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ON THE INTERGENERATIONAL SHARING OF COHORT-SPECIFIC SHOCKS ON PERMANENT INCOME

KENJI MIYAZAKI
Hosei University

MAKOTO SAITO
Hitotsubashi University

TOMOAKI YAMADA
Meiji University

This paper investigates the intergenerational sharing of shocks on the permanent income of new entry cohorts when prior-to-entry markets are missing. When Lucas trees are traded among generations, procyclical cohort-specific shocks are shared partially via the movement of asset prices; cohorts with lower endowments may benefit more from asset pricing dynamics than cohorts with higher endowments. Given a reasonable set of parameters concerning the Japanese labor market, the evaluated welfare loss ranges from 1% to 3% in terms of the certainty equivalence consumption level. The first-best outcome may be achieved by either a combination of subsidies and taxes or the introduction of prior-to-entry markets.

Keywords: Cohort-Specific Shock, Intergenerational Transfer, Capital Income Tax

1. INTRODUCTION

This paper investigates the intergenerational sharing of shocks on the permanent income of new entry cohorts when prior-to-entry markets are missing. We define cohort-specific permanent shocks as the risks received by the cohort members when they enter labor markets for the first time. Due to the absence of prior-to-entry markets, it is not possible to insure perfectly against those risks using financial claims after the cohort-specific permanent shocks are realized. We quantitatively evaluate the welfare losses from the partial sharing of the cohort-specific permanent shocks. In addition, we investigate which policy interventions restore

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a first-best allocation and whether such a first-best allocation can be attained by opening up prior-to-entry markets.

There are many empirical studies that support the existence of cohort-specific permanent effects on labor earnings. For example, Baker et al. (1994) studied the personnel data of managers in a single company and found that first-time wages after entry were positively correlated with the lifetime wages of the corresponding cohort. Using the National Longitudinal Survey of Youth (NLSY), Kahn (2007) investigated the career outcomes of college graduates who entered labor markets during the 1982 recession and concluded that graduation in a recession has significantly negative effects on lifetime income. Using the NLSY, Kletzer and Fairlie (2003) found that a job displacement at an early stage of a career leads to persistent wage losses. Card and Lemieux (2001) demonstrated that date of birth significantly affects the lifetime wage of each cohort, using microdata from the United States, the United Kingdom, and Canada.[1]

Despite the difficulties involved in accessing microdata in Japan, several papers have addressed cohort-specific effects on lifetime wages in the Japanese labor market. Ohtake and Inoki (1997), Ohta (1999), and Okamura (2000) found that not only the cohort size, but also the economic conditions of a graduation year yield long-run effects on both wage rates and tenure. Genda (1997), upon whose study our numerical exercises rely, demonstrated that cohort-specific economic conditions, as well as an increase in the number of college graduates, account for differences in wages between college and high-school graduates in Japan.

Assuming that cohort-specific effects on lifetime wages are present,[2] we theoretically investigate the sharing of cohort-specific shocks among different cohorts through active financial transactions. For our research, we adopt an analytical framework proposed by Huffman (1987). Within Huffman's framework, perishable consumption goods are endowed upon the entry cohort, whereas Lucas trees yield perishable consumption goods as dividends. In financial markets, claims on Lucas trees are traded among cohorts. Given this endowment and dividend structure, the inability of entry cohorts to trade financial claims prior to entry is a fundamental source of market incompleteness.

In this model, a shock on entry endowments can be regarded as a cohort-specific shock on capitalized lifetime labor income or permanent income. The above setup completely removes age-specific consumption patterns, the timing of labor income receipts (life-cycle wage profiles), idiosyncratic shocks on labor income, and any other type of financial constraints. Thus, any welfare loss computed under the above setup would arise as a consequence of missing prior-to-entry markets. If the estimated welfare loss is not negligible for market allocations, then we will consider whether such welfare losses can be avoided by either implementing policy interventions or opening up prior-to-entry markets.

We augment Huffman's (1987) setup with a class of nonexpected utility, known as Kreps–Porteus preferences, to separate the elasticity of intertemporal substitution from relative risk aversion. A major reason for this generalization is that existing papers, including Bohn (1998) and Krueger and Kubler (2003), have suggested that the effect arising purely from intertemporal substitution is significant in the determination of intergenerational allocation of resources and risks.

Given a reasonable set of parameters concerning the Japanese labor market, the welfare loss ranges from 1% to 3% in terms of the certainty equivalence consumption level. One interesting result is that when cohort-specific shocks are positively correlated with dividend shocks, generations with lower endowments benefit much more from asset pricing dynamics than cohorts with higher endowments. Although cohorts receive lower endowments when they enter an economy in a recession, they can expand their future consumption opportunities by purchasing Lucas trees at substantially cheaper prices.

As a possible optimal policy, we demonstrate that a subsidy to the entry generation financed by a 100% levy on dividends from Lucas trees would yield the first-best outcome as a consequence of the risk pooling of labor and capital income. As an alternative way to restore the optimal allocation, on the other hand, a new entry cohort may reach the first-best outcome by borrowing consumption goods from existing cohorts in prior-to-entry markets and repaying outstanding debts from every period's capital income (dividends from Lucas trees) until death. These arguments suggest that the optimal policy is concerned primarily *not* with the intertemporal transfer from cohorts with higher endowments to cohorts with lower endowments, *but* with the intratemporal risk sharing between endowment receivers (entry cohorts) and capital-income earners (existing cohorts).

Our analytical focus contrasts sharply with the existing papers in several respects. Many papers, including Gordon and Varian (1988), Bohn (1998), Shiller (1999), DeMange (2002), Krueger and Kubler (2003), and Ball and Mankiw (2007), have explored the intergenerational risk-sharing issue using stochastic overlapping generations models with incomplete markets. However, in contrast to our focus on permanent labor-income shocks faced by young cohorts, these authors were interested in the sharing of capital-income risks faced by old cohorts.[3] In addition, their focus was on mechanisms of redistribution from young to old consumers, such as pay-as-you-go social security systems, as opposed to our focus on income transfers from old to newly born generations. Finally, some of the above works considered only a two-period lifetime horizon for every cohort in order to limit financial transactions between generations, whereas our setup employs a multiperiod lifetime horizon to allow active transactions among generations.

The study by Campbell and Nosbusch (2007) is similar to our investigation in that these authors explored intergenerational risk sharing between labor and capital income in a multiperiod overlapping-generations model. However, they did not focus on the welfare loss associated with cohort-specific shocks and intertemporal substitution. Smetters's (2004) study is most closely related to our study, as he also analyzed the effect of incompleteness arising from the inability of the unborn to trade financial claims, and considered taxes (subsidies) on capital income to fix such incompleteness. He employed a two-period overlapping-generations model with endogenous capital accumulation and showed that the sign and magnitude of optimal capital income taxation depend crucially on the contemporaneous correlation between labor and capital income. On the other hand, as mentioned above, we focus on the intergenerational sharing of cohort-specific shocks on permanent income within an exchange economy with multiperiod overlapping generations. Thanks to a simple endowment structure, our model yields a clear prediction concerning an optimal policy to recover first-best outcomes.

Our paper is organized as follows. Section 2 presents our theoretical framework. In Section 3, we report on numerical exercises conducted under various assumptions regarding structural parameters. In Section 4, we derive the optimal subsidy/tax policy that results in a first-best allocation and attempt to restore such an allocation by opening up prior-to-entry markets. Section 5 offers concluding remarks.

2. THEORETICAL FRAMEWORK

We adopt Huffman (1987) as a baseline model to investigate the intergenerational sharing of cohort-specific permanent shocks through capital market transactions. This section describes the basic framework in detail.

2.1. Basic Setup

We consider an overlapping-generations economy. Identical consumers enter the economy at time t and live until time $t + N - 1$. The population of each cohort is constant over time and normalized to one. A consumer aged zero at time t is endowed with y_t (> 0) units of perishable consumption goods and receives no endowments afterward. Here, y_t follows a first-order Markov process. There is no insurance market for this entry shock on y_t. As discussed in the Introduction, this uninsured shock on y_t is regarded as the entry shock on capitalized labor income or permanent income. As explained below, consumers use a portion of their initial endowments to obtain parts of Lucas trees, thereby allocating consumption goods over N periods.

There exist fixed K units of nondepreciable physical capital or Lucas trees. In period t, consumers sell or buy physical capital at a market price P_t, measured in terms of consumption goods. When consumers hold physical capital at the beginning of period t, they receive d_t (> 0) units of perishable consumption goods per unit of capital as dividends. Again, d_t follows a first-order Markov process. As long as $N \geq 3$, the transaction of physical capital takes place among generations.

We assume that both endowments y_t and dividends d_t are generated by a four-state Markov chain z_t with state space $Z = \{z_{HH}, z_{HL}, z_{LH}, z_{LL}\}$, where the first (second) lower subscript denotes the realization of y_t (d_t). To be specific, income and dividend shocks are represented by

$$y_t = \begin{cases} (1+\epsilon_y)\bar{y} & \text{if } z_t \in \{z_{HH}, z_{HL}\} \\ (1-\epsilon_y)\bar{y} & \text{if } z_t \in \{z_{LH}, z_{LL}\} \end{cases}, \tag{1}$$

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$$d_t = \begin{cases} (1+\epsilon_d)\bar{d} & \text{if } z_t \in \{z_{HH}, z_{LH}\} \\ (1-\epsilon_d)\bar{d} & \text{if } z_t \in \{z_{HL}, z_{LL}\} \end{cases}, \tag{2}$$

where $\bar{y}$ and $\bar{d}$ are the average values and ϵ_y and ϵ_d represent the volatility of endowments and dividends.

