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Single-legged hopping robotics research—A review

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SUMMARY

Inspired by the agility of animal and human locomotion, the number of researchers studying and developing legged robots has been increasing at a rapid rate over the last few decades. In comparison to multilegged robots, single-legged robots have only one type of locomotion gait, i.e., hopping, which represents a highly nonlinear dynamical behavior consisting of alternating flight and stance phases. Hopping motion has to be dynamically stabilized and presents challenging control problems. A large fraction of studies on legged robots has focused on modeling and control of single-legged hopping machines. In this paper, we present a comprehensive review of developments in the field of single-legged hopping robots. We have attempted to cover development of prototype models as well as theoretical models of such hopping systems.

1. Introduction

Research on legged robots has been going on for over a century.[1] The reason behind such sustained interest in legged robots is due to the fact that most of the earth's land surface is inaccessible to wheeled or tracked systems. Legged animals can, however, be found everywhere. Thus, mankind has been fascinated with the idea of a mobile legged robot that can handle difficult terrain and be useful in the fields of transportation, forestry, agriculture, fire fighting, hazardous areas, defense (carrying weapons to soldiers, de-mining), police purposes, assistive devices for walking, entertainment (toy production), robotic pets, and ocean and space exploration.

The main advantages of legged locomotion can be summarized as:[2]

- Active suspension of body
- Use of isolated footholds
- Adaptation to uneven terrain
- Less damage to soil and vegetation as compared to wheeled or tracked vehicles

Some of the challenges in developing legged robots are:

- Legged robots have to carry the entire weight of the machine, including the weight of all the actuators. This leads to the requirement of stronger and heavier legs, requiring even bigger actuators. This has a multiplying effect on the weight of the machine.
- Payload-to-machine-weight ratio is less as compared to wheeled or tracked vehicles.
- Dynamics of legged robots tend to be more complex than stationary and wheeled mobile robots, especially due to the impact with the ground and the presence of distinct swing and stance phases.
- Control of walking is complex as legged robots are inherently nonlinear and each leg works in two distinct regimes where the speed and force/torque requirements are very different.
- There is energy loss due to joint friction and intermittent impacts. As mobile robots have to carry their power pack, energy efficiency is very important.
- In multi-legged robots, coordination of legs is complex. Different gait patterns are admissible in multi-legged systems; depending on the number of legs, terrain, and speed of locomotion, particular gaits are most efficient. Thus, the coordination pattern of legs may change as speed or terrain changes.

For all these reasons, building and analyzing legged robotic systems is relatively complex. Therefore, a large number of researchers have focused on single-legged systems. Single-legged systems have simpler configurations and admit only one gait, namely, hopping. These robots require dynamic balancing. One of the motivations to study one-legged robots is to gain a good understanding of system dynamics and extend it to human and animal locomotion.

The following issues appear to be important in understanding mobility by hopping:

- Active balance and dynamic stability
- Use of elastic muscles and tendons in enabling resonant mechanical oscillations associated with hopping.

Many researchers have simulated passive and controlled monopedal hopping motion, analyzed the stability of such systems, and developed hopping robot hardware. This paper is a review of different aspects of research work on hopping motion of single-legged robots. The paper is organized as follows. Section 2 describes the hardware developed by several researchers. Section 3 contains modeling and simulation aspects of such robotic systems. Section 4 describes the different control strategies that have been applied to achieve stable dynamic motion. Section 5 extends the single-legged robot research to multilegged robots. Section 6 provides the concluding remarks.

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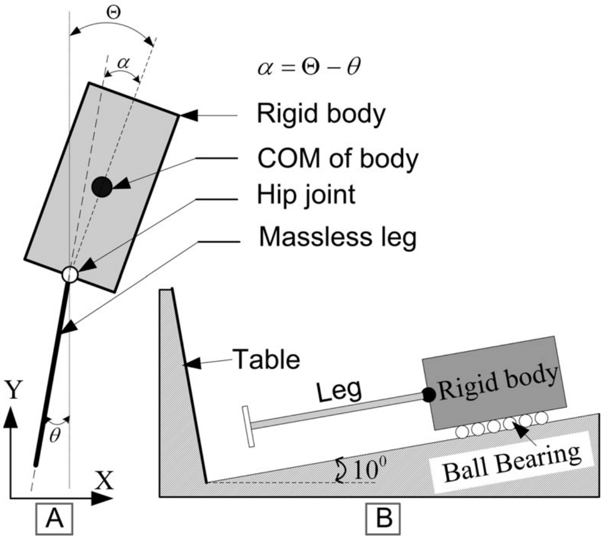

Fig. 1. Conceptual model (A) and the hardware set-up (B) of one-legged robot considered by Matsuoka (adapted[3]).

2. Hardware Development

Matsuoka[3] may be the first researcher to formulate a linearized two-link model of a hopping mechanism. The model consisted of a massless leg and an upper rigid body connected by a rotary hip joint without knee and ankle as shown in Fig. 1. For simplicity, he assumed that the stance duration is very short compared to the duration of ballistic flight phase. He stated that the hopping machine could balance itself in the plane of motion if gravity was reduced to about 0.2 g. He also constructed an actively balanced planar one-legged hopping machine,[4] which was constrained to move on a table, inclined 10° to the horizontal, using antifriction rolling elements (ball bearings) in order to reduce the effect of gravity. An electric solenoid was used to provide rapid thrust at the foot. The machine hopped in place at 1 hop/s and traveled back and forth on the table.

Raibert is a major contributor in the field of hopping robot research.[1, 5–12] The mechanism designed and built by Raibert and coresearchers (Fig. 2) became the standard for several researchers. The significant parts of the model were (i) a body with mass and mass moment of inertia, (ii) a compliant leg with mass and mass moment of inertia, and (iii) a compliant ground surface. The hopper had two actuated joints: a springy prismatic leg and a revolute hip joint. The actuated rotary joint propelled the leg to and fro, and thus, controlled the direction of motion. The telescopic leg (pneumatic cylinder) functioned as an energy restoring as well as a ground interacting element. A linear actuator located at the prismatic joint controlled the hopping height. The first experimental prototype built by Raibert and coresearchers was a one-legged machine[1, 6] as shown in Fig. 3. A long boom connected this machine to a central pivot post, so that it could hop along a circular path on the floor. It was capable of hopping in-place, hopping at various forward speeds, and leaping over small obstacles. Raibert's control strategy was based on the symmetry constraint.[9]

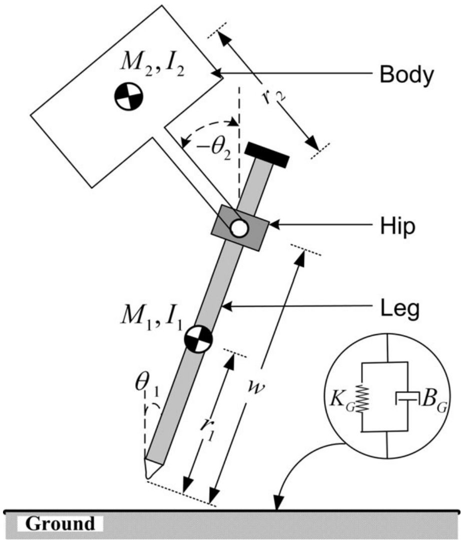

Fig. 2. Preliminary mechanism of one-legged hopping robot considered by Raibert (adapted[5]).

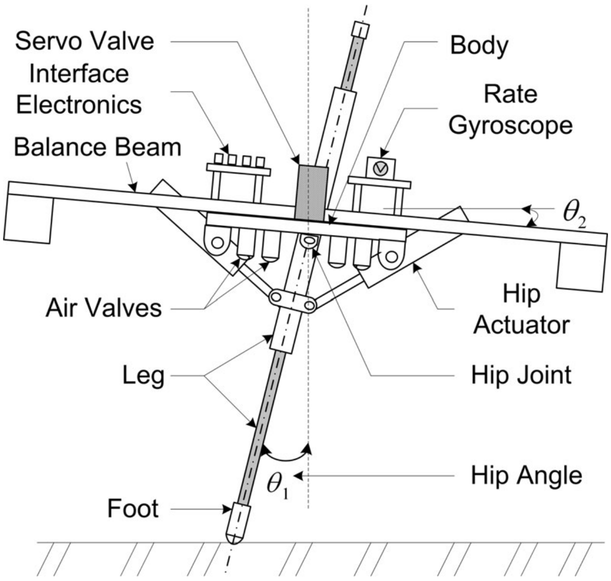

Fig. 3. Raibert's 2D prototype of one-legged hopping robot (adapted[6]).

Raibert extended this 2D hopper mechanism to a single-legged machine that could hop in 3D.[8] This machine (Fig. 4) became more autonomous as the boom carrying the umbilical was eliminated and umbilical cord was hung from the ceiling. It could hop about the room under the supervision of an operator with a simple joystick. Both these 2D and 3D machines used springy telescopic legs. Subsequently, Lee and Raibert[11] explored the use of an articulated leg with a hoof for a one-legged running robot, called the "Monopod" (Fig. 5). This planar prototype consisted of a body, a foot, and a hoof. The hip was offset from the center of mass. The foot was a fiberglass leaf spring, and a linear hydraulic actuator placed at the hip joint actuated the hopper through a rotary ankle joint and an inelastic tendon. Another hydraulic actuator enabled the hip to move to and fro. A retraction spring attached between the foot and the leg maintained tension in the tendon. The monopod was designed to run on its hoof without rolling and falling over. They placed joint angle sensors and potentiometers to measure the different angles and positions. This monopod could be made to hop

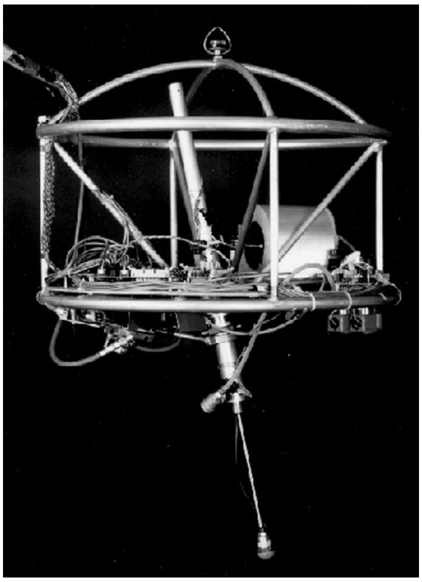

Fig. 4. Raibert's 3D experimental prototype of one-legged hopping robot.[12]

in-place or run in the forward and backward directions at a relatively low speed of 0.8 m/s on a treadmill.

Zeglin[13] and coresearchers constructed the hydraulically actuated one-legged hopping machine, "Uniroo" (Fig. 6). Motivated by kangaroo's locomotion, they selected a three-link leg structure with revolute joints as shown in Fig. 7. The constructed leg was kinematically similar to a real kangaroo and was proportional in size to a kangaroo of similar mass. The experimental prototype was confined to vertical plane by means of a long pivoted boom. Uniroo had an umbilical cable (running over the boom) for power and communications. This robot had successful trials of over 40 hops. Raibert *et al.*[10] used hydraulic and pneumatic actuators, while Zeglin[13] used only hydraulic actuators to realize the abovementioned prototypes. These actuators provided high power-to-weight ratios, thus, making the control task easier. However, the efficiency of the whole system remained unsatisfactory. It may be assumed that an energy-efficient realization was not their goal. Raibert also stated that direct implementation of the control concepts in small-scale (e.g., human sized) energy-efficient systems represented a serious engineering problem.[1] Due to the use of powerful actuators, accelerations/decelerations were large and active balancing was complex at higher speeds of locomotion.

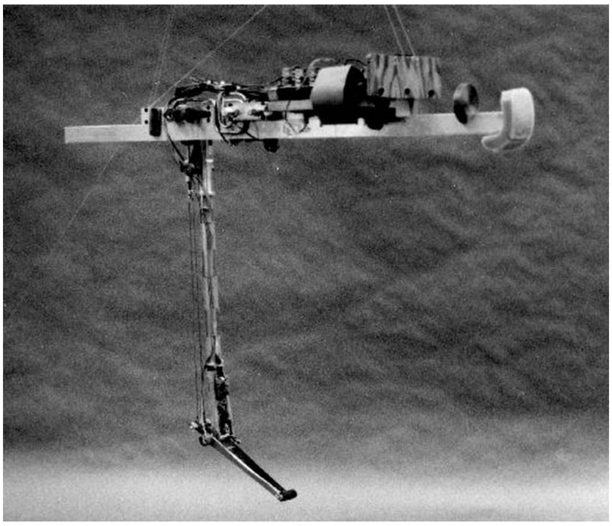

Fig. 5. Monopod—Raibert's one-legged robot with articulated leg with hoof.[11]

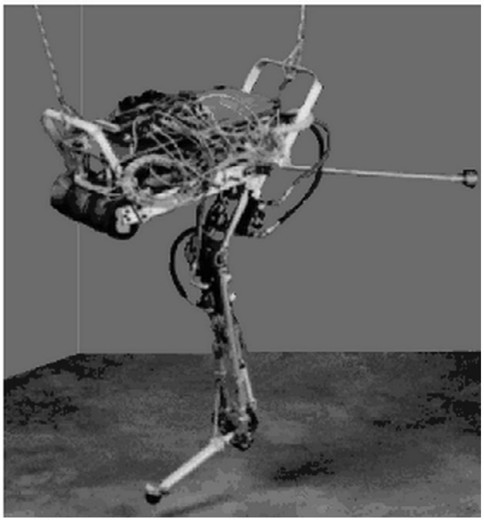

Fig. 6. Uniroo—Zeglin's first one-legged hopping robot.[12]

In this context, Papantoniou[14] implemented an electromechanical prototype of a one-legged planar hopping robot as shown in Fig. 8. It was an electrically powered and actively balanced one-legged planar machine, capable of operating with an average 48 W power requirement, at a maximum speed of 0.3 m/s, with a total mass of 7.5 kg. In his prototype, the leg design consisted of a four-bar linkage behaving like a telescopic leg and yet having the simplicity of an articulated leg. An electric leg actuator was placed on the body, rather than on the leg, in order to minimize the moment of inertia of leg with respect to the body during stance phase. In order to transfer the action of the electric actuator to the leg and to realize the bouncing motion of the robot with elastic energy storage during locomotion, he investigated

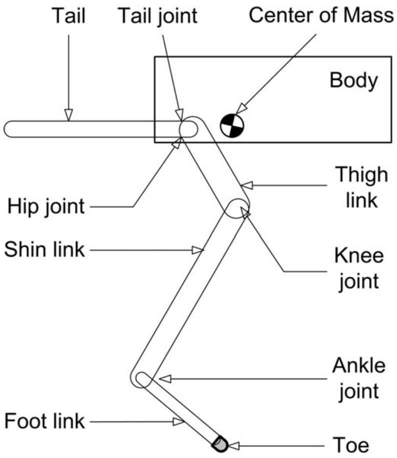

Fig. 7. Uniroo—Mechanical design similar to a real Kangaroo (adapted[13]).

