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Analysis of the synchronization error measurement via non-collinear cross-correlation

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Abstract

The method for measuring synchronization error of ultra-short pulses was introduced based on the principle of non-collinear cross-correlation. The analytical expression for the measurement was deduced according to the cross-correlation signal. The influences of angular error on the measurement were analyzed by simulated experiments. The incident angle and the angular error tolerance were both required to be considered and determined for the synchronization error measurement of ultra-short pulses. The results provide a theoretical basis for the measurement and control of the synchronization error in the coherent beam combination, plasma parameter diagnosis, etc.

Keywords: Coherent beam combination; Non-collinear cross-correlation; Plasma parameter diagnosis; Second harmonic generation; Synchronization error

1. INTRODUCTION

Ultra-short pulse is widely used in rapid diagnosis (Zewail, 1988), strong-field physics (Umstadter, 2001; Tajima & Mourou, 2002; Ditmirea *et al.*, 2004), etc. Seeking for extremely high-intensity has been the chief goal of laser technology since the invention of ultra-short pulse (Kong *et al.*, 2006; 2009; Banici & Ursescu, 2011). However, the intensity of single-beam ultra-short pulse is limited by the physical properties of materials, the pumping technique, nonlinear effects, etc. To further scale the intensity, an alternative approach is to coherently combine multi-beam ultra-short pulses, which is already planned in some large international facilities, for example, Exawatt Center for Extreme Light Studies (Bashinov *et al.*, 2014) and extreme light infrastructure (Chambaret *et al.*, 2010).

In the coherent combination of multi-beam ultra-short pulses, the synchronization error of different beams (i.e., the time delay of different beams, caused by environmental vibration, thermal changes, and so on) has to be measured and controlled for a good synthetic effect. As ultra-short pulses develop from picosecond to femtosecond magnitude, traditional electronic measurement techniques cannot meet the necessary precision requirements. For example, the measurement accuracy of synchronization error using a high-speed oscillograph equipped with phototubes is usually in the range of nanoseconds to tens of picoseconds, and it still stays at the level of picoseconds even using a high-precision streak camera (Qiao *et al.*, 2013). Compared with the above methods, the measurement accuracy has been improved significantly by applying a cross-correlation method, achieving a magnitude of femtoseconds. The cross-correlation method is widely used to characterize the time jitter between two mode-locked oscillators (Evans *et al.*, 1993; Wei *et al.*, 2001), the temporal duration, and phase structure of the laser pulses (Salin *et al.*, 1987; Brun *et al.*, 1991; Le Blanc *et al.*, 1991; Raghuramaiah *et al.*, 2001), etc. This method can be divided into two kinds: Multiple-shot and single-shot cross-correlation. The former is not well suited for a high-power laser facility since the facility usually has a relatively low repetition rate. Single-shot cross-correlation has been developed to avoid this problem.

Non-collinear cross-correlation is a kind of single-shot cross-correlation, which is used to measure the synchronization error of combined beams based on the fact that synchronization error causes a variation in the peak position of the cross-correlation signal. That is different from the balanced optical cross-correlator (BOC) (Shelton *et al.*, 2002; Kim *et al.*, 2008), which determines the synchronization error

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from the amplitude noise of the cross-correlation signal. Compared with BOC, non-collinear cross-correlation has a simpler structure which is beneficial to optical setup. Besides, the orthogonal polarizations of incident beams are not required. In this paper, a general analytical expression for the synchronization error measurement was deduced based on the non-collinear cross-correlation theory. Then the influences of angular error (i.e., the drift of the incident angle, caused by environmental vibration, thermal changes, and so on) on the measurement of synchronization error were analyzed. The above researches provide a theoretical basis for the measurement and control of the synchronization error in the coherent combination of multi-beam ultra-short pulses. Moreover, the results can be applied to the high-precise detection and control of the time delay between the main and probe pulses for studying the temporal evolution of plasma parameters.

