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THE EFFECTS OF NEGATIVE POPULATION GROWTH: AN ANALYSIS USING A SEMIENDOGENOUS R&D GROWTH MODEL

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This study investigates the rates of technological progress, total output growth, and per capita output growth when population growth is negative using a semiendogenous research and development (R&D) growth model. The analysis shows that within a finite time horizon, the employment share of the final goods sector reaches unity and that of the R&D sector reaches zero; accordingly, the rate of technological progress tends toward zero. In this case, the growth rate of per capita output asymptotically approaches a positive value.

Keywords: Technological Progress, Semiendogenous Growth model, Negative Population Growth

1. INTRODUCTION

Japan's first postwar experience of a fall in population occurred in 2005, with negative population growth rates following in 2009 and 2011. Similarly, concern about population decline has been increasing in Italy and Germany [World Bank (2013)]. Therefore, population decline is an urgent problem in developed economies.

In recent years, the negative effect of population decline has often been pointed out, with per capita gross domestic product (GDP) rather than GDP per se shown to be closely related to economic welfare. Accordingly, it is important to examine whether population decline negatively affects per capita GDP growth. Based on the foregoing, the present study investigates the rates of technological progress, total output growth, and per capita output growth when population growth is negative using a slightly modified Jones (1995) semiendogenous research and development (R&D) growth model.[1]

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In the literature on this topic, Futagami and Hori (2010) investigate the relationship between the low fertility rate in Japan and per capita output growth using an extended Jones model.[2] They measure economic welfare using per capita GDP and investigate how a low fertility rate affects the rate of technological progress, which then determines per capita GDP growth. In their model, the fertility rate (i.e., the population growth rate) is endogenously determined. They compare the dynamics of the market equilibrium fertility rate and the technological progress rate with the dynamics of the socially optimal fertility rate and the technological progress rate. They conclude that the market equilibrium values exceed the socially optimal values and, accordingly, that it is not necessarily desirable for the government to intervene in the private decisions of parents with regard to the number of children that they want to have. However, the endogenously determined population growth rate is positive. In other words, they investigate a decline in population growth but not negative population growth. In contrast, the present study investigates the case of negative population growth even though this is exogenously given.

Closely related to population decline is population aging. Many studies have examined how population aging affects economic growth. For example, Prettner (2013), using a semiendogenous growth model, shows that (1) increases in longevity have a positive impact on per capita output growth; (2) decreases in fertility have a negative impact on per capita output growth; and (3) which effect dominates depends on the relative change between fertility and mortality in the semiendogenous growth framework. However, similarly to Futagami and Hori (2010), he does not consider the case of negative population growth.[3]

Gruescu (2007) considers negative population growth in the context of population aging and economic growth. She surveys the relationship between population size, the population growth rate, and economic growth to investigate the effects of negative population using the Solow (1956) and Lucas (1988) models. However, she does not strictly investigate the trend of the long-run growth rate of per capita output when population growth is negative.

At first sight, few countries seem to have experienced negative population growth. However, we should consider the effect of immigrants. For instance, if we consider the rate of natural increase (i.e., the crude birth rate minus the crude death rate), several countries have experienced negative population growth. Indeed, according to United Nations (2013), the rates of natural population increase of 17 OECD countries were negative between 2005 and 2010.[4] Therefore, it is meaningful to consider negative population growth in economic growth models.

Nevertheless, negative population growth is only just beginning to be considered in the field of economic growth theory [Ferrara (2011)]. In this body of research, Christiaans (2011) is relevant, as he shows the importance of negative population growth using a simple growth model.[5] Consider a neoclassical growth model with a production function that exhibits increasing, but relatively small, returns to scale. When the population growth rate is negative, contrary to expectations, per capita output growth is positive. To obtain increasing returns to scale, he uses the externality that arises from capital accumulation. However, he does not explicitly consider endogenous technological progress.

Based on the preceding observation, we use the Jones R&D growth model, in which technological progress is endogenously determined, in order to investigate the growth rates of key variables when population growth is negative. In this model, labor allocation between the final-goods-producing sector and the R&D sector is endogenously determined; it then specifies the rates of technological progress and economic growth.[6]

Our analysis shows that when population growth is negative, the long-run rate of technological progress is zero, that of total output growth is negative, and that of per capita output growth is positive. That is, even though population growth is negative, the long-run growth rate of per capita output is positive.