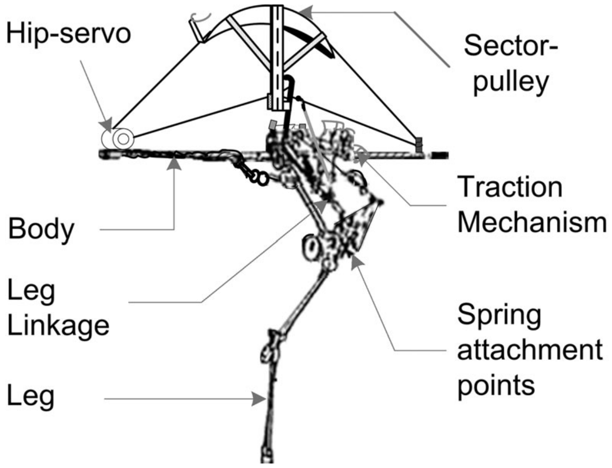

Fig. 8. Overall view of Papantoniou's final prototype (adapted[14]).

different mechanical designs and transmission mechanisms. Finally, he selected two special transmission mechanisms, namely, a cable-type transmission for the spring traction mechanism and a high ratio, reversible, toothed belt single-stage transmission based on the use of specially designed sector pulley for the leg-attitude actuator mechanism.

Prosser and Kam proposed[15, 16] and later built[17] an electromechanical prototype of a one-legged hopping machine. A vertically constrained experimental prototype is shown in Fig. 9. The physical prototype consisted of an elevated body, supported by a single springy telescopic leg and a leg actuator. On the body of the machine was a dc motor that drove an actuation mechanism and regulated the energy stored in the leg. The leg actuator was used to control the length of the leg. The leg and leg actuator consisted of a system of tubes and a mechanical spring, and were guided by a leg guide tube. This arrangement allowed smooth and repeatable hopping.

Buehler and co-researchers[18–24] developed two experimental prototypes of single-legged planar robots called "ARL Monopod I" (Fig. 10) and "ARL Monopod II" (Fig. 11). They avoided hydraulic and pneumatic actuation and focused on electrical actuation, since it is cleaner, safer, less expensive, and more appropriate for autonomous robots. Before the final implementation of ARL Monopod I, Rad and coresearchers[19] proposed and experimented the mechanical design of a prismatic robot leg (shown in Figs. 12 and 13). This design was optimized for electrical actuation and selected as the basic mechanical design for building both ARL monopods. Their objective was to achieve reliable locomotion with high torque-to-weight ratio using small electric motors. The selection of the components was based on maximizing the energy added during the short stance phase. They used an 80 W brushed dc motor (Maxon, 1.3 kg), a 5 mm/rev ball screw and a 4 kN/m spring. They also derived a dynamical model of the robot, including compliant ground and actuator dynamics. They validated their derived model by comparing simulations and experimental runs.

Gregorio *et al.*[20, 21, 23] observed that there was a need for modifying the thrust controller in their previously developed leg design (Fig. 13). So, they modified the hardware of the leg and also developed the full prototype (Fig. 14) called "ARL

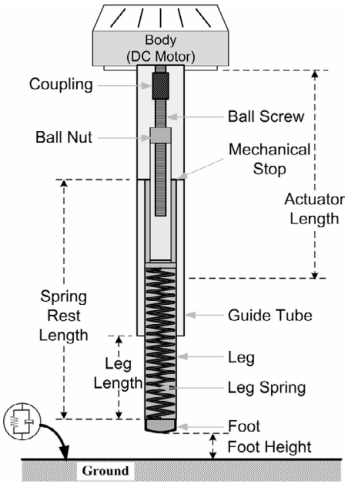

Fig. 9. Physical one-legged hopping robot developed by Prosser and Kam (adapted[15]).

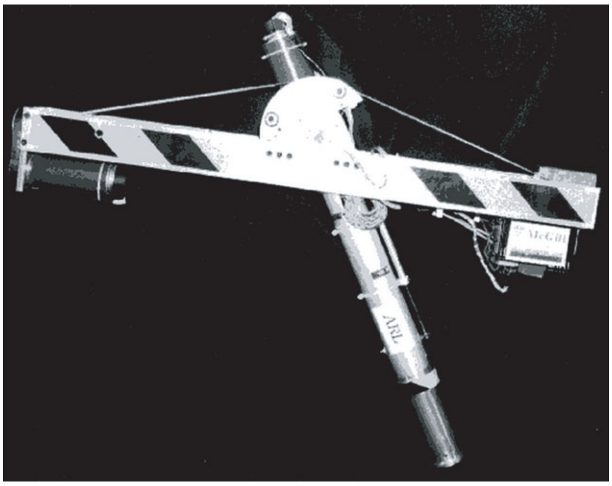

Fig. 10. ARL Monopod-I.[18]

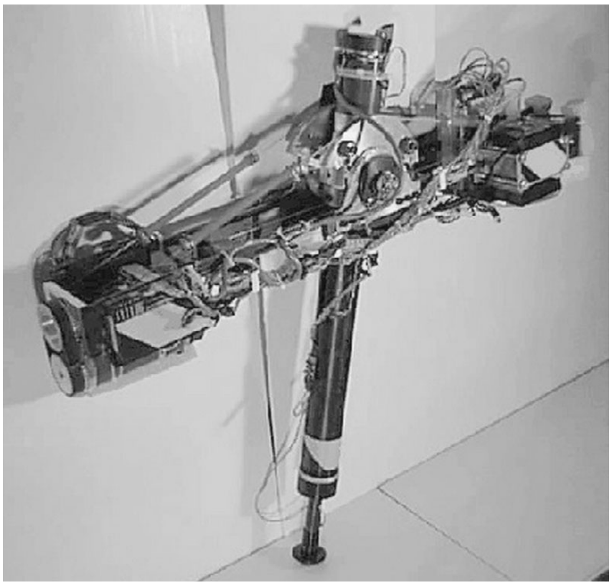

Fig. 11. ARL Monopod-II.[18]

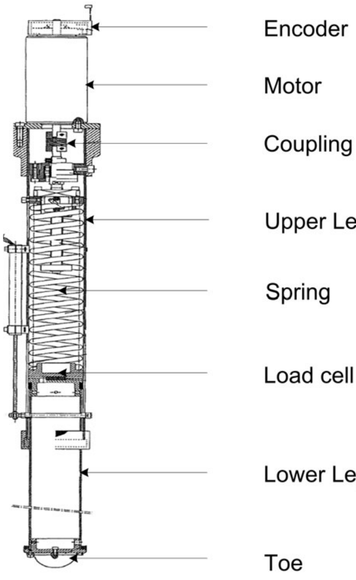

Fig. 12. Basic mechanical leg design of ARL Monopods (adapted[19]).

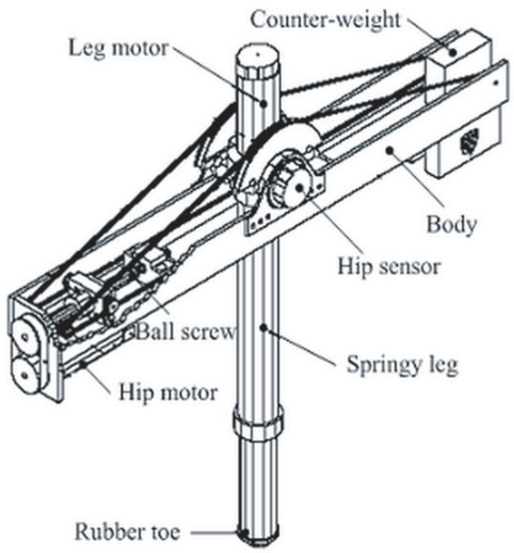

Fig. 14. Detailed mechanical assembly of ARL Monopod-I (adapted[20]).

Monopod I", which weighed 150 N. This hopping robot was confined to move in a vertical plane on a 5 HP treadmill through a Virtual Motion System. The ARL Monopod and Raibert's single-legged robot were similar in overall size and kinematics but differed significantly in their actuation and transmission systems. ARL Monopod was similar to Raibert hopper with electric actuators instead of hydraulics, and a metal spring instead of an air cylinder. The hopping machine consisted of two subsystems, namely, a telescopic leg and a robot body attached to the leg through an actuated hip. The body inertia was kept high compared to that of the leg in order to minimize the pitching motion of the body in response to leg swinging. Also, they kept body weight low to minimize energy consumption for vertical motion. The leg actuation was targeted to excite and maintain a sustained vertical oscillation. The hip actuation system was designed such that it controlled the leg angle in order to achieve stable running up to a speed of 1.2 m/s with an average power consumption of 125 W. They also realized that there was significant wastage of energy during swinging of leg in the previously developed ARL Monopod I.[22, 24] In order to build an energy-efficient prototype, they modified the ARL Monopod I by adding compliant elements as shown in Fig. 15 and called it "ARL Monopod II". It weighed 180 N, and total power consumption at maximum running speed (1.25 m/s) was only 48 W.

Mehrandezh *et al.*[25] presented a mechanical design for a 1D hopping robot (as indicated in Fig. 16). A dc motor (as a main body and actuator) plugged in series with a telescopic springy leg was able to add or remove energy from the system to regulate the jumping height.

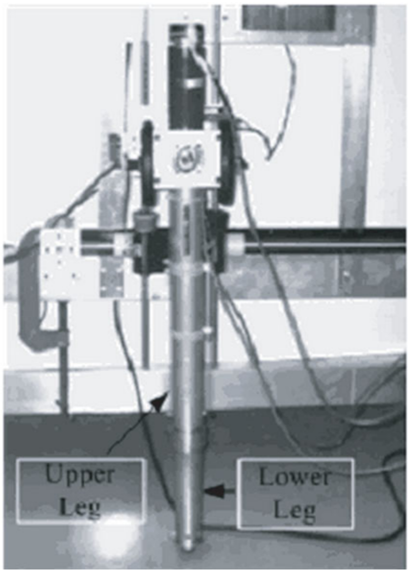

Fig. 13. Experimental prototype of robot leg proposed by Rad *et al.*[19]

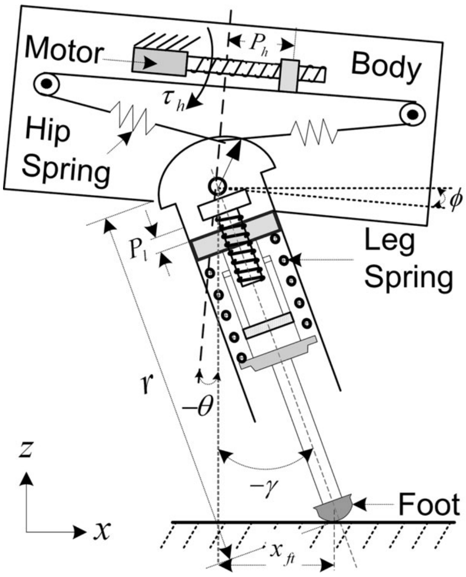

Fig. 15. Mechanical elements of ARL Monopod- II (adapted[22]).

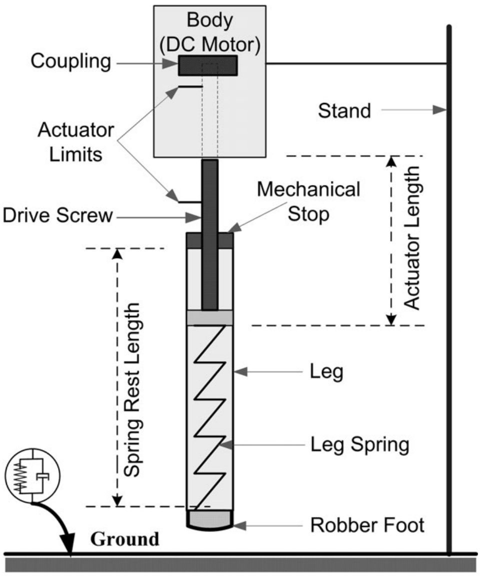

Fig. 16. Schematic of the hopping machine proposed by Mehrandezh *et al.* (adapted[25]).

In order to enhance the performance of the hopping robot in practical use (such as an increased payload capacity), it is necessary to use lightweight actuators. Inspired by this idea, Okubo *et al.*[26] constructed a prototype of a high-jump machine using small lightweight actuators and springs. The model and the prototype are shown in Fig. 17. This jumping machine had a body with center of gravity (CG) along the axis of a spring. The body of the system consisted of two links, symmetric to the springy leg. Each link had its own mass and a rotary actuator, and the body had an additional mass located at the top of the springy leg. The links were forced to move symmetrically to the leg and thus, the additional mass moved along the leg. This sequence of movements accumulated the mechanical energy in the spring sequentially. Depending upon the postural stability, the actuators could be moved to store energy. The notable features of the robot were, self-energizing spring without resonant oscillations, ability to overcome an obstacle and the use of small actuators. The experimental results were reported for a machine restricted to hop in-place.

Ringrose[27, 28] built a self-stabilizing one-legged robot called "Monopod". It was self-stabilizing in the sense that it was inherently stable and did not require sensory feedback to reject perturbations. This monopod consisted of a body mass, attached to a foot by a telescoping leg, and a spring and an actuator connected in series, as shown in Fig. 18. There was a damper in parallel with the actuator and spring, representing friction in the leg. The contact surface of the curved foot was covered with a rubber-sheet to prevent slippage. A set of linear bearings constrained the foot and the leg to move vertically, and a hinge mechanism kept the foot from twisting. A boom restricted the robot motion, such that the robot could move vertically and horizontally and could pitch around the boom. In an attempt to achieve self-stabilization, they actuated the central electric motor (actuator) with a delay cycle timer. A linkage transferred the rotary motion of electric motor to linear motion in the spring. This monopod could hop with self-stabilization in the presence of height, phase, and pitch disturbances.

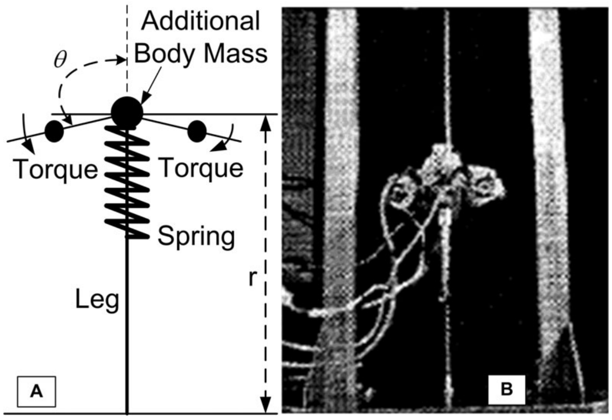

Fig. 17. Jumping machine model (A) and the prototype (B).[26]

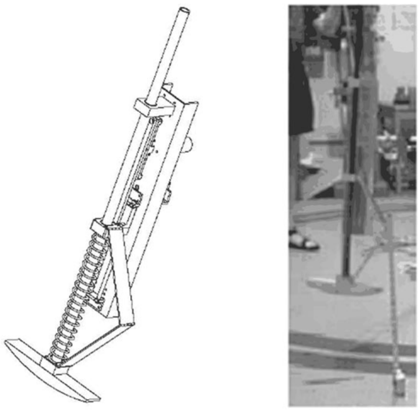

Fig. 18. Self-stabilizing "Monopod" (Left). Design (Right). Prototype.[27]

Zhang *et al.*[29] explored the animal-like locomotion using three-link and four-link articulated unipeds. They tested the feasibility of their approach with preliminary experiments on the prototype of a three-link uniped. They fabricated three-link structures (two prototypes) with different mechanical specifications. The design had two dc motor actuated joints and self-locking mechanism (gear reducer) to avoid undesirable folding of the link, as shown in Fig. 19. Of the

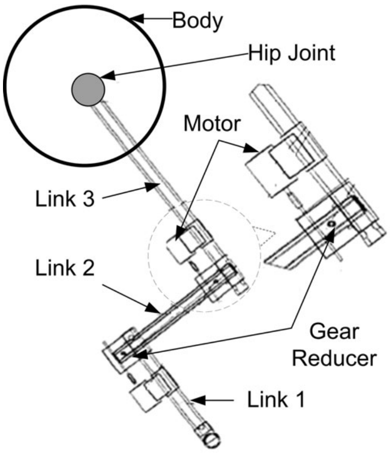

Fig. 19. Three-link Uniped design details (adapted[29]).

