Macroeconomic Dynamics, **21**, 2017, 2138–2157. Printed in the United States of America.
doi:10.1017/S136510051500111X

A NOTE ON NOMINAL GDP TARGETING AND THE ZERO LOWER BOUND

ROBERTO M. BILLI
Sveriges Riksbank

I compare nominal gross domestic product (GDP) level targeting with strict price level targeting in a small New Keynesian model, with the central bank operating under optimal discretion and facing a zero lower bound on nominal interest rates. I show that, if the economy is only buffeted by purely temporary shocks to inflation, nominal GDP level targeting may be preferable because it requires the burden of the shocks to be shared by prices and output. However, in the presence of persistent supply and demand shocks, strict price level targeting may be superior because it induces greater policy inertia and improves the tradeoffs faced by the central bank. During lower bound episodes, somewhat paradoxically, nominal GDP level targeting leads to larger falls in nominal GDP.

Keywords: Nominal Level Targets, Optimal Discretionary Policy, Inertial Taylor Rule

1. INTRODUCTION

With policy interest rates constrained by the zero lower bound (ZLB) and a weak global economy, alternatives to the monetary policy frameworks of major central banks are being proposed.[1] Some argue for a nominal gross domestic product (GDP) level target, which is conceptually appealing because the central bank then commits to making up for past shortfalls in economic activity. A shortlist of recent proponents includes Hatzius and Stehn (2011, 2013), Sumner (2011, 2014), Woodford (2012, 2013), and Frankel (2013), among others.[2] However, shedding light on such a proposal, this Note shows that a better alternative, which ensures greater policy stimulus during ZLB episodes, may be a strict price level target.

The Note compares the two targeting frameworks in a small New Keynesian model, with the central bank operating under optimal discretion and facing a ZLB on nominal interest rates. Before the evaluation of the frameworks, the model is calibrated to recent U.S. data, with the conduct of monetary policy described by a

I thank for valuable comments Bill English, Bob Hall, Keith Kuester, Scott Sumner, Carl Walsh, seminar participants at Danmarks Nationalbank, Sveriges Riksbank, and University of Glasgow Adam Smith Business School, as well as conference participants at CEF, EEA, and NASM. The views expressed herein are solely the responsibility of the author and should not be interpreted as reflecting the views of Sveriges Riksbank. Address correspondence to: Roberto M. Billi, Research Division, Sveriges Riksbank, SE-103 37 Stockholm, Sweden; e-mail: Roberto.Billi@riksbank.se.

 1365-1005/16

simple rule often used in policy analysis (that is, a version of the Taylor rule with interest rate smoothing). The frameworks are then ranked in terms of performance, based on the model's social welfare function.

In the model, three types of shocks buffet the economy. On the supply side of the model, technology shocks push output gaps and prices in the same direction, whereas cost–push shocks instead cause an inflation–output tradeoff. On the demand side of the model, adverse demand shocks and the ZLB constraint create a tradeoff between stabilizing current and future output, because it is desirable in a ZLB episode to promise to induce an economic expansion after the ZLB episode. The stylized model offers a clear illustration of such tradeoffs in the evaluation of the frameworks.

The model produces three main results, one for each type of shock. First, if the economy is only subject to technology shocks, nominal GDP level targeting is clearly a less effective framework because it fails to insulate the economy from technology shocks. In contrast, under strict price level targeting, as well as under the simple policy rule, the economy is fully insulated from technology shocks.

Second, faced with purely temporary shocks to inflation (namely cost–push shocks, which are assumed to follow a white-noise stochastic process), nominal GDP level targeting and the simple policy rule may be preferable because they require the burden of the shocks to be shared by prices and output. Strict price level targeting instead causes costly fluctuations in output. However, if shocks to inflation are persistent (cost–push shocks follow an autoregressive stochastic process), nominal GDP level targeting results in costly price fluctuations, and the two targeting frameworks may be similarly effective in terms of social welfare, whereas the simple policy rule is less effective and causes even larger changes in prices.

As a third result, if the economy is only buffeted by persistent demand shocks during ZLB episodes, nominal GDP level targeting may be an inferior targeting framework, because it induces less policy inertia and, ironically, leads to larger falls in nominal GDP. Compared to the targeting frameworks, the simple policy rule is less effective and causes larger fluctuations in output and prices. Further, accounting for all three types of shocks in the analysis, the ranking of the frameworks is shown to be robust to a wide range of alternate calibrations.

In the New Keynesian literature, the desirability of a nominal level target when the ZLB is a constraint was stressed by Eggertsson and Woodford (2003) and Svensson (2003), before the financial crisis and Great Recession. Related to this, Svensson (1999), Vestin (2006), and Giannoni (2014) argued in favor of price level targeting versus inflation targeting in the absence of the ZLB constraint. Jensen (2002) showed that nominal income *growth* targeting fails to insulate the economy from technology shocks. In the aftermath of the crisis, Billi (2011) and Coibion et al. (2012) studied the optimal rate of inflation in the presence of the ZLB. In this Note, the analysis is conducted in a stylized model that does not include the effects of positive trend inflation, nor balance sheet policies and fiscal spending, which involve additional tradeoffs for monetary policy. Thus, further study is needed to extend the results to a broader class of models.

Section 2 describes the model and baseline calibration to recent data. Section 3 presents the policy evaluation and considers a range of alternate calibrations. Section 4 concludes. The Appendix contains technical details on the model solution.

2. THE MODEL

I use a small New Keynesian model as described in Woodford (2010), but with a nominal level target in the central bank's objective function. I also describe the conduct of monetary policy with a version of the Taylor rule, to be used in the model calibration. At the same time, I take into account that the nominal policy rate occasionally hits the ZLB. After describing the features of an equilibrium that accounts for the ZLB and uncertainty about the evolution of the economy, I calibrate the model to U.S. data.

2.1. Private Sector

The behavior of the private sector is summarized by two log-linearized structural equations, namely an Euler equation and a Phillips curve, respectively describing the demand and supply sides of the economy. The equations of this basic model are linearized around zero inflation.

The Euler equation, which describes the representative household's expenditure decisions, is given by

$$y_t = E_t y_{t+1} - \varphi \left(i_t - r - E_t \pi_{t+1} - \upsilon_t \right), \tag{1}$$

where E_t denotes the expectations operator conditional on information available at time t. y_t is output measured as the log deviation from trend. π_t is the inflation rate, the log difference of prices between the current and previous period, $p_t - p_{t-1}$. And $i_t \geq 0$ is the short-term nominal interest rate, which is the instrument of monetary policy and is constrained by a ZLB. $r > 0$ is the steady-state interest rate, with zero inflation in steady state. Thus, $i_t - r - E_t \pi_{t+1}$ is the real interest rate in deviation from steady state. $\varphi > 0$ is the interest elasticity of real aggregate demand, capturing intertemporal substitution in household spending.

