

# Photon kinetic theory in plasmas and in optics

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## Abstract

Recent advances of the photon kinetic theory are illustrated here, with examples taken from plasma physics (forward Raman scattering), and from nonlinear optics (self-phase modulation). This theory can provide a unified view of nonlinear optical phenomena involving photon beams with an arbitrary spectrum.

**Keywords:** Optics; Photo kinetic; Plasma; Raman scattering; Self-phase modulation

## 1. INTRODUCTION

Nonlinear wave mixing was first considered in optics by Boembergen and others (Armstrong *et al.*, 1962) in the early 1960s and imported into plasma physics soon afterward, especially by Russian researchers. It was soon realized that, in contrast to optics, where the laser frequencies were well-defined quantities and the spectral bandwidth was negligible, in plasma physics we usually deal with broadband spectra. Photon kinetic theory is appropriate to deal with waves having a large spectral bandwidth, and the early versions of this theory can be found in books on plasma turbulence published in the late 1960s (Kadomtsev, 1965; Sagdeev & Galeev, 1969). This theory was derived in the framework of the random phase approximation, which is well justified for plasma turbulence problems, in contrast with the fixed phase approximation (or monochromatic wave approach) more fitted to optical problems. The main feature of this early work was that nonlinear effects are much stronger in the fixed phase approximation than in the random phase one, where they appear at a higher order of the nonlinear coupling parameters. In recent years, we have returned to the photon kinetic theory by using a different perspective and a better understanding of the physical meaning of the photon kinetic equations (Silva & Mendonça, 1998; Tsintsadze & Mendonça, 1998; Mendonça & Tsintsadze, 2000). The result of this new approach is that many of the effects associated with the fixed phase approximation can also be described by the kinetic equations, which provide a general

view of nonlinear effects in plasmas and in optics. Thus, the previous distinction between the two regimes (random and fixed phase) loses its sense, the rule played by the field phase is better understood, and intermediate cases can be treated as well. Finally, the resonant collective processes associated with Landau damping by quasi-particles become extremely relevant in a large variety of domains (Bingham *et al.*, 1997; Mendonça & Bingham, 2002). On the other hand, with the recent advent of very short laser pulses, the discovery of phase modulation effects and of supercontinuum radiation (Alfano, 1989), the study of broadband spectra became also very useful and important in nonlinear optics. The more recent versions of the photon kinetic theory are thus very appropriate to provide a unified description of nonlinear wave phenomena both in plasma physics and in optics. Here, we will briefly describe the contents of the photon kinetic equations and apply them to a specific problem in plasma physics (the Raman forward scattering process) and another similar problem in optics (self-phase modulation and the generation of a supercontinuum).

## 2. PHOTON KINETIC EQUATIONS

It is well known that a quantum system described by a wave function  $\psi(\mathbf{r}, t)$  can be represented in the classical phase space  $(\mathbf{r}, \mathbf{p})$  if we use the Wigner function (Wigner, 1932):

$$F_p(\mathbf{r}, t) = \int \psi(\mathbf{r} + \mathbf{s}/2, t) \psi^*(\mathbf{r} - \mathbf{s}/2, t) e^{-i\mathbf{p} \cdot \mathbf{s}} d\mathbf{s}. \quad (1)$$

This function behaves in many cases as a density distribution of point particles and, for that reason, is usually called a quasi-distribution. Similarly, an electromagnetic wave spec-

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trum described by the electric field  $\mathbf{E}(\mathbf{r}, t)$  can be represented in terms of particles, evolving along ray trajectories with momentum  $\mathbf{k}$ , if we introduce the corresponding Wigner function (Mendonça, 2001):

$$F_k(\mathbf{r}, t) = \int \mathbf{E}(\mathbf{r} + \mathbf{s}/2, t) \cdot \mathbf{E}^*(\mathbf{r} - \mathbf{s}/2, t) e^{-i\mathbf{k} \cdot \mathbf{s}} ds. \quad (2)$$

From Maxwell's equations it is possible to derive an evolution equation for this Wigner function, for waves propagating in a space and time varying medium, with dielectric constant  $\epsilon$ , in the form (Tsintsadze & Mendonça, 1998; Mendonça & Tsintsadze, 2000; Hall *et al.*, 2002):

$$\left( \frac{\partial}{\partial t} + \mathbf{v}_k \cdot \nabla \right) F_k + \frac{\partial \ln \epsilon}{\partial t} F_k = -\frac{\omega_k}{\epsilon} [\epsilon \sin \Lambda F_k], \quad (3)$$

where  $\mathbf{v}_k$  and  $\omega_k$  are the group velocity and the frequency of the field mode  $\mathbf{k}$ , and  $\Lambda$  is a differential operator, acting backwards on  $\epsilon$  and forward on  $F_k$ :  $\Lambda = (1/2) \leftarrow (\partial/\partial \mathbf{r} \cdot \partial/\partial \mathbf{k}) \rightarrow$ .