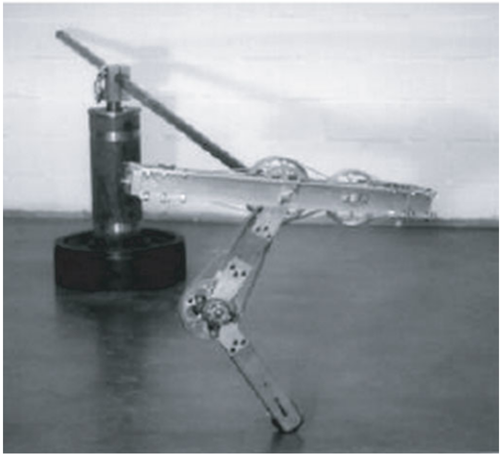

Fig. 20. "OLIE"—One-legged hopping robot with an articulated leg.[32]

two prototypes, the one with the steel gears and the stronger links performed superior as compared to the one with plastic gears. The uniped was able to jump in 2D under normal and reduced gravity.

De Man *et al.*[30–33] constructed a prototype of an electrically actuated one-legged hopping robot with an articulated leg (Fig. 20). They called it "OLIE" (One Leg Is Enough). This prototype of 11.66 kg mass and upright standing height of 0.65 m had three major parts, namely, body, upper leg, and lower leg. It was constrained to move on the surface of a cylinder by a boom connected to the hip of the robot. It consisted of an active part comprising of two electric motors regulating the hip and the knee. The passive part, consisting of two carbon steel torsional springs placed at the knee, exerted a torque depending upon the relative angle between the upper and lower leg. They presented some simulation results but experimental trials had not been reported.[32]

After successful implementation of "Uniroo," Zeglin built the planar bow-leg hopper[34, 35] as shown in Fig. 21. It was the first prototype of a hopping robot using the bow principle with a total machine mass of 2.5 kg. It was constrained to 3 DOF on the surface of the sphere using a radial boom. It operated in simulated 35% gravity provided by a counter-spring attached between the boom and the ceiling. In such reduced gravity, it hopped as high as 50 cm and as fast as 1 m/s. The machine used two hobby servomotors for control,[36] one to apply thrust by compressing the leg via the bow-string, and the other to position the leg using a pair of control strings. Motive power was supplied by an onboard battery pack of four Ni-Cd sub-C cells, which was sufficient for about 30–45 min of operation.

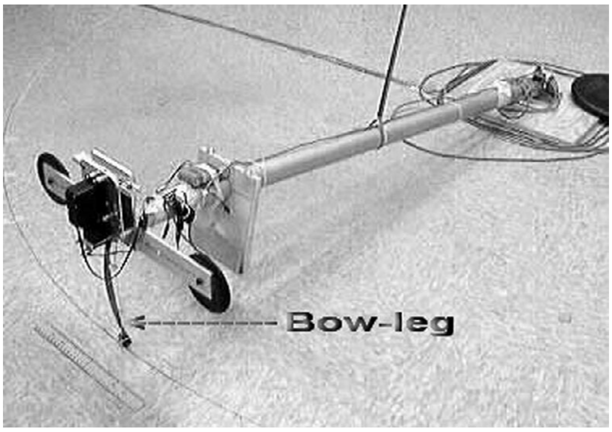

Fig. 21. Planar bow-leg hopper developed by Zeglin.[34]

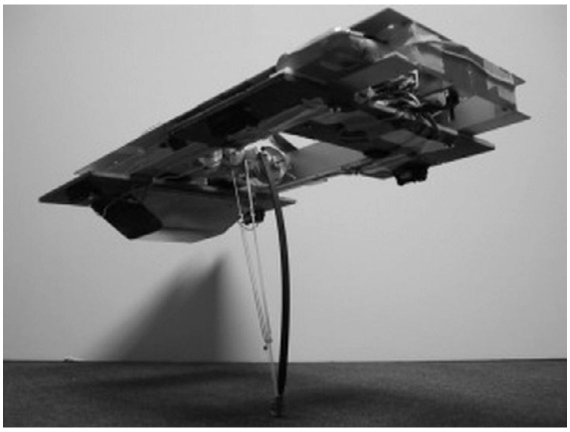

Fig. 22. 3D bow-leg hopper developed by Zeglin.[38]

The performance of the planar prototype motivated Zeglin and coresearchers for constructing a fully self-contained hopping robot based on the bow leg. Their first planar prototype demonstrated some limited hopping abilities. A fundamental question raised was whether a freely pivoting hip could passively stabilize body attitude in the 3D case. Their design objective was to develop the leg and body design in order to build a machine intended for remote control.[37, 38] They constructed a 3D machine as shown in Fig. 22. The planar machine had parasitic torques from the constraining boom that increased the body damping, while the 3D machine was only damped by air friction. Moving from 2D to 3D geometry, they modified the design of the body, hip, and leg positioner. Since the hip used a gimbal instead of a pin joint, the placement of the leg positioning and tensioning strings was especially a challenging problem.

In an exploration of single-legged mechanical systems incorporating neural network-based controllers, Berkemeier and Desai,[39, 40] designed and constructed a novel 2D, 2 DOF electrically actuated hopping leg (Fig. 23). It had decoupled

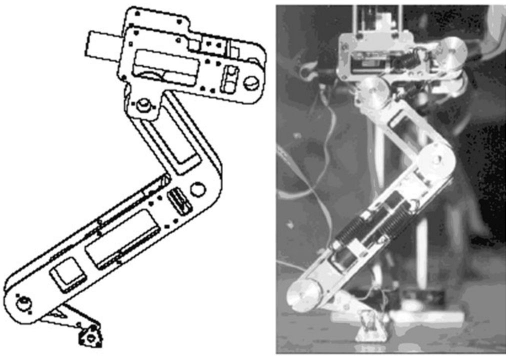

Fig. 23. Hopping leg design and prototype developed by Berkemeier and Desai.[39]

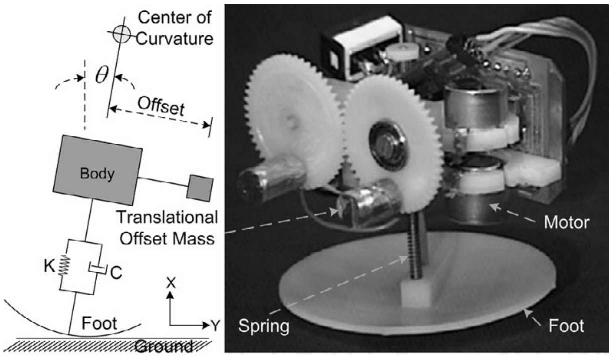

Fig. 24. Leg schematic and a 5-cm hopping monopod built by Wei *et al.*[41]

vertical and forward motions. Other specifications included aluminum leg links, coupled leg actuation mechanism (20 W dc motor with a drive spool, bevel gears, steel cables, and springs), hip actuation mechanism, controllers, and signal conditioning parts attached to the legs (PID motion control chip, amplifier, optical encoders, digital I/O board, etc.). They performed experiments on this hopping leg fitted with a boom constraint and a controller implemented on a real-time operating system computer. The counterweight was placed at the opposite end of the boom such that the leg was only lifting its own weight.

Wei *et al.*[41] constructed an autonomous one-legged hopper that could fit into a 5 cm cube. It was statically stable and passive-dynamically stable. Figure 24 shows the leg structure and robot prototype. The hopper was restricted to move in a plane and defined the motion with 5 DOF. Hopping was achieved by exciting a spring-mass system at its resonant frequency. 2D simulations were extensively used to choose the design parameters, before the construction of the robot. This hopper included a translational offset mass in order to cause the tipping action that helped in forward propulsion. The hopper did not have active directional control. It could travel at a rate of 7.75 cm/s or 1.5 body lengths per second. It could cover a distance of over 225 cm without tipping over in one direction, and climb a step of 1 mm. It could operate up to 45 min before depleting the energy stored in the batteries. Through experiments on the prototype, they observed that the performance could be improved if the hopper included some mechanism of directional control.

Sandia's Intelligent Systems and Robotics Center[42] constructed a lightweight hopping robot of the size of a coffee-can, as shown in Fig. 25, for space exploration applications. It was made to jump using an internal combustion-driven piston. This hopper could travel greater distances and clear larger obstacles using hops of more than 20 feet in height. It could make about 100 hops on a tank of hydrocarbon fuels. The hopper had been tested in a variety of conditions, and performed reliably against obstacles, mud, sand, and rough terrain.

In the same context, Peck[43] developed a prototype (Fig. 26) of a low-power, low-cost, dynamically stable, and electrically actuated autonomous hopping robot, based on the concept of controlled momentum gyroscopes. The robot consisted of an axially symmetric, momentum-bias stabilized body and a telescopic leg. The line of action of the leg force was passing through the robot's CG. It was observed that rover could spin for a long time imitating a momentum-biased top in stance phase and a hopping top (gyrostat) in flight phase. No experimental results were reported.

Fig. 25. Hopping robot developed in Sandia's National Lab.[42]

Based on the configuration of Pixar's Luxo jumping lamp, Albro and Bobrow[44] built a two-linked acrobat like experimental hopping robot. It had Plexiglas base and two aluminium links connected by two active joints. It was statically stable and had 5 DOF as shown in Fig. 27. The active joints were equipped with low-cost RC servomotors, which were controlled by Scenix SX series microcontroller. It had a fully actuated configuration during the stance phase and an underactuated configuration during the flight phase. The lampshade part of the hopper was clamped on the experimental robot as a payload. In order to understand and validate the dynamic stability of a single-legged vertical hopper model, Cham and coresearchers[45,46] constructed a pneumatically actuated vertical hopper called "Dashpod"

Fig. 26. Gyroscopic hopping robot prototype.[43]

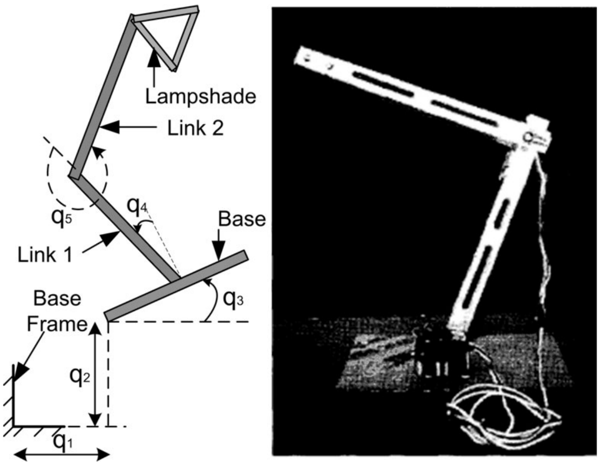

Fig. 27. Luxo hopper: model and prototype.[44]

(Fig. 28). The main parts of Dashpod were a low-stiction pneumatic piston, pneumatic cylinder, wide curved dish (foot), solenoid valve (actuator), and a spring. The spring was connected between the foot and the pneumatic cylinder. The solenoid valve allowed pressurized air to fill the cylinder and caused the Dashpod to push against the ground. Experimentally, they demonstrated periodic hopping motion when the valve was activated periodically with a square wave function. They conducted several experiments varying the mass attached to the dish and the thrust timing for activating solenoid valve. They categorized the results as "Period-2," "Max Work," and "Hop-Settle-Fire" type of trajectories based on the stride period and mass of the body.

As tendons store energy of locomotion in the form of potential energy during stance and also absorb the impulse shocks at touchdown, they play an important role in running or jumping motion of living creatures. Motivated from such studies, Hyon and coresearchers[47, 48] proposed a new alternative leg model for high-speed running. They focused on the geometric alignment of the muscle–tendon system of ankle joint and its performance (Fig. 29). The robot had articulated leg composed of three links. It used two hydraulic actuators as muscles and a linear spring as a tendon, as shown in Fig. 30. They also developed prototype hardware of one-legged running robot (Fig. 31), named "Kenken". The mass of the leg was relatively large (about 3.6 kg). Individual masses of the thigh, shank, and foot were 2.42, 0.75, and 0.43 kg, respectively. Using an empirical controller based on the passive dynamics of the model, the robot had succeeded in running several steps in a plane. The experiments showed that the selected leg mechanism was effective for running. There

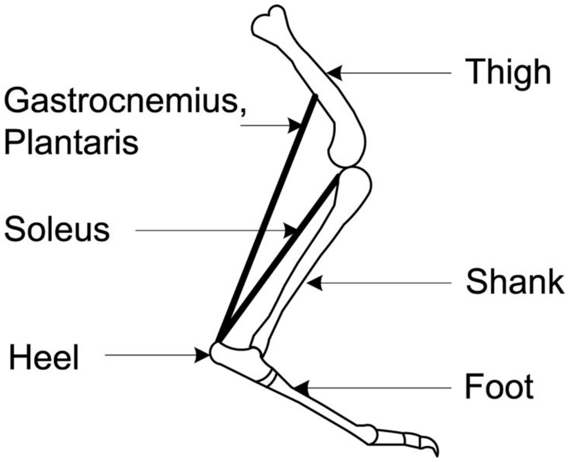

Fig. 29. Musclo-skeletal system of dog ankle (adapted[47]).

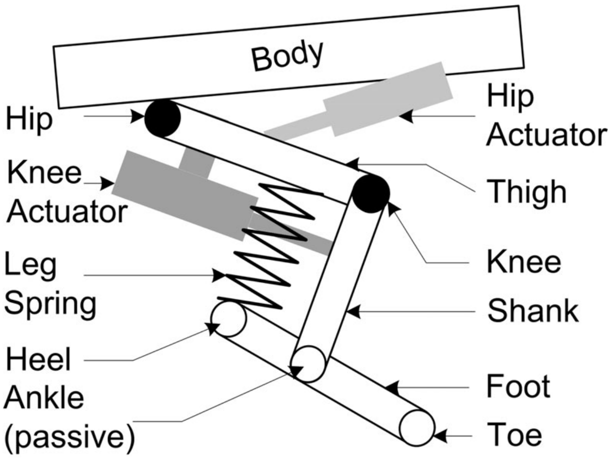

Fig. 30. One-legged robot with a new hind-limb mechanism (adapted[47]).

