system is nonholonomic.

Now an analysis which exactly parallels the one given above for kinematic nonholonomic systems can be applied to reduce the original $2n$ order mechanical system to a $2r$ order mechanical system together with m kinematic equations:

$$\dot{\mathbf{x}}_1=A^{**}(\mathbf{x}_2)\dot{\mathbf{x}}_2 \tag{7a}$$

$$\mathbf{F}=H(\mathbf{x}_2)\ddot{\mathbf{x}}_2+V_{cc}(\mathbf{x}_2,\dot{\mathbf{x}}_2)\dot{\mathbf{x}}_2+\mathbf{G}(\mathbf{x}_2) \tag{7b}$$

where $A^{**}=-J_1^{-1}Q$ and $\mathbf{F}=B(\mathbf{x}_2)\mathbf{T}$ for some matrix $B\in\Re^{r\times p}$ which depends only on $\mathbf{x}_2$ (because it is supposed that the inputs do not break the system symmetry). It is assumed that $p\geqslant r$ and B is full rank, so that any desired $\mathbf{F}$ can be realized through proper specification of $\mathbf{T}$ and the system (7b) is fully actuated. Note that (7b) is a $2r$ order differential equation which defines the evolution of the $2r$ states $(\mathbf{x}_2,\dot{\mathbf{x}}_2)$, and that the behavior of the remaining configuration coordinates $\mathbf{x}_1$ is completely determined by the kinematic relationship (7a). Moreover, an analysis virtually identical to the one summarized in Lemma 1 can be used to show that the reduced system (7b) retains the mechanical system structure of the original system.

Reduced representations have now been obtained for both kinematic and symmetric noholonomic mechanical systems. Examination of these reduced models reveals that the model (3) contains the model (7) as a special case (corresponding to the situation in which the Lagrangian $L(\mathbf{x},\dot{\mathbf{x}})$, distribution Δ, and input matrix are independent of $\mathbf{x}_1$). Thus, in what follows, we focus on the nonholonomic mechanical system (3), with the implicit understanding that all results apply to the system (7) as well.

The reduced representations obtained above for kinematic and symmetric nonholonomic mechanical systems can be employed to develop a useful and natural definition for kinematic redundancy in these systems. We wish to provide a definition for redundancy which captures the idea that redundant systems possess "extra" DOF beyond those required to satisfy the given task objectives. Note, however, that it may not be appropriate to treat all of the system DOF equally. Indeed, from a motion control perspective, there is a fundamental difference between the $\mathbf{x}_1$ and $\mathbf{x}_2$ coordinates for the system (3). More specifically, the evolution of the reducing outputs $\mathbf{x}_2$ can be controlled directly using the input $\mathbf{F}$, since (3b) is a fully actuated mechanical system, while the behavior of $\mathbf{x}_1$ can be influenced only indirectly through the control of $\mathbf{x}_2$. One consequence of this structural property of the system is that it is much harder to control $\mathbf{x}_1$ than $\mathbf{x}_2$; in fact, much of the difficulty in controlling nonholonomic systems is related to the problem of specifying, and then tracking, a desired trajectory $\mathbf{x}_{2d}$ for $\mathbf{x}_2$ which causes *both* $\mathbf{x}_1$ and $\mathbf{x}_2$ to evolve as desired.

These considerations lead us to a definition of redundancy that focuses on systems with extra DOF in the reduced space. More precisely, we have the following:

Definition: The nonholonomic system (3) is *kinematically redundant* for a given task if the task can be completed using only a subset $\mathbf{x}_{21}\in\Re^{r_1}$ of the reducing outputs $\mathbf{x}_2=[\mathbf{x}_{21}^T\ \mathbf{x}_{22}^T]^T$, where $\mathbf{x}_{22}\in\Re^{r_2}$ and $r_1+r_2=r$.

Implicit in this definition is the assumption that all of the coordinates $\mathbf{x}_1$ are relevant for the given task. Note that, if this is not the case, the control problem can be simplified by ignoring those elements of $\mathbf{x}_1$ which are unimportant for the task.

3. REDUNDANT NONHOLONOMIC SYSTEM CONTROL

In this section we present a class of motion control algorithms for nonholonomic mechanical systems which are redundant according to the definition given above. It is shown that the available redundancy can be effectively exploited both to simplify the control of these systems and to enhance their performance capabilities. Moreover, it is demonstrated that these results can be obtained in the presence of uncertainty regarding the system dynamic model and the nonholonomic constraints.

3.1 Preliminaries

The definition for kinematic redundancy given above is motivated, in part, by the claim that the reducing outputs $\mathbf{x}_2$ can be controlled directly since (3*b*) is fully actuated. This, in turn, implies that the redundant reducing outputs $\mathbf{x}_{22}$ can be conveniently utilized to improve performance or simplify the control problem. Thus we begin our discussion of redundant system control by exhibiting two algorithms for controlling the reduced system (3*b*), one for position tracking and one for velocity tracking. These algorithms will be utilized as subsystems in the complete redundant nonholonomic system controllers proposed later in the paper.

Recall that (3*b*) is shown in Lemma 1 to inherit all of the "nice" mechanical system structure of the original system. As a consequence, motion control for this reduced system can be accomplished using the *performance-based adaptive control* methodology recently proposed by the authors.[16,17] Thus, for example, the following adaptive control scheme can be used to track any desired *position* trajectory $\mathbf{x}_{2d}(t)$ for $\mathbf{x}_2$ without rate measurements or knowledge of the system dynamic model:

$$\begin{aligned}\mathbf{F}&=A(t)\ddot{\mathbf{x}}_{2d}+B(t)\dot{\mathbf{x}}_{2d}+\mathbf{f}(t)+k_1\gamma^2\mathbf{w}+k_2\gamma^2\mathbf{e}\\ \dot{\mathbf{w}}&=-2\gamma\mathbf{w}+\gamma^2\dot{\mathbf{e}}\end{aligned} \tag{8}$$

where $\mathbf{e}=\mathbf{x}_{2d}-\mathbf{x}_2$ is the trajectory tracking error, $\mathbf{w}$ provides a means of injecting damping into the closed-loop system without using rate measurements, k_1, k_2, γ are positive scalar constants, and $\mathbf{f}(t)\in\Re^r$, $A(t)\in\Re^{r\times r}$, $B(t)\in\Re^{r\times r}$ are (feedforward) adaptive gains which are adjusted according to the following simple update laws:

$$\begin{aligned}\dot{\mathbf{f}}&=-\sigma_1\mathbf{f}+\beta_1\mathbf{q}\\ \dot{A}&=-\sigma_2A+\beta_2\mathbf{q}\ddot{\mathbf{x}}_{2d}^T\\ \dot{B}&=-\sigma_3B+\beta_3\mathbf{q}\dot{\mathbf{x}}_{2d}^T\end{aligned} \tag{9}$$

where $\mathbf{q}=\dot{\mathbf{e}}+k_2\mathbf{e}/k_1\gamma-\mathbf{w}/\gamma$ represents a weighted and filtered error term and the σ_i and β_i are positive scalar adaptation gains.

The suitability of the position trajectory tracking controller (8), (9) is indicated by the following lemma.

Lemma 2: The adaptive controller (8), (9) ensures that (3*b*) evolves in such a way that $\mathbf{e}$, $\dot{\mathbf{e}}$, $\mathbf{w}$, $\mathbf{f}$, A, B are semiglobally uniformly bounded and the tracking error $\mathbf{e}$, $\dot{\mathbf{e}}$ converges exponentially to a neighborhood of the origin which can be made arbitrarily small.