Following Krueger and Kubler (2003), we characterize the transition matrix of z_t by

$$(1-\delta)\Pi + \delta I,$$

where I is a four-dimensional identity matrix, and $0 \leq \delta < 1$. Each row of Π corresponds to the stationary distribution of each state $(\pi_{HH}, \pi_{HL}, \pi_{LH}, \pi_{LL})$. If $\delta = 0$, then y_t and d_t are sequentially independent. An increase in δ makes shocks more serially correlated. The stationary distribution Π is assumed to be symmetric: $\pi_{HH} = \pi_{LL}$ and $\pi_{HL} = \pi_{LH}$. If $\pi_{HH} > (<)\ 0.25$, then y_t and d_t are positively (negatively) correlated with each other. When $\pi_{HH} = 0.5\ (0.0)$, y_t and d_t are perfectly positively (negatively) correlated with each other.

Let $c_{j,t+j}$ and $x_{j,t+j}$ be the consumption goods and capital holdings at the age of j of an agent born in t. Owing to the absence of bequest motives, agents exhaust capital at death. A consumer maximizes a utility function characterized by Kreps–Porteus preferences.[4] The agent's utility function at the age of $j\ (< N)$, denoted by $U_{j,t+j}$, is recursively defined as follows:

$$U_{j,t+j} = \left\{ c_{j,t+j}^{\frac{\sigma-1}{\sigma}} + \beta \left[E_{t+j} U_{j+1,t+j+1}{}^{1-\gamma} \right]^{\frac{\sigma-1}{\sigma(1-\gamma)}} \right\}^{\frac{\sigma}{\sigma-1}}$$

and $U_{N,t+N} = 0\ (\forall\, t)$, where E_{t+j} is the expectation operator conditional on information available at time $t+j$. $\beta\ (> 0)$ is a discount factor, $\sigma (> 0, \sigma \neq 1)$ is the elasticity of intertemporal substitution, and $\gamma\ (> 0, \gamma \neq 1)$ is the degree of relative risk aversion.

If $\sigma\gamma = 1$, then the above specification reduces to a constant relative risk-aversion preference. The case where $\sigma = \gamma = 1$ (a logarithmic preference) is the same as in Huffman (1987).

A consumer born at t chooses a plan of $\{c_{j,t+j}, x_{j,t+j}\}_{j=0}^{N-1}$ to maximize $U_{0,t}$ subject to

$$c_{0,t} + P_t x_{0,t} = y_t,$$

$$c_{j,t+j} + P_{t+j} x_{j,t+j} = \left(P_{t+j} + d_{t+j}\right) x_{j-1,t+j-1}, \text{ if } j = 1, \ldots, N-1,$$

$$x_{j,t+j} > 0, \text{ for } j = 0, \ldots, N-2, \text{ and } x_{N,t+N-1} = 0.$$

Let us define a competitive equilibrium for the above framework. There are two competitive markets (consumption goods and physical capital) in terms of the cross-sectional allocation. A competitive equilibrium at time t consists of consumers' optimal plans $\{c_{j,t}, x_{j,t}\}_{j=0}^{N-1}$ and an asset price P_t such that all markets are cleared: $\sum_{j=0}^{N-1} c_{j,t} = y_t + d_t K$ for the consumption goods market, and $\sum_{j=0}^{N-2} x_{j,t} = K$ for the physical capital market at time t. By Walras's law, if the capital market is cleared, then the consumption goods market is cleared.

As z_t follows a first-order Markov process, the optimal decision rules and the equilibrium price function can be represented by $c_{j,t} = c_j(\mathbf{x}_{t-1}, z_t)$, $x_{j,t} = x_j(\mathbf{x}_{t-1}, z_t)$, and $P_t = P(\mathbf{x}_{t-1}, z_t)$, where $\mathbf{x}_{t-1} \equiv (x_{0,t-1}, \ldots, x_{N-1,t-1})$ is a one-period lagged distribution of Lucas trees among generations. As shown later, the equilibrium process of capital prices is indeed influenced by the evolving cross-generational capital distribution. Substituting the optimal rules and the equilibrium price function into the lifetime utility function, we obtain the indirect lifetime utility of a consumer born at time t $[V(\mathbf{x}_{t-1}, z_t)]$.

2.2. Methods for Evaluating Risk-Sharing Performance

To evaluate the intergenerational allocation of cohort-specific permanent shocks, we adopt both unconditional and conditional welfare measures based on the lifetime utility. The conditional expected lifetime utility is the welfare evaluated after the realization of cohort-specific permanent shocks, whereas the unconditional expected lifetime utility is the welfare evaluated before their realization.

We make a welfare comparison under various combinations of structural parameters including σ and γ. For this purpose, we convert the absolute level of welfare into the certainty equivalence consumption (hereafter, the CEQ consumption) for the conditional and unconditional lifetime welfare. The CEQ consumption level $\bar{c}$ is computed such that the evaluated welfare may be equal to $U_0 \equiv \{(1 + \beta + \cdots + \beta^{N-1})\bar{c}^{\frac{\sigma-1}{\sigma}}\}^{\frac{\sigma}{\sigma-1}}$.

In addition, we report the following ratio of the CEQ consumption of a market allocation relative to that of a governmental allocation in which a social planner distributes the entire endowment available at time t according to age-specific weights:

$$\bar{c}_j(z_t) = \frac{\beta^j}{1 + \beta + \cdots + \beta^{N-1}}(y_t + d_t K).$$

We refer to the above intergenerational allocation as a *simple sharing rule*. As proved in Appendix A.1, the allocation delivered by this rule corresponds to the first-best allocation where a planner treats all generations with equal weights if (i) a preference is logarithmic ($\sigma = \gamma = 1$)[5] or (ii) a preference is time-additive ($\sigma\gamma = 1$) with $\beta = 1$.[6]

On the other hand, the above rule may not result in the first-best outcome in the case where Kreps–Porteus preferences apply because, unlike the case of time-additive preferences, marginal period utility depends not only on current consumption, but also on future consumption.[7] Accordingly, the welfare loss implied by the CEQ consumption ratio is underestimated by using the simple sharing rule as a reference point. Nevertheless, we evaluate the welfare loss based on the simple sharing rule for the following reasons. First, the allocation delivered by this rule always yields substantially higher welfare than does the market allocation, as shown in the numerical examples of the next section. Consequently, the extent to which the cohort-specific shock is shared effectively may be inferred from how the unconditional CEQ consumption of the market allocation is short of that of the simple sharing rule.

Second, as discussed in the Introduction, the separation of the welfare impact of intertemporal substitution from that of risk aversion is critically important in the context of an overlapping-generations setup. Only Kreps–Porteus preferences allow for this kind of thought experiment. We are particularly interested in the welfare comparison between the cohorts with high intertemporal substitution and those with low intertemporal substitution given a fixed risk aversion parameter. The effect of overall underestimation may be minimized in such a welfare comparison among those with different preference parameters.

Third, in the case of Kreps–Porteus preferences, optimal decision rules are not available without numerical computation. Thus, the welfare evaluation depends on the accuracy of numerical procedures. However, as examined in detail in Appendix A.3, the numerical computation errors have little effect on the comparison of welfare losses in this case.

2.3. Properties of the Logarithmic Preference Case

Optimal decision rules. In this section, we discuss several important properties concerning the equilibrium allocation for a logarithmic preference ($\sigma = \gamma = 1$). As shown in Huffman (1987), a closed form is available for optimal decision rules and equilibrium pricing in this case. The optimal consumption/saving decision rule is derived as follows:

$$c_{0,t} = \psi_0 y_t, \;\; x_{0,t} = (1-\psi_0)\frac{y_t}{P_t}, \tag{3}$$

and

$$c_{j,t+j} = \psi_j (P_{t+j} + d_{t+j}) x_{j-1,t+j-1}, \;\; x_{j,t+j} = (1-\psi_j)\frac{P_{t+j}+d_{t+j}}{P_{t+j}} x_{j-1,t+j-1}, \tag{4}$$

where $\psi_j = \frac{1}{1+\beta+\cdots+\beta^{N-1-j}}$ for $j = 0, 1, \ldots, N-2$ and $\psi_{N-1} = 1$. These optimal decision rules are called "myopic" in the sense that the rules depend only on the current state variables.[8] The consumption rule is rewritten as

$$c_{j,t+j} = \frac{\beta^j y_t}{1+\beta+\cdots+\beta^{N-1}} \prod_{i=1}^{j} \frac{P_{t+i}+d_{t+i}}{P_{t+i-1}}, \quad j = 1, \ldots, N-1. \tag{5}$$

The above optimal decision rule sharply contrasts with those of a standard overlapping-generations setup where labor income is yielded every period. In

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the current setup, zero-year-old consumers earn no dividends, and the other consumers receive no endowments. Only Lucas trees are traded among generations. As equation (4) implies, consumers consume a portion of capital every period, but always carry positive assets before they exhaust their assets at death. That is, consumers do not consume beyond their remaining capitalized labor income. On the other hand, when insurance markets are missing in a standard overlapping-generations setup, financial assets may serve as buffer stocks for negative income shocks, and transactions in risk-free assets may facilitate self-insurance for income fluctuations.[9]

Negative correlation in asset pricing. Given the capital-market-clearing condition, or $K = \sum_{i=0}^{N-2} x_{i,t}$, the equilibrium asset price $[P(\mathbf{x}_{t-1}, z_t)]$ is derived as

$$P(\mathbf{x}_{t-1}, z_t) = \frac{(1-\psi_0)y_t + d_t\xi(\mathbf{x}_{t-1})}{K - \xi(\mathbf{x}_{t-1})}, \tag{6}$$

where $\mathbf{x}_{t-1} = (x_{0,t-1}, \ldots, x_{N-2,t-1})$ and $\xi(\mathbf{x}_{t-1}) = \sum_{j=1}^{N-2}(1-\psi_j)x_{j-1,t-1}$. Note that $\xi(\mathbf{x}_{t-1}) < K$ always holds.