The *demand shock*, υ_t, represents other spending, such as government spending. Because of the ZLB constraint, the effects of the demand shock on the economy are asymmetric. A positive demand shock can be countered entirely by raising the nominal interest rate, whereas a large negative shock that leads to hitting the ZLB causes output and prices to fall.[3] As a result, in the model the central bank faces a tradeoff between stabilizing current and future economic activity. The reason for this tradeoff is that, during a ZLB episode, the central bank can lower the real interest rate and stimulate economic activity today by credibly promising to induce a surge in economic activity and inflation after the ZLB episode.[4]

The Phillips curve, which describes the optimal price-setting behavior of firms, under staggered price changes à la Calvo, is given by

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t + u_t, \qquad \textbf{(2)}$$

where $\beta \in (0, 1)$ is the discount factor of the representative household, determined as $1/(1+r)$. $x_t \equiv y_t - y_t^n$ is the output gap. y_t^n is the natural rate of output, or potential output, the output deviation from trend that would prevail in the absence of any price rigidities, which represents a *technology shock*. A positive technology shock implies slack in economic activity and downward pressure on prices, whereas a negative shock implies a strong economy and puts upward pressure on prices. Because this type of shock pushes output gaps and prices in the same direction, it does not entail any tradeoffs for the central bank in the model, as long as the nominal interest rate does not reach the ZLB.

Moreover, u_t is a cost–push shock, or a *mark-up shock* resulting from variation over time in the degree of monopolistic competition between firms. The mark-up shock creates an inflation–output tradeoff for monetary policy in the model. The reason for this tradeoff is that, if prices decrease because of a negative mark-up shock, the central bank needs to stimulate economic activity to place upward pressure on prices, whereas because of a positive shock, the central bank needs to discourage economic activity and put downward pressure on prices.

The slope parameter in the Phillips curve,

$$\kappa = \frac{(1-\alpha)(1-\alpha\beta)}{\alpha} \frac{\varphi^{-1} + \omega}{1 + \omega\theta} > 0,$$

is a function of the structure of the economy, where $\omega > 0$ denotes the elasticity of a firm's real marginal cost with respect to its own output level. $\theta > 1$ is the price elasticity of demand substitution among differentiated goods produced by firms under monopolistic competition.[5] Each period, a share $\alpha \in (0, 1)$ of randomly picked firms cannot adjust their prices, whereas the remaining $(1-\alpha)$ firms get to choose prices optimally.[6]

The three types of exogenous shocks are assumed to follow AR(1) stochastic processes,

$$y_t^n = \rho_y y_{t-1}^n + \sigma_{\varepsilon y} \varepsilon_{yt},$$

$$u_t = \rho_u u_{t-1} + \sigma_{\varepsilon u} \varepsilon_{ut},$$

$$v_t = \rho_v v_{t-1} + \sigma_{\varepsilon v} \varepsilon_{vt},$$

with first-order autocorrelation parameters $\rho_j \in (-1, 1)$ for $j = y, u, v$. And $\sigma_{\varepsilon j} \varepsilon_{jt}$ are the innovations that buffet the economy, which are independent across time and cross-sectionally, and normally distributed with mean zero and standard deviations $\sigma_{\varepsilon j} > 0$ for $j = y, u, v$.

The policy frameworks to be considered are evaluated based on the social welfare function, given by

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\pi_t^2 + \lambda \left(x_t - x^*\right)^2\right], \tag{3}$$

where λ is the weight assigned to stabilizing the output gap relative to inflation. x^* is the target level of the output gap, which stems from monopolistic competition and distortion in the steady state. Output subsidies are assumed to offset the monopolistic distortion so that the steady state is efficient, $x^* = 0$.[7] This social welfare function, as explained by Woodford (2010), can be derived as a second-order approximation of the lifetime utility function of the representative household. The utility function is approximated around zero inflation. The approximation of the utility function makes it possible to determine λ in terms of the structure of the model economy. Thus, λ is equal to κ/θ in this model.

2.2. Monetary Policy

The conduct of monetary policy is first described by a simple rule, to be used in the model calibration. It is then described by optimal discretion with a nominal-level target in the central bank's objective function.

The *simple policy rule* employed is a version of the Taylor rule subject to the ZLB constraint, along the lines of Taylor and Williams (2010):

$$i_t = \max\left\{0, \phi_i i_{t-1}^u + (1-\phi_i)\left[r + \phi_\pi \pi_t + \phi_x\left(y_t - y_t^n\right)\right]\right\}, \tag{4}$$

where ϕ_π and ϕ_x are positive response coefficients on inflation and the output gap, respectively. The rule incorporates smoothing into the behavior of the interest rate, through a positive value of the coefficient ϕ_i. And i_{t-1}^u denotes an unconstrained or notional interest rate, the preferred setting of the policy rate in the previous period that would occur absent the ZLB. Thus, the policy rate is kept below the notional interest rate following an episode when the ZLB is a binding constraint on policy. This approach implies that the central bank compensates to some extent for the lost monetary stimulus due to the existence of the ZLB, even though the central bank does not commit to making up for past shortfalls from a nominal level target.

Under optimal discretion, the policy maker has an objective function rather than a simple rule and reoptimizes its policy decision in each period, as described in Woodford (2010).[8] In such a setting, two monetary policy frameworks are considered. First, with *strict price level targeting*, the objective function is assumed to take the form

$$\min_{i_t \geq 0} E_t \sum_{j=0}^{\infty} \beta^j p_{t+j}^2,$$

where p_t is the log price level, which is equal to $p_{t-1} + \pi_t$. In this framework, the policy maker seeks to stabilize prices without concern for output stability and therefore transfers the entire burden of shocks onto output. The framework involves inertia in the behavior of policy, because the current policy decision depends on the past price level. To ensure price stability, the real interest rate is raised above its equilibrium value after shocks that put upward pressure on prices, whereas it is pushed below its equilibrium value following deflationary shocks.

Second, with *nominal GDP-level targeting*, the objective function becomes

$$\min_{i_t \geq 0} E_t \sum_{j=0}^{\infty} \beta^j n_{t+j}^2,$$

where n_t is nominal GDP measured as log deviation from trend, which is equal to the sum of the logs of prices and output, $p_t + y_t$. In this framework, the policy maker seeks to stabilize both prices and output, as opposed to focusing entirely on price stability, and thus requires the burden of shocks to be shared by prices and output. As a consequence, however, the current policy decision involves relatively less dependence on the past price level, and the policy maker acts less in accordance with a precommitment to price stability. To ensure nominal GDP stability, the real interest rate is raised following shocks that put upward pressure on nominal GDP, whereas it is lowered after shocks that put downward pressure on nominal GDP.