Proof: The proof follows immediately from the proof of Theorem 2 in reference 17 once it is observed that (3*b*) is a fully actuated mechanical system. ■

In a similar manner the performance-based adaptive control methodology can be used to derive a scheme for tracking any desired *velocity* trajectory $\mathbf{v}_{2d}(t)$ for $\mathbf{v}_2=\dot{\mathbf{x}}_2$ without knowledge of the system dynamic model. More specifically, consider the velocity tracking scheme

$$\begin{aligned}\mathbf{F}&=C(t)\dot{\mathbf{v}}_{2d}+D(t)[\mathbf{v}_2\mathbf{v}_{2d}]+\mathbf{g}(t)+k\mathbf{e}_\nu\\ \dot{\mathbf{g}}&=-\sigma_4\mathbf{g}+\beta_4\mathbf{e}_\nu\\ \dot{C}&=-\sigma_5C+\beta_5\mathbf{e}_\nu\dot{\mathbf{v}}_{2d}^T\\ \dot{D}&=-\sigma_6D+\beta_6\mathbf{e}_\nu[\mathbf{v}_2\mathbf{v}_{2d}]^T\end{aligned} \tag{10}$$

where $\mathbf{e}_\nu=\mathbf{v}_{2d}-\mathbf{v}_2$ is the velocity tracking error, $\mathbf{g}(t)\in\Re^r$ is an adaptive auxiliary signal, $C(t)\in\Re^{r\times r}$, $D(t)\in\Re^{r\times r^2}$ are adaptive feedforward elements, k is a positive constant, the σ_i and β_i are positive scalar adaptation gains, and the notation $[\mathbf{uw}]=[u_1w_1,\ \ldots,\ u_1w_r,\ u_2w_1,\ \ldots,\ u_rw_r]^T\in\Re^{r^2}$ is introduced for convenience. The stability properties of the proposed velocity tracker are summarized in the following lemma.

Lemma 3: The adaptive controller (10) ensures that $\mathbf{e}_\nu$, $\mathbf{g}$, C, and D are globally uniformly bounded and that the velocity tracking error $\mathbf{e}_\nu$ converges exponentially to a compact set which can be made arbitrarily small.

Proof: The proof follows immediately from the proof of Lemma 3 in reference 18 once it is observed that (3*b*) is a fully actuated mechanical system. ■

3.2 Motion control

We are now in a position to address the problem of controlling the motion of the redundant nonholonomic mechanical system (3). Our approach is to consider three subclasses of the general system (3), each of which possesses certain structural properties which simplify the redundancy utilization problem, and then provide a control method for each subclass. Note that, in view of the availability of the position tracking scheme (8), (9) and the velocity tracker (10) given above, it can be assumed that any desired trajectory for the reducing coordinates $\mathbf{x}_2$ or velocities $\dot{\mathbf{x}}_2$ can be accurately followed. Thus it is seen that the challenge is to utilize the available redundancy to improve the control of the $\mathbf{x}_1$ coordinates in some way, and we are led to focus our attention on the kinematic system (3*a*) in much of what follows.

To assist in the identification of useful subclasses of systems for subsequent study, let us rewrite the kinematic system (3*a*) as

$$\dot{\mathbf{x}}_{11}=A_3(\mathbf{x})\dot{\mathbf{x}}_{21}+A_4(\mathbf{x})\dot{\mathbf{x}}_{22} \tag{11a}$$

$$\dot{\mathbf{x}}_{12}=A_5(\mathbf{x})\dot{\mathbf{x}}_{21}+A_6(\mathbf{x})\dot{\mathbf{x}}_{22} \tag{11b}$$

In (11), the A_i are appropriate submatrices of A^*, $\mathbf{x}_1$ is partitioned as $\mathbf{x}_1=[\mathbf{x}_{11}^T\ \mathbf{x}_{12}^T]^T$, and the vectors $\mathbf{x}_{11}\in\Re^{r_2}$ and $\mathbf{x}_{12}\in\Re^{r_3}$ are chosen in such a way that $r_2+r_3=m$ and A_4 is nonsingular in the region of interest (such a partition is always possible, possibly with a reordering of the configuration coordinates).

Consider first the subclass of nonholonomic systems for which $A_5=A_6=0$ in (11). In this case the system possesses the same number of redundant reducing outputs $\mathbf{x}_{22}$ as "vertical" coordinates $\mathbf{x}_{11}=\mathbf{x}_1$ (so that $r_2=m$ and $r_3=0$). It should be noted that such systems are important in applications; for instance, the systems given in examples 1 and 2 belong to this class. In this situation, it turns out that it is possible to eliminate the nonholonomic nature of the control problem altogether, so that motion control is straightforward to accomplish and the available redundancy can be readily utilized to achieve a wide range of performance objectives.

Toward this end, observe first that in many applications

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the performance enhancement objectives can be formulated as a trajectory tracking problem for the (nonredundant) reducing outputs $\mathbf{x}_{21}$ and vertical coordinates $\mathbf{x}_1$ (see, for example, reference 10 for an analogous discussion for redundant manipulator systems). Now for nonredundant nonholonomic systems it is not possible to simultaneously track arbitrary trajectories for the reducing and vertical coordinates, because arbitrarily chosen trajectories will in general violate the nonholonomic constraints. In the present case, however, the presence of the redundant reducing outputs $\mathbf{x}_{22}$ can be used in a straightforward manner to permit any desired trajectories for $\mathbf{x}_{21}$ and $\mathbf{x}_1$ to be followed. More precisely, suppose that the desired evolution of $\dot{\mathbf{x}}_{22}$, denoted $\dot{\mathbf{x}}_{22d}$, is specified as follows:

$$\dot{\mathbf{x}}_{22d} = A_4^{-1}(\dot{\mathbf{x}}_{1d} - A_3\dot{\mathbf{x}}_{21d})$$

where $\mathbf{x}_{1d}$ and $\mathbf{x}_{21d}$ are the desired trajectories for $\mathbf{x}_1$ and $\mathbf{x}_{21}$, respectively. Then, if the desired velocity trajectories $\dot{\mathbf{x}}_{21d}$, $\dot{\mathbf{x}}_{22d}$ for the reducing outputs $\dot{\mathbf{x}}_{21}$, $\dot{\mathbf{x}}_{22}$ are accurately tracked (using the adaptive scheme (10), for example), it can be seen that $\mathbf{x}_{21}$ and $\mathbf{x}_1$ will evolve as desired despite the non-holonomic constraints. Indeed, in this case it can be seen that the control problem is no longer nonholonomic, so that control algorithm simplification and performance enhancement are achieved simultaneously. This approach to nonholonomic system control is illustrated in the following section through simulations with the systems in examples 1 and 2. It should be noted that the performance obtainable using the above strategy can be improved by introducing a "closed-loop" component to the equation defining $\dot{\mathbf{x}}_{22d}$:

$$\dot{\mathbf{x}}_{22d} = A_4^{-1}(\dot{\mathbf{x}}_{1d} + k_3(\mathbf{x}_{1d} - \mathbf{x}_1) - A_3\dot{\mathbf{x}}_{21d}) \tag{12}$$

where k_3 is a positive constant. It is easy to show that this "kinematic controller" guarantees accurate tracking of the desired vertical trajectory $\mathbf{x}_{1d}$ provided that the desired reducing output trajectories $\mathbf{x}_{21d}$ and $\mathbf{x}_{22d}$ are accurately tracked (using an analysis similar to that given in reference 19, for example); recall that accurate tracking of the reducing output trajectories can be achieved because these coordinates are fully actuated.

While the subclass of redundant nonholonomic systems identified above is important in applications, it is clear that not all nonholonomic systems have as many redundant reducing coordinates as vertical coordinates. For example, it will be shown that the fire truck in example 3 has three vertical coordinates and only one redundant reducing output. Thus we are led to examine more general situations. Consider again the kinematic system (11), but now suppose that $A_6=0$ and A_5 is nonzero but independent of $\mathbf{x}_{22}$. In this case the system possesses more vertical coordinates $\mathbf{x}_1$ than redundant reducing outputs $\mathbf{x}_{22}$, but the evolution of $\mathbf{x}_{22}$ only influences a subset $\mathbf{x}_{11}$ of the vertical coordinates $\mathbf{x}_1$. This class of systems is also important in applications; for instance, example 3 belongs to this class.