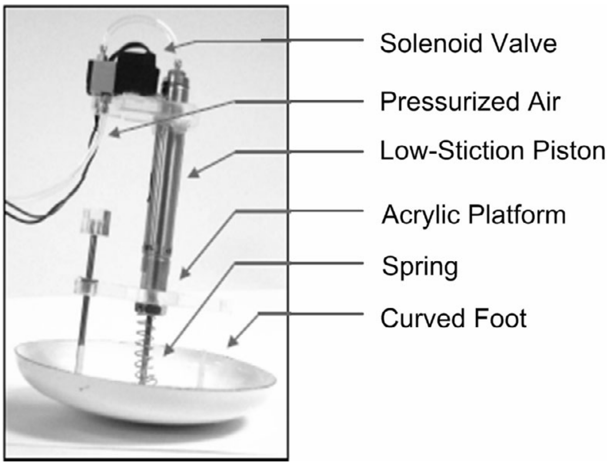

Fig. 28. Pneumatically-actuated vertical hopper "Dashpod".[45]

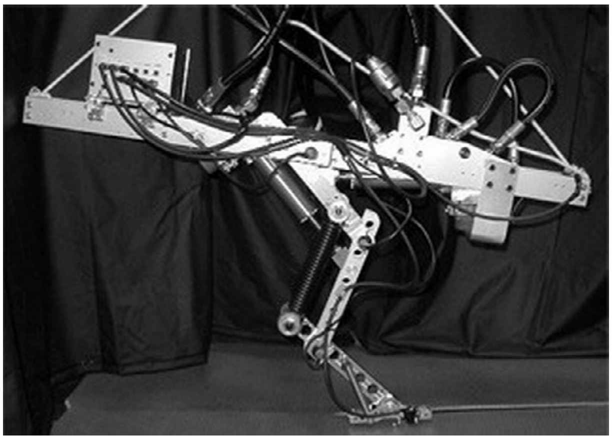

Fig. 31. Biologically-inspired hopping robot: "Kenken".[48]

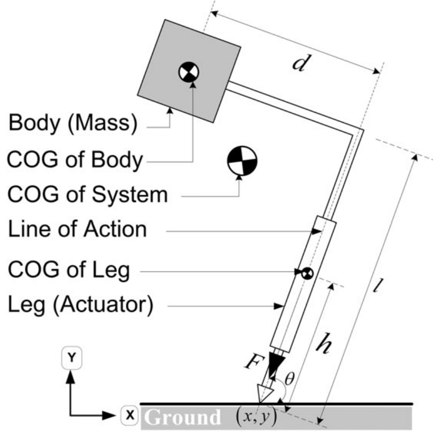

Fig. 32. Asymmetrical hopping model proposed by Kuswadi *et al.* (adapted[50]).

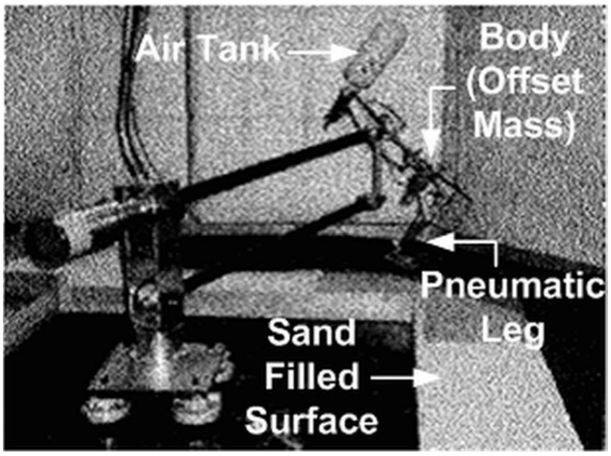

Fig. 34. Improved prototype of hopping robot developed by Funato *et al.*[53]

was no actuator at its foot (this meant that the toe could rotate on the ground during stance phase, acting as a free pivot). The most distinctive feature of this model was arrangement of the leg spring. The leg spring was attached between the thigh and heel parallel to the shank, which could be observed in the robot "Uniroo."[13] This arrangement facilitated the leg spring to store the potential energy during stance phase and to retract and extend the leg during flight phase. Thus, this leg spring arrangement played an important role in the hopping motion.

Kuswadi *et al.*[49–54] proposed an asymmetric model of a single-legged robot. The proposed model, as shown in Fig. 32, did not have the center of gravity on the line of action of the actuator. They realized that a sustained jumping motion was possible using a telescopic pneumatic leg and single pneumatic linear actuator. They also constructed the physical prototype (shown in Fig. 33). The robot consisted of an offset body and a leg, which were in contact with a sufficiently wide horizontal ground surface. They produced partially elastic landing using counter weight arrangement, sand-filled surface (covered with rubber sheet), and pneumatic cylinder (operated like a damper). Force actuation affected the orientation of the body by a moment of force that arose due to the mass of the body and the offset of the leg. Hence, both the orientation of the body and the height of a jump could be controlled by only one actuator. Funato *et al.*[53] experienced some hardware difficulties and conceptual problems in this prototype. They found difficulty to get enough data to validate their model of the robot. They implemented a new prototype (shown in Fig. 34), which had the original asymmetrical configuration except changes in robot parameters that were optimally determined from the "transition map."[54]

Based on the same proposal of self-stabilization with a mass-offset, a prototype of a 3D hopping robot was constructed.[55] It was able to hop in any direction and reject horizontal disturbances. As shown in Fig. 35, the prototype consisted of a springy leg, which was actuated electrically with a dc motor placed at the top of the leg. The rotary motion of the motor was converted to linear motion with the help of a slider-crank mechanism. Another servomotor along with a spur gear could rotate the weights, thus, influencing the offset mass system.

Uno *et al.*[56] modeled and developed a hopping robot with an impulsive actuator. The hopping robot shown in Fig. 36 was modeled as two masses and a spring. They analyzed the hopper based on the optimal timing of the impulsive input.

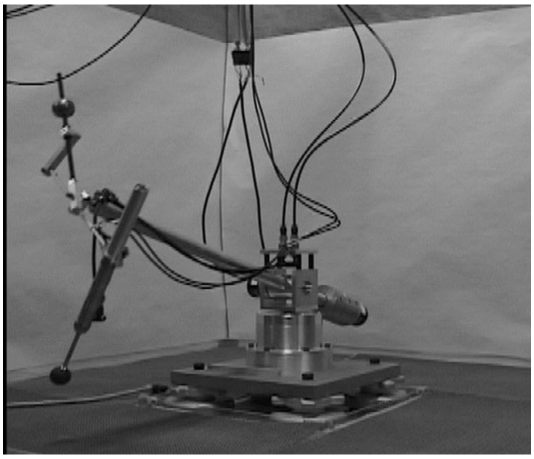

Fig. 33. First experimental prototype of hopping robot built by Funato *et al.*[53]

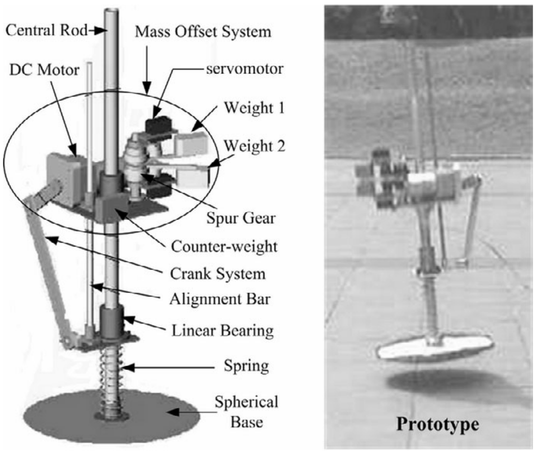

Fig. 35. An overall design and prototype of hopping robot.[55]

Fig. 36. Hopping robot prototype with impulsive actuator.[56]

A motor drove a mechanism for generating impulsive input. The disk and the frame attached to the base were connected with a loose wire as shown in Fig. 37. When the point of the disk attached to the wire came to the lowermost position, the wire was strained, the frame was pulled down, and an impulsive force was generated.

Akinfiev *et al.*[57] designed an in-place hopping robot with telescopic leg and electric drive. The kinematic configuration of the robot is shown in Fig. 38. The robot leg was anchored to a body, and a spring was installed between the body and the leg. An electric motor with a speed reducer was fixed on top of the body, and the motor was activated by a control system. The motor shaft was connected to a rotating cylinder, which was connected to the top of the leg through a flexible rope. A prototype was designed to jump up to 0.4 m and had body and leg weight as 3.5 and 0.15 kg, respectively. They observed that the frictional losses between the spring and tubular guide during touchdown and takeoff were significant. So, they replaced the single compression spring with four extension springs and observed more energy-efficient movements.

As seen in[39, 44] an acrobot-like hopping robot was realized using two-link structure in the hopping leg. In that context, Leavitt *et al.*[58] built an inexpensive and lightweight pneumatically actuated one-legged robot. It had two rigid links with circular (massless) wheel foot (Fig. 39). Pneumatically powered piston-cylinder system constituted the actuator and was placed at the knee joint. The piston force was converted to the torque at the hinge, which resulted in an acrobot-like underactuation.

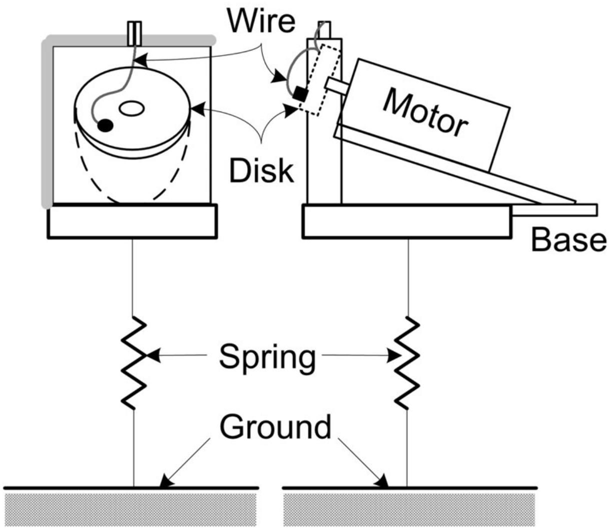

Fig. 37. Mechanism of generating impulsive force (adapted[56]).

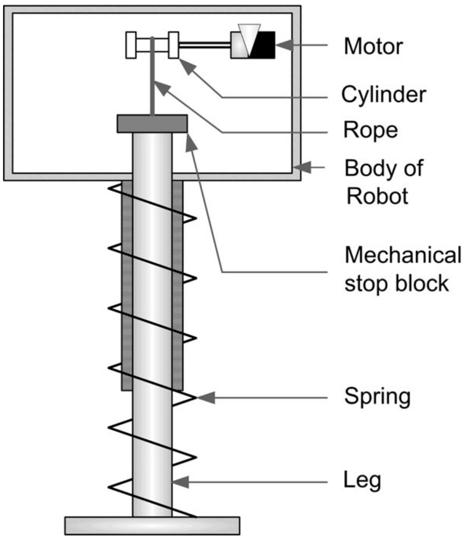

Fig. 38. Kinematic configuration of in-place hopping robot (adapted[57]).

The 2-DOF Spring-loaded Inverted Pendulum (SLIP) model (described in Section 3.1) is the minimal order model used for analysis and control of running legged robots. Sato and Buehler[59] exploited this model to implement a 2D hopping robot, called the ‘SLIP Hopper’, which had only one rotary actuator placed at the hip joint. This actuator (servomotor with gear reducers) rotated the leg to the desired angle at touchdown and take-off events. The spring deflection at ground contact produced the bouncing force. Thus, the robot was able to regulate both forward speed and hopping height using only one actuator. The “planarizer” (consisted of two beams and rotational base) as shown in Fig. 40, constrained the robot to planar motion. An aluminum block was placed at the top of the leg to form the platform for future installation of local controller. Infrared distance sensor was

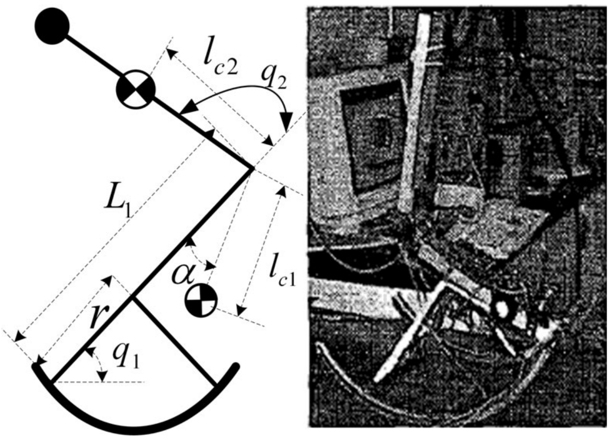

Fig. 39. Acrobot-like hopping robot—model and prototype.[58]

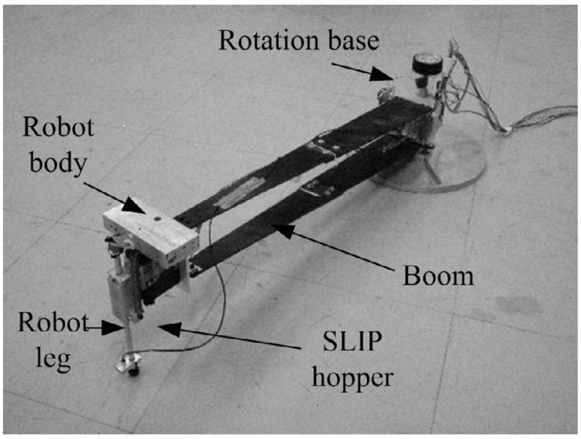

Fig. 40. SLIP hopper with planarizer.[59]

used to detect the touchdown and take-off events. The present robot was connected to the target computer via an interface card.

This section described only the mechanical functioning of hopping robot prototypes, incorporating a single leg. There were also a few special designs and mechanical hardware, which performed like the hopping robot, but did not use a leg to make the contact with the ground. These have not been covered in the present survey. In the next section, various aspects of modeling/simulation of these robots are discussed.

3. Modeling and Simulation of Hopping Leg Configurations

Several studies in hopping robots are inspired by the biological systems. For example, Lee and Raibert[11] introduced a hoof in an articulated legged robot inspired by horse locomotion, Zeglin[13] constructed “Uniroo” inspired by kangaroo locomotion and Hyon and Mita[47] implemented three-link leg structure similar to a dog hind limb. Mathematical models of hopping robots are useful to simulate the behavior, prior to building a physical prototype. Since design parameters can be varied easily in a simulation, design based on modeling and simulation can potentially yield better prototypes. Broadly, two types of legs have been used in single-legged hopping robots, namely, telescopic leg and articulated leg. These leg models can be further distinguished on the basis of negligible or nonzero foot/leg mass; absence or presence of ankle joint; point, flat, or curved foot; pneumatic, hydraulic, or electric leg actuation; kinematic configuration of the leg mechanism; etc.

3.1. Telescopic Leg

Spring-loaded Inverted Pendulum (SLIP) model has been used to study locomotion in legged creatures by many researchers in the biomechanics community as well as in Raibert’s pioneering work. Raibert and coresearchers[1, 27] designed numerous theoretical models as well as prototypes of one-legged hopping robot using a telescopic (SLIP) leg configuration. Also, this model attracted many researchers to develop controllers and analyze its stability. The idealized SLIP model consists of a body with mass concentrated at a point and a massless springy leg. If this model is restricted in 1D, then it has only 1 DOF (Fig. 41) and is, hereafter, referred to as 1-DOF SLIP model. If it has planar motion and pitching of body is ignored, then it has 2 DOF (Fig. 42) and is, hereafter, referred to as 2-DOF SLIP model. The planar 2, DOF model with pitching of body has 3 DOF (Fig. 43) and is, hereafter, referred to as 3-DOF SLIP model. The SLIP model having body and leg (with distributed mass) is, however, a more representative model of practical systems.

