A Problem for Confirmation Measure Z

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In this article, I present a serious problem for confirmation measure Z.

1. Confirmation Measure *Z*. Crupi, Tentori, and Gonzalez (2007) provide a very interesting set of theoretical and empirical arguments in favor of the following (piecewise) Bayesian measure of the degree to which evidence E confirms hypothesis H, relative to background knowledge K:[1]

$$Z(H, E|K) = \begin{cases} \dfrac{\Pr(H|E \,\&\, K) - \Pr(H|K)}{1 - \Pr(H|K)} & \text{if } \Pr(H|E \,\&\, K) \geq \Pr(H|K) \\[2ex] \dfrac{\Pr(H|E \,\&\, K) - \Pr(H|K)}{\Pr(H|K)} & \text{if } \Pr(H|E \,\&\, K) < \Pr(H|K) \end{cases}$$

I will not go into their arguments in favor of Z here. Instead, I will present what I take to be a serious problem with Z. This will require a brief digression into the notion of independent evidence.

2. Independent Evidence Regarding a Hypothesis. Fitelson (2001) offers the following Bayesian account of what it means for two pieces of evidence E_1 and E_2 to be *confirmationally independent*, regarding hypothesis H, according to a confirmation measure $\mathfrak{c}$.

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1. Several authors discussed/endorsed measure Z before Crupi et al. (2007). See, e.g., Rescher (1958) and Shortliffe and Buchanan (1975). However, Crupi et al. provide the most compelling and comprehensive theoretical and empirical arguments in its favor.

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Independence. E_1 and E_2 are confirmationally independent regarding H, according to $\mathfrak{c}$ (i.e., E_1, E_2 are $\mathfrak{c}$-independent regarding H) if and only if both $\mathfrak{c}(H, E_1|E_2) = \mathfrak{c}(H, E_1)$ and $\mathfrak{c}(H, E_2|E_1) = \mathfrak{c}(H, E_2)$.[2]

Intuitively, E_1 and E_2 are confirmationally independent regarding H, according to $\mathfrak{c}$, just in case the degree to which E_1 (E_2) confirms H (according to $\mathfrak{c}$) does not depend on whether E_2 (E_1) is already known.

As Fitelson shows, this notion can be applied in various useful confirmation-theoretic ways (e.g., to provide a Bayesian account of the value of varied/diverse evidence). I will not delve into Independence here. Rather, I will simply apply it to reveal that measure Z has a serious shortcoming when it comes to the handling of certain sorts of independent evidence.

3. A Problem for Measure *Z*. Sometimes, we have *conflicting evidence* regarding a hypothesis. That is to say, sometimes, the following property holds for a triple E_1, E_2, and H.

Conflict. E_1 and E_2 constitute conflicting evidence regarding H if and only if E_1 confirms H, while E_2 disconfirms H. Formally, E_1 and E_2 constitute conflicting evidence regarding H if and only if $\Pr(H|E_1) > \Pr(H)$ and $\Pr(H|E_2) < \Pr(H)$.

Intuitively, it should be possible for some triple E_1, E_2, and H to satisfy both Independence and Conflict. That is to say, intuitively, there sometimes exists independent, conflicting evidence regarding some hypotheses. More precisely, we have the following eminently plausible existence claim.

Existence. There exist some triples E_1, E_2, and H that satisfy both Independence and Conflict.

Indeed, Existence strikes me as so plausible as to require little justification. Having said that, it is worth giving a simple example that illustrates the intuitive plausibility of Existence.[3] Here, I borrow the following example, which belongs to a class of examples used by Fitelson (2001) to provide an intuitive illustration of Independence (with individual degrees of strength that can be tweaked via some simple parameters, which I have set here).

2. Here, $\mathfrak{c}(H, E)$ is shorthand for $\mathfrak{c}(H, E|\top)$, where $\top$ is a tautology. This can be read simply as “the degree to which E confirms H (unconditionally), according to $\mathfrak{c}$.”

3. I thank an anonymous referee for urging me to include an illustrative intuitive example of Existence.

The Urn Example. An urn has been selected at random from a collection of urns. Each urn contains some balls. In some of the urns (the H-urns) the proportion of white balls to nonwhite balls is 1/3, and in all the other urns (the $\sim H$-urns) the proportion of white balls to nonwhite balls is 2/3. The proportion of H-urns is 1/2. Balls are to be drawn randomly from the selected urn, with replacement.