2.3. Equilibrium

At equilibrium, the policy maker chooses a policy based on a response function $\mathbf{y}(\mathbf{s}_t)$ and a state vector $\mathbf{s}_t$. The corresponding expectations function takes the form

$$\mathbf{E}_t \mathbf{y}_{t+1}(\mathbf{s}_{t+1}) = \int \mathbf{y}(\mathbf{s}_{t+1})\, f(\varepsilon_{t+1})\, d(\varepsilon_{t+1}),$$

where $f(\cdot)$ is a probability density function of the future innovations that buffet the economy. Because there is uncertainty about the future state of the economy, the ZLB is an occasionally binding constraint among the endogenous variables in the model.

In such a setting, I provide the following equilibrium definition:

DEFINITION 1. *A stochastic rational-expectations equilibrium (SREE) is given by a response function and a corresponding expectations function,* $\mathbf{y}(\mathbf{s}_t)$ *and* $\mathbf{E}_t \mathbf{y}_{t+1}(\mathbf{s}_{t+1})$*, respectively, which satisfy the equilibrium conditions, derived in Appendix A.1.*

Ignoring the existence of uncertainty about the evolution of the economy, the model could be solved with a standard numerical method, as for example in Coibion et al. (2012). By contrast, as in Billi (2011), I use a numerical procedure that accounts for the ZLB and uncertainty about the future state of the economy.[9]

TABLE 1. Baseline calibration of the model

Definition	Parameter	Numerical value
Discount factor	β	0.99
Interest elasticity of aggregate demand	φ	6.25
Share of firms keeping prices fixed	α	0.66
Price elasticity of demand	θ	7.66
Elasticity of a firms' marginal cost	ω	0.47
Slope of aggregate supply curve	κ	0.024
Weight on output gap	λ	0.003
Taylor rule coefficients	$\phi_{\pi,x,i}$	1.5; 0.25; 0.85
Std. deviation of technology shock	σ_y	0.80%
Std. deviation of mark-up shock	σ_u	0.05%
Std. deviation of demand shock	σ_v	0.80%
AR(1) parameter of shocks	$\rho_{y,u,v}$	0.80

Note: Because in the model a period is one quarter, parameter values shown correspond to inflation and interest rates measured at a quarterly rate.

When the ZLB threatens, the mere possibility of hitting the ZLB causes expectations of a future decline in output and inflation, as shown by Adam and Billi (2006, 2007) and Nakov (2008).

2.4. Calibration

The model economy is calibrated to U.S. data for recent decades. To do so, monetary policy in the model is described by the inertial Taylor rule (4), which features prominently in Federal Reserve discussions. The values of the rule coefficients are taken from English et al. (2015), with $\phi_\pi = 1.5$, $\phi_x = 1/4$ (quarterly rates), and $\phi_i = 0.85$. The rule thus accounts for smoothing in the behavior of the policy interest rate.

The values of the structural parameters of the model are standard in the related literature, and are the same ones used by Billi (2011) and Giannoni (2014), among others. Regarding the calibration of the shocks, the first-order autocorrelation parameters $\rho_{y,u,v}$ are set to 0.8 to generate persistent effects on the economy. At the same time, the standard deviations of the technology shock and demand shock $\sigma_{y,v}$ are set to 0.8% (quarterly) to try to replicate the volatility of output and nominal interest rates, respectively, in the data. And the standard deviation of the mark-up shock σ_u is set to 0.05% (quarterly), to match the inflation volatility in the data. This baseline calibration is summarized in Table 1.

Overall, with the simple policy rule and baseline calibration, the model does a fairly good job in replicating the relevant features of U.S. data for the sample period 1984Q1–2014Q4, as Table 2 shows.[10] The model strikes a balance between replicating output volatility and nominal interest rate volatility in the data, and thus slightly overstates output volatility and understates nominal interest rate volatility.

TABLE 2. Fitting the model to the data

	Std. deviation (pa)			Autocorrelation		
	π	y	i	π	y	i
U.S. data[a]	1.04	4.01	2.96	0.78	0.99	0.99
Model results[b]	0.99	4.86	2.08	0.53	0.46	0.92

pa, Percent annual.
[a]Sample period 1984Q1 to 2014Q4.
[b]Baseline calibration and policy follow the inertial Taylor rule (4).

In fact, because in this basic model the nominal interest rate is the only available policy instrument, the model does not account for other policies used in actuality to stabilize output such as balance sheet policies and fiscal spending. Related, output and inflation are somewhat less persistent in the model results than in the data, because this basic model, for the sake of simplicity, does not allow for structural propagation mechanisms that give rise to output and inflation inertia.[11]

Estimating the shocks with a similar model, Adam and Billi (2006) find that mark-up shocks do not display any significant autocorrelation, because of an estimation procedure that allows inflation to inherit the persistence of its prediction and of a shorter sample period that excludes the recent decade of low and stable inflation. However, in this analysis, if mark-up shocks are assumed to have no persistence, the autocorrelation of inflation falls from its baseline value of 0.53 to 0.35 (not shown), which is then further below the autocorrelation of 0.78 in the data (Table 2). However, the policy evaluation in the next section illustrates the importance of shock persistence for the model results, whereas the policy ranking with persistent shocks is shown to be robust to a wide range of alternate calibrations.

3. POLICY EVALUATION

Employing the small New Keynesian model with a calibration to recent U.S. data, I compare the performance of nominal-GDP-level targeting and strict price-level targeting under optimal discretion, relative to the performance of an inertial Taylor rule. I first consider a version of the model in which the shocks have no persistence and illustrate key features of the two targeting frameworks. I then introduce persistence in the shocks. I also study a range of alternate calibrations to test the robustness of the findings.

3.1. White Noise Shocks

I start the evaluation by assuming the shocks have no persistence, setting $\rho_{y,u,v}$ to zero in the baseline calibration. Figure 1 shows the expected evolution of the economy after each of the three types of shocks considered in the model.[12] Shown are the responses of the real interest rate, price level, and nominal GDP level.

https://doi.org/10.1017/S136510051500111X Published online by Cambridge University Press