As equation (6) implies, $P(\mathbf{x}_{t-1}, z_t)$ depends on the realization of endowments (y_t), dividends (d_t), and the one-period lagged cross-generational capital distribution $\mathbf{x}_{t-1}$. $P(\mathbf{x}_{t-1}, z_t)$ is increasing in both y_t and d_t. The volatility of $P(\mathbf{x}_{t-1}, z_t)$ becomes higher when the volatility of y_t or d_t (ϵ_y or ϵ_d) rises, or when y_t and d_t are more positively correlated. Among the four possible states given $\mathbf{x}_{t-1}$, $P(\mathbf{x}_{t-1}, z_{HH})$ is the highest, whereas $P(\mathbf{x}_{t-1}, z_{LL})$ is the lowest. It is possible to prove that $P(\mathbf{x}_{t-1}, z_{HL}) > P(\mathbf{x}_{t-1}, z_{LH})$, if and only if $(1-\psi_0)\epsilon_y\bar{y} > \epsilon_d\bar{d}\xi(\mathbf{x}_{t-1})$. If the cross-generational capital distribution is skewed more toward younger generations, then the equilibrium price is higher.[10]

Due to the fact that equilibrium pricing is influenced by the cross-generational capital distribution $\mathbf{x}_{t-1}$, a negative serial correlation in asset prices emerges even when there is no serial correlation in either y_t or d_t. That is, a lower realization of d_t, leading to a decrease in the current asset price $P(\mathbf{x}_{t-1}, z_t)$, favors the entry generation over the older generations; the entry generation can purchase capital at a lower price than can the older generations. An increase in the saving of the entry generation yields stronger subsequent demand for capital, thereby raising asset prices in the next period. In this way, the asset prices become higher after they decrease due to a lower realization of d_t.[11]

The above asset-pricing behavior generates interesting properties of risk sharing among generations. Below, we demonstrate that the entry generation with low y_t may share risks with the entry generation with high y_t. Suppose that y_t and d_t are perfectly positively correlated, and there is no serial correlation in either y_t or d_t. Consequently, only z_{HH} and z_{LL} emerge. Given the optimal consumption/saving rule [equation (5)], the unconditional indirect utility

$V(\mathbf{x}_{t-1})\left[=\sum_{j=0}^{N-1}\beta^j \ln(c_{j,t+j})\right]$ is expressed as

$$V(\mathbf{x}_{t-1}) = \psi_0 \sum_{j=0}^{N-1} \beta^j \ln(\beta^j) + (1+\beta+\cdots+\beta^{N-1})\ln(y_t)$$
$$+ (\beta+\cdots+\beta^{N-1})\ln\frac{P_{t+1}+d_{t+1}}{P_t} + \cdots + \beta^{N-1}\ln\frac{P_{t+N-1}+d_{t+N-1}}{P_{t+N-2}}.$$

In computing the conditional expectations of indirect utility $V(\mathbf{x}_{t-1}, z_t)$, y_{t+j} and d_{t+j} can be replaced by their averages $\bar{y}$ and $\bar{d}$ because there is no serial correlation in endowments or dividends. It is assumed that the conditional average of P_{t+j} is constant over time. Thus, $V(\mathbf{x}_{t-1}, z_t)$ can be approximated by

$$V(\mathbf{x}_{t-1}, z_t) \approx \text{constant} + (1+\beta+\cdots+\beta^{N-1})\ln(y_t) - (\beta+\cdots+\beta^{N-1})\ln P_t.$$

If N is sufficiently large, and β is close to one, then $V(\mathbf{x}_{t-1}, z_t)$ is approximated by constant $N \ln \frac{y_t}{P_t}$.

Given the above approximation, it is possible to prove that if $\epsilon_y < \epsilon_d$, then $V(\mathbf{x}_{t-1}, z_{LL}) > V(\mathbf{x}_{t-1}, z_{HH})$.[12] That is, when the volatility of cohort-specific endowment shocks is small relative to that of aggregate shocks on dividends, the entry generation with low y_t can attain higher welfare than the entry generation with high y_t.

A major reason for the above welfare consequence of the market allocation is that, in a recession state (z_{LL}), the entry generation suffers from a low level of human capital, but its cohort members can purchase physical capital at cheap prices as a consequence of the low realization of dividends. In a boom state (z_{HH}), on the other hand, the entry generation enjoys a high level of human capital but is forced to purchase costly physical capital for future consumption. Given a relatively large ϵ_d, asset pricing is volatile, and P_t is low enough for the entry generation to purchase a large amount of capital during a recession period. Hence, a negative shock on permanent income borne initially by the entry cohort would be passed over to older generations (asset holders).

Equilibrium consumption profiles. One caveat to the above argument is that although the welfare conditional on entry is similar among all cohorts when ϵ_y is close to ϵ_d, the corresponding welfare remains short of that of the first-best outcome. In relation to this point, we make a brief remark on the equilibrium age profile of consumption. A gross return on capital, defined as $\frac{P_{t+1}+d_{t+1}}{P_t}$, is equal to

$$\frac{P_{t+1}+d_{t+1}}{P_t} = \frac{(1-\psi_0)y_{t+1} + d_{t+1}K}{(1-\psi_0)y_t + d_t\xi(\mathbf{x}_{t-1})},$$

whereas its average is

$$\frac{\bar{P}+\bar{d}}{\bar{P}} = 1 + \frac{1-\xi(\bar{\mathbf{x}})/K}{(1-\psi_0)\bar{y}/\bar{d}K + \xi(\bar{\mathbf{x}})/K}.$$

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The above equation implies that the average return on capital is greater than one. Thus, given zero time preferences ($\beta = 1$), the consumption profile is upward sloping on average in the market allocation [see equation (5)], whereas it is flat on average in the first-best allocation, which can be attained by the simple sharing rule. Hence, the degree of welfare loss is associated with the extent to which the consumption profile is upward sloping at market allocations.

3. QUANTITATIVE PROPERTIES OF MARKET ALLOCATION

Using the theoretical framework presented in the preceding section, this section investigates quantitative properties of the market allocation. We choose structural parameters based on the macroeconomic performance and labor market of the Japanese economy.

3.1. Parameter Settings

It is assumed that one period corresponds to two years, and that a consumer lives for 30 periods ($N = 30$) or 60 years. The amount of physical capital is standardized at $K = 100$.

The average dividend ratio $\bar{d}$ is set at the average ratio of aggregate capital income relative to physical capital. As reported in Hayashi and Prescott (2004), $\bar{d} = 0.099$ for the period between 1974 and 2000. Based on a method proposed by Tauchen (1986),[13] the two-state Markov process of d_t is approximated as

$$\begin{bmatrix} 0.89 & 0.11 \\ 0.11 & 0.89 \end{bmatrix}$$

at the annual frequency. Given the above approximated transition probability, ϵ_d is approximated to be 0.1411. Accordingly, d_t takes a value of either 0.08503 or 0.11297. In addition, the serial correlation coefficient in d_t is computed as $\delta = 0.608$, when one period is two years.[14]

In our framework, the capitalized labor income is endowed at the beginning of a lifetime. Thus, the average y_t/K can be regarded as the average of aggregate labor income relative to physical capital. As reported by Hayashi and Prescott (2004), the average of y_t ($\bar{y}$) is 34.9 for the period between 1974 and 2000. For simplicity, we assume that a cohort-specific permanent shock is perfectly positively correlated with dividend shocks; π_{HH} is equal to 0.5, and only z_{HH} and z_{LL} emerge. Hereafter, we call such a cohort-specific shock *procyclical* in the sense that dividends and labor endowments move in accordance with the total endowment.

The volatility of y_t (ϵ_y) is based on a finding of Genda (1997). Genda investigated the business-cycle-related changes in the wage profile between the ages of 25–29 and 40–44 for male university graduates. Controlling for aggregate effects on wages,[15] he found that the wage profile is steeper (flatter) for workers who graduated during booms (recessions). As a concrete number, we pick the slope (the 15-year wage growth) of the entry year 1963 (57%) as that of a boom period, and the slope of the entry year 1965 (52%) as that of a recession period.

