

For its intended readership – undergraduates just with a background in real analysis – this book contains within it a very readable fully fleshed-out treatment of the usual components of a first course in complex analysis with each new concept lucidly explained and contextualised. It is clear which sections are more appropriate to a second reading, indeed some of the more reflective portions which touch on history and pedagogy will also be of interest to those teaching such a course. My one reservation concerns the significantly high number of typos: many are just annoying, but there are also a number of unsettling ones, particularly in the worked examples.

The complex analysis course should be one of the highlights of every undergraduate mathematics course with aspects appealing to a wide spectrum of interests. The first edition of Stewart and Tall's book went through 13 reprintings in 35 years and I see no reason why this substantially revised second edition should not enjoy similar success.

#### References

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**Understanding topology, a practical introduction** by Shaun V. Ault, pp. 416, £74.00 (hard), ISBN 978-1-42142-407-1, Johns Hopkins University Press (2018).

This is a textbook aimed at undergraduate students reading a first course (module) in topology. Needless to say, the book covers much more material. The main chapters and their sub-chapters are: I. Euclidean topology (Introduction to topology, metric topology in Euclidean spaces, vector fields in the plane), II. Abstract topology with applications (Abstract point-set topology, Surfaces, Applications to graphs and knots), III. Basic algebraic topology (The fundamental group, introduction to homology). In addition there are two appendices: Review of set theory and functions, and Group theory and linear algebra.

Students in such a course will have read (linear) algebra and analysis (including in higher dimensions) and they will have some confidence in working with sets and mappings, although everyone teaching a first course in topology or measure theory knows that whenever it comes to infinite operations there are still problems.

The author does not use the word mapping, and since he also considers maps when dealing with the map colouring problem, all mappings are functions since they can not be maps—maybe not a helpful approach.

It is natural to start with Euclidean topology and revise basic concepts from higher dimensional analysis (metric, open and closed sets, compact sets, connected sets, continuous mappings, uniform convergence) as well as revising some central results such as the Bolzano-Weierstraß theorem, the Heine-Borel theorem, variants of the intermediate value theorem, images of compact sets under continuous mapping, etc. For me, 80 pages dealing mainly with these elementary results is too much, in particular when essentially no attempt is made to extend notions and results. When turning to vector fields the author faces some problems since he does not want to

assume some material from a course on differential equations. Thus he tries to provide these results in the text, and this is problematical. As a student I would not understand his way of introducing a flow or the way in which a differential equation corresponds to a vector field. A lot of the discussions in this chapter are vague, and many notions are introduced whose relevance for dynamics are not really discussed. Although needed, line integrals are not properly introduced. Eventually the index or winding number of a closed curve is (formally) introduced, Hopf's theorem is stated but not proved ... From my point of view less would have been more.

I am also not happy that the author often introduces non-standard notations. Examples are separation for splitting, or radially convex for star-shaped, but there are quite a few more. At least the more standard names should have been mentioned. Some formulations are questionable: "Now in order to apply topological reasoning in a vector field..." or "If  $C$  is any simple closed curve in a vector field...", just to mention some.

We come to the second main chapter, and eventually on page 120 the notion of a topology is introduced! The second chapter should be the core of the first half part of such a module and it should contain the main notions and results from "general topology". Many standard notions and results we do find, but too many are missing. We find a base of a topology, but no base of neighbourhoods and how to construct a topology given a base of neighbourhoods; the Hausdorff separation axiom is introduced but I could not find any further separation axiom as I could not find first and second countable spaces. Compactness is not treated to the extent it deserves, for example students must learn the existence of infinite dimensional spaces in which the bounded and closed sets are compact, e.g. Montel spaces, or different types of compactness, for example needed to prove the existence of Urysohn functions or a partition of unity. The subchapter on surfaces goes in the right direction. Indeed, topology is not only 'general topology' with its impact on analysis, it is a very geometric theory, and linking topology to geometry and discussing the role of topological invariants is important. The idea of triangulations is discussed in an appealing way, if not always rigorously, and the existence of a triangulation is stated but not proved. But we are seeing the author moving into areas he seems to appreciate more and this is reflected in the book. It is nice to see graphs treated within topology and in fact considered as tools in investigating topological spaces from a more general point of view. The same applies in principle for knots, but maybe here less material being treated in more detail would have been more to the benefit of the students. The problem here is that more connections to other mathematical subject areas are needed. Moreover, why, for example, do we need to introduce Jones polynomials when we are not doing anything with them?

I like very much the chapter 'Basic algebraic topology', and more precisely the introduction of the fundamental group and the discussion of its importance. Moreover, the author is giving examples and calculations often missing in other books (at least as detailed as here). Homology theory is maybe treated a bit short—more ideas and examples could be helpful. Given their importance in analysis, cohomology groups (at least de Rham cohomology groups) deserve some mention.

In summary, I have mixed feelings. I am not convinced by the first part ("general topology"), but when combining topology with geometry, the author provides an appealing treatment.

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