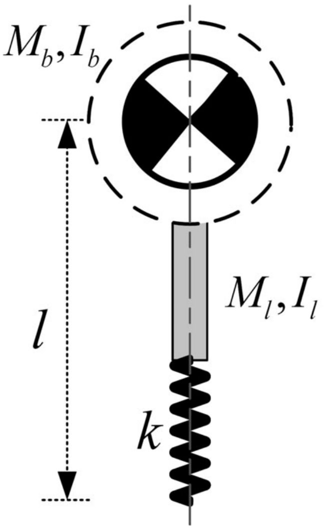

Fig. 41. 1-DOF SLIP model.

3.1.1. 1-DOF SLIP Model. Several researchers[15, 17, 19, 25, 57, 60–74] focused on the 1-DOF telescopic leg. These models were able to realize the in-place hopping/juggling/jumping motion. In order to analyze the behavior of Raibert’s vertical hopper, Koditschek and Buehler[61] formulated the modified Raibert’s model (Fig. 44). They considered a unit mass body, massless leg, and the environment with viscous friction. They arranged the body and the leg such that it simultaneously functioned as a prismatic joint and an energy-storing element (pneumatic spring). They presented two discrete dynamical models of

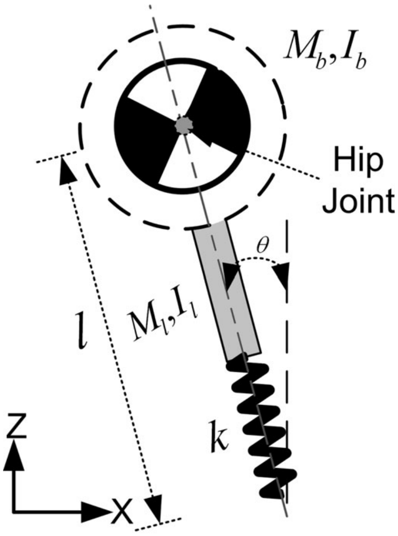

Fig. 42. 2-DOF SLIP model.

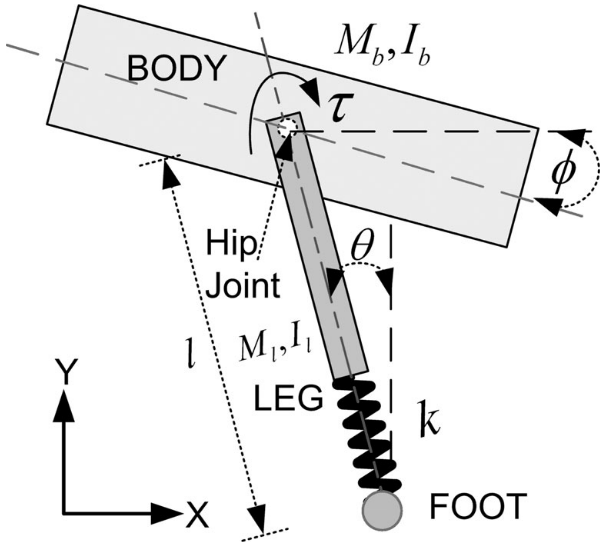

Fig. 43. 3-DOF SLIP model.

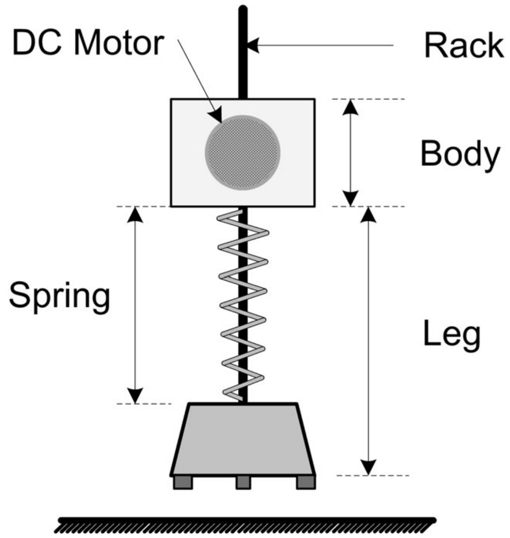

Fig. 45. Rotary actuated vertical hopping robot (adapted[73]).

this vertical hopper (juggler) as "linear spring model" and "nonlinear spring model."

As noticed in the Section 2, the body mass plays an important role as a source of charging the spring.[26] Kusano and Tsutsumi[73] also illustrated a similar result. The body (dc motor acting as the actuator) was placed above the leg (Fig. 45). The vertical travel of the body produced resonant oscillations in springy leg leading to hop in-place.

3.1.2. 2-DOF SLIP model. A planar 2-DOF hopper with a telescopic leg has a prismatic joint and may be connected to body with or without revolute hip joint. It can be controlled with linear actuator or rotary actuator, or both. A 2-DOF SLIP model is energetically conservative in nature, provided proper landing is achieved. This model had been practically exploited to achieve hopping gait with underactuation.[58] Several other researchers also studied this model,[27, 28, 41, 45, 46, 49–55, 59, 66, 75–85] mostly to analyze the stability. Harbick and Sukhatme[78] proposed 1D and 2D models of a pneumatically powered one-legged hopping robot (Fig. 46).

Almost all model configurations[75–84] had symmetric placement of body CG lying on the geometrical axis of the leg, except the asymmetric configurations considered by Wei,[41] Kuswadi *et al.*,[54] and Shanmuganathan.[85] Shanmuganathan[85] considered the configuration of single-legged hopping robot shown in Fig. 47. He termed it the "Springy Leg Offset Mass (SLOM) Hopper" wherein the spring force was offset from the center of mass, which gave rise to restoring rotational moments. Through simulation and phase-plane analysis, he demonstrated the possibility of achieving continuous jump due to rotational moments arising around CG during each cycle.

3.1.3. 3-DOF SLIP model. The planar 3-DOF SLIP model shown in Fig. 43 exploits the extra DOF of body pitching in balancing the robot.[5–9, 20–24, 42, 43, 86–114] Mombaur *et al.*[101] presented a different model of 2D hopping robot having a

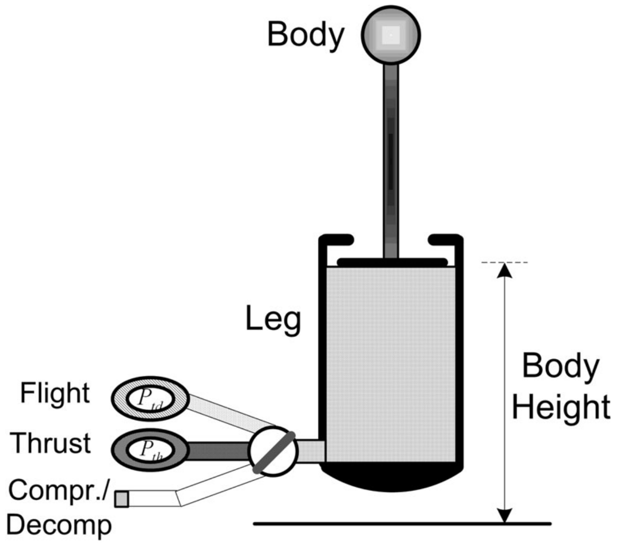

Fig. 44. Simplified model proposed by Buehler and Koditschek (adapted[61]).

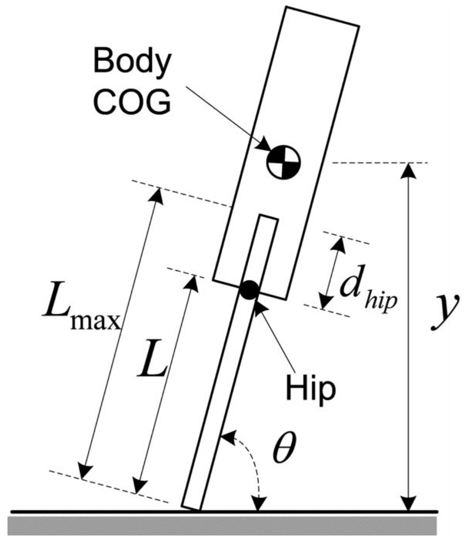

Fig. 46. 2D hopping model considered by Harbick and Sukhatme (adapted[76]).

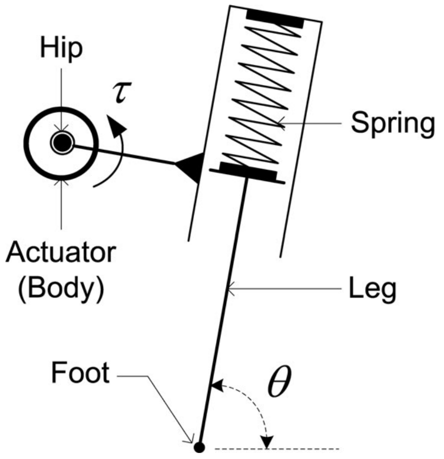

Fig. 47. Underactuated (rotary actuator at hip) springy-leg offset mass hopper (adapted[85]).

small circular foot and even a point foot. The robot shown in Fig. 48 consists of a toroidal body and a telescopic leg that are connected by an actuated hinge. The models described in[5–9, 42, 43, 86–105] have noncompliant hip, while many studies[20–24, 106–114] added a torsional spring at the hip to obtain energy-efficient hopping, such as shown in Fig. 49.

3.2. *Articulated Leg*

Gaits with articulated legs are representative of animal and human locomotion. Such a leg configuration constitutes multiple links and active or passive rotary joints, with or without spring and damping elements. We categorize such legs as springless and springy articulated legs, based on the use of springy elements.

3.2.1. Springless articulated leg model. The present day humanoid robots are stiff-legged, have complex structures, and do not use energy restoring element like pneumatic hydraulic cylinders or mechanical springs. Generally, such robots are statically stable, and the technique of balancing is based on the principle of conservation of kinetic energy and angular momentum. It is possible to achieve a hopping gait with the use of some active joints. A few researchers[29, 44, 115–124] tackled this problem.

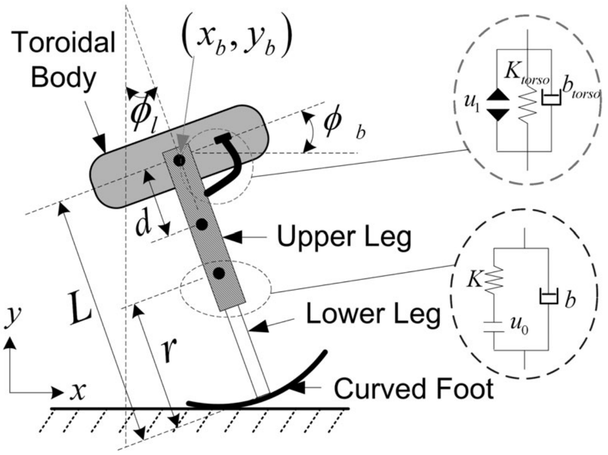

Fig. 48. Hopping model considered by Mombaur and coresearchers (adapted[102]).

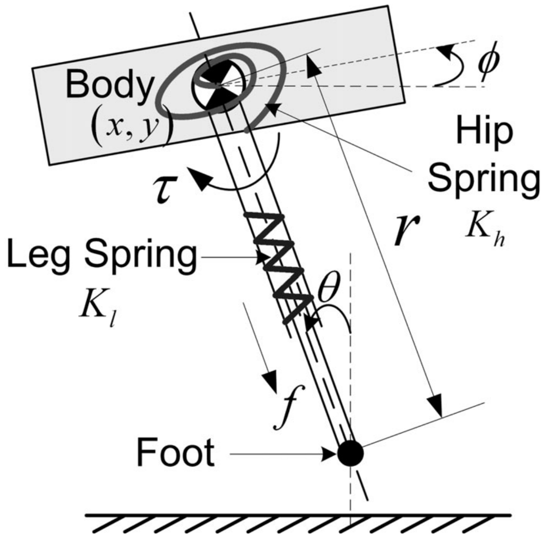

Fig. 49. 3-DOF SLIP model with hip spring.

Berkemeier and Fearing[115] achieved hopping and sliding walking gaits with a planar robot (Acrobot configuration) as shown in Fig. 50. It had two links and only one actuated joint. They discussed the problem of controlling both balance and thrust with only one actuated joint. The mathematical model of the system was equivalent to a single nonvertical pendulum. Their basic idea was to use internal motion to achieve a particular orientation during flight phase. In the same manner, Geng *et al.*[116–119] suggested a novel one-legged model (Fig. 51) having three revolute joints, two links, and two feet. They achieved locomotion through ballistic flipping.

Ohnishi *et al.*[120–123] modeled their robot (Fig. 52) as a massless two-link leg system, composed of two revolute joints one each at the hip and the knee. The thigh link is modeled as parallel link structure, in order to distribute the actuator's torque. The robot is constrained to move in a plane

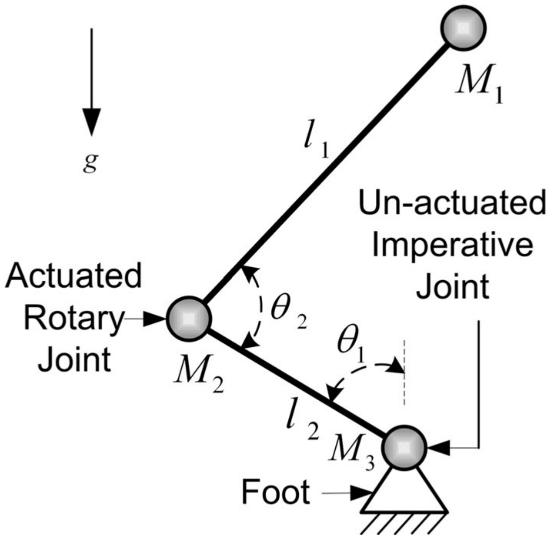

Fig. 50. Berkemeier and Fearing's planar acrobot (adapted[115]).

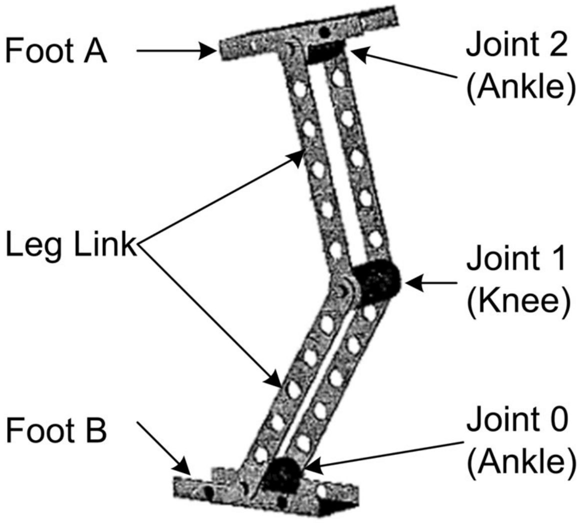

Fig. 51. Schematic one-legged model proposed by Geng and coresearchers.[116]

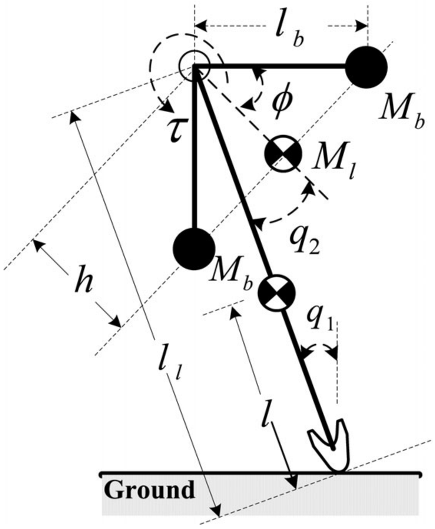

Fig. 53. Hopping robot having swinging body mechanism (adapted[124]).

with the help of a third link. They used the compliant ground model to demonstrate the soft-landing capability of their proposed robot. Nji and Mehrandezh[124] proposed a novel design of energy-efficient, underactuated, 2-DOF hopping robot. The robot, as shown in Fig. 53, consisted of a two-link (arm) swinging body and a massive leg. The effective CG of the body is located below the hip joint. The bowl-shaped swinging of the body around the leg helped to balance the posture of the robot.