We make the following assumptions to pin down the magnitude of business-cycle-related cohort shocks (ϵ_y). First, the entry wage level is identical for all cohorts. Second, the wage profile is steep up to the age of 49 and then flat up to the retirement age of 64.[16] Third, the applied discount rate is equal to the growth rate of overall wages.[17] Thus, the difference in lifetime income between the boom entry and the recession entry amounts to 5.24%.[18] Accordingly, the volatility of y_t is equal to $\epsilon_y = 0.0262$, and y_t takes a value of either 33.986 or 35.814. As discussed in the preceding section, given that ϵ_y ($= 0.0262$) is smaller than ϵ_d ($= 0.1411$), a cohort with low endowments is likely to be better off than a cohort with high endowments.

For numerical exercises of overlapping-generations models, a time preference β is often chosen such that the predicted life-cycle profile of asset accumulation is consistent with the observed profile among households. In our framework, however, such a life-cycle aspect is removed completely. Thus, we set $\beta = 1$ for simplicity.

This assumption about time preference yields reasonable predictions under logarithmic preference. First, as implied by the properties of this model (see Section 2.3), the individual consumption profile becomes upward; the average annual growth rate of consumption is 5.07%,[19] and it is comparable to the observed growth between the ages of 25 and 64.[20] Second, the economywide consumption inequality amounts to 0.1612 in terms of the standard deviation of logarithmic consumption. Ohtake and Saito (1998) reported that the consumption inequality reached 0.4911 in 1989 using the same measure. It follows that around one-third of the economywide consumption inequality can be explained by business-cycle-related cohort-specific shocks on labor income. Idiosyncratic shocks and population shocks, both of which are out of our consideration, may be responsible for another two-thirds of consumption inequality.

For a numerical procedure, we employ the algorithm proposed by Krusell and Smith (1998). The detailed procedure is discussed in Appendix A.2. In addition, Appendix A.3 carefully explores the accuracy of the approximated law of motion for asset pricing, partly because it is always one of the most essential issues in any numerical study on economies with heterogeneous agents, and partly because the welfare evaluation depends critically on how accurately the law of motion for asset pricing is approximated.

3.2. Numerical Results

We begin with the logarithmic preference case. As the first column (perfectly positive contemporaneous correlation between y_t and d_t, or $\pi_{HH} = 0.5$) of Table 1 shows, the unconditional CEQ consumption is 1.477, and the ratio relative to that of the first-best allocation is 99.0%. That is, the presence of uninsured cohort-specific

TABLE 1. Unconditional and conditional CEQ consumption with logarithmic preferences under different contemporaneous correlation coefficients between endowments and dividends

	Contemporaneous correlation coefficient between y and d		
	$+1.0$ $(\pi_{HH} = 0.5)$	0.0 $(\pi_{HH} = 0.25)$	-1.0 $(\pi_{HH} = 0.0)$
Unconditional CEQ (CEQ ratio)	1.477 (0.990)	1.477 (0.989)	1.477 (0.989)
CEQ conditional on high y and high d	1.452	1.452	—
CEQ conditional on high y and low d	—	1.524	1.523
CEQ conditional on low y and high d	—	1.431	1.432
CEQ conditional on low y and low d	1.503	1.503	—

shocks reduces welfare by 1.0% in terms of the CEQ consumption. Such a relative welfare loss is not at all negligible.

As suggested in the preceding section, given that ϵ_y (= 0.0262) is smaller than ϵ_d (= 0.1411) in our setup, the CEQ consumption conditional on z_{LL} (1.503) is greater than that conditional on z_{HH} (1.452) because the entry generation with low endowments can purchase physical capital at low prices. The procyclical movement of asset pricing favors the cohort with low endowments over the cohort with high endowments. As discussed in the preceding section, the serial correlation in asset pricing tends to decline in relation to the original dividend process. In our baseline case where $\delta = 0.608$, the serial correlation in asset pricing is 0.588. If δ is zero, then it falls to -0.016.

In the case with a weaker or negative contemporaneous correlation between endowments and dividends ($\pi_{HH} = 0.25$ and 0.0 in Table 1), the movement of asset pricing does not contribute to the risk sharing between the low-endowment and high-endowment entry cohorts. The CEQ consumption conditional on z_{LH} is much lower than that conditional on z_{HL} in the absence of procyclical asset pricing. Nevertheless, the unconditional CEQ consumption is higher in the cases where $\pi_{HH} = 0.25$ or 0.0 than in the case where $\pi_{HH} = 0.5$. A possible reason for this is that, in the latter case, the price movement excessively favors the entry cohort with low dividends, as ϵ_d is much larger than ϵ_y in our numerical setup.[21]

Next, we examine the impact of both intertemporal substitution (σ) and relative risk aversion (γ) on the CEQ ratio (the ratio of the unconditional CEQ consumption relative to that of the approximated first-best allocation) and asset pricing. As shown in Table 2, the unconditional CEQ consumption ratio is decreasing in σ given γ. For example, when $\gamma = 0.2$, the unconditional ratio decreases from 99.8% to 97.0% as σ increases from 0.2 to 5.0. The resulting welfare loss is serious when σ is large.

Table 3 shows that the average price of physical capital is more expensive as σ becomes larger. The reason for this is that stronger intertemporal motives

TABLE 2. Unconditional CEQ consumption ratios under various combinations of relative risk aversion and elasticity of intertemporal substitution

	$\gamma = 0.2$	$\gamma = 1.0$	$\gamma = 1.25$	$\gamma = 5.0$
$\sigma = 0.2$	0.998	0.993	0.992	0.994
$\sigma = 0.8$	0.992	0.991	0.992	0.990
$\sigma = 1.0$	—	0.990	—	—
$\sigma = 1.25$	0.987	0.987	0.987	0.987
$\sigma = 5.0$	0.970	0.970	0.970	0.970

TABLE 3. Averages and standard errors of logarithmic asset prices

	$\gamma = 0.2$	$\gamma = 1.0$	$\gamma = 1.25$	$\gamma = 5.0$
$\sigma = 0.2$	1.716 (0.140)	1.717 (0.141)	1.717 (0.141)	1.722 (0.141)
	1.848 (0.045)	1.850 (0.046)	1.850 (0.046)	1.855 (0.046)
	1.581 (0.039)	1.582 (0.040)	1.582 (0.039)	1.587 (0.040)
$\sigma = 0.8$	1.767 (0.058)	1.767 (0.058)	1.767 (0.058)	1.767 (0.058)
	1.824 (0.008)	1.824 (0.008)	1.824 (0.008)	1.824 (0.008)
	1.709 (0.008)	1.709 (0.008)	1.709 (0.008)	1.709 (0.007)
$\sigma = 1.0$	—	1.782 (0.050)	—	—
	—	1.831 (0.005)	—	—
	—	1.732 (0.005)	—	—
$\sigma = 1.25$	1.807 (0.041)	1.807 (0.041)	1.807 (0.041)	1.807 (0.041)
	1.848 (0.003)	1.848 (0.003)	1.847 (0.003)	1.847 (0.003)
	1.766 (0.003)	1.766 (0.003)	1.766 (0.003)	1.765 (0.003)
$\sigma = 5.0$	1.999 (0.018)	1.999 (0.018)	1.999 (0.018)	1.998 (0.018)
	2.017 (0.003)	2.016 (0.003)	2.016 (0.003)	2.015 (0.003)
	1.982 (0.003)	1.981 (0.003)	1.981 (0.003)	1.980 (0.003)

Note: (i) The numbers in parentheses are the standard errors of logarithmic asset prices. (ii) In each cell, the number in the top row corresponds to the unconditional average, whereas the numbers in the middle and bottom rows correspond to the averages conditional on high y and high d, and those on low y and low d, respectively.

promote the postponement of consumption and boost asset demand. Thus, it costs more to employ physical capital as a risk-sharing instrument. Accordingly, when σ is larger, the merit of procyclical asset pricing becomes smaller for the entry generation with low endowments; as shown in Table 4, an increase in σ narrows the difference in the conditional CEQ ratio between z_{HH} and z_{LL}.

On the other hand, risk-averse behavior would generate two opposite effects on asset pricing. First, high risk aversion promotes precautionary savings and accordingly has a positive impact on asset pricing. Second, risk-averse investors require premiums on risk assets and discount asset pricing heavily. According to Tables 2–4, when σ is less than one (weaker intertemporal motives), the former effect is dominant. That is, as γ increases, the average asset price becomes more

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TABLE 4. Conditional CEQ consumption ratios under various combinations of relative risk aversion and elasticity of intertemporal substitution

	$\gamma = 0.2$	$\gamma = 1.0$	$\gamma = 1.25$	$\gamma = 5.0$
$\sigma = 0.2$	0.898	0.897	0.897	0.893
	1.101	1.101	1.100	1.095
$\sigma = 0.8$	0.963	0.963	0.963	0.962
	1.021	1.021	1.021	1.020
$\sigma = 1.0$	—	0.968	—	—
	—	1.012	—	—
$\sigma = 1.25$	0.974	0.974	0.974	0.974
	1.001	1.001	1.001	1.001
$\sigma = 5.0$	0.977	0.977	0.977	0.978
	0.962	0.963	0.963	0.963

Note: In each cell, the number in the top row corresponds to the CEQ consumption ratio conditional on high y and high d, whereas the number in the bottom row corresponds to the CEQ consumption ratio conditional on low y and low d.