2. NON-COLLINEAR CROSS-CORRELATION PRINCIPLE

By non-collinear phase matching and sum frequency generation in a nonlinear crystal, the non-collinear cross-correlation method transforms the time characteristics of incident ultra-short pulses to a spatial distribution of the cross-correlation signal (Salin *et al.*, 1987; Brun *et al.*, 1991; Le Blanc *et al.*, 1991; Raghuramaiah *et al.*, 2001). So by analyzing the spatial distribution of the cross-correlation signal, the time characteristics of incident ultra-short pulses can be obtained. The principle of the synchronization error measurement of ultra-short pulses based on non-collinear cross-correlation is shown in Figure 1.

When two ultra-short pulses enter in a nonlinear crystal with an included angle of $\alpha_1 + \alpha_2$, a cross-correlation signal is generated at the overlapping region in time and space of the incident ultra-short pulses, and the signal's intensity is proportional to the product of the local intensity of the incident ultra-short pulses. The intensity distribution of the cross-correlation signal in the z-direction is detected using a linear array charge-coupled device (CCD). For ultra-short pulses, $I_1(t)$ and $I_2(t)$, the cross-correlation signal S is (Salin *et al.*, 1987; Brun *et al.*, 1991; Raghuramaiah *et al.*, 2001):

$$S \propto \int_{-\infty}^{+\infty} I_1(t) I_2(t) dt a. \quad (1)$$

Supposing the cross-correlation signal is S_0 when there is no relative time delay between the two ultra-short pulses, and the cross-correlation signal moves from S_0 to S_1 in the z-direction when there is a relative time delay t_d. In this case, there is also a relative motion Δz for the peak position of the cross-correlation signal, which moves from z_0 to z_1 (see Fig. 1). Therefore, the synchronization error of the ultra-short pulses could be determined by measuring Δz.

Figures 2a and 2b show the variation in peak position of the cross-correlation signal when a time delay is induced by ultra-short pulses 2 and 1, respectively. The velocity of light in vacuum is denoted as c, the refractive index of the nonlinear crystal is n, and the velocity of light in the nonlinear crystal is $u = c/n$. Supposing the angles outside and inside the nonlinear crystal in the x-direction of ultra-short pulse 1 are α_1 and β_1, and the angles outside and inside the nonlinear crystal in the x-direction of ultra-short pulse 2 are α_2 and β_2, respectively. Then we have $\sin\beta_1 = \sin\alpha_1/n$ and $\sin\beta_2 = \sin\alpha_2/n$. According to Figure 2a, the variation of Δz in the peak position of cross-correlation signal S caused by the time delay t_d is:

$$\Delta z = O_2'P = O_2'O_2 \bullet \cos\beta_1. \quad (2)$$

Since

$$O_2'O_2 = \frac{MN}{\sin(\beta_1 + \beta_2)}, \quad MN = ut_d, \quad (3)$$

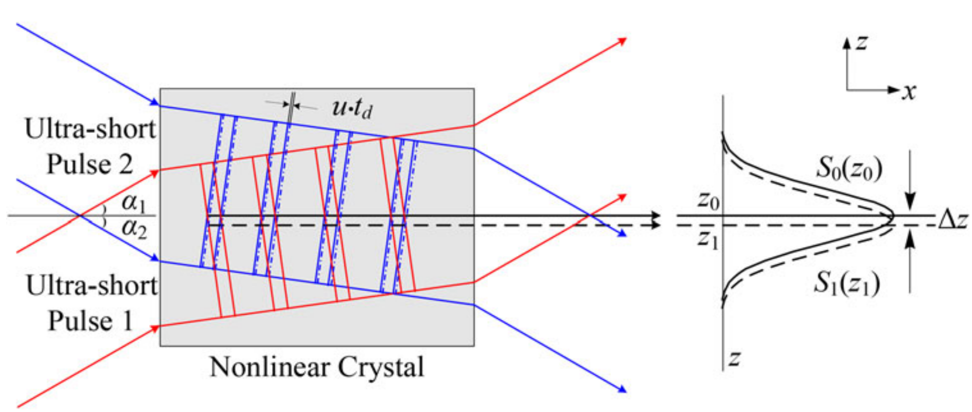

Fig. 1. Schematic diagram of non-collinear cross-correlation of ultra-short pulses.