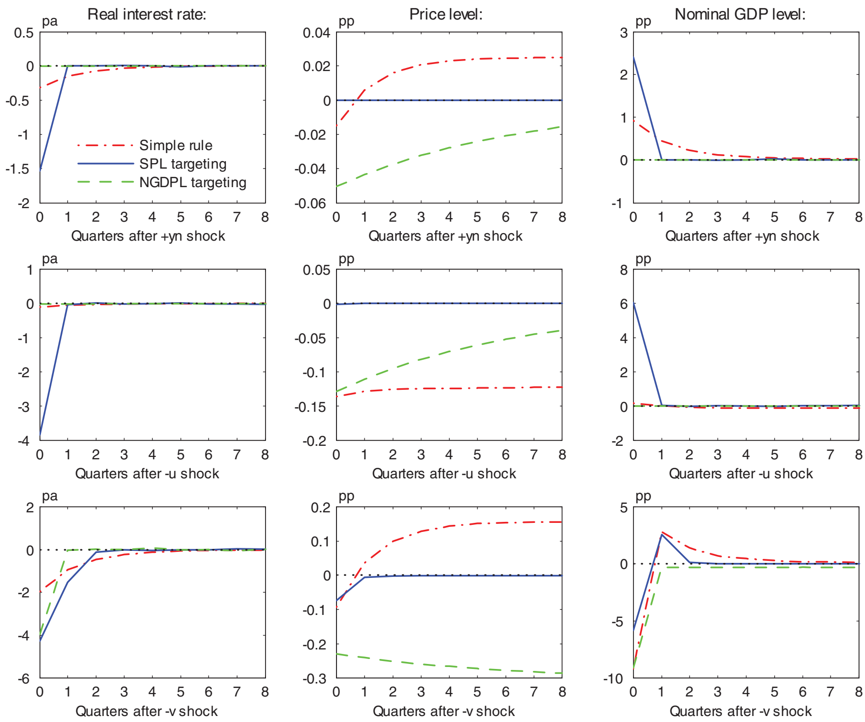

FIGURE 1. Evolution of the economy, with purely temporary shocks. Shown are the expected paths after three-standard-deviation shocks, for each type of shock as in Table 3, using the baseline calibration in Table 1 but with the persistence of the shocks set to zero.

The top panel of Figure 1 shows the response to a positive technology shock, whereas the middle panel shows the response to a negative mark-up shock. Both types of supply shock put downward pressure on prices in the model, but the outcome depends on the monetary policy framework. With nominal GDP level targeting (dashed green lines), the real interest rate is nearly unchanged, prices fall, and nominal GDP is stabilized. With strict price-level targeting (solid blue lines), the real interest rate falls, nominal GDP rises, and prices are stabilized. However, with the simple policy rule (dash-dotted red lines), the real interest rate edges down, whereas prices and nominal GDP fluctuate. Each targeting framework can fully achieve its intended goal in response to supply shocks. However, to achieve the goal, a nominal-GDP-level target requires the burden of the shocks to be shared by prices and output, whereas a strict price-level target requires the entire burden of the shocks to be transferred onto output (not shown).

The bottom panel of the figure shows the response to a negative demand shock, which exerts downward pressure on output and prices in the model. Given the size of the shock, under the targeting frameworks, the central bank cuts the nominal policy rate (not shown) all the way to the ZLB. In both targeting frameworks,

https://doi.org/10.1017/S136510051500111X Published online by Cambridge University Press

TABLE 3. Economic performance, with purely temporary shocks[a]

	ZLB episodes		Welfare loss[b]		
	Freq.[c]	Duration[d]	π	x	Tot.
	Simple policy rule				
Technology shock only	0.0	0.0	0.0	0.0	0.0
Mark-up shock only	0.0	0.0	0.8	0.0	0.8
Demand shock only	0.0	0.0	1.3	13.0	14.3
	Strict price-level targeting				
Technology shock only	0.0	0.0	0.0	0.0	0.0
Mark-up shock only	0.0	0.0	0.0	5.3	5.3
Demand shock only	8.0	1.1	0.1	0.7	0.8
	Nominal-GDP-level targeting				
Technology shock only	0.0	0.0	0.1	0.8	0.9
Mark-up shock only	0.0	0.0	0.8	0.0	0.8
Demand shock only	7.9	1.1	0.2	1.3	1.5

[a]Baseline calibration (Table 1) but with persistence of shocks set to zero.
[b]Permanent consumption loss (basis points).
[c]Expected percentage of time at the ZLB.
[d]Expected number of consecutive quarters at the ZLB.

during the ZLB episode, the real interest rate falls, prices fall, and nominal GDP falls. However, with strict price-level targeting the real interest rate stays below its equilibrium value for a longer time, which implies a greater degree of monetary policy stimulus and therefore a smaller downturn in the economy. The reason for this better performance is that, as noted earlier, a strict price-level target implies a greater dependence of current policy decisions on past policy actions, and thus a surge in economic activity and inflation after the ZLB episode. In contrast, during the ZLB episode, nominal-GDP-level targeting provides less policy stimulus and, ironically, leads to a larger fall in nominal GDP. The fall in nominal GDP is large also under the simple policy rule.

To rank the monetary policy frameworks, Table 3 summarizes their performance. The table reports the expected frequency and duration of ZLB episodes, as well as the welfare loss due to business cycles.[13] Each line shows the outcome for one type of shock only. The top panel shows the results for the simple policy rule, the middle panel shows the outcome for strict price-level targeting, and the bottom panel reports results for nominal-GDP-level targeting. Regarding technology shocks, the simple policy rule and strict price-level targeting are more effective frameworks because they fully insulate the economy from technology shocks. In contrast, nominal-GDP-level targeting fails to insulate the economy from technology shocks and therefore results in a welfare loss due to fluctuations in prices and output.

Regarding the outcome of mark-up shocks, as the table shows, strict price level targeting is the least effective framework, because to offset inflationary pressures it causes relatively costly fluctuations in output. The simple policy rule and

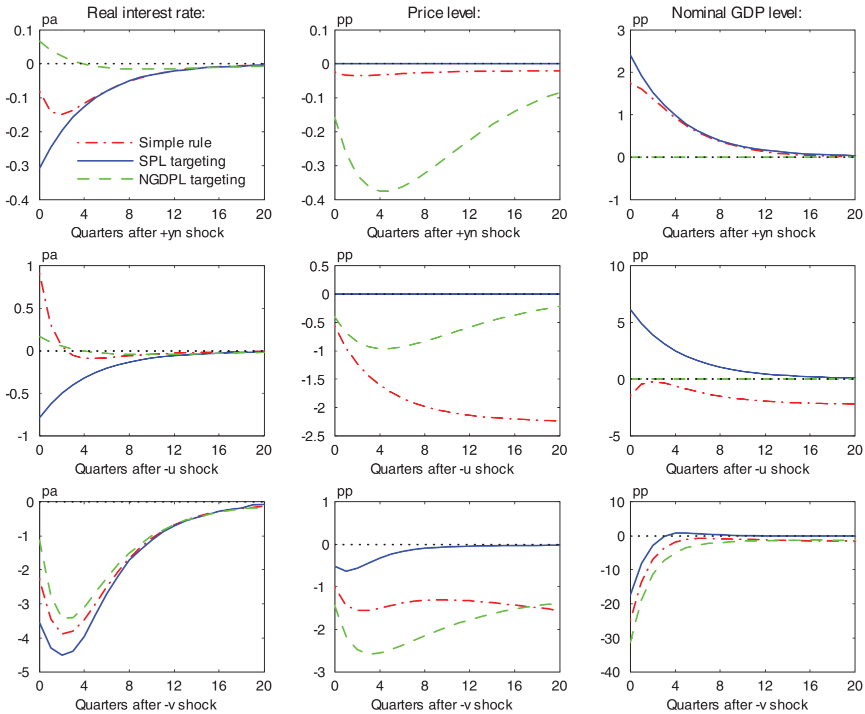

FIGURE 2. Evolution of the economy, with persistent shocks. Shown are the expected paths after three-standard-deviation shocks, for each type of shock as in Table 4, using the baseline calibration in Table 1.

nominal-GDP-level targeting instead improve the inflation–output tradeoff faced by the central bank and result in a smaller total welfare loss. Moreover, regarding the results for demand shocks, the two targeting frameworks are similarly effective in dealing with the ZLB and the related tradeoff between stabilizing current and future output, whereas the simple policy rule causes relatively costly fluctuations in output. However, as shown in the next section, the policy ranking depends on the persistence of the shocks.