TABLE 5. Effects of the contraction of trading opportunities ($N = 30$ versus $N = 6$)

	$\gamma = 0.2$	$\gamma = 1.0$	$\gamma = 1.25$	$\gamma = 5.0$
$\sigma = 0.2$	0.998 ⇒ 0.996	0.993 ⇒ 0.992	0.992 ⇒ 0.991	0.994 ⇒ 0.989
$\sigma = 0.8$	0.992 ⇒ 0.989	0.991 ⇒ 0.989	0.992 ⇒ 0.989	0.990 ⇒ 0.988
$\sigma = 1.0$	—	0.990 ⇒ 0.987	—	—
$\sigma = 1.25$	0.987 ⇒ 0.984	0.987 ⇒ 0.984	0.987 ⇒ 0.984	0.987 ⇒ 0.984
$\sigma = 5.0$	0.970 ⇒ 0.965	0.970 ⇒ 0.965	0.970 ⇒ 0.965	0.970 ⇒ 0.966

Note: In each cell, the number on the left-hand side (right-hand side) corresponds to the unconditional CEQ consumption ratio in the case of $N = 30$ ($N = 6$).

expensive, and the unconditional and conditional CEQ ratios decrease slightly. When σ is larger than one, the former effect is almost canceled out by the latter, and the risk aversion coefficients do not have significant effects on either asset pricing or the unconditional CEQ ratio.

Now, we evaluate the effect of the contraction of trading opportunities. In contrast to the previous setup ($N = 30$), a 60-year lifetime is divided into six periods ($N = 6$).[22] Table 5 compares the unconditional CEQ ratios between the two cases. As this table demonstrates, when intertemporal substitution is large, the contraction of trading opportunities reduces the CEQ ratio to some extent. For example, when $\gamma = 0.2$ and $\sigma = 5.0$, the CEQ ratio decreases from 97.0% to 96.5%. In other words, the expansion of trading opportunities favors those with strong intertemporal saving motives.

In summary, the level and movement of asset pricing play key roles in sharing cohort-specific shocks between cohorts with high endowments and cohorts with low endowments. That is, the intergenerational risk sharing through financial transactions is effective to the extent that physical capital as a risk-sharing instrument is cheaply available. In other words, the capital market allocation is less efficient in the presence of strong demand for physical assets. In addition, the expansion of trading opportunities favors young cohorts with stronger incentives to postpone consumption.

4. ON THE OPTIMAL POLICY INTERVENTION AND THE INTRODUCTION OF PRIOR-TO-ENTRY MARKETS

As documented in the preceding section, the capital market transactions among generations contribute to the sharing of cohort-specific permanent shocks. However, the shocks remain partially uninsured, particularly under a large elasticity of intertemporal substitution and limited transaction opportunities, because physical capital as a risk-sharing instrument is quite costly in these cases. Therefore, there may be an opportunity for a government to directly intervene in the intergenerational allocation.

The current section demonstrates that the optimal allocation may be attained by pooling resources between endowment receivers (entry cohorts) and capital-income earners (existing cohorts). Concretely, when preference is logarithmic, the first-best allocation is attainable through a subsidy to the entry generation financed by a 100% levy on dividend income. In addition, this section examines whether the first-best allocation can be achieved by opening up prior-to-entry markets.

4.1. Optimal Policy Intervention

Suppose that a government provides a transfer to the youngest (entry) generation. To finance this transfer, the government levies taxes on the dividends of the other generations at a rate τ. In this case, we can obtain the optimal rule of consumption and saving by replacing y_t and d_t with $\tilde{y}_t = y_t + \tau d_t K$ and $\tilde{d}_t = (1-\tau)d_t$.

Given a 100% levy on dividends ($\tau = 1$), the following optimal consumption and saving rules of age j at period t are derived from equations (3) and (4):

$$c_{0,t} = \psi_0(y_t + d_t K), \quad x_{0,t} = (1-\psi_0)\frac{y_t + d_t K}{P_t},$$

$$c_{j,t} = P_t \psi_j x_{j-1,t-1}, \quad x_{j,t} = (1-\psi_j)x_{j-1,t-1}, \quad j = 1, \ldots, N-2.$$

From equation (6), we obtain

$$P_t = (1-\psi_0)\frac{y_t + d_t K}{K - \xi(\mathbf{x}_{t-1})},$$

where $\xi(\mathbf{x}_{t-1}) = \sum_{j=1}^{N-2}(1-\psi_j)x_{j-1,t-1}$.[23]

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Substituting the above pricing equation into $x_{0,t} = (1 - \psi_0)\frac{y_t + d_t K}{P_t}$ yields $x_{0,t} = K - \xi(x_{t-1})$. As $x_{j,t} = (1 - \psi_j)x_{j-1,t-1}$ for $j \geq 1$, the cross-generational capital distribution $\mathbf{x}_t = (x_{0,t}, \ldots, x_{j-1,t})$ is independent of y_t and d_t, and depends only on $\mathbf{x}_{t-1}$. Hence, the capital distribution is constant over time. We denote the time-invariant capital distribution by $\bar{\mathbf{x}} = (\bar{x}_0, \ldots, \bar{x}_{N-2})$. Thus, $c_{j,t} = P_t \psi_j \bar{x}_{j-1}$ for $j \geq 1$. Note that the consumption profile, as well as the asset pricing, is proportional to the aggregate outcome $y_t + d_t K$. We can show that $\bar{x}_{j-1} = \beta^j \frac{1}{\psi_j} \frac{\psi_0}{1-\psi_0}[K - \xi(\bar{\mathbf{x}})]$.[24] Accordingly, we obtain

$$c_{j,t} = \psi_0 \beta^j (y_t + d_t K), \quad j = 0, \ldots, N-1.$$

The above consumption allocation corresponds to the allocation under the optimal sharing rule (the first-best outcome).

By conducting intensive numerical calculations, we confirmed that the above subsidy to the entry generation financed by a 100% levy on dividends would almost yield the first-best allocation even if preference were not logarithmic. In the case where $\sigma = \gamma = 5.0$, for example, the unconditional CEQ consumption ratio is 97.0% without any policy intervention, but it reaches 99.7% under the above combination of subsidies and taxes.

4.2. Opening up Prior-to-Entry Markets

Given the above optimal policy, cohort-specific shocks and dividend shocks are pooled completely, and all generations are exposed only to aggregate risks (proportional to $y_t + d_t K$) through the movement of asset prices P_t. The next question is whether the first-best allocation can be achieved by introducing another market.

Here, we introduce a prior-to-entry market into the current setup. To imitate the resource allocation delivered by the optimal policy combination of subsidies and taxes, we suppose that the entry cohort borrows an amount equivalent to the aggregate capital income in the above prior-to-entry market and repays its outstanding debts using the entire capital income earned at every age until death. Is this borrowing contract arbitrage-free under the first-best allocation?

As discussed in the preceding section, $c_{j,t} = \psi_0 \beta^j (y_t + d_t K)$ holds at the first-best allocation and, accordingly, a stochastic discount factor between time t and $t + j$ $(M_{t,t+j})$ can be characterized as $\beta^j \frac{c_{0,j}}{c_{j,t+j}}$. Then, for $M_{t,t+j}$, we obtain

$$M_{t,t+j} = \frac{y_t + d_t K}{y_{t+j} + d_{t+j} K}.$$

As discussed before, the cross-generational capital distribution is constant over time under the first-best allocation. A capital holding at age j is defined as $\bar{x}_j$. Thus, for the above borrowing contract, the following arbitrage condition should be satisfied prior to entry:

$$E_{t-1}[d_t K] = E_{t-1}[M_{t,t+1} d_{t+1} \bar{x}_0 + M_{t,t+2} d_{t+2} \bar{x}_1 + \cdots + M_{t,t+N-1} d_{t+N-1} \bar{x}_{N-2}].$$

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With some manipulation, the above condition is rewritten as

$$E_{t-1}\left[\frac{d_t K}{y_t + d_t K}\right]$$
$$=E_{t-1}\left[\frac{d_{t+1}}{y_{t+1}+d_{t+1}K}\bar{x}_0+\frac{d_{t+2}}{y_{t+2}+d_{t+2}K}\bar{x}_1+\cdots+\frac{d_{t+N-1}}{y_{t+N-1}+d_{t+N-1}K}\bar{x}_{N-2}\right].$$

As mentioned above, the capital distribution $\mathbf{x}_t = (x_{0,t}, \ldots, x_{j-1,t})$ is fixed over time in a first-best allocation. In addition, we have $\sum_{j=0}^{N-2}\bar{x}_j = K$ from market-clearing conditions. Thus, the above equality implies that if the ratio of dividends to entire endowments, $\frac{d_t}{y_t+d_t K}$, is sequentially independent ($\delta = 0$), and its expectation is constant, then the arbitrage condition holds.[25]

Given that $\delta = 0$, the first-best allocation is attainable in a decentralized manner as long as the entry cohort can arrange the above type of borrowing contract in the prior-to-entry market. The preceding argument suggests that the allocation inefficiency arising from Huffman's (1987) setup is indeed a consequence of the inability of the entry cohort to make short positions at the prior-to-entry market. Viewed from a different angle, the inefficiency associated with the capital market allocation arises *not* from the failure of an intertemporal transfer from a cohort with high endowments to one with low endowments, *but* from insufficient intratemporal risk sharing between receivers of labor endowments (entry cohorts) and capital-income earners (existing cohorts).