As might be expected, in this situation it is not possible to achieve as much as with the first class of systems identified above. However, in this case it is still possible to control the system in such a way that $\mathbf{x}_{21}$ and $\mathbf{x}_{12}$ are driven to their desired final values and $\mathbf{x}_{11}$ is made to track any user-specified trajectory. This latter trajectory tracking capability can be used to improve overall system performance. One method of achieving this control objective is given in the following algorithm:

(i) For the system $\dot{\mathbf{x}}_{12} = A_5(\mathbf{x})\dot{\mathbf{x}}_{21}$ (i.e., (11*b*) with $A_6=0$), utilize nonholonomic control methods to determine a trajectory for $\dot{\mathbf{x}}_{21}$ which would drive both $\mathbf{x}_{21}$ and $\mathbf{x}_{12}$ to their desired values (this is always possible because the original system is controllable).

(ii) Given the desired evolution for $\mathbf{x}_{11d}$, generate a desired trajectory for $\dot{\mathbf{x}}_{22d}$ using

$$\dot{\mathbf{x}}_{22d} = A_4^{-1}(\dot{\mathbf{x}}_{11d} + k_3(\mathbf{x}_{11d} - \mathbf{x}_{11}) - A_3\dot{\mathbf{x}}_{21}) \tag{13}$$

(iii) Track the desired velocity trajectories for $\dot{\mathbf{x}}_{21}$ and $\dot{\mathbf{x}}_{22}$ simultaneously using the adaptive scheme (10).

This approach to nonholonomic system control is illustrated in the following section through simulations with the fire truck in example 3.

Finally, we turn to our third subclass of redundant nonholonomic systems. Consider the class of systems for which there is a time scale separation between the task to be completed by the $\mathbf{x}_{21}$ coordinates and the task which defines the evolution of all of the other coordinates. More precisely, suppose that the task associated with the $\mathbf{x}_{21}$ outputs must be performed quickly, while the task defined in terms of $\mathbf{x}_1$ and $\mathbf{x}_{22}$ can be completed more slowly. This situation is very common in applications. For instance, many tasks involving mobile manipulators (as in example 2) belong to this class. A common objective with mobile manipulators is to obtain the performance of a high bandwidth manipulator with a large workspace, and this goal can be realized if the end-effector of the manipulator can be positioned quickly even while the platform is positioned more slowly. Observe that this is the perspective often taken with "compound" robotic manipulators.[20] One approach to controlling such systems is to regard *all* of the elements of $\mathbf{x}_1$ and $\mathbf{x}_{22}$ to be redundant, so that the primary concern is with $\mathbf{x}_{21}$ and the motion control of the remaining coordinates is viewed as a secondary task. Since the $\mathbf{x}_{21}$ coordinates are fully actuated it is easy to control them quickly, using (8), (9) for example, and the other coordinates can then be driven to their goal configuration more slowly. More specifically, we propose the following algorithm:

(i) Quickly drive $\mathbf{x}_{21}$ to the goal $\mathbf{x}_{21d}$ using the adaptive tracking scheme (8), (9).

(ii) Hold $\mathbf{x}_{21}$ at $\mathbf{x}_{21d}$ (using (8), (9)) and simultaneously drive $\mathbf{x}_1$ and $\mathbf{x}_{22}$ to appropriate values. The motion control for $\mathbf{x}_1$ and $\mathbf{x}_{22}$ can be realized using the stabilization strategy proposed by the authors in reference 18, for example.

Note that even if the system is controllable it may not be possible to control $\mathbf{x}_1$ and $\mathbf{x}_{22}$ to their desired values while holding $\mathbf{x}_{21}$ at $\mathbf{x}_{21d}$, because this latter requirement can be a strong constraint. However, in those cases in which the task is achievable in this way the proposed algorithm has the desired effect of ensuring that the system is driven to the goal and the "important" coordinates are controlled more quickly than the "unimportant" ones.

3.3 Compensation for kinematic uncertainty

The control strategies summarized above provide methods for controlling the motion of certain classes of redundant nonholonomic mechanical systems. These schemes can be used both to simplify the control problem and to enhance system performance. Moreover, the algorithms can be implemented despite the presence of uncertainty regarding the mechanical system dynamics (3*b*). Note, however, that implicit in the development of these control strategies is the assumption that there is no uncertainty associated with the kinematic relationship (3*a*) (or, equivalently).[11] This can be a reasonable assumption for many nonholonomic systems but is certainly not always the case. Consider, for example, the situation in which the nonholonomic constraints are a consequence of a symmetry of the system dynamics. In this case the kinematic constraint (11) depends on the inertial parameters of the system, and it is often desirable to permit these parameters to be uncertain. In view of this observation, we now briefly consider the situation in which there is uncertainty in the constraint (11). Here we concentrate on the first subclass of redundant nonholonomic systems discussed above, in which $A_5 = A_6 = 0$, and present a method of compensating for kinematic uncertainty for this case; similar methods can be developed for the other cases.[21]

Consider the kinematic system (11) with $A_5 = A_6 = 0$; in this case $\mathbf{x}_{11} = \mathbf{x}_1$ and the system model (11) can be written

$$\dot{\mathbf{x}}_1 = A_3(\mathbf{x}, \mathbf{p})\dot{\mathbf{x}}_{21} + A_4(\mathbf{x}, \mathbf{p})\dot{\mathbf{x}}_{22} \tag{14}$$

where we have made explicit the dependence of the kinematic system on a vector of system parameters $\mathbf{p}$. Let $\hat{\mathbf{p}}$ denote an estimate for $\mathbf{p}$ and define the associated *predicted value* of $\dot{\mathbf{x}}_1$, denoted $\hat{\dot{\mathbf{x}}}_1$, as follows:

$$\hat{\dot{\mathbf{x}}}_1 = \hat{A}_3\dot{\mathbf{x}}_{21} + \hat{A}_4\dot{\mathbf{x}}_{22} \tag{15}$$

where the notation $\hat{A}_i = A_i(\mathbf{x}, \hat{\mathbf{p}})$ is introduced. In what follows it is supposed that the matrix $\hat{A}_4$ is invertible for all values of the parameter estimate $\hat{\mathbf{p}}$ (recall that it is invertible for $\hat{\mathbf{p}} = \mathbf{p}$); this can be ensured by restricting the possible values for this estimate, for example. In this case, application of the kinematic control scheme

$$\dot{\mathbf{x}}_{22d} = \hat{A}_4^{-1}(\dot{\mathbf{x}}_{1d} + k_3(\mathbf{x}_{1d} - \mathbf{x}_1) - \hat{A}_3\dot{\mathbf{x}}_{21}) \tag{16}$$

to the system (15) leads to the result

$$\dot{\mathbf{e}}_1 + k_3\mathbf{e}_1 = \hat{\dot{\mathbf{x}}}_1 - \dot{\mathbf{x}}_1 \tag{17}$$

where $\mathbf{e}_1 = \mathbf{x}_{1d} - \mathbf{x}_1$. Observe that, since $\hat{\dot{\mathbf{x}}}_1 \neq \dot{\mathbf{x}}_1$ in general (because $\hat{\mathbf{p}} \neq \mathbf{p}$), this approach will not achieve the desired objective of accurate tracking of $\mathbf{x}_{1d}$ by $\mathbf{x}_1$ (note, of course, that any given $\mathbf{x}_{21d}$ can be accurately followed because $\mathbf{x}_{21}$ is directly actuated). However, we can use the error associated with this basic method to improve our estimate for $\mathbf{p}$ and thereby asymptotically realize this trajectory tracking goal.