3.2. Persistent Shocks

I now introduce persistence into the shocks and use the baseline calibration. Figure 2 shows the expected evolution of the model economy with persistent shocks.[14] As shown in the top and middle panels, even though the shocks are persistent, each targeting framework still fully achieves its intended goal in response to technology and mark-up shocks. Regarding the response to demand shocks that push the economy into a ZLB episode, as shown in the bottom panel, the real interest rate still falls by more under a strict price level target, which implies a

TABLE 4. Economic performance, with persistent shocks[a]

	ZLB episodes		Welfare loss[b]		
	Freq.[c]	Duration[d]	π	x	Tot.
Simple policy rule					
Technology shock only	0.0	0.0	0.0	0.0	0.0
Mark-up shock only	0.0	0.0	10.3	0.3	10.6
Demand shock only	2.4	3.1	13.0	27.4	40.4
Strict price-level targeting					
Technology shock only	0.0	0.0	0.0	0.0	0.0
Mark-up shock only	0.0	0.0	0.0	5.3	5.3
Demand shock only	17.5	3.0	1.0	4.0	5.0
Nominal-GDP-level targeting					
Technology shock only	0.0	0.0	0.8	0.5	1.3
Mark-up shock only	0.0	0.0	4.9	0.5	5.4
Demand shock only	14.2	2.3	6.5	9.7	16.2

[a]Baseline calibration (Table 1).
[b]Permanent consumption loss (basis points).
[c]Expected percentage of time at the ZLB.
[d]Expected number of consecutive quarters at the ZLB.

greater degree of monetary policy stimulus to the economy. As a result, if the analysis takes shock persistence into account, the fall in nominal GDP during a ZLB episode is substantially larger under a nominal GDP level target, relative to strict price-level targeting.

Next, Table 4 summarizes the performance of the monetary policy frameworks with persistent shocks. It shows that, even if technology shocks are persistent, the simple policy rule and strict price-level targeting are still the more effective frameworks because they fully insulate the economy from such shocks. Under nominal-GDP-level targeting, economic performance deteriorates to a certain extent. If mark-up shocks are persistent, the two targeting frameworks are now similarly effective in terms of overall welfare, whereas the simple policy rule becomes the least effective framework. The reason is that persistent mark-up shocks give rise to costly fluctuations in inflation under nominal-GDP-level targeting, which are now comparable in size to the costs of output volatility under strict price-level targeting. However, inflation volatility is even larger under the simple policy rule.

Finally, if demand shocks are persistent, as the table shows, strict price-level targeting is now the most effective framework. This result occurs because persistent demand shocks cause costly fluctuations in both inflation and output under nominal-GDP-level targeting and cause an even greater increase in inflation and output volatility under the simple policy rule. In sum, for all types of shocks considered, introducing shock persistence into the analysis leads to a certain deterioration in economic performance under nominal-GDP-level targeting, relative to strict price-level targeting.

TABLE 5. Economic performance, alternate calibrations

	ZLB episodes		Welfare loss[a]		
	Freq.[b]	Duration[c]	π	x	Tot.
Simple policy rule					
Baseline	2.3	3.1	25.7	29.1	54.8
Lower steady-state real rate ($\beta = 0.993$)[d]	3.7	3.3	27.8	38.6	66.4
Smaller demand elasticity ($\varphi = 1$)	0.1	2.2	26.1	8.4	34.5
Prices less sticky ($\alpha = 0.5$)	3.1	2.9	27.2	23.2	50.4
Prices more sticky ($\alpha = 0.75$)	2.4	3.3	42.2	31.8	74.0
Less competition ($\theta = 5$)	2.5	3.1	15.7	27.5	43.2
More competition ($\theta = 10$)	2.3	3.2	37.0	30.0	67.0
Purely temporary mark-up shocks ($\rho_u = 0$)	2.4	3.1	13.8	27.4	41.2
Larger mark-up shocks ($\sigma_u = 0.075$)	4.0	3.3	47.7	32.9	80.6
Supply shocks only ($\sigma_v = 0$)	0.0	0.0	10.4	0.4	10.8

[a]Permanent consumption loss (basis points).
[b]Expected percentage of time at the ZLB.
[c]Expected number of consecutive quarters at the ZLB.
[d]Smoothing coefficient ϕ_i raised to 0.9.

3.3. Alternate Calibrations

As the next step in the analysis, I consider a number of deviations from the baseline calibration of the model economy. For each change in the calibration, the model-implied parameters (β, κ, and λ) are adjusted accordingly. Tables 5 and 6 summarize the performance of the simple policy rule and targeting frameworks, respectively, with the parameter changes.[15]

I start with changes on the demand side of the economy. First, the equilibrium rate of interest is lowered substantially ($\beta = 0.993$), which implies that monetary policy is now more severely constrained by the ZLB.[16] As a consequence, as Table 6 shows, inflation and output volatility rise under both targeting frameworks relative to the baseline, but the welfare loss increases by more under nominal-GDP-level targeting. This result occurs because, as noted earlier, such a targeting framework is less effective in dealing with demand shocks that push the economy into a ZLB episode. Second, the interest elasticity of real aggregate demand is lowered substantially ($\varphi = 1$), which entails that changes in the nominal interest rate have smaller effects on output. At the same time, the supply side of the economy is also affected. As the Phillips curve becomes steeper (κ rises), changes in output have larger effects on prices. On net, monetary policy is now more effective, as inflation and output volatility fall under both targeting frameworks relative to the baseline, but strict price-level targeting is still a more effective framework in terms of overall welfare.