5. CONCLUDING REMARKS

This paper evaluates the intergenerational sharing of a procyclical cohort-specific shock on permanent income through capital market transactions, given the absence of prior-to-entry markets. The level and movement of asset pricing play a key role in sharing those shocks. The market allocation is effective to the extent that physical capital is cheaply available as a risk-sharing instrument. Conversely, the capital market fails to share the cohort-specific permanent shocks effectively in the presence of strong capital demand for intertemporal reasons.

Our numerical investigation shows that, given a reasonable set of parameters concerning the Japanese labor market, the market incompleteness with respect to the cohort-specific permanent shock would result in welfare losses of between 1% and 3% in terms of the CEQ consumption level. Such inefficiency in the capital market transactions is not a consequence of a failure of risk sharing between a cohort with high endowments and a cohort with low endowments. The risk sharing between these cohorts may be achieved at least partially by the movement of asset prices as long as cohort-specific shocks are procyclical.

As a possible optimal policy, we demonstrate that a subsidy to the entry generation financed by a 100% levy on dividends from Lucas trees would yield the first-best outcome as a consequence of the effective risk pooling of labor

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and capital income. In addition, the first-best outcome may be attained in a decentralized manner when the entry cohort can make short positions at the prior-to-entry market.

Given these implications from our theoretical and numerical investigation, an optimal policy response to uninsured cohort-specific shocks on permanent income should reflect *not* the intertemporal transfer from high-endowment cohorts to low-endowment cohorts, *but* the intratemporal risk sharing between labor-endowment receivers (entry cohorts) and capital-income earners (existing cohorts).

One of the theoretical limitations of our setup is that capital supply is exogenous and fixed. If this assumption is relaxed, as in Bohn (1998), Krueger and Kubler (2003), and Smetters (2004), then capital accumulation or decumulation may serve as an additional hedge device for cohort-specific permanent shocks. We leave this extension to future research.

NOTES

1. Von Wachter and Bender (2006) argued that cohort effects are not permanent, but temporary. Allowing for job mobility among firms and employees, they found that an early job displacement as well as graduation during a recession have only transitory wage effects in Germany.

2. Kahn (2007) showed that a theory of task-specific human capital [Gibbons and Waldman (2006)] was consistent with her empirical finding about cohort effects on wages.

3. In focusing on the intergenerational sharing of capital-income risks, some of the above authors assumed that human capital, yielding labor income, is a riskless asset. This assumption may be justified when labor income has a small cohort-specific risk and a weak correlation with capital income [Campbell et al. (2001)]. However, Benzoni et al. (2007) discovered that labor and capital income are cointegrated, and that the relationship between the two magnifies labor-income risk in the long run. Consequently, cohort-specific shocks on permanent labor income can no longer be regarded as negligible.

4. See Kreps and Porteus (1978) and Epstein and Zin (1989).

5. As the next section shows, in case (i), the marginal propensity to consume out of wealth (MPCW) in the simple sharing rule exactly corresponds to the MPCW of the optimal consumption decision rule in the market equilibrium. This aspect guarantees the simple sharing rule as the optimal policy under logarithmic preferences regardless of β.

6. In case (ii), marginal period utility depends only on current consumption and is independent of future consumption. When β is equal to one in time-additive preferences, the individual weight over period utility is exactly equal to the planner's weight over each lifetime utility. Thanks to these features, the simple sharing rule generates the first-best outcome under time-additive preferences with $\beta = 1$.

7. For exactly the same reason, it is extremely difficult to compute first-best outcomes even numerically, particularly in the presence of aggregate risks.

8. The main reason for this property of the optimal decision rules is that logarithmic preferences balance wealth and substitution effects so that a decision maker may ignore any effect of future uncertainty.

9. If a risk-free assets market is introduced into the current setup, transactions of risk-free assets may emerge among cohorts. That is, some cohorts may have short positions in risk-free assets. However, as Constantinides and Duffie (1996) and others have suggested, it may be hard to self-insure against permanent shocks on lifetime income using noncontingent claims (risk-free assets). In this regard, risk-free assets may not play an active role in self-insuring against permanent cohort-specific shocks. As shown in Section 4, on the other hand, once a prior-to-entry market opens, an entry cohort may issue a particular type of contingent claim for a risk-sharing purpose.

10. Note that $1 - \psi_j$ is decreasing in age j.

11. It is difficult to directly relate this theoretical property of asset pricing to any empirical observations of actual financial markets, but it may be possible to interpret it as a potential source of mean reversion in stock prices. Using financial data on market economies, Poterba and Summers (1988) and others found that declines in stock prices tend to be followed by increases in stock prices at a relatively low frequency (longer than one year).

12. From equation (6), we obtain $y_t/P_t = \frac{K-\xi(\mathbf{x}_{t-1})}{(1-\psi_0)+\xi(\mathbf{x}_{t-1})d_t/y_t}$ and

$$E(V_t|z_{LL}) > E(V_t|z_{HH}) \Leftrightarrow \frac{(1-\epsilon_y)\bar{y}}{P(\cdot, z_{LL})} > \frac{(1+\epsilon_y)\bar{y}}{P(\cdot, z_{HH})} \Leftrightarrow \frac{(1-\epsilon_y)\bar{y}}{(1-\epsilon_d)\bar{d}} > \frac{(1+\epsilon_y)\bar{y}}{(1+\epsilon_d)\bar{d}}.$$

Then, we establish $\epsilon_y < \epsilon_d$ as the condition under which $E(V_t|z_{LL}) > E(V_t|z_{HH})$ holds.

13. Tauchen (1986) proposed a method for approximating a transition probability matrix of a finite number of discrete state variables from the first-order autoregression model with normally distributed errors.

14. Concretely, δ is chosen such that, after the transition probability matrix is raised to the second power, the first element 0.804 may be equal to $0.5(1-\delta)+\delta$.

15. Genda (1997) controlled aggregate shocks by subtracting the growth of the average wage of male university graduate workers.

16. Such a pattern of wage profiles is observed for the monthly cash earnings of male university graduates in Table 2 from the Ministry of Labour and Welfare (2004).

17. This assumption itself is rather heroic, but for our purposes, it is not so audacious as it looks. Here, we are interested not in calculating the present value of lifetime income, but in computing a difference in the present value between boom cohorts and recession cohorts. A misspecified discount rate may have similar impacts on the computation of lifetime income of both cohorts; consequently, the computed difference between the two cohorts is relatively free of the impact of misspecification of discount rates.

18. More concretely, the annual wage X_t begins with X_0, grows at the annual rate of μ for 20 years (between the ages of 25–29 and 45–49), and then becomes constant for 15 years (up to the ages of 60–64). That is,

$$X_t = \begin{cases} X_0 \exp(\mu t) & t \in [0, 20), \\ X_0 \exp(20\mu) & t \in [20, 35]. \end{cases}$$

In this case, the lifetime income is derived as

$$X_0 \left[\int_0^{20} \exp(\mu t) dt + 15 \exp(20\mu) \right] = \frac{X_0}{\mu} [\exp(20\mu) - 1 + 15\mu \exp(20\mu)].$$

Suppose that μ_a is the annual growth rate of a boom cohort, whereas μ_b is that of a recession cohort ($\mu_a > \mu_b$). A difference in logarithmic lifetime income between the two cohorts reduces to

$$\ln \left[\frac{\mu_b}{\mu_a} \frac{\exp(\mu_a x) - 1 + \mu_a y \exp(\mu_a x)}{\exp(\mu_b x) - 1 + \mu_b y \exp(\mu_b x)} \right].$$

According to Genda (1997), the estimated 15-year wage growth was 57% (52%) for the boom (recession) cohort. Substituting $\mu_a = 0.57/15$ and $\mu_b = 0.52/15$ into the above equation, the logarithmic difference is equal to 5.24%.

19. This result suggests that the implied risk-free rate is about 5% per year under logarithmic preferences with $\beta = 1$. As in other asset-pricing models, the current model is subject to the overestimation of risk-free rates or the so-called risk-free rate puzzle.

20. The computed growth rate of the expenditure per household is based on the data grouped according to the age of household heads in Table 6 from the Ministry of Internal Affairs and Communications (2000).

21. We numerically confirm that the unconditional CEQ consumption is highest in the case where $\pi_{HH} = 0.5$ when $\epsilon_y = \epsilon_d$.

22. Given a lifetime horizon, an increase (decrease) in N expands (contracts) the opportunity for intergenerational financial transactions as dividend shocks arrive with higher (lower) frequency.

23. Given a 100% levy on dividends, the value of Lucas trees is not backed by dividends as income rises. However, older cohorts can sell their Lucas trees to younger cohorts. Consequently, Lucas trees are valued by principals in such intergenerational asset transactions.

24. Using $\bar{x}_0 = K - \xi(\bar{\mathbf{x}})$, $\bar{x}_j = (1-\psi_j)\bar{x}_{j-1}$, and $1-\psi_{j-1} = \frac{\beta\psi_{j-1}}{\psi_j}$, we obtain

$$\bar{x}_{j-1} = (1-\psi_{j-1})(1-\psi_{j-2})\cdots(1-\psi_1)[K-\xi(\bar{\mathbf{x}})] = \beta^{j-1}\frac{\psi_1}{\psi_j}[K-\xi(\bar{\mathbf{x}})] = \beta^j\frac{1}{\psi_j}\frac{\psi_0}{1-\psi_0}[K-\xi(\bar{\mathbf{x}})].$$

The last equality is established by $\psi_1 = \frac{\beta\psi_0}{1-\psi_0}$.