3.2.2. Springy articulated leg model. Animals use muscles, whose elastic properties help in conserving energy and providing shock tolerance. Various hardware attempts[11, 13, 14, 30–40, 47, 48, 56, 58] used the articulated leg configuration in their prototypes.

Dummer and Berkemeier[125] analyzed the passive dynamics of a one-legged hopping robot having two point masses. The robot model (shown in Fig. 54) consisted of a point mass located at the hip and connected to the leg through a torsional spring, constituting an articulated joint. Another point mass was located at a fixed distance from the hip joint by a massless rod. This second mass was connected to the massless toe by a translational spring constituting a telescopic joint. A hardware embodiment of this model is shown in Fig. 23. The use of torsional spring served to balance the leg. The combination of these two springs gave rise to a natural resonance in the behavior of the model. They concluded that, to have passive forward motion, there was a need to select suitable initial conditions that could return the leg to its original position after a cycle of motion. In order to have self-sustained passive hopping, they put constraint on symmetry, maximum hopping height, and landing impact.

Schwind *et al.*[126,127] described two one-legged multijoint hoppers, which can be viewed as extensions of 2-DOF SLIP hopper. The first model, called “Springy Loaded Small Knee Monoped” (Fig. 55), had a 2-DOF revolute leg. This monoped had massless ankle and knee, passive ankle and

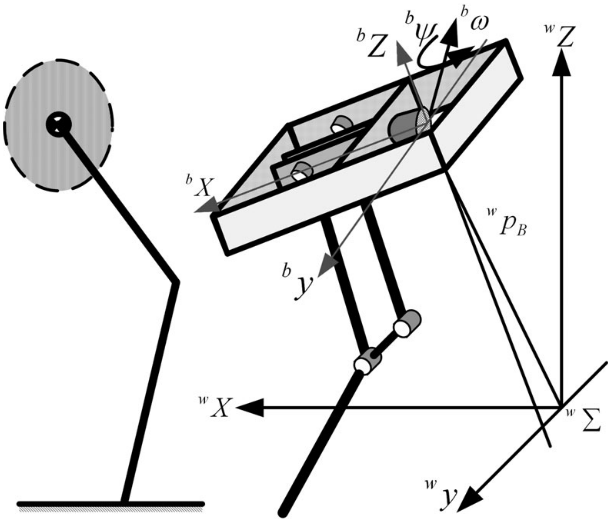

Fig. 52. Two-link schematic one-legged model proposed by Ohnishi *et al.* (adapted[120]).

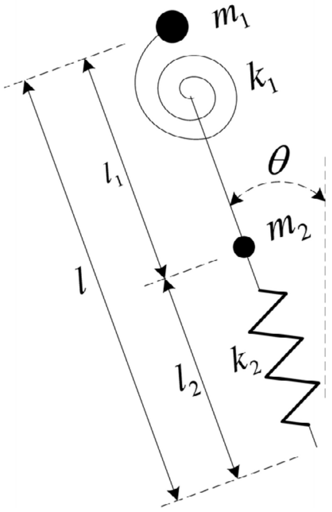

Fig. 54. Robot leg considered by Dummer and Berkemeier (adapted[125]).

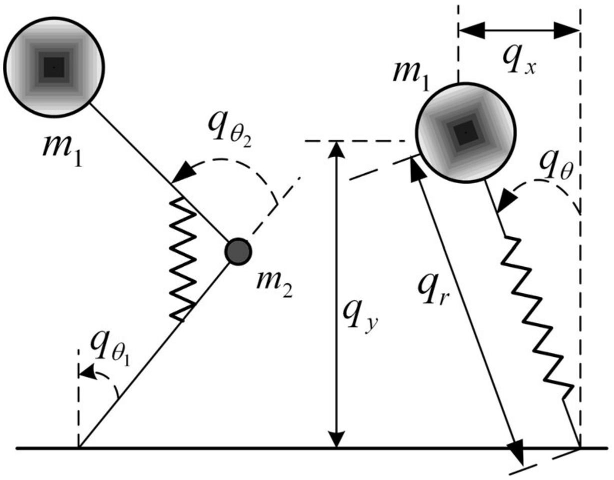

Fig. 55. Springy Loaded Small Knee (SLSK) Monoped and equivalent SLIP model (adapted[126]).

knee joints, and body placed at an active revolute hip joint. The second model, called "Ankle-Knee-Hip (AKH) Monoped" (Fig. 56), modeled the human/kangaroo as a 4-DOF articulated leg. This monoped had revolute ankle, knee, hip joints, and massless toe. The rotary actuators achieved the desired landing posture and virtual spring stiffness. Another four-link model as shown in Fig. 57, similar to AKH model, used two elastic actuators connected in series, one between the foot and the shank, and the other between the shank and the thigh.[128] It added joint compliance to absorb the shocks.

Waden and Ekeberg[129] presented a neuro-mechanical model. The mechanical part consisted of a two-link (thigh and shank) revolute leg structure. The robot had active hip and knee joints, and a passive ankle joint. The neuro-muscular part consisted of linear viscous elastic muscle, which was directed by a moto-neuronal output. The central controller (brain) could regulate the static torque of rotary actuator and stiffness of the spring. Two-link articulated robot proposed by Rummel *et al.*[130] had a hip actuator, which positioned the thigh link. The shank link was connected to the thigh link with a compliant elastic element.

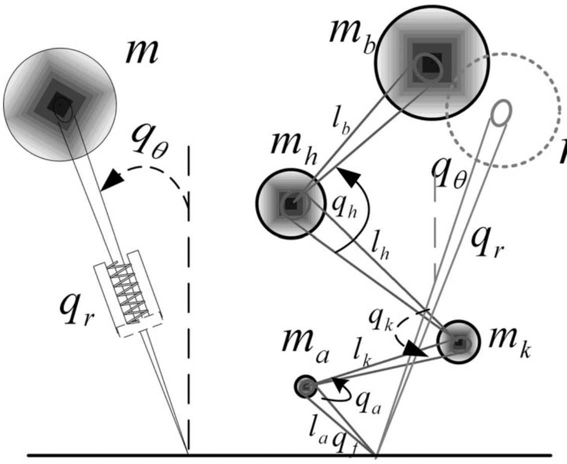

Fig. 56. Ankle-Knee-Hip (AKH) Monoped and equivalent SLIP model (adapted[127]).

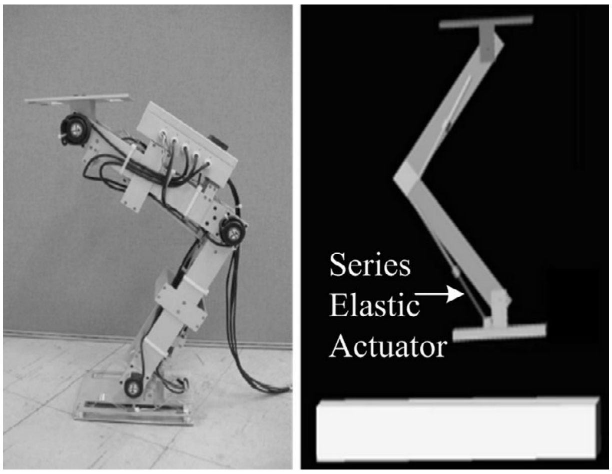

Fig. 57. Articulated four-link hopping robot with series elastic actuators.[128]

4. Control Strategies and Gait Stability

Single-legged robots have to be dynamically stabilized and pose a challenging control problem. Numerous attempts have been reported that aim to maintain some desired hopping height, forward speed, body orientation, etc., at certain phases of the hopping cycle. Stabilization of such robots may include active or passive dynamics. Also, the derived controller/s may be active either during stance phase or flight phase or both phases. From a control perspective, classification of such strategies may be given as the state feedback strategy, multirate feedback strategy, neural-fuzzy-genetic strategy, adaptive feedback strategy, dead-beat control, hybrid control, and variable structure control. Some literature also mentioned open-loop control, energy efficient control, and motion planning strategies. In this section, we are focusing on different approaches to stabilize the single-legged hopping robot.

4.1. Raibert's Decoupled Control Strategies

The computer algorithms that controlled the motion of Raibert's one-legged robot[1, 5–8, 10] were decomposed into two basic control problems. The control objectives of the Raibert's "vertical motion controller" were to initiate and terminate the hopping and to control its height. Lengthening and shortening the length of the leg using leg position actuator accomplished these objectives. This leg actuation was based on changing the energy (based on maximum height error as feedback) of the resonant mass-spring system formed by the leg and the body in order to maintain the predetermined height. The control objectives of the Raibert's "horizontal controller" were to control the forward velocity and to maintain the balance of body posture or body attitude to prevent the robot from tipping. This was done using a look-up table. The foot placement algorithm (to achieve the desired forward speed) used a simple PD controller.

Based on Raibert's 2D hopper, Koditschek and Buehler[60] carried out simulations of two models of juggler. They used the discrete feedback control law for controlling the hopping motion. They acknowledged that the pneumatic cylinder in these models was to be modulated by an adjustable "spring constant," and a "spring law." Their simulation, for the linear spring based hopper model, guaranteed globally attractive stable periodic gait trajectories for any initial conditions in the vicinity of the trajectories of the practical hopper (Raibert's 2D hopper). Simulation for nonlinear spring-based hopper model guaranteed to have stable periodic gait trajectories for particular choice of initial conditions. Due to impulsive thrust applied during ground contact, they observed that periodic motion of period one was not the only stable hopping gait. Vakakis and coresearchers[62, 64] showed that with proper selection of control parameters and for a finite duration of thrust, the robot would exhibit a globally stable uniform hopping motion for a large range of the physical model parameters.

M'Closkey and Burdick[66] extended the work done by Vakakis *et al.*[64] by including forward running dynamics. They called this analytical model as "2-DOF" hopper. Discrete dynamical system theory was applied for analysis of simplified hopping robot models, which were analogous to Raibert's experimental machines. A 2D model was presented, which included both forward and vertical hopping dynamics and a foot placement algorithm. These systems were analyzed using a Poincaré return map, and hopping behavior was investigated by constructing the return map bifurcation diagrams with respect to system parameters. The bifurcation diagrams exhibited period doubling, which resulted in the global dynamical hopping behavior. Similar results could be seen in the work of other researchers such as Ostrowski and Burdick.[65]

Schwind and Koditschek[80] considered a simplified 2-DOF model of Raibert's planar single-legged hopper, to control only the forward velocity. They modified Raibert's forward velocity feedback controller and characterized the fixed points of this closed-loop system. After studying the local stability of these fixed points, they concluded that this stability was dependent upon the specific control law. A new proportional coupled control law, which was *ad hoc* and inspired by system dynamic consideration, gave a better regulation and large regions of attraction of fixed points defined previously.

Zeglin[13] introduced a control algorithm similar to Raibert's three-part controller for controlling the forward speed, hopping height, foot placement, and transition between various gaits for "Uniroo" (Fig. 6). Zeglin and coworkers stabilized pitch of the hip by applying continuous control during the flight and stance phases. Cycle-to-cycle stability was achieved using discrete control to stabilize forward speed. Also, they ensured convergence of the machine to a given forward speed by controlling event-wise foot placement algorithm. Zeglin also used tri-partitioned Raibert's approach in the 2D model.[34]

Harbick and Sukhatme[74–76] proposed both height and speed controller based on Raibert's approach. They described a model-based height controller that allowed a wide range of apex heights to be selected and achieved. They noticed that setting the leg to an appropriate length during flight could change the height, and changing the leg length effectively changed the air spring constant. In this way, they analyzed that energy could be added or removed from the system to change the apex height.

Li and He[63] analyzed the stability of Raibert's hopper using energy-balance method and perturbation theory. They decomposed the motion dynamics into Hamiltonian and nonHamiltonian parts. The hopping height was controlled by fixed leg extension during the stance phase. Both linear and nonlinear models were simulated and compared. Francois and Samson[79] also used the Hamiltonian approach to simulate forward running in 2D hopper with constant energy based on symmetry constraints. They predicted a necessity of hopper landing with a proper touchdown angle and regulating the spring force adequately to stabilize the motion in a given environment with or without gravity.

4.2. *Adaptation-Based Control Strategies*

Prosser and Kam[15,16] performed numerical simulations to identify the performance of their machine in tracking a desired height, which was changed in stepwise fashion. They proposed a near-inverse controller with height feedback, which was based on a hop-to-hop model of the robot. This control algorithm was estimated from approximation of functional relationship between previous hopping heights, the height of the next hopping cycle, and the control signal. This control algorithm calculated a voltage signal, which drove a dc motor and changed the length of the springy leg. In order to enhance the performance of the system, they[17] used a recursive least-squares parameter estimator that continually tuned a previously proposed, near-inverse controller with height feedback. In the near-inverse controller with integral error feedback, the control input for the next hop was determined from the error between present and previous hopping heights with hopping height feedback to update the error estimate. They evaluated the transient and steady-state behavior for each controller. Their simulations suggest that the resulting algorithm was both computationally feasible and robust to unknown disturbances and time-varying parameters. Due to such enhancement of their system, it became relatively insensitive to drift in machine parameters as compared to previously proposed schemes by Raibert,[1] Sznaier and Domborg[86] and Helferty *et al.*[67]

Mehrandezh *et al.*[25] examined three methods to control the jumping height. Open-loop control (method 1) that used the offline simulation was sensitive to the perturbations of the system's parameters. To avoid this, they used a modified PI controller (method 2), which resulted in steady-state error. Method 3 used an offset and a control action band, in addition to the PI controller, to achieve near-zero steady-state error and a shorter settling time. The third control law showed minimal sensitivity to system parameters, such as spring type or ground surface characteristics.

Raibert *et al.*[8] reported that the horizontal controller resulted in a steady-state velocity error or offset, which depended on the forward velocity and the parameters of the robot model. In this context, Sznaier and Domborg[86] proposed an adaptive horizontal control algorithm that eliminated the offset problem in the forward velocity observed by Raibert. Also, it was robust in the sense of satisfactory performance under widely varying conditions. This algorithm was composed of two parts; one applied during transient and the other applied during steady state.

They also proposed an adaptive control law based upon an online numerical minimization of a performance criterion for the horizontal controller. They analyzed different online minimization techniques. Out of those, they found a random search technique to be superior to a deterministic minimization technique.

4.3. Energy-Efficient Control Strategies

Autonomous robots can be designed with electrical actuators. Also, power consumption of the hopper decides the actuator size, and in turn influences the hopper design. So, it is necessary to achieve the hopping gait with minimal actuation/power consumption. Such efforts have been revisited in this section.