This is similar to the Wigner–Moyal equation of quantum mechanics (Hillary *et al.*, 1984) and it has been discussed for the case of a dispersive dielectric medium (Mendonça & Tsintsadze, 2000), and more specifically for a plasma (Tsintsadze & Mendonça, 1998). This equation can be seen as a kinetic equation for the field quasi-distribution  $F_k$ . It should be noticed that it is exactly equivalent to the propagation equation for the electric field  $\mathbf{E}$ , but it is of little use because of its complexity. It is then more convenient to introduce some simplifying assumptions, namely that the medium is slowly varying in space and time in comparison with the space and time scales of the field. We are now left with a much simpler form of kinetic equation, where diffraction effects are completely neglected:

$$\left( \frac{\partial}{\partial t} + \mathbf{v}_k \cdot \nabla + \frac{d\mathbf{k}}{dt} \cdot \frac{\partial}{\partial \mathbf{k}} \right) N_k = 0. \quad (4)$$

The quantity  $N_k$  is the photon number density, defined in terms of the Wigner function as  $N_k = (\epsilon_0/8)(\partial R/\partial \omega)_k F_k$ , where  $R(\omega, \mathbf{k}) = 0$  denotes the linear dispersion relation of the medium. The electromagnetic energy density can then be simply written as the product of the photon frequency with the number density  $W_k = \omega_k N_k$ . This equation is nothing but the photon number conservation, valid in the geometric optics limit. Here we should stress the physical meaning of the third term, which was mostly ignored in the past and represents the photon acceleration effects leading to frequency shift and energy nonconservation. These effects are associated with the force  $d\mathbf{k}/dt = -\partial \omega_k / \partial \mathbf{r}$ , acting on the photon field. This new kinetic equation will then be applied to two illustrative examples in the following two sections.

### 3. FORWARD RAMAN SCATTERING IN A PLASMA

If a laser beam, described here as a bunch of photons, propagates in a plasma, electron plasma waves can be excited that modulate the propagating medium and induce a perturbation in the photon population, which in turn increase the electron plasma wave perturbation, thus leading to an instability process. The electron plasma oscillations are driven by the ponderomotive force associated with the laser beam inhomogeneities. The electron plasma density perturbations  $\tilde{n}$  are then described by the following propagation equation:

$$\left( \frac{\partial^2}{\partial t^2} + \omega_{p0}^2 \right) \tilde{n} = 4 \frac{\omega_{p0}^2}{m} \nabla^2 \int \frac{\tilde{N}_k}{\omega_k^2 (\partial R / \partial \omega)_k} \frac{d\mathbf{k}}{(2\pi)^3}, \quad (5)$$

where the right-hand side represents the ponderomotive force effect. On the other hand, the perturbed photon number density  $\tilde{N}_k$  is determined by the kinetic Eq. (3), where the force term is determined by the electron density perturbation  $\tilde{n}$ , according to:  $d\mathbf{k}/dt = -(e^2/2m\epsilon_0\omega_k)\nabla\tilde{n}$ . For electron and photon density perturbations,  $\tilde{n}$  and  $\tilde{N}_k$ , of the form  $\exp(i\mathbf{k} \cdot \mathbf{r} - i\omega t)$ , we can derive the dispersion relation:  $1 + \chi_e + \chi_{ph} = 0$ , where  $\chi_e$  is the usual electron susceptibility and  $\chi_{ph}$  is the new term associated with the photon field. If we consider the simplest case of a mono-energetic photon beam, such that the unperturbed distribution is  $N_{k0} = (2\pi)^2 N_0 \delta(k_\perp) \delta(k - k_0)$ , the dispersion relation reduces to

$$1 - \frac{\omega_{p0}^2}{\omega^2} - \frac{\omega_{p0}^2 \Omega^2}{\omega^2 (\omega - ku_0)^2} = 0, \quad (6)$$