Toward this end, observe that any uncertainty in the kinematic constraints which is associated with *inertial* parameters can always be linearly parameterized by slightly modifying (14) as follows:

$$\dot{\mathbf{x}}_1 = W(\mathbf{x}, \dot{\mathbf{x}})\mathbf{p} \tag{18}$$

where the matrix W is a known function. Indeed, the existence of such a linear parameterization is a direct consequence of the well-known property that the inertial parameters appear linearly in the dynamics of mechanical systems. It should be mentioned that this linear parameterization property also holds for many systems with uncertain *kinematic*, rather than inertial, parameters. As this property of (14) is obviously inherited by the prediction system (15), we can also rewrite that kinematic model in the same way:

$$\hat{\dot{\mathbf{x}}}_1 = W(\mathbf{x}, \dot{\mathbf{x}})\hat{\mathbf{p}} \tag{19}$$

The models (18) and (19) can be used to develop an algorithm for updating our estimate $\hat{\mathbf{p}}$ for the parameter vector $\mathbf{p}$. Consider the following simple estimation scheme:

$$\dot{\hat{\mathbf{p}}} = \alpha W^T\mathbf{E} \tag{20}$$

where $\mathbf{E} = \dot{\mathbf{x}}_1 - \hat{\dot{\mathbf{x}}}_1$ and α is a positive constant. If this estimation strategy is combined with the kinematic controller (16), then accurate tracking of the desired vertical trajectory $\mathbf{x}_{1d}$ can be obtained despite the kinematic model uncertainty. More precisely, we have:

Lemma 4: The kinematic controller (16) and estimation scheme (20) ensure that the vertical coordinates $\mathbf{x}_1$ accurately track the desired trajectory $\mathbf{x}_{1d}$.

Proof: Differencing the kinematic models (18) and (19) yields

$$\mathbf{E} = W(\mathbf{x}, \dot{\mathbf{x}})\phi \tag{21}$$

where $\phi = \mathbf{p} - \hat{\mathbf{p}}$. Differentiating the Lyapunov function candidate $V = \phi^T\phi/2$ along (20), (21) and simplifying the resulting expression gives $\dot{V} = -\alpha\,\|\mathbf{E}\|^2$, from which it may be concluded that $\mathbf{E}$ is bounded and converges to zero. This fact can then be combined with (17) to establish the claims of the lemma. ■

The performance of this approach to compensating for uncertainty in the kinematic model is illustrated in the next section.

4. CASE STUDIES

In this section we consider the three example systems introduced previously in light of our definition of kinematic redundancy, apply the proposed motion control strategies to these systems, and illustrate the control law simplification and performance enhancement obtainable with the proposed approach.

4.1 Space robot

Consider first the space robot presented in example 1 and shown in Figure 1. As summarized above, the space robot is an example of a symmetric system with nonholonomic constraint arising from angular momentum conservation. The system has configuration coordinates $\mathbf{x} = [\phi\ x\ y\ \theta_1]^T \in \Re^4$, with reducing outputs $\mathbf{x}_2 = [x\ y\ \theta_1]^T$ and vertical coordinate $\mathbf{x}_1 = \phi$. The (reduced) dynamic model for the system is of the general form (7) and is given in reference 21. Suppose that the task to be completed by the space robot involves tracking a user-specified trajectory with the manipulator end-effector while simultaneously

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ensuring that the platform orientation tracks some desired trajectory. Observe that this capability can be used to achieve many performance improvement objectives, such as maintaining a fixed platform location while completing tasks with the end-effector or placing the platform in an optimal configuration for a given operation. In this case, $\mathbf{x}_{21} = [x\ y]^T$ and $\mathbf{x}_{22} = \theta_1$ is the redundant coordinate according to our definition. Moreover, it can be seen that the system belongs to the first class considered above since it possesses one vertical coordinate and one redundant reducing output.

We begin our simulation study for the space robot by assuming that the system dynamic model (3*b*) is uncertain but the kinematic model (11) is accurately known. In this case we can apply the kinematic control scheme (12) to generate the trajectory $\dot{\mathbf{x}}_{22d}$ for $\dot{\mathbf{x}}_{22}$ which ensures that $\mathbf{x}_1$ and $\mathbf{x}_{21}$ closely track $\mathbf{x}_{1d}$ and $\mathbf{x}_{21d}$, respectively, and then use the adaptive control law (10) to track the desired velocity trajectories $\dot{\mathbf{x}}_{21d}$ and $\dot{\mathbf{x}}_{22d}$. This control approach is applied to the mathematical model of the space robot through computer simulation with a sampling period of two milliseconds. The system model parameters are defined as $m = J = 10$ and $l = 1$. The adaptive gains $\mathbf{g}$ C, and D are set initially to zero, and the controller parameters are set as $k = 10$, $k_3 = 2$, and $\sigma_i = 0.1$, $\beta_i = 10$ for all i. It is noted that no attempt was made to "tune" the gains to obtain the best possible performance for this simulation. The control strategy was tested using a wide range of initial conditions and desired trajectories; sample results are given in Figures 4a and 4b, and demonstrate the feasibility of the method.

It is interesting to note that, if the task requirements involve the special case of platform motion in which the platform remains stationary during the task, an alternative control approach can be employed. More specifically, because the space robot is redundant the desired motion can be achieved by controlling the end-effector using only the actuators at the second and third joints of the manipulator, so that the output of the actuator at the first joint is zero and the platform does not move from its initial configuration. It can be shown that $\mathbf{x}_{21}$ is fully actuated even with the actuator at the first joint turned off.[21] Thus $\mathbf{x}_{21}$ can be made to track $\mathbf{x}_{21d}$ using the adaptive scheme (8), (9), and since the actuator at the first joint is not utilized for this motion the platform remains in its original configuration. Sample results obtained using this approach are given in Figures 5a and 5b, and illustrate that this method can be used to move the end-effector without moving the platform.

Finally, we consider the problem of controlling the space robot in the presence of uncertainty regarding *both* the dynamic model (3*b*) and the kinematic model (11). In this case we can utilize the kinematic control scheme (16), (20) to generate the desired trajectory $\dot{\mathbf{x}}_{22d}$ for $\dot{\mathbf{x}}_{22}$ which ensures that $\mathbf{x}_1$ and $\mathbf{x}_{21}$ closely track $\mathbf{x}_{1d}$ and $\mathbf{x}_{21d}$, respectively, and then use the adaptive control law (10) to track the desired velocity trajectories $\dot{\mathbf{x}}_{21d}$ and $\dot{\mathbf{x}}_{22d}$. This control approach is applied to the mathematical model of the space robot through computer simulation with a sampling period of two milliseconds. The system model parameters are defined as above. The adaptive gains $\mathbf{g}$, C, and D are set to zero initially, and the controller parameters are set as $k = 10$, $k_3 = 2$, $\alpha = 1$, and $\sigma_i = 0.1$, $\beta_i = 10$ for all i. It is noted that no attempt was made to tune the gains to obtain the best possible performance for this simulation. The control strategy was tested using a wide range of initial conditions, desired trajectories, and initial estimates for $\mathbf{p}$; sample results are given in Figures 6a and 6b, and demonstrate the feasibility of the method.