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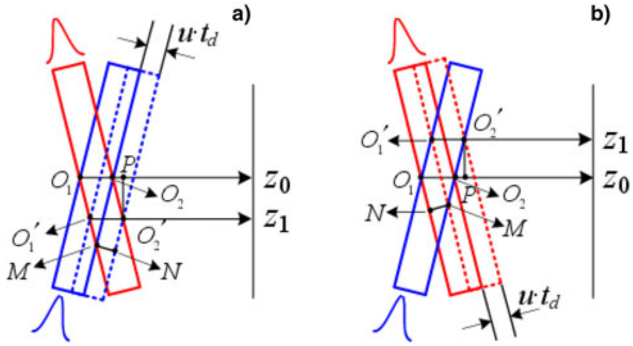

Fig. 2. Influence of time delay on the peak position of the cross-correlation signal: (a) time delay induced by ultra-short pulse 2, (b) time delay induced by ultra-short pulse 1.

we acquire

$$\Delta z = ut_{\mathrm{d}} \frac{\cos \beta_1}{\sin(\beta_1 + \beta_2)}. \tag{4}$$

When the pixel size of the linear array CCD is represented as d_{pixel}, Δz is therefore expressed (by pixel) as

$$\Delta z_{\mathrm{pixel}} = \frac{ut_{\mathrm{d}}}{d_{\mathrm{pixel}}} \frac{\cos \beta_1}{\sin(\beta_1 + \beta_2)}. \tag{5}$$

Let k represent the time resolution of the system, from which:

$$k = \frac{d_{\mathrm{pixel}}}{u} \frac{\sin(\beta_1 + \beta_2)}{\cos(\beta_1)}. \tag{6}$$

Therefore, the synchronization error, that is, the time delay t_{d}, can be calculated by Eq. (7) according to the variation Δz in the peak position of the cross-correlation signal S:

$$t_{\mathrm{d}} = k \Delta z_{\mathrm{pixel}} \tag{7}$$

when $\alpha_1 = \alpha_2 = \alpha$, the peak position of cross-correlation signal S in the z-direction lies on the bisector of angle $\alpha_1 + \alpha_2$. The time delay t_{d} induced by ultra-short pulse 1 is deduced in a similar way, whereas the $\cos\beta_1$ term in Eqs (3)–(6) is substituted by $\cos\beta_2$. It is not difficult to find that the smaller the incident angles are, the higher the time resolution of the system is, and the more the variation in the peak position of the cross-correlation signal is. So the measurement accuracy of the synchronization error is higher. When using this method for synchronization error measurement, the time resolution should be calibrated first.

3. SIMULATED RESULTS AND DISCUSSION

Supposing that incident ultra-short pulses, $I_1(t)$ (ultra-short pulse 1) and $I_2(t)$ (ultra-short pulse 2), obey Gaussian distribution in time and uniform distribution in space. The corresponding intensities can be written as

$$I_1(t) = A\,e^{-2(t/T)^2}, \qquad I_2(t) = A\,e^{-2(t-t_{\mathrm{d}}/T)^2}, \tag{8}$$

where A is the peak intensity, t_{d} is the time delay induced by ultra-short pulse 2, and T is $1/\sqrt{2\ln 2}$ of pulse width at the half-level in intensity. The simulation is analyzed under the following conditions: $A = 1$, $T = 30$ fs, $t_{\mathrm{d}} = 10$ fs, $d_{\mathrm{pixel}} = 0.125$ μm. To simplify the process, we suppose that angular error only exists in one ultra-short pulse beam and the phase-matching condition of these two beams is always satisfied.

3.1. Variation in Peak Position of Cross-Correlation Signal with Incident Angles of Ultra-Short Pulses

Figures 3a and 3b show the variation in the peak position of the cross-correlation signal with incident angles of ultra-short pulses under the conditions of $\alpha_2 \geq \alpha_1$ and $\alpha_1 > \alpha_2$,