I now consider a range of parameter values on the supply side of the model economy.[17] In the model, if firms change prices less frequently or face more competition ($\alpha = 0.75$ and $\theta = 10$, respectively), the Phillips curve becomes

https://doi.org/10.1017/S136510051500111X Published online by Cambridge University Press

TABLE 6. Economic performance, alternate calibrations

	ZLB episodes		Welfare loss[a]		
	Freq.[b]	Duration[c]	π	x	Tot.
Strict price-level targeting					
Baseline	15.4	3.1	1.1	9.3	10.4
Lower steady-state real rate ($\beta = 0.993$)	27.6	4.5	3.4	16.1	19.5
Smaller demand elasticity ($\varphi = 1$)	13.1	3.6	0.3	2.6	2.9
Prices less sticky ($\alpha = 0.5$)	16.3	2.9	1.7	3.4	5.1
Prices more sticky ($\alpha = 0.75$)	15.0	3.1	0.7	27.3	28.0
Less competition ($\theta = 5$)	15.4	3.2	0.8	6.4	7.2
More competition ($\theta = 10$)	15.2	3.1	1.2	12.2	13.4
Purely temporary mark-up shocks ($\rho_u = 0$)	18.0	2.8	1.1	8.2	9.3
Larger mark-up shocks ($\sigma_u = 0.075$)	15.5	3.1	1.1	15.7	16.8
Supply shocks only ($\sigma_v = 0$)	0.0	0.0	0.0	5.3	5.3
Nominal-GDP-level targeting					
Baseline	11.0	2.1	11.9	10.9	22.8
Lower steady-state real rate ($\beta = 0.993$)	21.7	2.8	21.2	25.2	46.4
Smaller demand elasticity ($\varphi = 1$)	15.8	2.6	5.6	1.8	7.4
Prices less sticky ($\alpha = 0.5$)	10.8	1.9	8.4	6.2	14.6
Prices more sticky ($\alpha = 0.75$)	10.7	2.1	20.6	13.7	34.3
Less competition ($\theta = 5$)	10.9	2.0	6.9	9.7	16.6
More competition ($\theta = 10$)	11.0	2.1	17.6	11.8	29.4
Purely temporary mark-up shocks ($\rho_u = 0$)	13.2	2.2	8.0	10.3	18.3
Larger mark-up shocks ($\sigma_u = 0.075$)	10.2	2.0	17.9	11.8	29.7
Supply shocks only ($\sigma_v = 0$)	0.0	0.0	5.7	1.0	6.7

[a] Permanent consumption loss (basis points)
[b] Expected percentage of time at the ZLB.
[c] Expected number of consecutive quarters at the ZLB.

flatter (κ falls), and changes in output have smaller effects on prices. As a result, monetary policy becomes less effective. As Table 6 shows, inflation and output volatility generally rise under both targeting frameworks relative to the baseline, but the total welfare loss is higher under nominal-GDP-level targeting. Conversely, if firms change prices more frequently or face less competition ($\alpha = 0.5$ and $\theta = 5$, respectively), inflation and output volatility generally fall under both frameworks relative to the baseline, but strict price-level targeting is still a more effective framework.

Next, I consider the types of shocks that buffet the economy. First, as Table 6 shows, if mark-up shocks are assumed to have no persistence ($\rho_u = 0$), inflation volatility falls under nominal-GDP-level targeting, but not enough to change the policy ranking. Second, mark-up shocks are assumed to be substantially larger ($\sigma_u = 0.075$).[18] As a result, whereas output volatility rises under strict price-level targeting and inflation volatility rises under nominal-GDP-level targeting, the

https://doi.org/10.1017/S136510051500111X Published online by Cambridge University Press

increase in the welfare loss is roughly the same for both targeting frameworks and therefore the policy ranking is not affected.[19] Third, if the economy is only buffeted by supply shocks ($\sigma_v = 0$), which in this analysis implies that monetary policy is not constrained by the ZLB, inflation and output volatility fall substantially under both targeting frameworks, but strict price-level targeting is still the more effective framework.[20]

Overall, as the results in Table 6 show, the ranking of the two targeting frameworks with persistent supply and demand shocks is robust to a wide range of alternate calibrations of the model. Finally, a comparison of the results in Tables 5 and 6 shows that, for all the calibrations considered, both targeting frameworks perform better, in terms of social welfare in the model, than the simple policy rule.

4. CONCLUSION

Shedding light on recent proposals directed at major central banks to adopt a nominal-GDP-level target, this Note compares nominal-GDP-level targeting to strict price-level targeting in a standard model often used for monetary policy analysis. In the model, the central bank operates under optimal discretion and faces a ZLB constraint, and the economy is buffeted by supply and demand shocks. The stylized model is calibrated to recent U.S. data and offers a clear illustration of the tradeoffs faced by the central bank. The two targeting frameworks are ranked in terms of performance, based on the model's social welfare function.

The analysis suggests that, if the economy is only buffeted by purely temporary shocks to inflation, nominal-GDP-level targeting may be preferable because it requires the burden of the shocks to be shared by prices and output, whereas strict price-level targeting causes costly fluctuations in output. However, in the presence of persistent supply and demand shocks, strict price-level targeting may be superior because it induces greater policy inertia and improves the tradeoffs faced by the central bank. During ZLB episodes, ironically, nominal-GDP-level targeting leads to larger falls in nominal GDP. Such results are shown to be robust to a wide range of alternate calibrations. Still, as the analysis is conducted in a stylized model, further study is needed to extend the results to a broader class of models.

NOTES

1. This Note adopts the standard practice of referring to a zero lower bound for nominal interest rates, but recent experience with negative nominal interest rates in Denmark, Sweden, Switzerland, and the eurozone suggests that the effective lower bound is somewhat below zero. See Svensson (2010) for a discussion.
2. There is also an extensive literature on the notion of nominal income *growth* targeting, at first suggested by Meade (1978) and Tobin (1980) and then studied by Bean (1983), Taylor (1985), West (1986), McCallum (1988), Hall and Mankiw (1994), Jensen (2002), and Walsh (2003), among others.
3. The fall in prices stems from the supply side of the economy; see equation (2).

4. The promise is credible if the central bank commits to making up for past shortfalls from the target, as is the case under an inertial Taylor rule or under optimal discretion with a nominal-level target.

5. The seller's desired markup is $\theta/(\theta - 1)$.

6. The implied duration between price changes is $1/(1 - \alpha)$.

7. As a result, in the analysis there is no inflation bias, but there is a stabilization bias due to discretionary policy.

8. In this analysis, as in Woodford (2010), the outcome under optimal discretion corresponds to a Markov perfect equilibrium of the noncooperative game among successive policy makers, which implies that the central bank rationally accounts for how the current state of the economy affects future decisions.

9. See Appendix A.2 for a description of the algorithm.

10. The inflation rate is measured as the continuously compounded rate of change in the seasonally adjusted personal consumption expenditures chain-type price index less food and energy (source BEA). Output is measured as the log deviation from trend in seasonally adjusted real gross domestic product (source BEA). And the nominal interest rate is measured as the average effective federal funds rate (source Fed Board). Because the funds rate is on the average about 4 percent annual over the sample period, the discount factor β is set to 0.99 in the baseline.