4.2 Mobile manipulator

Consider next the mobile manipulator system presented in example 2 and shown in Figure 2. The mobile manipulator provides an example of a nonholonomic system arising from the presence of explicit constraints on the system kinematics, in this case corresponding to the rolling constraint of the wheeled mobile platform. Let d denote the distance from the point (z_1, z_2) on the platform to the point (x_1, x_2) at the end-effector of the manipulator. Note that controlling the system so that the distance d remains in a desired range provides a simple means of ensuring that the manipulator is maintained in a useful posture. The elements of $\mathbf{x} = [d\ x_1\ x_2\ z_1\ \phi]^T \in \Re^5$ define configuration coordinates for the system, with $\mathbf{x}_1 = d$ and $\mathbf{x}_2 = [x_1\ x_2\ z_1\ \phi]^T$ the vertical coordinate and reducing outputs, respectively. The (reduced) dynamic model for the mobile manipulator is of the general form (3) and is given in reference 21. Suppose that the task specification for the mobile manipulator involves driving the manipulator end-effector to some goal location while simultaneously controlling the platform in such a way that the distance d remains at its initial value; this task represents a simple model for applications involving end-effector placement while maintaining a useful manipulator configuration. In this case it can be seen that $\mathbf{x}_{21} = [x_1\ x_2]^T$ and the elements of $\mathbf{x}_{22} = [z_1\ \phi]^T$ are redundant by our definition. This system belongs to the first class of redundant nonholonomic systems because the number of vertical coordinates is one and the number of *task relevant* redundant coordinates is also one: since motion of the redundant coordinate ϕ has no impact on the distance d, z_1 is the only task relevant redundant coordinate.

In our simulation study of the mobile manipulator, we assume that the system dynamic model (3*b*) is uncertain but the kinematic model (11) is accurately known. Thus we define desired trajectories for $\mathbf{x}_{21d}$, $\mathbf{x}_{1d}$, and ϕ_d which will accomplish the task objectives, and use (12) to generate the desired trajectory $\dot{\mathbf{x}}_{22d}$ for $\dot{\mathbf{x}}_{22}$. Using the adaptive control law (10) to track the desired velocity trajectories $\dot{\mathbf{x}}_{21d}$ and $\dot{\mathbf{x}}_{22d}$ ensures that $\mathbf{x}_1$, $\mathbf{x}_{21}$ and ϕ closely track $\mathbf{x}_{1d}$, $\mathbf{x}_{21d}$, and ϕ_d, respectively. This control approach is applied to the mathematical model of the mobile manipulator through computer simulation with a sampling period of two milliseconds. The system model parameters are defined as $M = J = 10$, $l = 1$, and $m = 10$. The kinematic controller (12) and adaptive velocity tracker (10) are implemented exactly as in the space robot simulation, despite the fact that the two systems have quite different properties. This choice for the controller terms is made to demonstrate that these gains need not be tuned for a particularly system to obtain good performance. The control strategy was tested using a wide range of initial conditions and desired trajectories. The results of these simulations are quite similar to those given in the previous section and hence are not shown.

As stated in the definition, the idea of redundancy for nonholonomic systems is strictly connected with the definition of the task the system is supposed to achieve. For example, if we are only interested in the end effector location of the mobile manipulator, the vehicle portion of the robot would be redundant and the control problem would be straightforward (since the end effector coordinates are reducing outputs and can be controlled directly). On the other hand, if we are interested in the final location of both the end effector and the mobile platform, so that we can gain a virtually infinite workspace, the system would not be redundant at all; in this case we are led to a classical

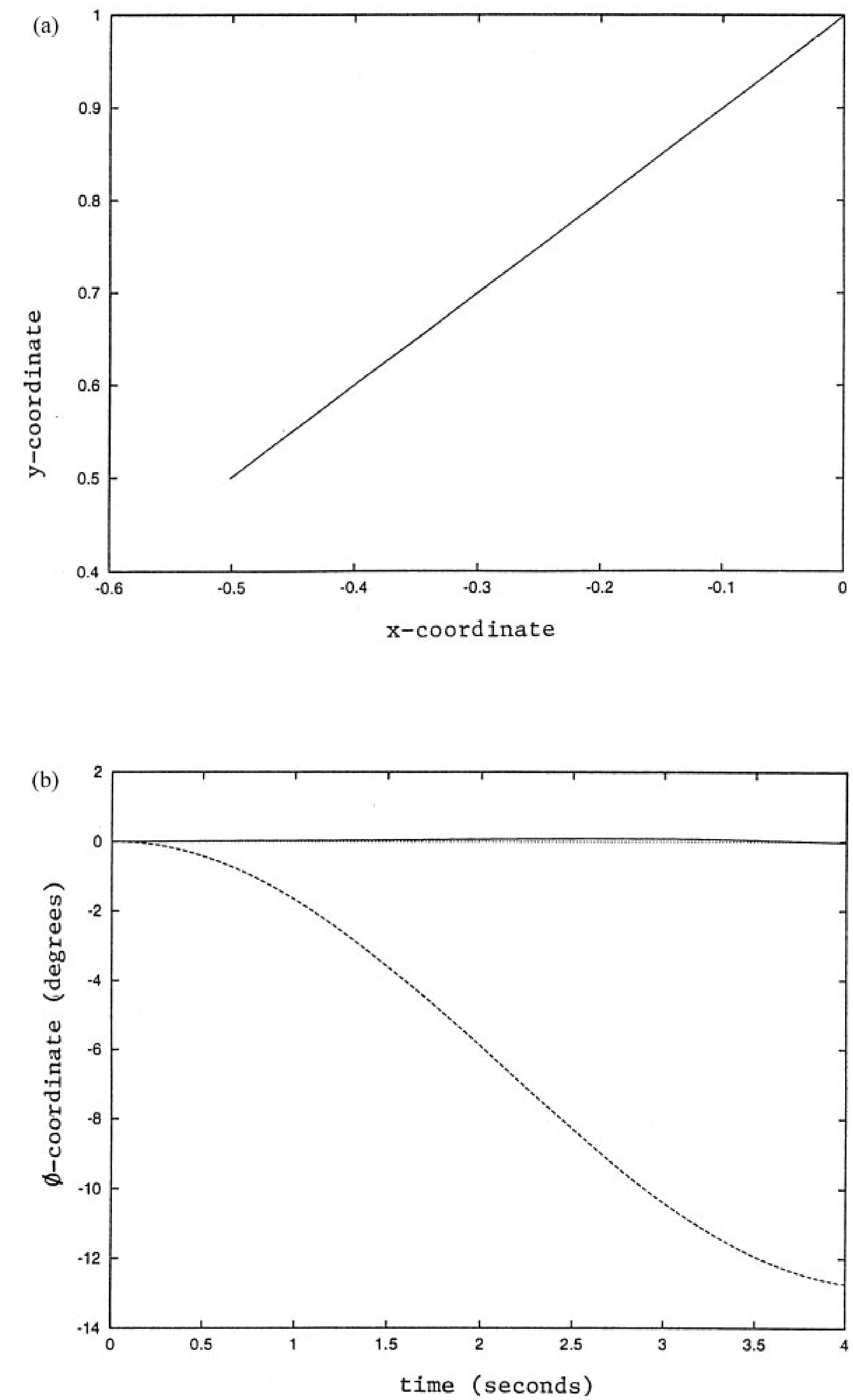

Fig. 4. (a) Response of end-effector coordinates (x, y) of space robot in first simulation. (b) Desired (dotted) and actual (solid) response of the space robot platform angle ϕ in first simulation. Dashed line depicts response of platform angle when uncontrolled.

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nonholonomic control problem with the limitations and difficulties mentioned in the Introduction. For example, in this second case, the overall system could require a considerable amount of time to get to a desired location because of the nonholonomic constraint on the mobile platform.