The remainder of the paper is organized as follows. Section 2 presents the framework of our model, derives the system of differential equations, and briefly investigates the dynamics when population growth is positive. Section 3 investigates the dynamics of the model when population growth is negative. Section 4 concludes the paper.

2. THE MODEL

A closed economy with no government consists of three sectors: the final goods-producing, capital goods-producing, and R&D sectors. The production function of the final goods sector is given by

$$Y = L_Y^{1-\alpha} \int_0^A x_i^{\alpha}\, di, \quad 0 < \alpha < 1, \tag{1}$$

where Y denotes the output of final goods; L_Y, the employment of the final goods sector; x_i, the input of capital goods; A, the number of capital goods; and α, a positive parameter. Final goods are used as numéraire.

The market for final goods is perfectly competitive. Profits of final goods-producing firms are given by

$$\Pi_Y = L_Y^{1-\alpha} \int_0^A x_i^{\alpha}\, di - w_Y L_Y - \int_0^A p_i x_i\, di, \tag{2}$$

where w_Y denotes the final goods sector's wage rate and p_i, the rental price of the capital good that the ith capital good firm produces. From the profit maximization condition and equation (1), we obtain

$$w_Y = (1-\alpha)\frac{Y}{L_Y}, \tag{3}$$

$$p_i = \alpha L_Y^{1-\alpha} x_i^{\alpha-1}. \tag{4}$$

The market for capital goods is monopolistically competitive. The ith capital good is produced only by the ith capital good firm. The ith capital good firm buys a blueprint from the R&D sector and produces finished capital goods by borrowing unfinished capital goods at an interest rate r. Unfinished capital goods can be converted into finished capital goods at zero cost. Accordingly, profits of capital goods-producing firms are given by

$$\Pi_i = p_i(x_i)x_i - rx_i, \tag{5}$$

where $p_i(x_i)$ is the inverse demand function for the ith capital good and is given by equation (4). Considering symmetric equilibrium, from the profit maximization condition, we obtain

$$p_i = p = \frac{r}{\alpha}, \tag{6}$$

$$x_i = x = \left(\frac{\alpha L_Y^{1-\alpha}}{p}\right)^{\frac{1}{1-\alpha}}. \tag{7}$$

Substituting equations (6) and (7) into equation (5), we obtain

$$\Pi_i = \Pi = \alpha(1-\alpha)\frac{Y}{A}. \tag{8}$$

Total capital stock K is the sum of capital goods:

$$K = \int_0^A x\,di = Ax. \tag{9}$$

Substituting equation (9) into equation (1), we obtain the aggregate production function as follows:

$$Y = K^{\alpha}(AL_Y)^{1-\alpha}. \tag{10}$$

Thus, with equations (4), (6), (9), and (10), the interest rate is given by

$$r = \alpha^2\frac{Y}{K}. \tag{11}$$

The market for blueprints is perfectly competitive. At equilibrium, the price of blueprints P_A is equal to the discounted present value of profits that new capital goods produce. Accordingly, the following nonarbitrage condition holds:

$$\frac{\Pi}{r} = P_A. \tag{12}$$

Here, for simplicity, we consider a situation in which there is neither capital gain nor capital loss, that is, $\dot{P}_A = 0$ ($\dot{x} = dx/dt$ hereafter).

Let L_A be employment in the R&D sector. Then the full employment condition leads to

$$L_Y + L_A = L, \tag{13}$$

where L denotes total population. We assume that the growth rate of total population n is constant and can be positive ($n > 0$) or negative ($n < 0$).

The production function of the R&D sector is given by

$$\dot{A} = \delta L_A, \quad \text{where } \delta = A^{\gamma}, \quad 0 < \gamma < 1, \tag{14}$$

where δ denotes externalities specific to knowledge production. An individual firm takes δ as given to maximize profits. The aggregate production function of the R&D sector is given by

$$\dot{A} = L_A A^{\gamma}, \tag{15}$$

where γ is the degree of externalities.