Let H be the hypothesis that the proportion of white balls in the urn is 1/3 (i.e., that the sampled urn is an H-urn). Let W_i state that the ball drawn on the i^{th} draw ($i \geq 1$) is white. Intuitively, $\sim W_1$ and W_2 are confirmationally independent regarding H; that is, the triple $\langle \sim W_1, W_2, H \rangle$ instantiates Independence.[4]

Surprisingly, according to measure Z, Existence is false (a fortiori). That is, according to measure Z, it is conceptually impossible for any pair of evidence E_1, E_2 to be both independent regarding H and conflicting regarding H (for any hypothesis H).

Problem. According to measure Z, Existence is (analytically) false.

Proof. Suppose, for reductio, that there does exist some triple E_1, E_2, H that satisfies both Independence (according to measure Z) and Conflict. Then, we may reason as follows.

(1) $\Pr(H \mid E_1) > \Pr(H)$	Assumption (Conflict)
(2) $\Pr(H \mid E_2) < \Pr(H)$	Assumption (Conflict)
(3) $Z(H, E_1) = Z(H, E_1 \mid E_2)$	Assumption (Z-Independence)
(4) $Z(H, E_2) = Z(H, E_2 \mid E_1)$	Assumption (Z-Independence)
(5) $\dfrac{\Pr(H \mid E_1) - \Pr(H)}{1 - \Pr(H)} = \dfrac{\Pr(H \mid E_1 \,\&\, E_2) - \Pr(H \mid E_2)}{1 - \Pr(H \mid E_2)}$	(1), (3), definition of Z
(6) $\dfrac{\Pr(H \mid E_2) - \Pr(H)}{\Pr(H)} = \dfrac{\Pr(H \mid E_1 \,\&\, E_2) - \Pr(H \mid E_1)}{\Pr(H \mid E_1)}$	(2), (4), definition of Z

4. Fitelson (2001) would be committed to a claim far stronger than mere Existence here (note that Conflict is obviously true in this case). He would be committed to the stronger claim that $\sim W_1$ and W_2 should have equal and opposite degrees of confirmation, which exactly cancel each other out, so that the total degree to which the conjunction $\sim W_1 \,\&\, W_2$ confirms H is zero. This is because he accepts the likelihood-ratio measure of degree of confirmation, which satisfies (not only Existence but) a strong independence-additivity requirement. Of course, we do not need to go along with Fitelson (2001) on that stronger/more specific claim. All we need this example to do is make Existence somewhat plausible (i.e., not a conceptual impossibility). As I point out below, among all the measures of confirmation that have been proposed and defended in the literature, Z is the only measure that entails the conceptual impossibility of Existence. Indeed, a plenitude of examples satisfying Existence are easily described for all other confirmation measures in the literature.

Now, let $x = \Pr(H|E_1)$, $y = \Pr(H|E_2)$, $z = \Pr(H)$, and $u = \Pr(H|E_1 \,\&\, E_2)$. Then, (5) and (6) can be rewritten as the following pair of algebraic equations.

(5) $\frac{x-z}{1-z} = \frac{u-y}{1-y}$

(6) $\frac{y-z}{z} = \frac{u-x}{x}$

Algebraically (assuming only that x, y, z, and u are real numbers), (5) and (6) entail that either (7) $x = z$ or (8) $y = z$. But, this contradicts our assumption that both (1) $x > z$ and (2) $y < z$. QED

In closing, it is worth noting that it seems to be the piecewise nature of Z that causes Problem. For it can be shown that none of the non-piecewise-defined confirmation measures that have been discussed in the literature (see, e.g., Crupi et al. [2007] and Crupi and Tentori [2014] for recent surveys) have this Problem (proof omitted). Finally, because Problem only rests on ordinal features, it will plague any measure that is ordinally equivalent to Z.[5]

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5. An anonymous referee points out that the following (formally similar) piecewise confirmation measure (on which see Mura [2006, 2008] and Crupi and Tentori [2014] for further discussion), which takes its theoretical inspiration from Törnebohm (1966), is not ordinally equivalent to Z:

$$Z^{\star}(H, E|K) = \begin{cases} \dfrac{\log[\Pr(H|E \,\&\, K)] - \log[\Pr(H|K)]}{-\log[\Pr(H|K)]} & \text{if } \Pr(H|E \,\&\, K) \geq \Pr(H|K) \\[2ex] \dfrac{\log[\Pr \sim H|E \,\&\, K)] - \log[\Pr(\sim H|K)]}{-\log[\Pr(\sim H|K)]} & \text{if } \Pr(H|E \,\&\, K) < \Pr(H|K) \end{cases}$$

I have been unable to determine whether $Z\star$ also falsifies Existence (because $Z\star$ involves logarithms, this question cannot be answered using standard algebraic techniques; Fitelson 2008). This is an interesting open question.

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