where  $u_0 = \omega_0/k_0 \simeq c$  is the laser group velocity, and  $\Omega^2 = (k^4 c^2 / mn_0) (\omega_{p0}/\omega_0)^2 N_0$ . For  $ku_0 \leq \omega_{p0}$  we can have an instability at a frequency  $\omega = ku_0$ , with a growth rate  $\gamma \sim (\Omega/\omega_{p0})ku_0$ . This means that we can recover the usual growth rate of the forward Raman scattering, proportional to the field amplitude,  $N_0^{1/2}$ . However, the above dispersion relation shows that the maximum growth rate is attained at  $\omega \simeq ku_0 = \omega_{p0}$ , with a growth rate proportional to  $N_0^{1/3}$ :

$$\gamma = \frac{\sqrt{3}}{2^{4/3}} (\omega_{p0} \Omega^2)^{1/3}. \quad (7)$$

Our kinetic approach could also be used to study the photon beam instabilities created by laser beams with a frequency or an angular spread (Silva *et al.*, 2000; Mendonça, 2001), thus allowing for a global description of instabilities created by spectra with arbitrary shape.

### 4. SELF-PHASE MODULATION IN OPTICAL MEDIA

Let us now apply the photon kinetic theory to one of the most paradigmatic effects of nonlinear optics, which is usually associated with phase modulation. Here, in contrast, the

field phase will be completely ignored (Silva & Mendonça, 2001). We will now use the kinetic Eq. (4), with the photon acceleration force determined by:  $d\mathbf{k}/dt = -\nabla[kc/n_0 + n_2 I(\mathbf{r}, t)]$ , where  $n_0$  and  $n_2$  are the linear and the nonlinear refractive indices of the medium (e.g., an optical fiber or a piece of common glass) and  $I(\mathbf{r}, t) = \int \omega_k N_k(\mathbf{r}, t) d\mathbf{k}/(2\pi)^3$  is the intensity of the laser beam. The beam spectrum will change along propagation, due to the local gradient of the nonlinear refractive index, as determined by

$$\omega(\eta, t) = \omega_0 \exp \left[ \frac{2n_2 c}{n_0^2} \frac{\partial I(\eta)}{\partial \eta} t \right], \quad (8)$$

where we have assumed propagation along Ox and used  $\eta = x - ct/n_0$ . For a Gaussian laser pulse of width  $\sigma$ , as determined by the intensity profile  $I(\eta) = I_0 \exp(-\eta^2/\sigma^2)$ , we can obtain two maxima of the up-shifted and the down-shifted spectra, which grow exponentially with time:

$$\omega_{\pm} = \omega_0 \exp \left[ \pm \frac{2\sqrt{2}n_2 c}{n_0 \sigma \sqrt{e}} I_0 t \right]. \quad (9)$$

For small time durations, this can be simplified to:  $\omega_{\pm} = \pm \omega_0 (2\sqrt{2}n_2/n_0 \sigma \sqrt{e}) I_0 t$ . This is the usual result for the optical theory of self-phase modulation, which gives a frequency shift proportional to the time of propagation of the laser pulse inside the nonlinear medium. We can see from here that the photon kinetic theory can not only lead to more exact expression for the frequency shifts, but also show that the same effects remain when we completely ignore the field phase. This process can then be more appropriately described as a nonlinear photon acceleration process in the optical medium.

## 5. CONCLUSION

Here we have given a brief description of the photon kinetic theory. Photon kinetic equations can be derived from Maxwell's equations and, in their more rigorous versions, they are formally identical to the Wigner–Moyal of quantum mechanics. If diffraction effects are neglected, they can take the simpler and more suggestive form of an equation of conservation for the photon number density.

Examples of application in plasmas and in other optical media were discussed. We have shown that beamed photon distributions propagating in a plasma can drive electron plasma waves, by a process known as forward Raman scattering. We can say that our photon kinetic approach has the

ability to describe forward scattering instabilities, not only for mono-energetic photon beams, but also for laser beams with an arbitrary spectral width. When applied to optics, the photon kinetic theory can provide a very accurate description of the processes known as phase modulation, thus showing that the field phase (absent in the photon kinetic description) is not an essential ingredient. We can then conclude that the photon kinetic theory provides a very powerful method to understand the spectral changes in plasma physics and in optics, and can provide a unified view of a large variety of physical processes.

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