Profits of the R&D sector are given by

$$\Pi_A = P_A \delta L_A - w_A L_A. \tag{16}$$

From the profit maximization and free entry conditions, we obtain

$$w_A = P_A \delta. \tag{17}$$

Using equations (14) and (17), we obtain

$$w_A = P_A A^{\gamma}. \tag{18}$$

Equalizing the wage rate of the final goods sector with that of the R&D sector from equations (3) and (18), that is, $w_Y = w_A$, we obtain

$$\alpha \frac{Y}{L_Y} = P_A A^{\gamma}. \tag{19}$$

From equations (8), (11), and (12), we can eliminate the interest rate r:

$$\frac{K}{P_A A} = \frac{\alpha}{1-\alpha}. \tag{20}$$

We now turn to consumers' behavior. Consumers solve the following utility maximization problem:

$$\max_{C,S} U = C^{1-s} S^{s}, \quad 0 < s < 1, \tag{21}$$

$$\text{s.t. } C + S = Y, \tag{22}$$

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where C denotes consumption of final goods and S, savings. From this, we obtain

$$C = (1 - s)Y, \tag{23}$$

$$S = sY. \tag{24}$$

From the final goods-market-clearing condition, total savings S are equal to investment I:

$$\dot{K} = I = sY, \quad 0 < s < 1, \tag{25}$$

where we assume that the rate of depreciation is zero for simplicity.

Eliminating P_A from equations (19) and (20), and substituting equation (10) into the resultant expression, we obtain

$$\sigma = \left(\frac{\alpha^2}{1-\alpha}\right)^{\frac{1}{\alpha}} A^{\frac{2-\gamma-\alpha}{\alpha}} K^{-\frac{1-\alpha}{\alpha}} L^{-1} = \sigma(\underset{+}{A}, \underset{-}{K}, \underset{-}{L}), \tag{26}$$

where $\sigma = L_Y/L$ denotes the employment share of the final goods sector. Accordingly, the employment share of the R&D sector is given by $1 - \sigma = L_A/L$. Equation (29) states that if A, K, and L are given, the value of σ is determined.

The dynamics of our model is summarized as the following three equations:

$$\frac{\dot{K}}{K} = sK^{\alpha-1}(A\sigma L)^{1-\alpha}, \tag{27}$$

$$\frac{\dot{A}}{A} = (1 - \sigma)LA^{\gamma-1}, \tag{28}$$

$$\sigma = \sigma(\underset{+}{A}, \underset{-}{K}, \underset{-}{L}). \tag{29}$$

When $n > 0$, there exists a balanced growth path (BGP hereafter) along which A and K grow at constant rates and σ stays constant. In the following analysis, g_x denotes $\dot{x}/x$. By calculating $\dot{g}_K/g_K$ and $\dot{g}_A/g_A$ from equations (27) and (28) and letting the resultant expressions be zero, we obtain the BGP growth rates of A and K as follows:

$$g_A^* = \phi n > 0, \quad \phi \equiv \frac{1}{1-\gamma} > 1, \tag{30}$$

$$g_K^* = (1 + \phi)n > 0, \tag{31}$$

where the asterisk denotes the BGP value of a variable. Accordingly, A and K continue to increase at constant rates. In this case, from equation (29), we know that σ stays constant. Based on equations (30) and (31), we introduce the following

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scale-adjusted variables:

$$a \equiv \frac{A}{L^{\phi}}, \tag{32}$$

$$k \equiv \frac{K}{L^{1+\phi}}. \tag{33}$$

In addition, from equation (29), σ is rewritten as follows:

$$\sigma = \left(\frac{\alpha^2}{1-\alpha}\right)^{\frac{1}{\alpha}} a^{\frac{2-\gamma-\alpha}{\alpha}} k^{\frac{\alpha-1}{\alpha}} = \sigma(\underset{+}{a}, \underset{-}{k}). \tag{34}$$

Accordingly, when a and k are given, σ is determined. The growth rate of σ is given by

$$\frac{\dot{\sigma}}{\sigma} = \frac{2-\gamma-\alpha}{\alpha}\frac{\dot{a}}{a} - \frac{1-\alpha}{\alpha}\frac{\dot{k}}{k}. \tag{35}$$

By summarizing the preceding discussions, we obtain the following system of differential equations:

$$\dot{k} = k\left[sk^{\alpha-1}a^{1-\alpha}\sigma(a,k)^{1-\alpha} - (1+\phi)n\right], \tag{36}$$

$$\dot{a} = a\left\{[1-\sigma(a,k)]a^{\gamma-1} - \phi n\right\}. \tag{37}$$

When $n > 0$, we can show that there exist steady state values such that $k^* > 0$ and $a^* > 0$.[7] From $\dot{k} = \dot{a} = 0$, we obtain

$$k^* = \frac{[s(1-\alpha)]^{\frac{2-\gamma-\alpha}{(1-\alpha)(1-\gamma)}}}{[\alpha^2(1+\phi)n]^{\frac{\alpha}{1-\alpha}}\left\{[s\phi(1-\alpha)+\alpha^2(1+\phi)]n\right\}^{\frac{2-\gamma}{1-\gamma}}} > 0, \tag{38}$$

$$a^* = \left\{\frac{s(1-\alpha)}{\left[s(1-\alpha)\phi + \alpha^2(1+\phi)\right]n}\right\}^{\frac{1}{1-\gamma}} > 0. \tag{39}$$