These considerations suggest a compromise: if we are interested in a final location of the end effector inside the range reachable without moving the platform, we can move the arm to the goal by a fast and efficient movement and then move the platform to the desired location while holding the end-effector position fixed. While this strategy doesn't

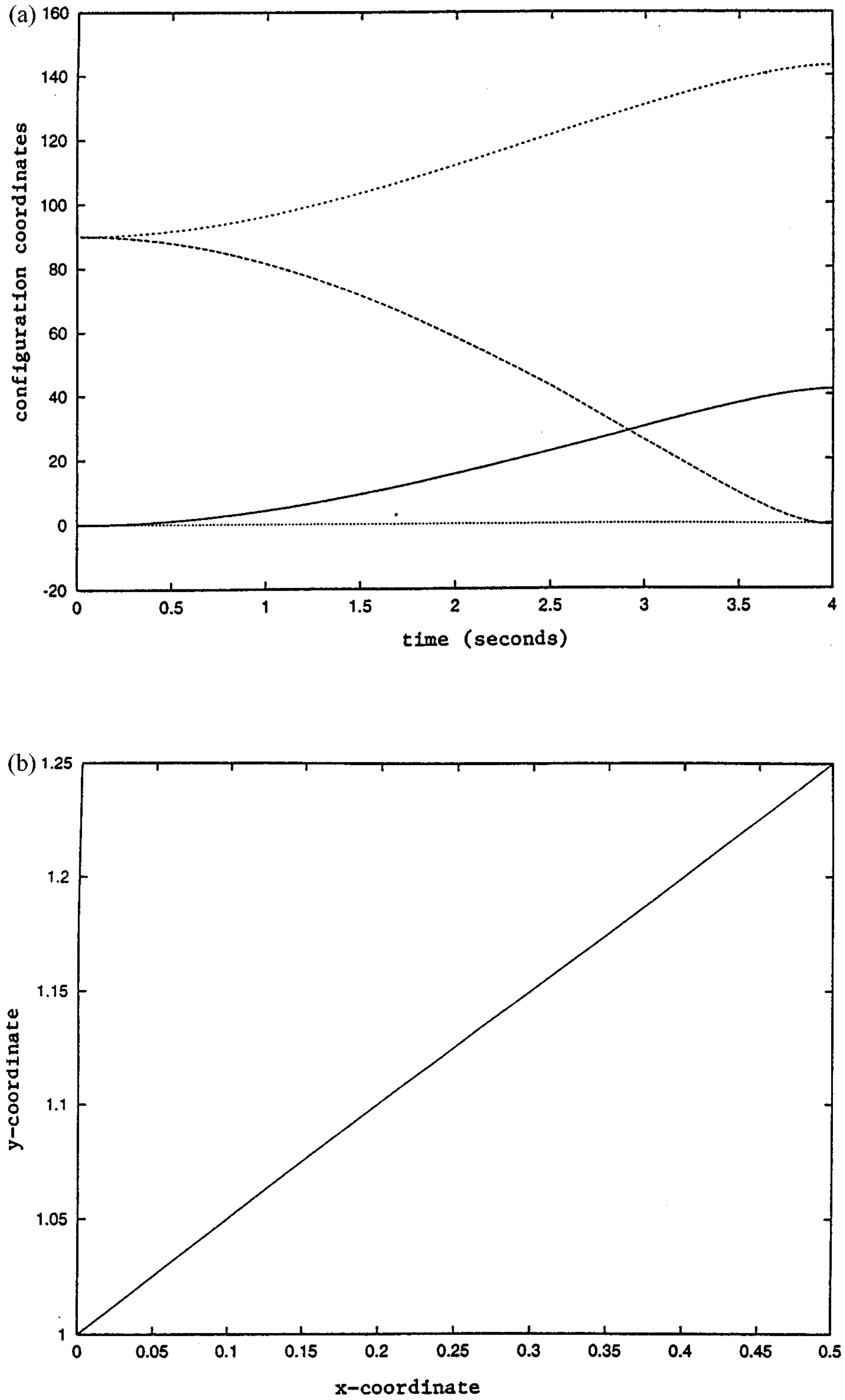

Fig. 5. (a) Response of configuration coordinates θ_1 (solid), θ_2 (large dash), θ_3 (small dash) and ϕ (dotted) of space robot in second simulation. (b) Response of end-effector coordinates (x, y) of space robot in second simulation.

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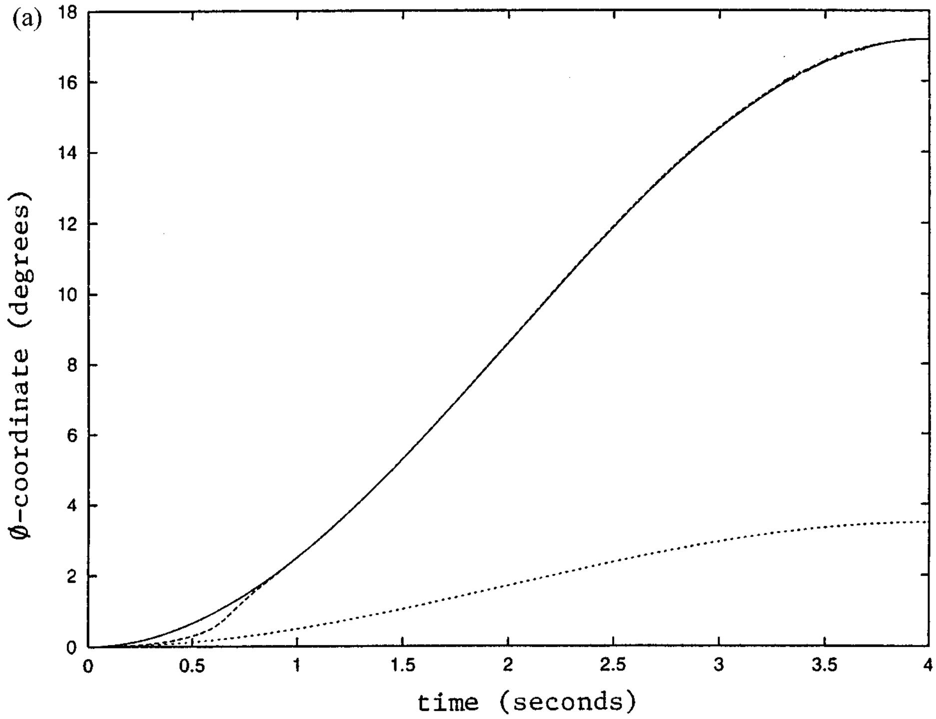
provide a virtually infinite workspace, it allows efficient motion while simultaneously retaining the possibility for future efficient motions; thus the effective workspace for multiple tasks is increased significantly. An algorithm to realize this objective is:

(i) Quickly drive $\mathbf{x}_{21}$ to the goal $\mathbf{x}_{21d}$ using the adaptive tracking scheme (8), (9).

(ii) Hold $\mathbf{x}_{21}$ at $\mathbf{x}_{21d}$ (using (8), (9)) and simultaneously drive $\mathbf{x}_1$ and $\mathbf{x}_{22}$ to values which place the manipulator in a desired configuration relative to the platform. The motion control for $\mathbf{x}_1$ and $\mathbf{x}_{22}$ can be realized using the stabilization strategy proposed by the authors in reference 18, for example.

(a)



(b)