Thompson and Raibert[106] utilized a compliant hip and leg to simulate the periodic gait without actuators. But, it needed suitable initial conditions and also resulted in unstable passive gait. Related to ARL Monopod I (without hip compliance), Buehler *et al.*[19, 20] used Raibert's three-part control algorithms with slight modification to distribute thrust over the hopping cycle to accommodate lower power electric motors. They controlled hopping height with an open-loop controller. For controlling the forward speed, they modified Raibert's control law by adding an integral position error term. The pitch control algorithm was unchanged. A comparison showed that the ARL Monopod I with its 125 W average power consumption was more energy efficient than previously built robots.[1, 13, 14]

The other design (ARL II) used hip compliance[20, 23, 24] to restore the energy during the swinging of the body about hip joint. They achieved vertical motion stability with a high degree of accuracy by means of a simple adaptive and energy-based controller. They referred this type of motion as "controlled passive dynamic running." The specific resistance (measure of energy efficiency) of ARL Monopod I and II was approximately 0.7 and 0.22, respectively, even though Monopod II had more weight than Monopod I.

Francois and Samson[109] addressed the problem of energy-efficient control of running legged mechanisms with a case study of the planar one-legged hopper. They showed that it was possible to maintain balance without spending much actuation energy by selecting a proper hip spring. They derived a new class of simple controllers (used in nonlinear oscillating systems), which were different from Raibert's controllers and were capable of stabilizing passive periodic motions. With the help of a discrete Poincaré map, they simplified the nonlinear error model and used it to derive the feedback control law (approximated linear flow) such that it could stabilize (at least locally) the model with minimum actuation. Although stability analysis was not complete,[109] the robustness of the proposed approach was illustrated via simulation.

Schammass and coresearchers[110] presented a similar strategy to control a one-legged robot (similar to Raibert's 2D model) with the perspective of reducing the energy expended by the system. They discussed two classical methods for controlling the speed and orientation. The first method (without hip compliance) was similar to the strategy presented by Raibert.[1] The second method (including hip compliance) exploited the passive dynamics of the system, yielding the desired leg trajectory with lesser control effort. In both the simulated methods, the controlled system achieved the desired speed. It was observed through simulation that the second method yielded approximately 67% total energy saving as compared to the first method.

De Man and coworkers[30–33] constructed a mechanical prototype of an electrically actuated one-legged hopping robot with one articulated leg, "OLIE" (Fig. 20). This robot was an underactuated system with holonomic and nonholonomic constraints. Their core goal was to set allied objective parameters of prototype (like forward speed during flight, vertical hopping height, orientation at takeoff and touchdown, angular momentum) with minimal power consumption. These objectives were grouped in a suitable way to explore the possibility of robot locomotion over uneven terrain. But they experienced that every combination of these control objectives did not result in energy minimization due to limitation on actuator dynamics and hopper geometry. They observed that the overall energy consumption was within the specified range of the actuator.

Hyon *et al.*[111–114] simulated a controller for an energy-efficient passive one-legged hopping robot with a compliant hip. First, based on the dynamics of this nonlinear hybrid system, they found passive orbits using a new gait-searching algorithm and then evaluated their gait stability.[112] They found that those passive orbits were unstable. So, they proposed two stabilizing controllers, called local feedback controller and energy preserving controller (Non-Dissipative Touchdown Controller). The latter controller resulted in quasi-stable periodic orbits, which could be seen in some Hamiltonian systems. Next, they proposed additional adaptive controller, which used two-parameter adaptation laws. First law helped in stabilizing (asymptotically) from the quasi-periodic gaits to periodic gaits of arbitrary period. The other adaptive law was useful for spring stiffness adaptation, and it minimized control inputs. Simulation results showed that the robot eventually hopped without any control inputs, especially for "periodic-one" gait.

Cherouvim and Papadopoulos[83, 84] analytically showed that there existed only a particular passive gait, which was energy-efficient in the ideal 2-DOF SLIP hopper subjected to electromechanical losses. They validated their analytical prediction for the realistic 2-DOF SLIP model. By adding a simple low-energy control strategy based on sensory feedback, Dummer and Berkemeier[125] analyzed that a moderate range of stable forward speeds might be achievable. A simple control strategy used by them added a small thrust during the stance phase, which kept the path of the leg near its natural resonance in vertical direction. They found that the overall system was robust to 10% variation in a single parameter value. Also, the system maintained its stability and showed tracking ability when the forward velocity changed from one hop to another.

4.4. Strategies Based on Internal Motion Dynamics

Based on the fundamental principle of conservation of angular momentum, Li and Montgomery[91] proposed a "closed-loop" strategy that could optimally control the body orientation of a one-legged robot during flight phase using the internal motion of the leg. The angular momentum constraint, as a nonholonomic constraint, was used to state the problem as a nonholonomic motion-planning problem. Then Chow's theorem[63] was applied to verify the system's controllability, and the concept of holonomy was used to construct an optimal path. Finally, they used "linearization" control in the internal motion space to realize the desired path.

Based on the same principle, Lapshin[87] analyzed the motion control problem of a one-legged hopper in the flight phase. He considered a linearized model, similar to Raibert's hopping model, and elaborated only the vertical motion problem without considering the weight of the leg. He investigated how the body and leg orientation could be changed during the flight phase by applying perturbations (as control input) based on the difference between the actual and desired orientation. Stretching or shortening the leg during the flight could alter the moment of inertia and effectively vary the angular motion of the body. He showed that the vertical motion was periodic only for small perturbations. He also discussed the vertical and horizontal motion control problem[88] of a one-legged hopper. Open and closed-loop control algorithms were studied for vertical motion analysis considering linear springy telescopic leg. Velocity control and position control methods were proposed for horizontal motion control during the stance phase. The counter weight was positioned suitably to change the CG of the body.

Peck[43] realized the necessity of conservation of angular momentum and internal energy dissipation for stabilizing the attitude and hopping sequences. In that context, he selected the "Kane Damper," as an energy dissipation model, which represented viscous interaction between a spherical body and a main rigid body. Onboard wheel momentum controller with approximate open-loop controller stabilized his 3D gyroscopic robot. In 3D case, the control strategy to increase body stability using a body-stabilizing gyroscope was employed by Zeglin and Brown.[37] Other possible strategies included actively adjusting the center of CG and using aerodynamic effects to produce small correction torques on the body.

Rehman and Michalska[92] stated that the kinematical model of a hopping robot in the flight phase was a fully controllable nonholonomic system, but its controllability algebra was infinite dimensional according to Lie algebra. Such systems required some advanced nonlinear control techniques (like time-varying feedback control technique and discontinuous feedback control technique). So, they introduced a novel approach for the synthesis of time-varying stabilizing feedback control, which was based on the trajectory intersection idea and primarily applied to systems whose controllability Lie algebra was finite dimensional. So, they approximated their original model of the hopping robot, whose controllability algebra was infinite dimensional, to a simplified model whose controllability Lie algebra was finite dimensional. Then they constructed a time-varying stabilizing feedback law[92, 94] for this simplified model. This law construction appeared as a composition of a standard stabilizing time-invariant feedback control for an extended Lie algebraic system and a periodic continuation of a parameterized solution to a finite horizontal trajectory interception problem in logarithmic coordinates. This composed stabilizing feedback law showed satisfactorily large stability robustness margin for the extended controlled system. It also ensured that the constructed feedback control was also stabilizing the original model.

In continuation of his earlier work,[92, 93] Rehman suggested a second practical approach, i.e., discontinuous feedback control technique[94] for controlling a hopping robot in flight phase. His approach to construct a stabilizing feedback control law was dependent on the selection of a Lyapunov function. The constructed Lyapunov function was the sum of two semipositive definite functions, which were determined using Lie algebraic methods and Frobenius theorem. Also, the constructed Lyapunov function was asymptotically converging to zero. The proposed control law was uniformly bounded and piece-wise constant in nature.

Berkemeier and Fearing[115] described closed-loop control schemes, for a planar acrobot (Fig. 50). The control during stance phase involved the tracking of special trajectories that kept the robot inverted while periodically accelerating the center of mass vertically. For the flight phase, rotating the leg an integral number of times could typically enable the robot to land in the same configuration in which it took off. This was due to the holonomy associated with the internal motion. Simulation results showed that the derived control strategies were effective. The robot could also slide (movement without breaking ground contact) along the ground and hop.

4.5. *Strategies Based on Fuzzy-Neural-Genetic Algorithms*

Fuzzy-neural-genetic learning techniques, resembling the simplified models of biological brains, are combined to achieve adaptive control. These techniques are knowledge-based biological intelligence mechanisms and have the capability of performing incremental and exploratory learning expected in successful locomotion control. Such evolutionary controllers were explored in several studies for monopedal robots.[29, 39, 50–54, 67, 68, 70, 73, 77, 78, 81, 90, 118, 129]

Helferty *et al.*[67, 68] presented two neural network strategies for the control of a one-legged hopping robot. They considered Raibert's one-legged model and focused only on the vertical motion to achieve a stable vertical hopping. In order to maintain a fixed vertical hopping height, they made corrections to the motion of the robot that served to maintain a fixed level of energy and also to minimize energy losses. The control of the robot was achieved by using the associative search element network or adaptive critic element network of artificial neural networks with a continuous learning memory. Through continuous learning and reinforcement via multilayer connectionist network of past successes and failures, the control system achieved a stable vertical hopping. The proposed control strategy was robust, since the formulation of control signals did not require precise knowledge of a current state and the mathematical model of the robot leg. Also, it did not require knowledge of the energy losses due to impact.

Zhang *et al.*[29] proposed a multiagent neuro-fuzzy control approach. The approach was based on the efficient inverse dynamics of Uniped and geometrical learning rate discovered by interpolation and extrapolation. They observed that for the same jumping height, different take-off angles would result in different jump distances. After analyzing the limitations of "reflex control" and "periodic forcing control" for energy pumping, Berkemeier and Desai[39] formulated "adaptive periodic forcing" control strategy governed by Central Pattern Generator (CPG). This strategy required minimal sensory information for generating stable vertical hopping. The experimental data showed that the performance depended on the sampling rate of CPG.

Waden and Ekeberg[129] proposed a two-level hierarchical control system to produce stable stepping patterns for a large range of velocities and parameter variations. The higher level control center gathered sensory information and initiated locomotion with appropriate adjustments. Further, Neural Phase Generator (NPG), ensuring that only appropriate responses got activated, filtered these control signals and guided the fast feedback paths. The NPG consisted of propulsive, lift-off, swing, and touchdown phase modules. The overall system was able to generate rhythmic motion of the leg with short interval decisive control torque applied to the hip actuator.

Maier[81] investigated a two-layered feedback neuro-controller based on back-propagation with momentum learning algorithm. The raw data for learning purpose was generated after simulating the hopper in 3D flat surface. Using these data for learning, the neuro-controller decided appropriate motor control action (output signal) to drive the controllable states of the robot. The robot was able to hop in place, along a square, spiral, and eight-shaped trajectory, including on uneven terrains.

Similarly, Tedrake and Seung[90] proposed the three-layer feedback neural network to control hopping height and forward speed of Raibert's 2D model (Fig. 3). To maximize the stability region, they selected the back-propagation algorithm for pre-training and heuristic linear search algorithm to search the basin of attraction in control parameter space. With this controller, the hopper was able to perform stable motion for 5 s. Geng *et al.*[118] used a multilayer neural network strategy to initiate the flipping cyclic gait.

Kusano and Tsutsumi[73] investigated the learning control strategy composed of two learning modes, called Reinforcement Learning (RL) and Neural Network (NN) mode. RL mode identified the changing environment (discrete state space) and optimized the solution (action) using Q-learning method. Then, the NN mode generalized all optimum solutions using the back-propagation method based on nonlinear interpolation and selected the most desired action. This strategy regulated the hopper at different hopping heights and was efficient even when hopping over obstacles.

Harbick and Sukhatme[75] used Raibert's tri-portioned feedback controllers (PD controllers) for height and speed controller. But, they observed that these controllers were sensitive to parametric variations and accumulating steady-state error. So, they modified the speed controller[77, 78] for a hopping robot using the neural network to model the "neutral point" (Raibert's terminology—meaning, the landing point which generated net zero acceleration/deceleration) as a function of running speed and hopping height. The network was trained offline using Levenberg–Marquardt optimization algorithm for the training data, taken from a simulated hopper that was manually controlled by a human. Simulation experiments of hopping in the sagittal plane had shown improved performance over the PD controller of Raibert, which used a linear approximation for the neutral point. Simulation results showed that the control system performed soundly even in the noisiest case, with a relative error of approximately 20% and hence depicted robustness.

Kuswadi and coresearchers[50–54] investigated different control strategies for a one-legged hopping robot having a single actuator. The robot model (shown in Fig. 32) was considered as a discrete system, in which one cycle of motion was regarded as one sampling interval, the angular velocity of the robot in a standard position at the stance phase as a state variable, and the thrust timing as the control input. Initially,[54] they applied the state feedback controller to stabilize the system, and observed poor performance (reproducibility) due to large disturbance of the robot mechanism. Hence, in search of an improved control system, they used feedback error learning scheme.[50] Adaptive fuzzy network was used in this scheme, and back-propagation technique was utilized for learning algorithm. By simulation, continuous hopping gait was realized. They further applied model reference adaptive fuzzy controller for controlling the one-actuator hopping robot.[51, 52] The adaptive fuzzy control used in this scheme consisted of a linear state feedback servo controller as a nominal controller, and adaptive fuzzy networks to handle nonlinearity in the robot parameters.

Yoshida *et al.*[70] applied genetic algorithm to achieve the stable hopping in 1D hopper. The control system consisted of the CPG as a feedback controller, which decided the control input (current) for electrical actuator (dc motor). The genetic operators initially tuned the control input at every hopping interval, ensuring minimization of the error between the desired and actual height. In order to perform online tuning, it used the genetic algorithm with advanced operators. The results were validated for different operating conditions.

4.6. Hybrid/Switched Control Strategies

Leavitt *et al.*[58] used two optimal controllers (feedback linearized force controllers) to regulate the airflow from servo-valve to the pneumatic actuator. They also tested the linear quadratic regulator controller and found that it was inadequate for their highly nonlinear model. So, they selected H_∞ controller as a robust balancing controller, and it improved the stability of the system. Sato and Buehler[59] used two hybrid PD controllers (reflex control) to stabilize the 2D SLIP Hopper, each controller operated separately in flight and stance phase, with suitable selection of controller gains.

Michalska *et al.*[69] presented a new feedback control strategy for vertical motion control of a one-legged hopping robot. Their previous work[20] had shown that the stability analysis was dependent on the suitable Poincaré map and governed by a fourth-order actuator dynamics. According to them, calculation of an accurate control law for such higher order and variable structure systems was subjected to serious inaccuracies. Their proposed control law was aimed at stabilizing the robotic system to a desired limit cycle under the influence of the constraints on the system. This used a variable structure controller (VSC) and the proposed feedback law followed from a simple phase-plane analysis of a reduced order model, and its convergence was clear. There was no need of constructing the Poincaré map or fixed-point analysis, as required in previous attempts. Due to VSC strategy, the system was robust to modeling errors and disturbances.

4.7. Discrete Multirate Strategies

Chelouah *et al.*[95] discussed digital control of nonholonomic systems for two cases including planar one-legged hopping robot. They proposed a multirate feedback control strategy for solving the motion-planning problem. After the transformation of kinematical system equations into specific canonical one-chained form (after some parameterization steps), digital control in velocity and acceleration was possible for desired locomotion. The simulation results presented concerned exact steering and path following for a sequence of points. Also, optimization and robustness criteria were considered in this multirate strategy.