We now turn to the stability analysis. The elements of the Jacobian matrix **J** that corresponds to equations (36) and (37) are given by

$$J_{11} = \frac{\partial \dot{k}}{\partial k} = -\frac{(1-\alpha)(1+\phi)n}{\alpha} < 0, \tag{40}$$

$$J_{12} = \frac{\partial \dot{k}}{\partial a} = \frac{(1-\alpha)(2-\gamma)(1+\phi)n}{\alpha}\frac{k^*}{a^*} > 0, \tag{41}$$

$$J_{21} = \frac{\partial \dot{a}}{\partial k} = \frac{\alpha(1+\phi)n}{s}\frac{a^*}{k^*} > 0, \tag{42}$$

$$J_{22} = \frac{\partial \dot{a}}{\partial a} = -\frac{(1-\gamma)\phi n}{1-\sigma^*} - \frac{\alpha(2-\gamma)(1+\phi)n}{s} < 0. \tag{43}$$

All elements are evaluated by the steady state values. In this simplified Jones model, both k and a are state variables. Accordingly, the necessary and sufficient conditions for the local stability of the steady state are that the trace of $\mathbf{J}$ is negative and the determinant of $\mathbf{J}$ is positive. From equations (40) and (43), we can easily find that $\operatorname{tr}\mathbf{J} = J_{11} + J_{22} < 0$. The determinant of $\mathbf{J}$ is given by

$$\det\mathbf{J} = J_{11}J_{22} - J_{12}J_{21} = \frac{(1-\alpha)(1-\gamma)\phi(1+\phi)n^2}{\alpha(1-\sigma^*)} > 0. \tag{44}$$

Accordingly, the sign of $\det\mathbf{J}$ is positive. Therefore, the local stability condition is satisfied: k and a converge to their respective steady state values from arbitrary initial values k_0 and a_0.

The production function for final goods is rewritten as

$$Y = \sigma^{1-\alpha}a^{1-\alpha}k^{\alpha}L^{1+\phi}. \tag{45}$$

Hence, the growth rate of per capita output $y = Y/L$ is given by

$$g_y = (1-\alpha)\frac{\dot{\sigma}}{\sigma} + (1-\alpha)\frac{\dot{a}}{a} + \alpha\frac{\dot{k}}{k} + \phi n. \tag{46}$$

In the steady state, $\dot{k} = \dot{a} = \dot{\sigma} = 0$. Accordingly, the BGP growth of per capita output leads to

$$g_y^* = \phi n > 0. \tag{47}$$

Therefore, the BGP growth rate of per capita output is proportional to population growth.

3. ANALYASIS WHEN POPULATION GROWTH IS NEGATIVE

In this section, we show that if $n < 0$, the BGP never exists, and then investigate what happens when $n < 0$.[8]

When $n < 0$, the right-hand sides of equations (36) and (37) are always positive. Hence, we find that there never exists a situation in which $\dot{k} = \dot{a} = 0$. Because $\sigma(a, k)$ is restricted to the range $\sigma \in [0, 1]$, the growth rate of a is always positive even if σ takes any value. Thus, a continues to increase over time and, consequently, the growth rate of a asymptotically approaches $-\phi n > 0$ because $a^{-(1-\gamma)}$ in equation (37) approaches zero with $\gamma < 1$.

At this stage, the dynamics of $\sigma(a, k)$ is uncertain: σ may increase, decrease, or converge to a constant value. First, if σ continues to decrease and reaches $\sigma = 0$, then the growth rate of k becomes $-(1+\phi)n > 0$. Next, we consider a case in which σ continues to increase and reaches $\sigma = 1$ and a case in which σ converges to a constant value. In both cases, we can prove that the term $k^{\alpha-1}a^{1-\alpha}$ of equation (36) converges to zero in the long run. Define $z = k^{\alpha-1}a^{1-\alpha}$. By differentiating

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both sides with respect to time, we obtain

$$\dot{z} = (1-\alpha)[(1-\sigma)a^{\gamma-1} - s\sigma^{1-\alpha}z + n]z. \tag{48}$$

Note that σ in equation (48) is unity or a constant. For $t \to \infty$, we have $a^{\gamma-1} \to 0$ because g_a is always positive. Accordingly, we rewrite equation (48) as follows:

$$\dot{z} = -(1-\alpha)(s\sigma^{1-\alpha}z - n)z. \tag{49}$$

Because $n < 0$, we have $s\sigma^{1-\alpha}z - n > 0$, and hence, the steady state value is $z^* = 0$. Moreover, because $d\dot{z}/dz|_{z^*} = (1-\alpha)n < 0$, the steady state is locally stable. Therefore, the term $z = k^{\alpha-1}a^{1-\alpha}$ approaches zero in the long run. In these cases, the growth rate of k also becomes $-(1+\phi)n > 0$.