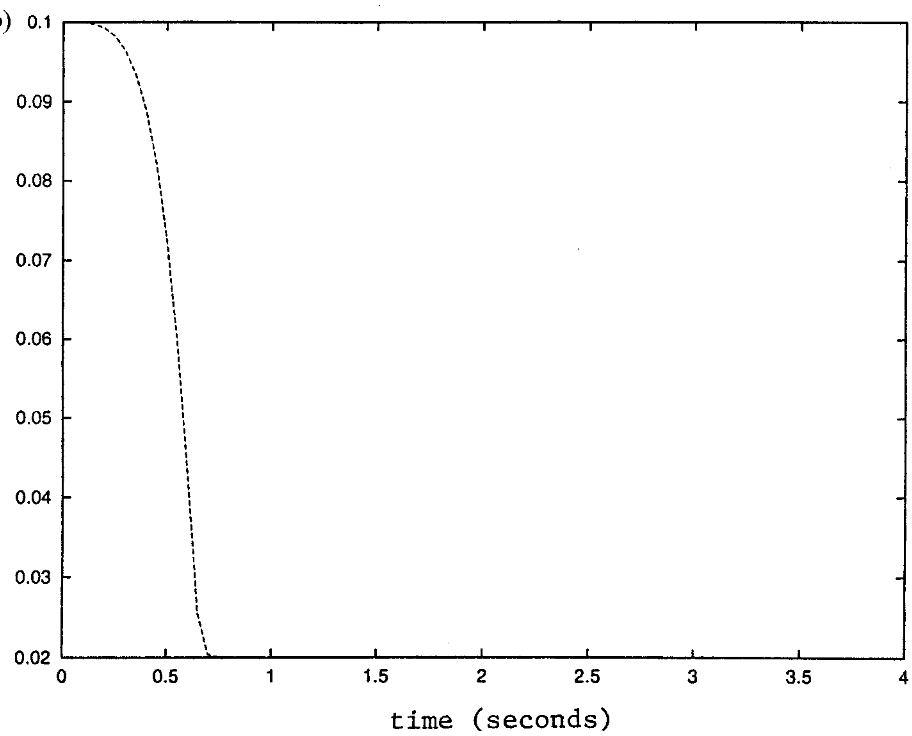


Fig. 6. (a) Desired (solid) and actual (dashed) response of space robot platform angle ϕ in uncertain kinematics simulation. Dotted line depicts response of platform angle if kinematic uncertainty is not compensated. (b) Evolution of parameter estimate $\hat{\mathbf{p}}$ for space robot in uncertain kinematics simulation.

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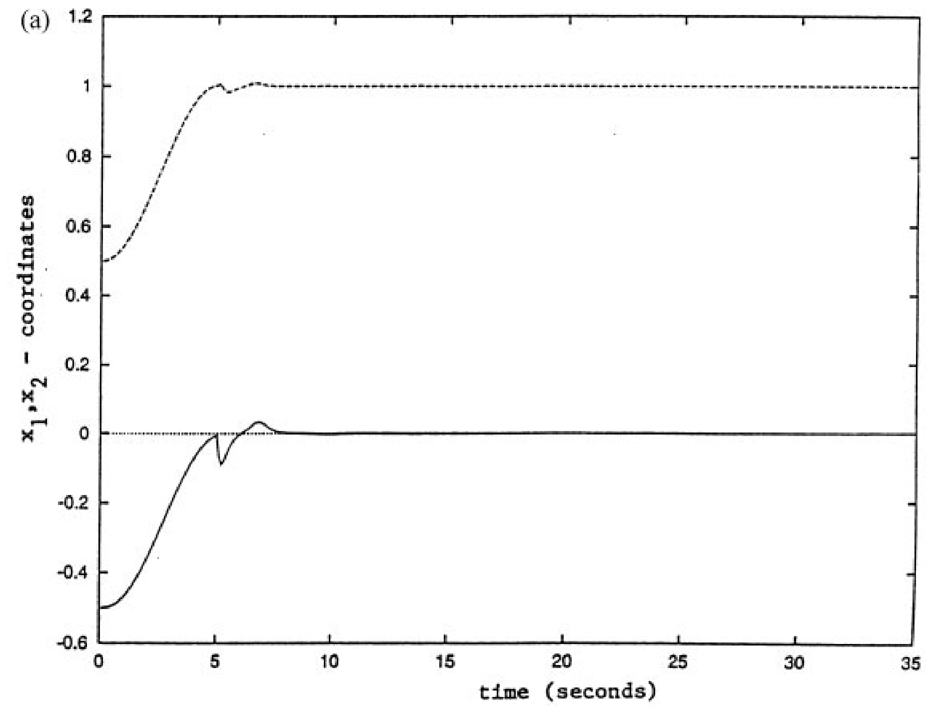
This algorithm is applied to the mathematical model of the mobile manipulator through computer simulation with a sampling period of two milliseconds. The system model parameters are defined as above. The adaptive gains **f**, A, and B are set to zero initially, and the controller parameters are set as $k_1=k_2=10$, $\gamma=2$, and $\sigma_i=0.1$, $\beta_i=10$ for all i. It is noted that no attempt was made to tune the gains to obtain the best possible performance for this simulation. The control strategy was tested using a wide range of initial conditions and desired trajectories. Sample results obtained using this approach are given in Figures 7a and 7b, and demonstrate the feasibility of the method. In this simulation both the end effector and the platform are commanded to move to the origin, but while the time for the overall system to get to the goal is large, the end-effector arrives to the goal in a shorter time, ready for a new task.



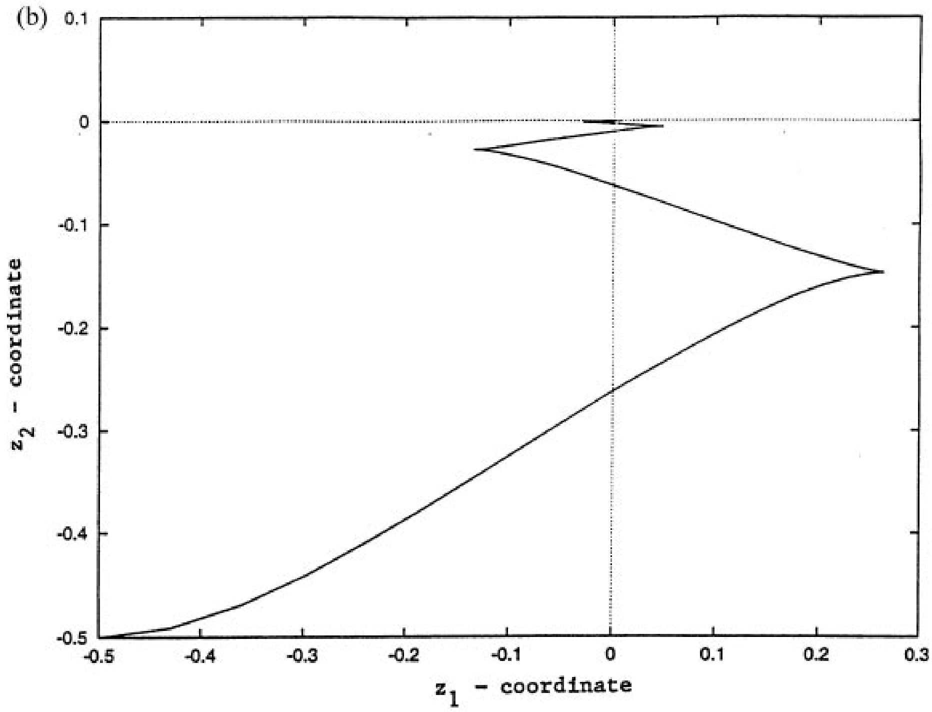


Fig. 7. (a) Response of end-effector coordinates (x_1, x_2) of mobile manipulator in a simulation study. (b) Response of platform coordinates (z_1, z_2) of mobile manipulator in a simulation study.

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4.3 Fire truck

Finally, consider the fire truck presented in example 3 and shown in Figure 3. The fire truck is another example of a system with explicit nonholonomic constraints on its kinematics, corresponding to the rolling constraints of the wheels. As noted above, the presence of two steering inputs provides this vehicle with enhanced maneuverability compared with single steering input tractor trailer systems. The fire truck has configuration coordinates $\mathbf{x} = [x\ y\ \theta_1\ \phi_1\ \theta_2\ \phi_2]^T \in \Re^6$, with vertical coordinate $\mathbf{x}_1 = [x\ y\ \theta_2]^T$ and reducing outputs $\mathbf{x}_2 = [\phi_1\ \theta_1\ \phi_2]^T$ (see Figure 3). The (reduced) dynamic model for the fire truck is of the general