25. Without $\delta = 0$, the arbitrage condition no longer holds. Appendix A.4 proves that, if $\epsilon_d > \epsilon_y$ as in our numerical setting, and $0 < \delta < 1$, then

$$E_{t-1}\left(\frac{d_t K}{y_t + d_t K}\right) < E_{t-1}\left(\frac{d_{t+1}}{y_t + d_t K}\bar{x}_0 + \cdots + \frac{d_{t+N-1}}{y_t + d_t K}\bar{x}_{N-2}\right) \text{ when } z_{t-1} = z_{HH},\ z_{HL},$$

$$E_{t-1}\left(\frac{d_t K}{y_t + d_t K}\right) > E_{t-1}\left(\frac{d_{t+1}}{y_t + d_t K}\bar{x}_0 + \cdots + \frac{d_{t+N-1}}{y_t + d_t K}\bar{x}_{N-2}\right) \text{ when } z_{t-1} = z_{LH},\ z_{LL}.$$

Thus, the arbitrage condition does not hold. When δ is greater than zero, a more sophisticated investment strategy in the prior-to-entry market may deliver first-best outcomes. However, we cannot find such a strategy in an explicit form.

26. We set the lower bound of x at a small but positive number because, except at the time of death, the equilibrium asset holding never reaches zero to avoid positive infinity of marginal period utility.

27. We adopt Newton's method and a grid search method for optimization procedures. Because there is not any noticeable difference in numerical results between the two methods, we report the results based on Newton's method.

28. Table A.1 reports the minimum R^2 among four possible cases, where z_t is either z_{HH} or z_{LL}, and z_{t+1} is either z_{HH} or z_{LL}, because there is no noticeable difference in R^2 among the four cases.

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APPENDIX

A.1. PROOF OF THE OPTIMALITY OF A SIMPLE SHARING RULE

In this appendix, we prove that when a social planner treats each cohort equally, a simple-sharing-rule allocation, $c_{j,t} = \beta^j (y_t + d_t K)/(1 + \beta + \cdots + \beta^{N-1})$ for $j = 0, 1, \ldots, N - 1$, is the first-best allocation if a preference is logarithmic, or if it is time-additive ($\sigma\gamma = 1$) with $\beta = 1$.

We obtain the first-best allocation by solving a social planner's problem as of time 0. Let $U_{0,t}$ be the lifetime utility function of a consumer born at time t. When $\sigma\gamma = 1$, $U_{0,t}$ is represented as $U_{0,t} = \sum_{j=0}^{N-1} \beta^j u(c_{j,t+j})$ for $t \geq 0$, and $U_{0,t} = \sum_{j=-t}^{N-1} \beta^j u(c_{j,t+j})$ for $-1 + N \leq t < 0$, where $u(c)$ is either $\ln(c)$ if $\gamma = 1$, or $c^{1-\gamma}/(1 - \gamma)$ if $\gamma \neq 1$. The social planner maximizes the welfare function

$$E_0 \left[\sum_{t=-N+1}^{\infty} \lambda_t U_{0,t} \right],$$

subject to the feasibility constraint $\sum_{i=0}^{N-1} c_{i,t} = y_t + d_t K$ for each period t, where $\lambda_t > 0$ is a welfare weight.

The first-best consumption plan is characterized by a set of first-order conditions, or

$$\mu_t = \lambda_{t-j} \beta^j u'(c_{j,t}),$$

for $j = 0, \ldots, N - 1$, where μ_t is the Lagrange multiplier associated with the feasibility condition at period t.

We assume that the planner's weight on the period t entry generation (λ_t) is identical among all generations. Thus,

$$u'(c_{0,t}) = \beta u'(c_{1,t}) = \cdots = \beta^{N-1} u'(c_{N-1,t}).$$

The allocation $c_{j,t} = \beta^j (y_t + d_t K)/(1 + \beta + \cdots + \beta^{N-1})$ for $j = 0, 1, \ldots, N - 1$ satisfies the above condition if a preference is logarithmic, or if it is time-additive ($\sigma\gamma = 1$) with $\beta = 1$.

A.2. NUMERICAL PROCEDURES

In this appendix, we briefly describe the numerical procedure adopted in our paper. We basically follow the algorithm proposed by Krusell and Smith (1998). In this economy, the optimal consumption rule in period t depends on the exogenous state variables z_t as well as the asset pricing P_t; P_t may summarize the information of the one-period lagged cross-generational distribution of capital holdings $\mathbf{x}_{t-1} = (x_{0,t-1}, \ldots, x_{N-2,t-1})$. We assume that the decision rules of consumers at age j in period t depend on z_t, P_t, and their own capital holding $x_{j,t-1}$. In addition, we assume that all consumers predict the price of Lucas trees in period $t + 1$ using the forecasting rule

$$\ln P(z_{t+1}) = b_0 (z_t, z_{t+1}) + b_1 (z_t, z_{t+1}) \ln P(z_t), \quad \textbf{(A.1)}$$

where coefficients b_0 and b_1 depend on the current and future states z_t and z_{t+1}.

The Krusell–Smith algorithm proceeds as follows.

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(1) Make a grid of points on both individual capital holdings x and capital price P for each state of $z \in \{z_{HH}, z_{HL}, z_{LH}, z_{LL}\}$. We make 50 grid points in x and 100 grid points in P at equal intervals. The lower bound of x is set at 0.01,[26] whereas its upper bound is set at 40 for $N = 6$, and 20 for $N = 30$. For each value of N, the lower (upper) bound of P is half (1.5 times) as large as the asset price that is computed under the assumption of logarithmic preferences.

(2) Choose the case of a logarithmic preference without any uncertainty as the initial value of the capital distribution ($\{x_{j,0}\}_{j=0}^{N-1}$) and the forecasting rule of capital pricing (A.1).

(3) Given the forecasting rule, solve the maximization problem of each age group by a backward induction at each point in the grid.[27] Starting from $\hat{V}_N = 0$, $\hat{V}_j$ is computed based on the following Bellman equation:

$$\hat{V}_j(x, z, P) = \max_{x'} \left\{ f(x, x', z, P) + \beta \left[E\hat{V}_{j+1}(x', z', P')^{1-\gamma} \right]^{\frac{\sigma-1}{\sigma(1-\gamma)}} \right\}^{\frac{\sigma}{\sigma-1}},$$

where

$$f(x, x', z, P) = \begin{cases} [y(z) - Px']^{\frac{\sigma-1}{\sigma}} & \text{if} \quad j = 0 \\ [Px + d(z)x - Px']^{\frac{\sigma-1}{\sigma}} & \text{otherwise.} \end{cases}$$

$y(z)$ and $d(z)$ are determined by equations (1) and (2). The forecasting rule sets one-period-ahead asset prices P'. The value of $\hat{V}_{j+1}$ for any point other than the grid points is computed by the two-dimensional piecewise linear interpolation. In this way, the decision rule of the age j cohort $[x_{j,t} = \hat{g}_j(x_{j-1,t-1}, z_t, P_t)]$ is approximated.

(4) Generate exogenous state variables $\{z_t\}$ for 41,000 periods. Given the initial capital distribution $\{x_{j,0}\}_{j=0}^{N-1}$ and the forecasting rule of capital prices, compute $\{\{x_{j,t}\}_{j=0}^{N-1}, P_t\}_{t=1}^{41,000}$ using the approximated decision function $x_{j,t} = \hat{g}_j(x_{j-1,t-1}, z_t, P_t)$, and the market-clearing condition $\sum_{j=0}^{N-1} x_{j,t} = K$. Given P_t found by the bisection method, $\{x_{j,t}\}_{j=0}^{N-1}$ and $c_{j,t}$ can be calculated.

(5) Discard the first 1,000 observations of the above simulated sample. Update the prediction rule by regressing P_{t+1} on P_t for a given (z_t, z_{t+1}).

(6) Repeat steps 3–5 until the forecasting rule converges.

A.3. ACCURACY OF THE APPROXIMATED LAW OF MOTION FOR ASSET PRICING

As mentioned in Appendix A.2, the forecast of logarithmic asset prices of Lucas trees is based on equation (A.1). The underlying presumption of this forecasting rule is that the current price of Lucas trees summarizes well the information concerning the infinite-dimensional cross-sectional capital distribution.

In general, the accuracy of the approximated law of motion is one of the most essential issues in any numerical study on economies with heterogeneous agents. Particularly in our context, the welfare evaluation depends critically on how accurately the law of motion is approximated. That is, if the law of motion is approximated inaccurately, the resulting decision rules are likely to be less optimal. When the welfare evaluation is based on the market allocation that is generated by such less optimal decision rules, welfare losses may be overestimated seriously.

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TABLE A.1. R^2-based accuracy tests for the law of motion of asset pricing

	Relative risk aversion (γ)			
EIS (σ)	0.2	1.0	1.25	5.0
0.2	0.951	0.950	0.950	0.947
0.8	0.960	0.960	0.960	0.960
1.25	0.935	0.935	0.935	0.947
5.0	0.989	0.989	0.989	0.989

Note: The reported R^2 corresponds to the minimum R^2 among four possible cases, where z_t is either z_{HH} or z_{LL}, and z_{t+1} is either z_{HH} or z_{LL}. There is not any noticeable difference in R^2 among the four cases.