Given the growth rates of a and k, from equation (35), the growth rate of σ becomes $-n > 0$ in the long run: σ continues to increase in the long run. Note that σ is restricted to the range $\sigma \in [0, 1]$. Accordingly, within a finite time horizon, $\sigma = 1$. This means that within this finite time horizon, the employment share of the R&D sector becomes zero and that of the final goods sector becomes unity.

We confirm the preceding discussions by using numerical simulations with regard to the dynamics of σ. For this purpose, we need to set the parameters of the model. We use parameters drawn from empirical observations and previous work; accordingly, we confine our analysis to Japan, Italy, and Portugal, because all the parameter values can be obtained for these countries.

First, the values of n are the annual rates of natural increase during the period 2005–2010, which are defined as the crude birth rate minus the crude death rate and taken from United Nations (2013): −0.01%, −0.02%, and −0.04% for Japan, Italy, and Portugal, respectively.

Second, the values of s are the average gross national saving rates, taken from OECD (2013), for the period 2005–2010: 0.253, 0.180, and 0.113 for Japan, Italy, and Portugal, respectively.

Third, the value of α is set to 0.3 because this can be interpreted as capital's share of income.

Fourth, the value of γ is set to 0.5, taken from Futagami and Hori (2010).

Finally, the initial values of $k(t)$ and $a(t)$, namely $k(0)$ and $a(0)$, are set to $k(0) = a(0) = 0.01$ so that the initial value of $\sigma(t)$ will not exceed unity.

Figures 1–3 show the results of the numerical simulations for Japan, Italy, and Portugal, respectively. In every case, the employment share of the final goods sector continues to increase and reaches unity within a finite time horizon. Because the Jones model is effective within the interval $\sigma \in (0, 1)$, after $\sigma = 1$, the Jones model will degenerate into the standard Solow model.

Therefore, within this finite time horizon, the employment share of the R&D sector becomes zero and hence the growth rate of A becomes zero. Accordingly, we can say that $t_1 \in (0, \infty)$, so that we have $\sigma \in (0, 1)$ during $t \in [0, t_1)$ and we have $\sigma = 1$ during $t \in [t_1, \infty)$. In what follows, we call the situation in which $\sigma \in (0, 1)$ the Jones regime, whereas the situation in which $\sigma = 1$ is termed the

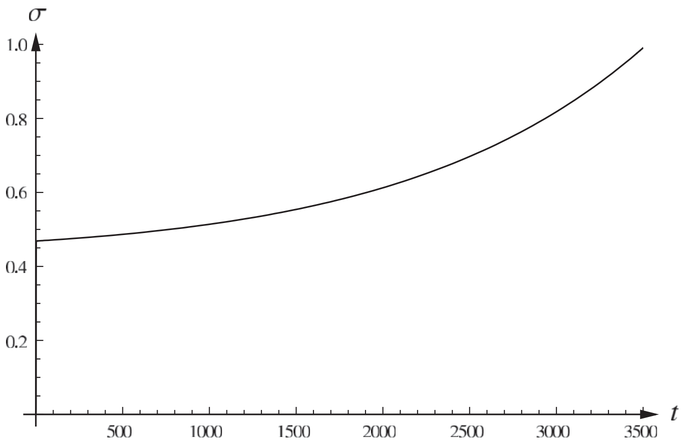

FIGURE 1. Estimated employment shares of the final goods sector in Japan over time.

