Testing War in the Error Term

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Researchers interested in the causes of war should note that the proof in "War Is in the Error Term" contains a mistake. Correcting the logic appears to reduce the advantage of large samples in testing rational explanations for war. The proof, presented by Erik Gartzke in the Summer 1999 issue of *International Organization*, uses a numerical example, a special case of James Fearon's conflict model that demonstrated how incomplete information with incentives to misrepresent can lead rational states to war despite the existence of a peaceful settlement both sides prefer to war.[1]

Gartzke chose plausible values for the parameters in Fearon's model to show how it generates a probabilistic prediction. Even with all of the necessary conditions present, the onset of war in the model depends on a stochastic process, as Gartzke put it, just like an error term.

Gartzke's example seemed to result in a 50 percent chance for war when the necessary conditions are met, and on this basis he laid out tentative empirical implications.[2] Case studies could not show how necessary conditions lead to war 50 percent of the time, but statistical tests could demonstrate whether a large sample of states satisfying the war conditions "fight about as often as they do not," or whether "the sample of wars carries with it an equal and opposite 'shadow sample' of 'not wars.'"[3]

Unfortunately, the probability of war in Gartzke's numerical example is calculated incorrectly; it is not 50 percent. Furthermore, the probability depends on the magnitude of the demanding state's war cost as well as the shape of the probabil-

We would like to thank the previous and current editors at *International Organization* for their assistance as we formulated this correction. This article in no way reflects the opinions, standards, or policy of the United States Air Force Academy or the United States government.

1. See Gartzke 1999; and Fearon 1995.
2. Gartzke 1999, 581, n22.
3. Ibid.

International Organization 57, Spring 2003, pp. 445–448

DOI: 10.1017/S0020818303572083

ity distribution for the defending state's war cost. In the model, these war costs are related to the resolve of states in a crisis. Practically speaking, it will be difficult for researchers to measure such costs well enough to produce the kind of clean point prediction that erroneously comes out of Gartzke's example. Given a large sample of crises fitting the conditions for war, researchers will not know which probability of war verifies the model.

One way to show the correct calculation for the probability of war is to recall Fearon's original proof at the point where Gartzke's takes a tempting shortcut. Gartzke asserts that the demanding state (state A) "calculates its best response to each type of player B weighted by the probability of encountering any given type of state B." He then goes on to show that "the weighted average reservation price" for state B produces a demand that generates a 50 percent chance of war.[4]

The problem with this strategy is that state A knows its demand will result in a positive chance for war, so it has to take its own war costs into account; it cannot simply push all possible players B back to their expected value for war. When Fearon reached this point in his equilibrium proof—of which he had to maximize state A's utility with respect to its demand—he took the tried and true path: write out state A's utility expression as a function of its demand, take the first derivative, and set it equal to zero.[5]

We replicate these steps for Gartzke's example where d represents the level of state A's demand.

$$U_A(d) = \Pr(\text{war})[\text{expected value from war}] + [1 - \Pr(\text{war})][\text{value from accepted demand}].$$

In Gartzke's example, state A and state B both have a .5 probability of winning a prize worth 100. The costs of fighting a war are $c_A = 20$ for state A and c_B distributed according to a uniform probability density function, $f(c)$, for state B.[6] State A's expected utility from war is $50 - c_A$; state B's expected utility from war is $50 - c_B$. As state A's demand increases above 50, state B is receiving less from settlement until the value of settlement dips below the value for war. If $d > 50 + c_B$, then state B prefers war to settlement. The probability of war, then, is the probability that a demand goes too far beyond the "peace line" at 50:[7] $\Pr(d > 50 + c_B) = \Pr(c_B < d - 50) = F(d - 50)$, where $F(c)$ is the cumulative proba-

4. Gartzke 1999, 585.
5. Fearon 1995, 411.
6. f(c) is taken as uniform between 0 and C-bar, where 0 ≤ C-bar ≤ 50. We will use C here instead of C-bar.
7. As long as both sides' war costs are greater than zero, both will prefer a settlement at 50 to the expected value for war.