Giamberardino *et al.*[96] discussed the efficiency of multirate sampling to design a control law when smooth continuous-time feedback law did not provide satisfactory solution. Generally, one-legged hopping robots had nonlinear continuous-time dynamics with nonholonomic constraints. So, they applied digital control strategy and concluded that multirate sampling technique was a good tool to compute such discontinuous feedback law. In multirate sampling, the differential equations governing the dynamics were integrated for different values of input signals. Also, sampling could be done for different time values, and it was not necessary to perform the integrations with fixed time duration. They stated that it was possible to find the discontinuous feedback law for a closed-loop control system and their approach was applicable for controllable and uncontrollable systems. The discussion was illustrated by the design of a multirate control strategy to focus on the effectiveness and robustness of the method to produce the desired behavior of a one-legged hopping robot.

4.8. Path/Motion Planning Control Strategies

Zeglin's 2D prototype[35] stabilized body attitude passively and was energy-efficient enough to run onboard batteries. It was controlled by a real-time motion planner and demonstrated crossing of simple artificial terrain, including stepping stones and shallow stairs. The planner used heuristics to discretize the continuous control space and estimated path costs. Paths were generated in real time as needed in conjunction with a feedback controller that rejected local disturbances. The machine was a form of "programmable mechanism" configured by leg position and stored energy during flight to control the evolution of the bounce dynamics.

Albro and Bobrow[44] used optimal feedback control to stabilize their 5-DOF hopper. The first step consisted of pre-defining optimal segments (motion primitives) of the motion sequences based on a path optimization, called static parameter optimization. The second step was to exploit these pre-defined data in experimental setup (for path planning of joint angles) with efficient use of actuators. They used servomotor with PD controller to decide the real trajectory utilizing the pre-defined trajectory.

Sung and Youm[128] simulated path-planning control approach in one-legged articulated robot, focusing only on the take-off motion. Using kinematic boundary conditions instead of dynamics, the robot was moved from stationary state to take-off state only with actuated revolute joints. Inverse and forward dynamic simulations validated the approach. In a similar fashion, Geng *et al.*[116–119] formulated the trajectory planning strategy for flipping robot. The fastest locomotive trajectory was decided optimally so that it would be adaptive to various nonlinear constraints. As compared with other one-legged robots that had to regulate their attitudes while in flight, this robot required much less energy. It used a nonlinear feedback control law during stance phase.

Larin[104, 105] synthesized the stabilization strategy for statically unstable one-legged robot, called "Hopping Apparatus." The proposed algorithm used linear matrix inequalities approach to solve the discrete Riccati equation that represented the periodic motion dynamics. The forward motion control problem decided the required landing states within the fixed sized step duration. Flight trajectory was formulated at every cycle and defined within the stationary (present) co-ordinates. The rotary actuator (placed at hip joint) was forced to generate adequate torque and moment in the hip jointed body.

Ohnishi *et al.*[120–123] investigated different motion planning strategies to control the stiff legged robot (as described in Section 3.2.1). The overall objective was to achieve a soft-landing trajectory by suppressing the energy loss due to hard landing by shaping the flight trajectory. The approach was based on the principle of conservation of kinetic energy. They used variable compliance control to ensure soft landing.[122, 123] While, for controlling hopping height and velocity, they steered the posture during the flight phase based on law of conservation of angular momentum.

Kumar and Singh[97] developed predictor-modifier path planning method. The overall objective/motion of the system during the starting and end points was pre-defined. The method scanned the dynamics in between the motion, which were likely to violate the constraints. Such part of dynamics was assured to fall in the permissible and reachable state space, with the help of a "forward projection" forecasting technique, subjected to the violating constraints. Once the constraints were forecasted and driven to the safe state space, they were optimally played to achieve the desired goal, like obstacle avoidance. Similarly, Liu *et al.*[98] transformed the nonholonomic hopping robot systems into the partial linear state space by co-ordinate transformation. Then, they applied the reduced order controllability technique to avoid the singularity problem. They demonstrated the usefulness of the said approach in context to the hopping robot, where they discussed the set-point control problem. The approach was effective in jumping over an obstacle with the help of an optimally selected collision free flight trajectory.

Wang[99] developed a nonholonomic motion planning strategy to steer the hopping robot between two end configurations. The control problem was tackled using a polynomial fitting approach.

4.9. Other Feedback Control Strategies

After satisfying the controllability and stability test, Matsuoka's model[3] permitted him to derive a time optimal state feedback controller to prove dynamic stability. He proved that the foot placement and a certain velocity were necessary to ensure the gait stability. His analysis made use of difference equations that model the rapid contraction and stretching of the leg with impulsive forces.

Lee and Raibert,[11] who implemented the "Monopod" (Fig. 4), experienced a major problem of hoof rolling while stabilizing the prototype. Their core control objectives were regulation of the forward speed at lift-off and forward acceleration during the stance phase. In order to prevent the hoof from rolling, they investigated the cause of this problem by means of force analysis. Finally, they concluded that the main sources of hoof rolling were the hip torque used for the body attitude correction and the inertial loading due to the changes in momentum of the body. Hence, they proposed two solutions: coordinating the hip torque with the downward force at the hoof and limiting the inertial loading at the hoof. But first approach did not work well as it disturbed the body attitude. Hence, they developed the control strategy based on the second approach with limited forward acceleration.

Okubo *et al.*[26] proposed a self-energizing mechanism for jumping. It transferred the rotary motion of body links to linear motion and was able to prepare the robot for jumping over an obstacle. Uno *et al.*[56] experimented on a simple mechanism to achieve minimum energy loss during landing. They used two point masses and an impulsive linear actuator to regulate the potential energy in the spring.

Akinfiev *et al.*[57] used a simple control law based on a fixed compression of the leg during each cycle for in-place hopping of their robot. An energy loss in robot due to impulsive touchdown and friction were compensated during the flight phase. They utilized motor angular displacement and an on–off type foot contact sensor in the control system. They realized stable vertical movements even in the presence of significant disturbances.

Cherouvim and Papadopoulos[89] analyzed the concept of distributing the energy associated with a single actuated (rotary) DOF to the remaining unactuated (linear) DOF. They utilized the energy transfer mechanism to derive a static feedback controller. The analytical results showed that the controller stabilized the realistic model of 3-DOF SLIP model for wide variations in initial conditions and parameters.

4.10. Open-loop Control Strategies

Thompson and Raibert[106] analyzed such hopping gaits (passive) for one-legged hopping robot (simulated model). They observed that this recurrent motion was not stable, since hopper did not have any actuation mechanism to stabilize in case the system got disturbed. Mehrandezh *et al.*[25] implemented an open-loop control law for hopping height control based on top-to-top tracking of the robot. But, they observed steady-state error for small parameter variations and so they switched to the other feedback controller.

Ringrose[27, 28] might be the first person, who achieved stable hopping without sensory feedback through simulation and demonstrated it in the monopod prototype. His simplified model used impulsive periodic thrust to change the length of springy leg. He observed hopping height stability and cyclic stability, assuming low damping in the leg. He predicted[28] that the hopper was unstable if the thrust occurred at maximum compression. Berkemeier and Desai[39] reported similar results, when the spring-mass type vertical hopper was controlled with a controller, called periodic forcing controller. But the overall system appeared partly as open-loop controlled system, as it had much reduced reliance on sensory information. Seyferth *et al.*[82] also investigated the necessity of proper adjustments of leg stiffness and touchdown angle for self-stabilizing springy type of hopping robots.

Lapshin's[88] approach to stabilize the hopping height was based on the fixed length compression of spring during the stance phase. It was not necessary to determine the required compression length for every hop. This approach resulted in globally stable vertical oscillations. Rad *et al.*[19] proposed a continuous and exactly implementable open-loop control algorithm for achieving constant hopping height. They obtained a stable limit cycle for their model with underlying fourth order intermittent robot dynamics. Wei *et al.*[41] designed an autonomous hopper (5-cm monopod), which was dynamically passive and stable (except the additional tail for directional control). But, the design mostly relied on the circular foot and offset-mass. Komsuoglu and Koditschek[71] stabilized a clock-driven vertical hopper (coupled oscillating system). The controller/oscillator generated the binary spaced control input, which varied the leg stiffness during the "forced stance mode."

Shanmuganathan[85] found that, due to an asymmetry in the configuration, the hopper was able to perform forward periodic sustained oscillations for some finite duration without any external actuation. He realized the need of starting the motion with proper initial conditions, which seemed to be practically difficult. So, he used a single rotary actuator (as shown in Fig. 47) to stabilize the passive forward-hopping motion of SLOM hopper. He reported the possibility of p-hop stabilization (periodic repetition after every p hopping cycles) and was able to extend the duration of hopping motion. This composite stabilization strategy was based on the alternative use of desired forward and vertical velocity determined from the nominal hop. He demonstrated 3-hop periodic motion in which the forward velocity of the nominal hop was used as the desired value for earlier two hops and the vertical velocity of the nominal hop was used as the desired value for every third hop. This strategy realized limping gait, and despite the back-and-forth hopping, there was a net forward motion. But the limitation of this stabilization approach was that the angular velocity of the rotary actuator became unbounded.

Cham and Cutkosky[46] observed open-loop stability in the vertical hopper, Dashpod. They reported the sufficient conditions for steady and stable gaits, when the hopper was activated by motor pattern (pre-defined input signal). They validated their analytical study with experimental results/data. With open-loop control and at fixed thrust time, multiple steady-state trajectories existed near the solutions of desired hopping height but finally, the system converged to stable and suboptimal (in the neighborhood to the desired) trajectories. Unstable steady-state trajectories resulted due to perturbations in the timing of thrust activation. These results are in conformity with the previous work.[28, 39] Mombaur *et al.*[101−103] presented a self-stabilizing strategy for their specific theoretical hopping robots (Fig. 48), which were dynamically stable but not statically stable. The strategy was decided optimally, using tailored SQP algorithm. The rotary actuator placed at joints received the periodic torque signal from the controller and served to stabilize the motion. The two-link leg model of hopping robot investigated by Rummel *et al.*[130] had a rotary actuator (position controlled motor) at hip joint. This actuator received sinusoidal input signal from the central controller. The hopper exhibited open-loop stability at different forward speeds on a circular surface.

5. Extension of Single-legged Robot Research to Multi-legged Robot

An emphasis of studying one-legged systems is to better understand the more complex multilegged systems. Many studies[10, 19, 39, 47, 114] have focused on one-legged systems, as an investigation platform. For example, Altendorfer *et al.*[131] explained how the dynamic behavior of the hexapod (Rhex) could be captured by simplified 3-DOF SLIP model (as shown in Fig. 43). This model captured the Rhex's pitching dynamics. They compared the performance of the SLIP model with a 24-DOF numerical model (SimSect model), when both models were actuated with the same open-loop controller settings. From those results, it could be observed that the SLIP and SimSect model had good correlation, with regard to the stability properties. Thus, it seems that proper kinematic one-legged robot model can function as a template model for multilegged runner dynamics, provided a proper tool for fitting the template model to a realistic model is used.

6. Conclusion

Among the numerous legged robots that have been constructed and studied, single-legged hopping robots have particularly fascinated researchers. Research in single-legged robots has been focused in understanding the dynamics, control strategies, and balancing principles. In this paper, we have presented a comprehensive survey of single-legged robots. It includes different theoretical and practical models, as well as, design and optimization aspects considered by different researchers. Apart from mechanical design aspects, we have also included stabilization and control issues for such legged systems.

Hopping is a special type of running gait in multilegged systems but is the only way a one-legged robot can locomote. Some variations are, however, possible. For example, hopping need not be a simple periodic phenomenon, and researchers have reported multihop periodic gaits as well as flipping hopping gaits. In general, single-legged robots require dynamic stabilization with the exception of robots equipped with a large foot. Several researchers have made use of this feature; however, this feature tends to negate an important potential advantage, namely, higher speed of locomotion, if the hopper is statically stabilized at each hop.

Most of the researchers have used relatively light telescopic leg mechanism attached to a massive body. In planar configurations, a revolute joint is used between the body and the leg. Some researchers have extended this to hopping robots in 3D by using a spherical joint between the body and the leg. Such configurations have been mostly symmetrical about the axis of the telescopic leg. Some researchers have departed from symmetric configurations. It has been claimed that asymmetric configuration may lead to a natural hopping motion as compared to a natural juggling motion (hopping in place) in symmetric configurations. Other hopping robots include the use of articulated leg and bow-leg configurations. Most researchers have used a pointed foot while some have used a curved foot. In cases when ankle joint is present, researchers have considered both active joint as well as passive springy joint. Some researchers have also used a tail for control and stabilization purpose.

Most of the configurations analyzed and built have utilized separate actuators for controlling leg bounce and for controlling leg orientation. Both these actuators are required to achieve hopping motion for symmetric configurations, whereas it has been shown that a single actuator could suffice for asymmetric configurations. Asymmetric models have been demonstrated with a single solenoid actuator as well as with a single reaction wheel actuator.

Hopping motion is repetitive and, therefore, several researchers have made use of Poincaré sections in analyzing the dynamics. Hopping dynamics is also highly nonlinear as there are stance and flight phases with sudden transitions. The dynamics of the two phases are also very different. Some researchers have studied the richness of the dynamics of these systems by analyzing chaos and bifurcation diagrams. For studying the controllability of these systems, Lie algebra approach has been used. An approach to establish the stability of the system is based on a two-part Lyapunov function corresponding to the two phases of motion.

Purely passive systems cannot sustain hopping motion on level ground indefinitely due to energy losses. Energy loss occurs due to impacts, as well as due to friction in joints. Several researchers have used open-loop energy compensation mechanism. Use of an actuator to pre-charge the leg spring appears a popular way of achieving this in a juggler. The latch mechanism used to store energy in the spring is released when the leg hits the ground leading to the additional pre-charge energy becoming available for the next cycle. It is conceivable that such energy-compensated systems would undergo sustained hopping even in the absence of active control.

Control has been applied in stance phase of motion or flight phase or both. The stance phase control can utilize the reaction forces for reorientation, which is not available in the flight phase control. No work has been reported on use of reaction jets as actuators. Flight phase motion of the center of mass of the system as a whole is ballistic. However, leg reorientation in flight phase can be achieved by relative motion of different parts of the robot.

Different approaches have been used in designing controllers for single-legged robots. A popular approach in practical systems has been based on heuristics. The concepts of neutral points and zero-moment points have been successfully exploited. Many researchers have designed separate controllers for height control and speed control. Some researchers have designed the controller based on neural fuzzy logic and genetic algorithms.

Portability of single-legged robot systems is a critical requirement for development of commercial applications. No portable single-legged robot systems have been demonstrated yet. However, several researchers are addressing issues that are important in this regard. The first step in this direction is the use of only electric actuators. Several researchers have made use of linear and torsional springs in providing passive dynamics to the system that approximates hopping motion. These have the potential to reduce the requirement of energy needed for actuator control and, hence, help make the hopping systems more realizable in practice. Other researchers have focused on reducing the energy losses. For example, impact losses can be minimized if foot mass is minimum and the landing velocity is in the direction of the leg.

Perhaps the focus of research in the coming years will be on systems that are minimally controlled, i.e., which utilize the natural passive dynamics for locomotion and require smaller control effort for energy loss compensation and stabilization. Such systems may be underactuated and may become practically realizable, if the power pack requirements are small enough. Research in practical systems would also have to deal with real issues of dealing with the environmental factors like uneven ground or obstructions and also with issues of sensor's input and fusion.

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