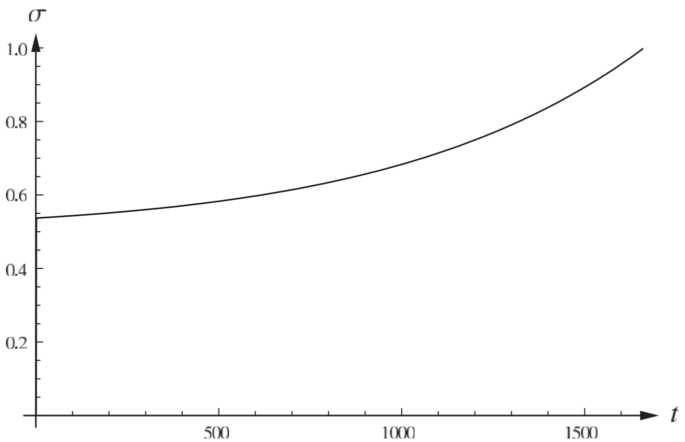

FIGURE 2. Estimated employment shares of the final goods sector in Italy over time.

Solow regime. Hence, the system of differential equations is decomposed into the following two subsystems:

$$\text{Jones regime : for } t \in [0, t_1) \begin{cases} \dot{k} = k\left[sk^{\alpha-1}a^{1-\alpha}\sigma(a,k)^{1-\alpha} - (1+\phi)n\right] \\ \dot{a} = a\left\{[1-\sigma(a,k)]a^{\gamma-1} - \phi n\right\}, \end{cases} \quad (50)$$

$$\text{Solow regime : for } t \in [t_1, \infty) \begin{cases} \dot{k} = k\left[sk^{\alpha-1}a^{1-\alpha} - (1+\phi)n\right] \\ \dot{a} = -\phi n a. \end{cases} \quad (51)$$

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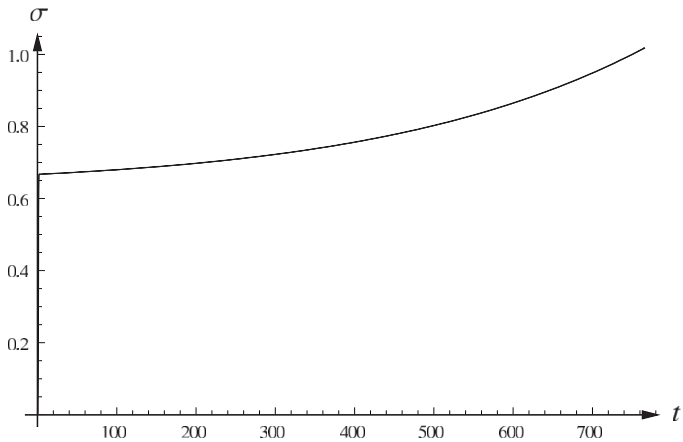

FIGURE 3. Estimated employment shares of the final goods sector in Portugal over time.

The Jones regime corresponds to $t \in [0, t_1)$, whereas the Solow regime corresponds to $t \in [t_1, \infty)$. Thus, at time t_1, the Jones regime switches to the Solow regime. By investigating the Solow regime, we find that the growth rates of k and a asymptotically approach $-(1+\phi)n > 0$ and $-\phi n > 0$, respectively, in the long run. In this case, from equation (46), the growth rate of per capita output $y = Y/L$ is given by

$$g_y = -(1-\alpha)(\phi n) - \alpha[(1+\phi)n] + \phi n = -\alpha n > 0. \qquad \textbf{(52)}$$

That is, even if population growth is negative, the growth rate of per capita output is positive in the long run.[9]

Suppose that a certain economic policy could keep the employment share of the final goods sector, σ, constant over time. In this case, the long-run growth rate of per capita output would also asymptotically approach $g_y = -\alpha n > 0$.

Therefore, when population growth is negative, per capita output continues to increase at a rate of $-\alpha n > 0$ in the long run. In this case, the rates of economic growth, capital accumulation, and technological progress are given by $g_Y = (1-\alpha)n < 0$, $g_K = 0$, and $g_A = 0$, respectively.

PROPOSITION 1. *Suppose that the population growth rate is constant and negative. Then, in a semiendogenous R&D growth economy, the growth rates of total output, technological progress, and per capita output are negative, zero, and positive, respectively.*

Figure 4 shows the long-run relationship between population growth and per capita output growth. The higher the absolute value of population growth, the faster per capita output grows.

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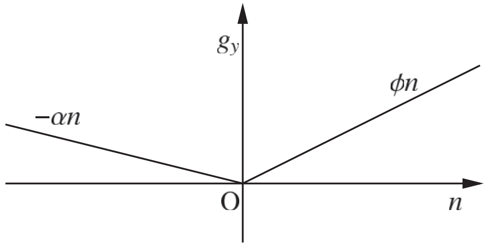

FIGURE 4. Relationship between population growth and per capita output growth.