In the literature, the accuracy of the approximated law of motion is usually evaluated by the fitness of one-period-ahead forecasting by equation (A.1), R-squared (R^2) is most often adopted for this purpose. On the other hand, den Haan (2007) pointed out several drawbacks of R^2-based tests and proposed an alternative accuracy test. In particular, he pointed out that R^2-based tests are based on the in-sample fitness using actual observations as explanatory variables. To overcome this weakness, a test proposed by den Haan (2007) compares the observed series in equilibrium with the simulated series based *only* on the approximated law of motion [equation (A.1)] without any update, given the initial asset price and the time series of realized shocks. The comparison is made in terms of the maximum and average of the difference between the observed and simulated values.

Table A.1 reports the results of R^2-based tests for our numerical exercises with various combinations of preference parameters.[28] R-squared ranges between 0.935 and 0.989. These levels of R-squared are a little lower than those of Krusell and Smith (1998), who studied the infinite-horizon economy with heterogeneous agents. A possible reason for this difference is that the degree of heterogeneity is more serious owing to cross-sectional differences in the remaining time horizon in our overlapping generations setup.

Using the accuracy test proposed by den Haan (2007), Table A.2 reports the results for numerical exercises with the degree of relative risk aversion (γ) fixed at one. In any case, the maximum absolute error is three or four times as large as the average absolute error. However, the two measures exhibit similar patterns; the error is decreasing substantially with the elasticity of intertemporal substitution (σ).

As mentioned above, our primary concern is that welfare losses may be overestimated as a result of inaccurately approximated laws of motion. However, this is not the case at all in our numerical exercises. As Table 2 reports, when γ is equal to one, the estimated welfare loss increases with σ; for example, it is 0.7% (1 − 0.993) for $\sigma = 0.2$, but it is 3.0% (1 − 0.970) for $\sigma = 5.0$. On the other hand, as reported in Tables A.1 and A.2, both R^2-based and den Haan accuracy tests indicate that the approximated law of motion is most accurate when $\sigma = 5.0$. Therefore, it is hard to imagine that the evaluated welfare loss is influenced heavily by the inaccuracy of the approximated law of motion.

TABLE A.2. Den Haan's (2007) accuracy tests for the law of motion of asset pricing

	$N = 30$		$N = 20$	
EIS (σ)	Maximum absolute error	Average absolute error	Maximum absolute error	Average absolute error
0.2	0.0593	0.0157	0.0610	0.0160
0.8	0.0122	0.0035	0.0110	0.0034
1.25	0.0054	0.0017	0.0052	0.0016
5.0	0.0022	0.0007	0.0022	0.0007

Notes: (i) The degree of relative risk aversion (γ) is set at one. (ii) The absolute error is defined as the absolute difference in logarithmic asset prices between the observed price and the simulated price.

A.4. PROOF OF THE ARBITRAGE CONDITION IN FOOTNOTE 5

Let $Q \equiv (1-\delta)\Pi + \delta I$, where $0 \le \delta < 1$, $\Pi = (\pi\ \pi\ \pi\ \pi)'$, $\pi = (\pi_{HH}, \pi_{HL}, \pi_{LH}, \pi_{LL})'$, and I is a four-dimensional identity matrix. Then

$$\begin{aligned} Q^n &= \{(1-\delta)\Pi + \delta I\}^n = (1-\delta)^n \Pi^n + \cdots + (1-\delta)\delta^{n-1}\Pi^n + \delta^n I \\ &= \Pi + \delta^n (I - \Pi). \end{aligned}$$

Let $a_t = d_t/(y_t + d_t)$, and $Ea_t = \pi_{HH}a_{HH} + \pi_{HL}a_{HL} + \pi_{LH}a_{LH} + \pi_{LL}a_{LL} = \bar{a}$. Then we have

$$\begin{aligned} E_{t-1}a_{t+i} &= (1-\delta^{i+1})\bar{a} + \delta^{i+1}a_{t-1}, \\ E_{t-1}a_t K &= (1-\delta)\bar{a}K + \delta a_{t-1}K = \bar{a}K - \delta(a_{t-1} - \bar{a})K, \\ E_{t-1}(a_{t+1}\bar{x}_0 + \cdots + a_{t+N-1}\bar{x}_{N-2}) &= \bar{a}K - (a_{t-1} - \bar{a})(\delta^2\bar{x}_0 + \cdots + \delta^N \bar{x}_{N-2}). \end{aligned}$$

If $\delta = 0$, $E_{t-1}a_t K = E_{t-1}(a_{t+1}\bar{x}_0 + \cdots + a_{t+N-1}\bar{x}_{N-2})$, and otherwise,

$$E_{t-1}a_t K - E_{t-1}(a_{t+1}\bar{x}_0 + \cdots + a_{t+N-1}\bar{x}_{N-2}) = (a_{t-1} - \bar{a})(\delta^2\bar{x}_0 + \cdots + \delta^N \bar{x}_{N-2} - \delta K).$$

As $\sum x_i = K$ and $0 < \delta < 1$, the term $(\delta^2\bar{x}_0 + \cdots + \delta^N \bar{x}_{N-2} - \delta K)$ is always negative. If $\delta = 0$, $E_{t-1}a_t K = E_{t-1}(a_{t+1}\bar{x}_0 + \cdots + a_{t+N-1}\bar{x}_{N-2})$, and otherwise,

$$\begin{aligned} E_{t-1}a_t K &> E_{t-1}(a_{t+1}\bar{x}_0 + \cdots + a_{t+N-1}\bar{x}_{N-2}) \quad \text{if } a_{t-1} < \bar{a}, \\ E_{t-1}a_t K &< E_{t-1}(a_{t+1}\bar{x}_0 + \cdots + a_{t+N-1}\bar{x}_{N-2}) \quad \text{if } a_{t-1} > \bar{a}. \end{aligned}$$

Recall that $a_{t-1} = d_{t-1}/(y_{t-1} + d_{t-1}K)$ holds. If $\delta = 0$, then

$$E_{t-1}\left(\frac{d_t K}{y_t + d_t K}\right) = E_{t-1}\left(\frac{d_{t+1}}{y_t + d_t K}\bar{x}_0 + \cdots + \frac{d_{t+N-1}}{y_t + d_t K}\bar{x}_{N-2}\right),$$

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and if $0 < \delta < 1$, then

$$E_{t-1}\left(\frac{d_t K}{y_t + d_t K}\right)$$
$$> E_{t-1}\left(\frac{d_{t+1}}{y_t + d_t K}\bar{x}_0 + \cdots + \frac{d_{t+N-1}}{y_t + d_t K}\bar{x}_{N-2}\right) \text{ if } \frac{d_{t-1}K}{y_{t-1} + d_{t-1}K} < \frac{\bar{d}K}{\bar{y} + \bar{d}K},$$
$$E_{t-1}\left(\frac{d_t K}{y_t + d_t K}\right)$$
$$< E_{t-1}\left(\frac{d_{t+1}}{y_t + d_t K}\bar{x}_0 + \cdots + \frac{d_{t+N-1}}{y_t + d_t K}\bar{x}_{N-2}\right) \text{ if } \frac{d_{t-1}K}{y_{t-1} + d_{t-1}K} > \frac{\bar{d}K}{\bar{y} + \bar{d}K}.$$

In the case where $\epsilon_d > \epsilon_y$,

(1) If $z_{t-1} = z_{HH}$, $d_{t-1}K/(y_{t-1} + d_{t-1}K) = \bar{d}K/(\frac{1+\epsilon_y}{1+\epsilon_d}\bar{y} + \bar{d}K) > \bar{d}K/(\bar{y} + \bar{d}K)$.
(2) If $z_{t-1} = z_{HL}$, $d_{t-1}K/(y_{t-1} + d_{t-1}K) = \bar{d}K/(\frac{1+\epsilon_y}{1-\epsilon_d}\bar{y} + \bar{d}K) > \bar{d}K/(\bar{y} + \bar{d}K)$.
(3) If $z_{t-1} = z_{LH}$, $d_{t-1}K/(y_{t-1} + d_{t-1}K) = \bar{d}K/(\frac{1-\epsilon_y}{1+\epsilon_d}\bar{y} + \bar{d}K) < \bar{d}K/(\bar{y} + \bar{d}K)$.
(4) If $z_{t-1} = z_{LL}$, $d_{t-1}K/(y_{t-1} + d_{t-1}K) = \bar{d}K/(\frac{1-\epsilon_y}{1-\epsilon_d}\bar{y} + \bar{d}K) < \bar{d}K/(\bar{y} + \bar{d}K)$.

Hence, we obtain

$$E_{t-1}\left(\frac{d_t K}{y_t + d_t K}\right) < E_{t-1}\left(\frac{d_{t+1}}{y_t + d_t K}\bar{x}_0 + \cdots + \frac{d_{t+N-1}}{y_t + d_t K}\bar{x}_{N-2}\right) \quad \text{if } z_{t-1} = z_{HH},\ z_{HL},$$
$$E_{t-1}\left(\frac{d_t K}{y_t + d_t K}\right) > E_{t-1}\left(\frac{d_{t+1}}{y_t + d_t K}\bar{x}_0 + \cdots + \frac{d_{t+N-1}}{y_t + d_t K}\bar{x}_{N-2}\right) \quad \text{if } z_{t-1} = z_{LH},\ z_{LL}.$$