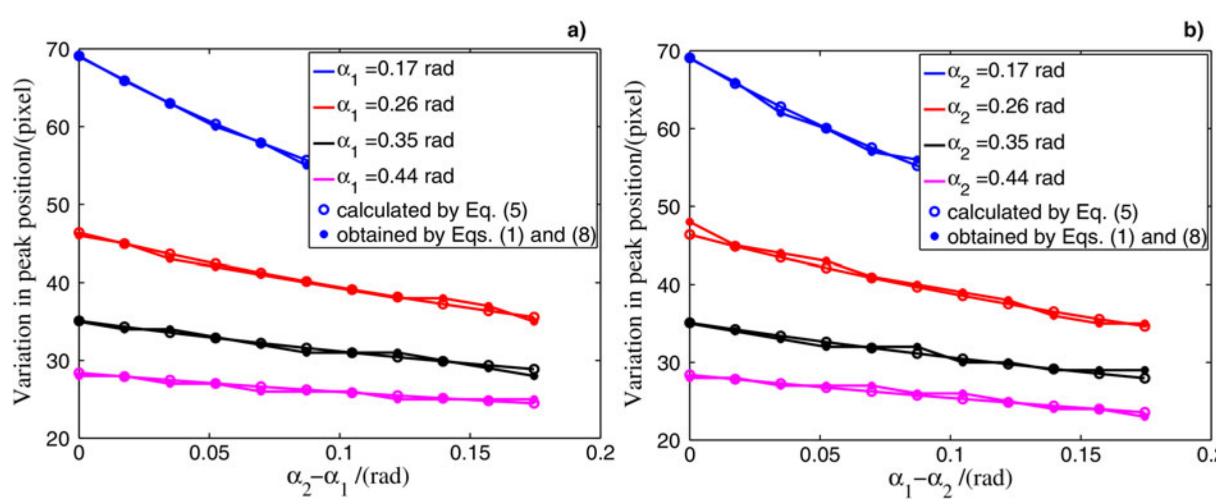

Fig. 3. Variation in peak position of cross-correlation signal with incident angles of ultra-short pulses. (a) $\alpha_2 \geq \alpha_1$, (b) $\alpha_1 > \alpha_2$.

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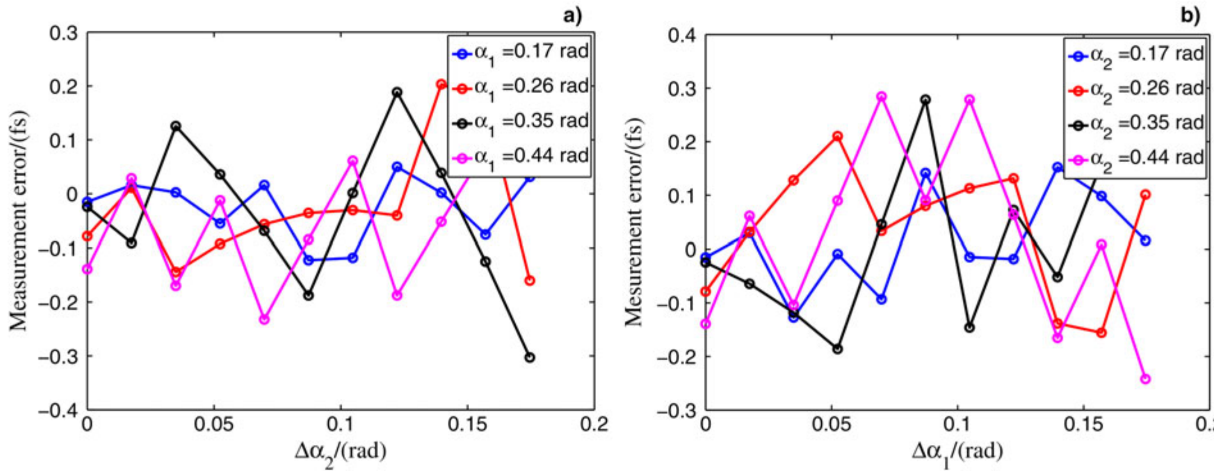

Fig. 4. Measurement error of the synchronization error due to limited pixel size: (a) angular error exists in ultra-short pulse 2, (b) angular error exists in ultra-short pulse 1.

respectively. Here, "o" represents the value calculated by Eq. (5), and "*" represents the result obtained by Eqs (1) and (8). It can be seen that the former vary slightly around the latter. This proves the accuracy of Eq. (5). Thus, the synchronization error of ultra-short pulses can be calculated by Eq. (7), according to the variation in peak position of the cross-correlation signal. Moreover, it can be seen from Figure 3 that the smaller the incident angles are, the more variation in the peak position of the cross-correlation signal is. That is because the smaller the incident angles are, the higher the time resolution of the system is.