Based on Figure 4, we consider the policy implications with regard to population growth. From the restrictions of $0 < \alpha < 1$ and $\phi > 1$, the slope of the graph when $n > 0$ is steeper than that of the graph when $n < 0$.

Suppose that the population growth rate is zero. Then, if a certain economic policy could change the population growth rate, increasing this rate would be more favorable than decreasing it in order to increase the growth rate of per capita output as long as the increment and decrement were the same.

In contrast, suppose that the population growth rate were negative. In this case, the long-run growth rate of per capita output would increase to a greater degree when n decreased than when n increased as long as n continued to be negative after n changed. However, because the effect of a change in n on g_y is larger when n is positive than when n is negative, it is more favorable to make n positive than it is to make n negative. In addition, our model does not consider the possible negative effects of population decline on pension and social security systems. Consequently, we should be cautious when interpreting the result that a decrease in n when n is negative leads to a higher g_y. The main point here is that the long-run growth rate of per capita output is positive even if population growth is negative as long as we use a standard growth model.

In order to understand why we obtain these results, from the production function of the final goods sector, we rewrite per capita output growth as follows:

$$g_y = \underbrace{(1-\alpha)\left(\frac{\dot{\sigma}}{\sigma} + \frac{\dot{A}}{A}\right)}_{\text{RD effect}} + \underbrace{\alpha\frac{\dot{\tilde{k}}}{\tilde{k}}}_{\text{CD effect}}, \tag{53}$$

where $\tilde{k} = K/L$ denotes per capita capital stock. Thus, per capita output growth is decomposed into two effects: the R&D effect (RD effect) and the capital deepening effect (CD effect). After $t = t_1$, that is, after the economy enters into the Solow regime, we have $\sigma = 1$ and $g_A = 0$; thus, the RD effect vanishes and only the CD effect lasts. The CD effect is given by

$$\frac{\dot{\tilde{k}}}{\tilde{k}} = \frac{\dot{K}}{K} - n = s\bar{A}^{1-\alpha}\tilde{k}^{\alpha-1} - n > 0, \tag{54}$$

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where $\bar{A}$ denotes the constant value of A after t_1. The CD effect is always positive and thus $\tilde{k}$ increases indefinitely. Hence, $g_{\tilde{k}}$ converges to

$$\lim_{\tilde{k}\to\infty} g_{\tilde{k}} = -n > 0. \tag{55}$$

From equation (53), we obtain $g_y = -\alpha n > 0$.

We explain the preceding results intuitively. In equation (20), the employment share of the final goods sector σ increases with the passage of time. However, the maximum value of σ is unity, and thus, $\dot{\sigma}/\sigma$ in the RD effect becomes zero within finite time. In addition, because population growth is negative, the level of employment in the R&D sector decreases over time, and accordingly, the growth rate of knowledge converges to zero, that is, $\dot{A}/A \to 0$. Hence, the RD effect vanishes in the end. The Jones regime switches to the Solow regime and only the CD effect remains. From equation (21), we can see that the growth rate of per capita capital stock $\tilde{k}$ is positive when population growth is negative. The growth rate of total capital stock K approaches zero, but L continues to decrease. Accordingly, $\tilde{k}$ continues to increase. Therefore, the CD effect is always positive. Summarizing the preceding discussions, we can state that when population growth is negative, the long-run growth rate of per capita output is positive because the capital deepening effect is always positive although technological progress stops within finite time.

From Figure 4, when population growth is zero, the long-run growth rate of per capita output is minimized, that is, zero. When population growth is zero, the long-run growth rate of knowledge becomes zero and the growth rate of the employment share of the final goods sector also becomes zero. Accordingly, the RD effect in equation (20) vanishes in the end. Moreover, when population growth is zero, the long-run growth rate of per capita capital stock becomes zero; that is, the CD effect in equation (20) vanishes in the end. Therefore, when population growth is zero and population is constant, the long-run growth rate of per capita output is zero.

To what extent is our model plausible? The long-run growth rate of per capita output depends only on α and n; hence, we can obtain the theoretical values of g_y and compare these with the actual values for the annual average growth rates of per capita output during 2005–2010 for Japan, Italy, and Portugal from World Bank (2013).