11. As a consequence, the stylized model may understate the frequency and duration of ZLB episodes. With the baseline calibration and the conduct of monetary policy described by the simple policy rule, the model predicts that the policy rate hits the ZLB about 2 percent of the time, and the expected duration of a ZLB episode is about three quarters (Table 5). In actuality, the federal funds rate has been near the ZLB since the end of 2008.

12. Shown are expected paths after three-standard-deviation shocks. The expected paths are obtained by averaging across 10,000 stochastic simulations.

13. To calculate the welfare loss, first the value of the objective function (3) is obtained by averaging across 10,000 stochastic simulations, each 1,000 periods long after a burn-in period. This value is then converted into a permanent consumption loss, as explained in Appendix A.3.

14. Again shown are expected paths after three-standard-deviation shocks.

15. Changes to ω are not reported, because varying ω as much as ± 50 percent makes no noticeable difference for the implied parameters and results.

16. The equilibrium interest rate was lowered from 4 to 3 percent annually. At the same time, because the numerical procedure then failed to converge under the simple policy rule, the smoothing coefficient ϕ_i in the rule was raised a little, from 0.85 to 0.9, to ensure greater policy stimulus and obtain a numerical solution.

17. In the baseline calibration, the duration between price changes is three quarters ($\alpha = 0.66$) and the desired markup is 15 percent ($\theta = 7.66$). In the alternate calibrations reported in Tables 5 and 6, the price duration ranges from two to four quarters ($\alpha = 0.5$ and 0.75, respectively) and the markup ranges from 11 to 25 percent ($\theta = 10$ and 5, respectively).

18. The standard deviation of the mark-up shock was raised by 50 percent relative to the baseline.

19. Conversely, if the economy is not buffeted by mark-up shocks (not shown), the fall in the welfare loss is roughly the same for both targeting frameworks and the policy ranking does not change.

20. Conversely, if demand shocks or technology shocks are larger (not shown), the increase in the welfare loss is greater for nominal-GDP-level targeting, relative to strict price-level targeting.

REFERENCES

Adam, Klaus and Roberto M. Billi (2006) Optimal monetary policy under commitment with a zero bound on nominal interest rates. *Journal of Money, Credit and Banking* 38, 1877–1905.

Adam, Klaus and Roberto M. Billi (2007) Discretionary monetary policy and the zero lower bound on nominal interest rates. *Journal of Monetary Economics* 54, 728–752.

Bean, Charles R. (1983) Targeting nominal income: An appraisal. *Economic Journal* 93, 806–819.

Billi, Roberto M. (2011) Optimal inflation for the U.S. economy. *American Economic Journal: Macroeconomics* 3, 29–52.
Coibion, Olivier, Yuriy Gorodnichenko, and Johannes Wieland (2012) The optimal inflation rate in New Keynesian models: Should central banks raise their inflation targets in light of the ZLB? *Review of Economic Studies* 79, 1371–1406.
Eggertsson, Gauti B. and Michael Woodford (2003) The zero bound on interest rates and optimal monetary policy. *Brookings Papers on Economic Activity* 34, 139–235.
English, William B., J. David Lopez-Salido, and Robert J. Tetlow (2015) The Federal Reserve's framework for monetary policy: Recent changes and new questions. *IMF Economic Review* 63, 22–70.
Frankel, Jeffrey (2013) Nominal-GDP targets, without losing the inflation anchor. In Lucrezia Reichlin and Richard Baldwin (eds.), *Is Inflation Targeting Dead? Central Banking after the Crisis*, pp. 90–94. London: CEPR, VoxEU.org.
Giannoni, Marc P. (2014) Optimal interest rate rules and inflation stabilization versus price-level stabilization. *Journal of Economic Dynamics and Control* 41, 110–129.
Hall, Robert E. and N. Gregory Mankiw (1994) Nominal income targeting. In N. Gregory Mankiw (ed.) *Monetary Policy*, pp. 71–93. Chicago: University of Chicago Press.
Hatzius, Jan and Sven Jari Stehn (2011) The Case for a Nominal GDP Level Target. *Goldman Sachs US Economics Analyst* 11/41, October 14.
Hatzius, Jan and Sven Jari Stehn (2013) A nominal GDP level target in all but name. *Goldman Sachs US Economics Analyst* 13/03, January 20.
Jensen, Henrik (2002) Targeting nominal income growth or inflation? *American Economic Review* 92, 928–956.
McCallum, Bennet T. (1988) Robustness properties of a rule for monetary policy. *Carnegie-Rochester Conference Series on Public Policy* 29, 173–204.
Meade, James E. (1978) The meaning of internal balance. *Economic Journal* 88, 423–435.
Nakov, Anton (2008) Optimal and simple monetary policy rules with zero floor on the nominal interest rate. *International Journal of Central Banking* 4, 73–127.
Sumner, Scott B. (2011) Re-targeting the Fed. *National Affairs* (Fall, 9), 79–96.
Sumner, Scott B. (2014) Nominal GDP targeting: A simple rule to improve Fed performance. *Cato Journal* 34, 315–337.
Svensson, Lars E.O. (1999) Price-level targeting versus inflation targeting: A free lunch? *Journal of Money, Credit and Banking* 31, 277–295.
Svensson, Lars E.O. (2003) Escaping from a liquidity trap and deflation: The foolproof way and others. *Journal of Economic Perspectives* 17, 145–166.
Svensson, Lars E.O. (2010) Monetary policy and financial markets at the effective lower bound. *Journal of Money, Credit and Banking* 42 (Supplement), 229–242.
Taylor, John B. (1985) What would nominal GNP targeting do to the business cycle? *Carnegie-Rochester Conference Series on Public Policy* 22, 61–84.
Taylor, John B. and John C. Williams (2010) Simple and robust rules for monetary policy. In Benjamin M. Friedman and Michael Woodford (eds.), *Handbook of Monetary Economics*, Vol. 3, Chap. 15, pp. 829–859. Amsterdam: Elsevier.
Tobin, James (1980) Stabilization policy ten years after. *Brookings Papers on Economic Activity* 11, 19–90.
Vestin, David (2006) Price-level versus inflation targeting. *Journal of Monetary Economics* 53, 1361–1376.
Walsh, Carl E. (2003) Speed limit policies: The output gap and optimal monetary policy. *American Economic Review* 93, 265–278.
West, Kenneth D. (1986) Targeting nominal income: A note. *Economic Journal* 96, 1077–1083.
Woodford, Michael (2010) Optimal monetary stabilization policy. In Benjamin M. Friedman and Michael Woodford (eds.), *Handbook of Monetary Economics*, Vol. 3, Chap. 14, pp. 723–828. Amsterdam: Elsevier.

Woodford, Michael (2012) Methods of Policy Accommodation at the Interest-Rate Lower Bound. Presented at the Federal Reserve Bank of Kansas City Economic Policy Symposium, Jackson Hole, WY.