3.2. The Influences of Angular Error on the Measurement of Synchronization Error

In practice, factors such as limited pixel size of CCD, angular error of incident ultra-short pulse, also have effects on the synchronization error measurement. To analyze the influences of angular error on the synchronization error measurement, the effects caused by limited pixel size have to be determined and eliminated first. In this case, the time resolution of the system could be calculated by Eq. (6). Figures 4a and 4b show the measurement error of synchronization error caused by limited pixel size when angular error exists in ultra-short pulses 2 and 1, respectively.

Angular error has an effect on the determination of the time resolution of the system, thus affecting the synchronization error measurement. After eliminating the measurement error of the synchronization error due to limited pixel size, we acquire that caused by angular error, as shown in Table 1. Table 1 shows that, with a constant incident angle of ultra-short pulse 1, the measurement error increases along with the increase of angular error of ultra-short pulse 2, and with a constant angular error of ultra-short pulse 2, the measurement error decreases as the incident angle of ultra-short pulse 1 increases. The same results can also be derived from Table 1. In addition, when the incident angles of

Table 1. *Measurement error (fs) of the synchronization error due to angular error: (a) angular error exists in ultra-short pulse 2; (b) angular error exists in ultra-short pulse 1*

	$\Delta\alpha_2$ (rad)										
α_1, (rad)	0	0.017	0.035	0.052	0.070	0.087	0.105	0.122	0.140	0.157	0.175
(a)											
0.17	0	−0.465	−0.886	−1.263	−1.623	−1.919	−2.213	−2.525	−2.767	−2.979	−3.23
0.26	0	−0.305	−0.582	−0.849	−1.101	−1.337	−1.558	−1.764	−2.007	−2.189	−2.291
0.35	0	−0.219	−0.435	−0.630	−0.811	−0.977	−1.166	−1.353	−1.488	−1.609	−1.717
0.44	0	−0.168	−0.321	−0.479	−0.611	−0.759	−0.904	−1.008	−1.144	−1.278	−1.41
(b)											
0.17	0	−0.479	−0.901	−1.309	−1.659	−2.038	−2.316	−2.602	−2.917	−3.153	−3.359
0.26	0	−0.326	−0.638	−0.936	−1.191	−1.453	−1.701	−1.936	−2.097	−2.296	−2.553
0.35	0	−0.245	−0.477	−0.694	−0.926	−1.158	−1.304	−1.522	−1.683	−1.895	−2.107
0.44	0	−0.201	−0.387	−0.581	−0.776	−0.934	−1.122	−1.259	−1.382	−1.556	−1.658

incident ultra-short pulses are smaller, the measurement accuracy is higher, as mentioned previously. So the incident angle and the angular error tolerance of ultra-short pulses are both required to be considered and determined for practical application. It should be noted that the above results are applicable not only to the pulses with uniform distribution in space, but also the ones with other spatial profile.

4. CONCLUSIONS

In the synchronization error measurement of ultra-short pulses using the non-collinear cross-correlation method, angular error has an effect on the time resolution of the system, and affects the synchronization error measurement. The analytical expression for the measurement is deduced according to the cross-correlation signal, and the influences of angular error on the measurement are analyzed by simulated experiments.

The results show that with a constant incident angle of the first ultra-short pulse beam, the measurement error increases along with the increase of the angular error of the second pulse beam, with a constant angular error of the second ultra-short pulse beam, the measurement error decreases as the incident angle of the first beam increases, and as the incident angles of two beams decrease, the measurement accuracy is improved. In practical application, the incident angle and the angular error tolerance of ultra-short pulses are both required to be considered and determined. To take the measurement error less than 1 fs as an example, the angular error should be less than 0.035 rad when the incident angles are 0.17 rad. The results can be applied to the initial design of the measurement and the control of synchronization error of ultra-short pulses. They provide guidance for the applications such as the coherent combination of ultra-short pulses and parameter diagnosis of plasma.

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