Table 1 shows the results of the numerical experiments when n is set to -0.01%, -0.02%, and -0.04% for Japan, Italy, and Portugal, respectively, and capital's share of income α changes from 0.3 to 0.8. Futagami and Hori (2010) use $\alpha = 0.8$ because α is the mark-up rate. The bottom row of Table 1 shows the actual growth rates of per capita output. Except for Italy, whose actual value is negative, the theoretical values are much lower than the actual values. Note that the theoretical growth rates are obtained after enough time elapses. From the analysis of the Solow regime, we further find that the growth rate of per capita output continues

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TABLE 1. Comparison of the theoretical and actual per capita income growth rates

	Theoretical growth rate (%)		
α	Japan	Italy	Portugal
0.3	0.03	0.06	0.12
0.4	0.04	0.08	0.16
0.5	0.05	0.10	0.20
0.6	0.06	0.12	0.24
0.7	0.07	0.14	0.28
0.8	0.08	0.16	0.32
Actual growth rate	0.38	−0.88	0.42

to decline over time and converges to $g_y = -\alpha n$. Therefore, we expect the growth rates of per capita output in Japan and Portugal to decrease in the future.

4. CONCLUSIONS

In the present study, using the Jones semiendogenous growth model, we investigated the long-run growth rates of per capita output when population growth is negative. Our results showed that when population growth is negative, the technological growth rate is zero, the growth rate of total output is negative, and that of per capita output is positive in the long run. Therefore, incorporating negative population growth into growth models is more complicated than simply replacing a positive population growth rate with a negative one.

Clearly, the OECD is heading in the direction of negative population growth. However, our analysis suggests that the hype about slow or negative population growth in OECD member nations (as often advocated by the *Economist*) is exaggerated. Therefore, the productivity effects of low fertility may not be as bad as often stressed in the news.

Our analysis focused only on the long-run relationship between negative population growth, economic growth, and technological progress. In particular, we only investigated growth rates after a sufficiently long time had passed. Accordingly, any analysis of transitional dynamics along which growth rates approach constant values is inadequate. Hence, detailed analysis of transitional dynamics will be left for future research. In addition, our analysis neglected the effects of negative population growth on population composition, the social security system, and so forth. These effects should also be included in future research.

NOTES

1. Some studies criticize the empirical validity of Jones's (1995) semiendogenous growth model. For example, Abdih and Joutz (2006) and Madsen (2008) conduct empirical analyses and conclude that Romer's (1990) endogenous growth model is more realistic than Jones's (1995) semiendogenous

growth model. Strulik et al. (2013) empirically show that there is a negative correlation between population growth and total factor productivity growth. Moreover, Sasaki (2011) states that the relationship between population growth and per capita real income growth differs for developed and developing countries.

2. The related study of Futagami and Nakajima (2001) investigates how population aging influences economic growth using an extended Romer (1986) AK model. However, these authors do not consider the case of negative population growth.

3. The survey of Prettner and Prskawetz (2010) is useful for understanding how population aging influences economic growth.

4. Japan, Belarus, the Czech Republic, Hungary, the Republic of Moldova, Romania, the Russian Federation, Ukraine, Estonia, Latvia, Lithuania, Bosnia and Herzegovina, Croatia, Italy, Portugal, Serbia, and Germany.

5. Sasaki (2015) builds a small-open-economy non-scale-growth model with negative population growth and investigates the relationship between trade patterns and per capita consumption growth. He shows that the home country is better off under free trade than under autarky in terms of per capita consumption growth irrespective of whether the population growth is positive or negative.

6. In some Schumpeterian models such as Dinopoulos and Thompson (1998, 1999), Peretto (1998), and Young (1998), the long-run growth rate of per capita output depends positively on both population growth and the employment share of the R&D sector. Hence, they remove the growth effect of population as Jones does (1995) but obtain endogenous growth as Romer does (1990). These Schumpeterian models generate endogenous growth without scale effects by introducing localized intertemporal R&D spillovers [Dinopoulos and Thompson (1999)].

7. If we consider the dynamic optimization of consumers, the Euler equation for consumption appears. In addition, if we consider capital gain and capital loss, the differential equation for P_A also appears. Hence, the Jones model consists of four differential equations. The dynamic stability of the Jones model in this case is fully analyzed by Arnold (2006).

8. As we will show later, when population growth is negative, the steady state in the usual sense ($\dot{k} = 0$ and $\dot{a} = 0$) does not exist, and accordingly, the BGP in the usual sense does not exist. For this issue, see Christiaans (2011).

9. Note that with the passage of time, the growth rate of per capita output approaches $g_y = -\alpha n$ but never reaches it. For this reason, $g_y = -\alpha n$ can be called the asymptotic BGP rate of growth of per capita output.

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