Woodford, Michael (2013) Inflation targeting: Fix it, don't scrap it. In Lucrezia Reichlin and Richard Baldwin (eds.), *Is Inflation Targeting Dead? Central Banking after the Crisis*, pp. 74–89. London: CEPR, VoxEU.org.

APPENDIX

A.1. EQUILIBRIUM CONDITIONS

I first derive the equilibrium conditions and then summarize them in a table.

Strict price-level targeting. To solve the model, recall the definition of the price level:

$$p_t \equiv p_{t-1} + \pi_t. \tag{A.1}$$

Using this identity, the problem can be written as

$$V(\mathbf{s}_t) = \max\left[-p_t^2 + \beta E_t V(\mathbf{s}_{t+1})\right]$$
$$\text{subject to (1), (2), (A.1), and } i_t \geq 0$$
$$\text{and } \mathbf{E}_t \mathbf{y}_{t+1}(\mathbf{s}_{t+1}) \text{ given.}$$

Write the period Lagrangian:

$$\begin{aligned} L_t = &-p_t^2 + \beta E_t V(\mathbf{s}_{t+1}) \\ &+ m_{1t}\left[\pi_t - \beta E_t \pi_{t+1} - \kappa\left(y_t - y_t^n\right) - u_t\right] \\ &+ m_{2t}\left[-y_t + E_t y_{t+1} - \varphi\left(i_t - r - E_t \pi_{t+1} - v_t\right)\right] \\ &+ m_{3t}\left[-p_t + p_{t-1} + \pi_t\right] \\ &\text{and } \mathbf{E}_t \mathbf{y}_{t+1}(\mathbf{s}_{t+1}) \text{ given.} \end{aligned}$$

The Kuhn–Tucker conditions are

$$\partial L_t/\partial \pi_t = m_{1t} + m_{3t} = 0, \tag{A.2}$$

$$\partial L_t/\partial y_t = -\kappa m_{1t} - m_{2t} = 0, \tag{A.3}$$

$$\partial L_t/\partial i_t \cdot i_t = -\varphi m_{2t} \cdot i_t = 0, \quad m_{2t} \geq 0, \quad i_t \geq 0, \tag{A.4}$$

$$\begin{aligned} \partial L_t/\partial p_t = &-2p_t + \beta \partial E_t V(\mathbf{s}_{t+1})/\partial p_t \\ &- (\beta m_{1t} - \varphi m_{2t}) \cdot \partial E_t \pi_{t+1}/\partial p_t + m_{2t} \cdot \partial E_t y_{t+1}/\partial p_t - m_{3t}, \end{aligned} \tag{A.5}$$

whereas the envelope condition is

$$\partial V(\mathbf{s}_t)/\partial p_{t-1} = m_{3t},$$

which implies that

$$\beta \partial E_t V(\mathbf{s}_{t+1}) / \partial p_t = \beta E_t m_{3t+1}.$$

Nominal-GDP-level targeting. Similarly, the problem can be written as

$$V(\mathbf{s}_t) = \max\left[-(p_t + y_t)^2 + \beta E_t V(\mathbf{s}_{t+1})\right]$$
$$\text{subject to (1), (2), (A.1), and } i_t \geq 0$$
$$\text{and } \mathbf{E}_t \mathbf{y}_{t+1}(\mathbf{s}_{t+1}) \text{ given.}$$

The Kuhn–Tucker conditions and envelope condition then give

$$\partial L_t / \partial \pi_t = m_{1t} + m_{3t} = 0. \quad \textbf{(A.6)}$$

$$\partial L_t / \partial y_t = -2(p_t + y_t) - \kappa m_{1t} - m_{2t} = 0. \quad \textbf{(A.7)}$$

$$\partial L_t / \partial i_t \cdot i_t = -\varphi m_{2t} \cdot i_t = 0, \quad m_{2t} \geq 0, \quad i_t \geq 0. \quad \textbf{(A.8)}$$

$$\partial L_t / \partial p_t = -2(p_t + y_t) + \beta E_t m_{3t+1}$$
$$- (\beta m_{1t} - \varphi m_{2t}) \cdot \partial E_t \pi_{t+1} / \partial p_t + m_{2t} \cdot \partial E_t y_{t+1} / \partial p_t - m_{3t}. \quad \textbf{(A.9)}$$

The equilibrium conditions are summarized as follows:

Policy framework	Equilibrium conditions	State vector $\mathbf{s}_t$
Simple policy rule	(1), (2), and (4)	$(y_t^n, u_t, v_t, i_{t-1}^u)$
Strict price-level targeting	(1), (2), (A.1), and (A.2)–(A.5)	$(y_t^n, u_t, v_t, p_{t-1})$
Nominal-GDP-level targeting	(1), (2), (A.1), and (A.6)–(A.9)	$(y_t^n, u_t, v_t, p_{t-1})$

A.2. NUMERICAL PROCEDURE

I find a numerical solution, as in Billi (2011), as a fixed point in the equilibrium conditions. To do this, the state vector is discretized into a grid of interpolation nodes, with a support of ±4 standard deviations for each state variable, which is large enough to avoid erroneous extrapolation. If the state is not on this grid, the response function is evaluated with multilinear interpolation. The approximation residuals are evaluated on a finer grid, to ensure the accuracy of the results. The expectations function is evaluated with Gaussian–Hermite quadrature and the derivatives are evaluated with a standard two-sided approximation. The initial guess is the linearized solution that ignores the ZLB constraint.

A.3. PERMANENT CONSUMPTION LOSS

I obtain the permanent consumption loss as in Billi (2011). The expected lifetime utility of the representative household is validly approximated by

$$E_0 \sum_{t=0}^{\infty} \beta^t U_t = \frac{U_c \overline{C}}{2} \frac{\alpha\theta(1 + \omega\theta)}{(1-\alpha)(1-\alpha\beta)} L, \quad \textbf{(A.10)}$$

where $\overline{C}$ is steady-state consumption; $U_c > 0$ is steady-state marginal utility of consumption; and $L \geq 0$ is the value of objective function (3).

https://doi.org/10.1017/S136510051500111X Published online by Cambridge University Press

At the same time, a steady-state consumption loss of $\mu \geq 0$ causes a utility loss of

$$E_0 \sum_{t=0}^{\infty} \beta^t U_c \overline{C} \mu = \frac{1}{1-\beta} U_c \overline{C} \mu. \qquad \textbf{(A.11)}$$

Equating the right sides of (A.10) and (A.11) gives

$$\mu = \frac{1-\beta}{2} \frac{\alpha\theta\,(1+\omega\theta)}{(1-\alpha)\,(1-\alpha\beta)} L.$$

https://doi.org/10.1017/S136510051500111X Published online by Cambridge University Press