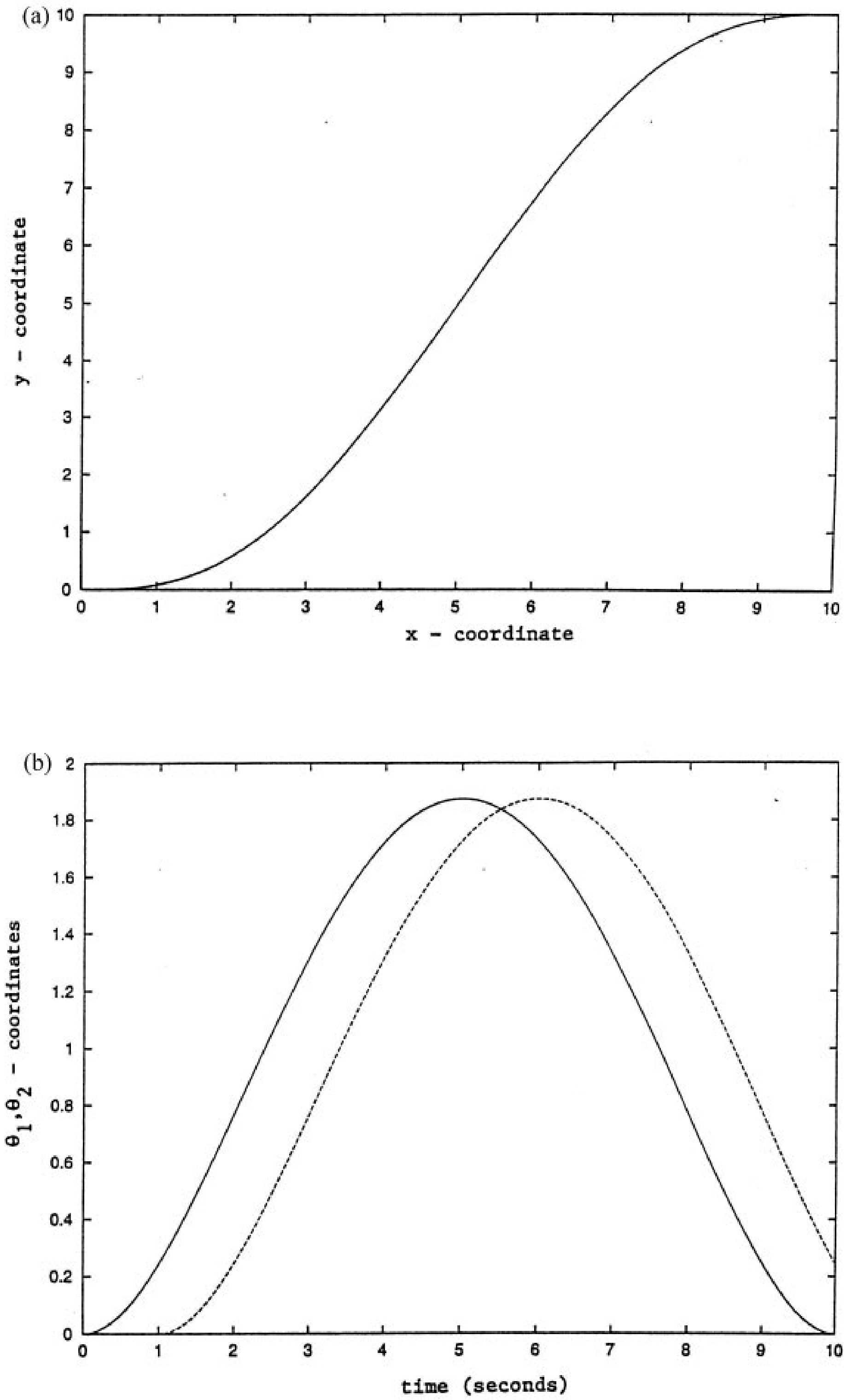

Fig. 8. (a) Response of tractor coordinates (x, y) in fire truck simulation. (b) Response of tractor angle θ_1 (solid) and trailer angle θ_2 (dashed) in fire truck simulation.

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form (3), and the simple version of the model used in this simulation study is given in reference 21. Suppose that the task to be completed by the fire truck involves maneuvering the system to some goal location in such a way that the potential for collisions with environmental obstacles is reduced. Since the rear steering input is not required to drive the system to the goal, and is only used to maneuver the system more effectively, it is seen that $\mathbf{x}_{21}=[\phi_1\ \theta_1]^T$ and $\mathbf{x}_{22}=\phi_2$ is the redundant coordinate. Moreover, note that the system belongs to the second class considered above since it possesses more vertical coordinates (three) than redundant reducing outputs (one).

To achieve the objective of maneuvering the fire truck while reducing the potential for collisions with obstacles, we adopt the following simple strategy: drive the tractor portion of the system to the goal in a collision-free manner, and use the available redundancy to cause the trailer to follow the path of the tractor as closely as possible. Thus the trailer may be visualized as "slithering" after the tractor in the same way that the body of a snake slithers after the snake's head, and it is seen that in this way the potential for collisions between the trailer and environmental obstacles is reduced. We present algorithms for realizing this slithering motion for highly redundant robotic arms in [20], and now consider implementing this idea with the fire truck.

Consider the following chained form representation of the kinematics of the fire truck (see reference 13 for a derivation of this representation):

$$\begin{aligned}\dot{z}_0&=\nu_1\\ \dot{x}_0&=\nu_2\\ \dot{x}_1&=x_0\nu_1\\ \dot{x}_2&=x_1\nu_1\\ \dot{y}_0&=\nu_3\\ \dot{y}_1&=y_0\nu_1\end{aligned}\tag{22}$$

We can use this kinematic model together with ideas from *differential flatness*[22] to plan a trajectory for the (non-redundant) reducing outputs $\mathbf{x}_{21}$ which drives both $\mathbf{x}_{21}$ and $\mathbf{x}_{12}=[x\ y]^T$ to their desired values. Roughly, differential flatness refers to a kind of kinematic reduction in which a set of outputs, equal in number to the number of inputs, is sufficient to describe the motion of the full system unambiguously. If a system is flat and a set of flat outputs is identified then the entire system state can be reconstructed from these outputs without resorting to integration of the system dynamic model. As a consequence, the trajectory generation problem is easily solved for flat systems; see references 9 and 22 for the details of this process. In the case of the firetruck it can be verified from (22) that the coordinates (z_0, x_2, y_1) are flat outputs, so that this approach can be used. Once the appropriate desired trajectory for $\mathbf{x}_{21}$ has been determined using flatness, the kinematic control law (13) can be utilized to generate the trajectory for $\dot{\mathbf{x}}_{22d}$ which will ensure that $\mathbf{x}_{11}=\theta_2$ closely tracks any desired trajectory. In the present case the desired trajectory for θ_2 is defined on-line in such a way that the slithering objective is realized.[21] The adaptive control law (10) can then be used to track the desired velocity trajectories $\dot{\mathbf{x}}_{21d}$ and $\dot{\mathbf{x}}_{22d}$.

This control approach is applied to the mathematical model of the fire truck through computer simulation with a sampling period of two milliseconds. The system model parameters used in the simulation are given in reference 21. The kinematic controller and adaptive velocity tracker are implemented exactly as in the space robot simulation, despite the fact that the two systems have quite different properties. This choice for the controller terms is made to demonstrate that these gains need not be tuned for a particular system to obtain good performance. Sample results obtained using this approach are given in Figures 8a and 8b. Figure 8a shows an example trajectory which takes the fire truck to the goal (in this case the origin) while Figure 8b depicts the body angles for the tractor and trailer and illustrates how the trailer slithers after the tractor.

5. CONCLUSIONS

This paper considers kinematically redundant nonholonomic mechanical systems, identifies some of the interesting properties which result because of the presence of the redundancy, and initiates a study of the control and application of these systems. It is shown that kinematic redundancy in nonholonomic systems can be exploited both to simplify the problem of controlling these systems and to enhance their performance capabilities. Moreover, it is demonstrated that these results can be obtained even in the presence of considerable uncertainty regarding the system model. The ideas are illustrated through case studies involving a space robot, a mobile manipulator, and a tractor-trailer system with two steering inputs.

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