bility distribution for the probability density function, $f(c)$.[8] The expected utility for state A in terms of its demand, d, is

$$U_A(d) = F(d-50)[50-c_A] + [1-F(d-50)][d].^9$$

Taking the derivative with respect to d, and setting it to zero, we have

$$U_A'(d) = f(d-50)[50-c_A] + 1 - \{F(d-50) + d[f(d-50)]\};$$
$$0 = f(d^*-50)[50-c_A-d^*] + 1 - F(d^*-50);$$
$$F(d^*-50) = 1 - f(d^*-50)[d^*-50+c_A].^{10}$$

It follows from $f(c)$ being uniform between 0 and C that $f(c) = (1/C)$ and $F(c) = (c/C)$ for c between 0 and C. Substituting in these special conditions for the expressions above, we have

$$(d^*-50)/C = 1 - [(d^*-50+20)/C].^{11}$$

This equation is satisfied for $d^* = 40 + C/2$. Notice that this is 10 less than what is claimed in Gartzke's example, and that the optimal demand would equal $50 + C/2$ if c_A were zero instead of 20.[12] State A demands less because with imprecise information about the other side's reservation price, and a positive chance that it may demand too much, state A must take its own war costs into account. The probability of war, $F(d^* - 50) = 1/2 - 10/C$, is also less than the 50 percent claimed in Gartzke's proof.

As state A's cost for war increases, the probability of war declines in Fearon's incomplete information model.[13] If the value of c_A is unknown, then one does not know what percentage of war outcomes to look for in a large sample of crises that fit the criteria for war. Direct statistical tests do not have the advantages described in Gartzke's 1999 piece.

8. In Fearon 1995, these probability functions are designated $H(z)$ and $h(z)$, respectively. Recall that if $f(c)$ is uniform between 0 and C, $F(c)$ will be the area under the rectangle determined by (0,0), (0, c), ($c, f(c)$), and ($0, f(c)$), for c between 0 and C.

9. This equation is a numerical example of the same equation in Fearon 1995, 411. We solve it the same way.

10. Again, this is the same condition we find in Fearon 1995, 411 for noncorner solutions where the optimal demand, d^*, is between 50 and 100, inclusive. Simply rewrite the equation that appears there with $H(x-p)$ on the left-hand side.

11. Recall that Gartzke sets c_A equal to 20. Gartzke 1999, 586.

12. Gartzke 1999, 586.

13. In fact, Fearon showed that if c_A increases to the point where $f(0) > (1/c_A)$, state A demands nothing beyond the "peace line" of 50, and the probability of war goes to zero. Fearon 1995, 411.

Certain types of states, such as those with democratic regimes, might have a higher than average cost for war. However, before concluding that these challengers ought to go to war less often, an additional parameter should be examined. The derivative of state A's utility shows that state A's optimal demand and the probability of war also depend on the shape of the distribution for state B's costs. If the uniform distribution truncates at a different value, or if the distribution takes on the equally plausible shape of a chi-squared curve rather than a rectangle, the prediction of Fearon's model changes. With no guarantee that the two parameters, c_A and the shape of the distribution for c_B, vary independently from one another in large samples of crises, even comparative statics become problematic.

Clarification of the difficulties in testing a highly influential incomplete information model comes at an auspicious moment for research on rationalist explanations for war. More models are emerging that address assumptions besides incomplete information.[14] Attempts to conceptualize war beyond Fearon's single-shot lottery for allocating an infinitely divisible prize may pave the way for a breakthrough. A fuller accounting of drawbacks introduced by Gartzke does not sound the death-knell for all rationalist conflict models.[15] Rather, it prepares the way for next-generation theories that will be easier to verify empirically.

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14. See Goemans 2001; Smith and Stam 2001a and 2001b; Wagner 2000; and Garfinkel and Skaperdas 2000. For an informal critique of the Fearon 1995, incomplete information explanation, see Kirshner 2000.

15. Gartzke 1999.