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A semantic characterisation of the correctness of a proof net

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The purpose of this note is to show that the correctness of a multiplicative proof net with mix is equivalent to its semantic correctness: a proof structure is a proof net if and only if its semantic interpretation is a clique, where one given finite coherence space interprets all propositional variables.

This is just an example of what can be done with these kinds of semantic techniques; for more information and further results, the reader is referred to Retoré (1994).

1. Presentation and warning

This note demonstrates the use of coherence semantics – more precisely, of the so-called *experiment method* of Girard (1987) – for the analysis of proof structures and proof nets. We limit our study to the cut-free, constant-free multiplicative fragment of linear logic enriched with the *mix* rule.

This allows our note to be self-contained, but we must warn the reader that the definitions we give are adapted to the particular case under consideration, and consequently, would be incomplete in the general setting. In the case of the formal definition of a proof structure or net, this is just a question of precision, but for the semantics of a proof structure, one should be aware that in the non-cut-free case, the set of all the results of the experiments has to be restricted to the set of the results of the *successful* experiments. For simplicity we will speak about 'proof structures (or nets)' instead of 'constant-free and cut-free multiplicative proof structure (or net) with *mix*' and of 'experiments' instead of 'successful experiments' – as the two notions agree in this restricted case.

We give here a semantic, but nevertheless algorithmic criterion for a proof structure to be a proof net. Notice that, despite this, the result applies to non-cut-free proofs, since, as far as *correctness* is concerned, a cut between A and $A^{\perp}$ may be viewed as a `tensor` rule or link between premises A and $A^{\perp}$ and conclusion $A \otimes A^{\perp}$.

In Retoré (1994) we extended these methods in two directions. First, dealing directly with non-cut-free proof structures allows a semantic characterisation of another interesting property of proof structures or proof expressions, which is called acyclicity in Abramsky (1993), and which corresponds to the deadlock freeness notion of Lafont (1990) when the considered interaction nets are multiplicative proof structures. Second, we extended the characterisations of correctness and deadlock freeness to the pomset calculus of Retoré (1993) and Retoré (1997); in the latter case, the proofs are harder, but they entail the simpler result exposed here.

2. Reminder

2.1. *Language and sequent calculus for cut-free and constant-free multiplicative linear logic with the* mix *rule*

The multiplicative formulae are generated from a set of propositional variables $\mathscr{P} = \{a, b, \ldots\}$ by the linear negation $(\_)^{\perp}$ and two binary connectives `par` $(\_)\wp(\_)$ and `tensor` $(\_)\otimes(\_)$. Let us call this set of formulae $\mathscr{F}$.

As this calculus allows, we shall consider formulae up to the De Morgan laws, namely $(A^{\perp})^{\perp}$, $(A\otimes B)^{\perp} \equiv A^{\perp}\wp B^{\perp}$ and $(A\wp B)^{\perp} \equiv A^{\perp}\otimes B^{\perp}$. Thus the formulae that we consider may be defined as $\mathscr{M} ::= \mathscr{P} \mid \mathscr{P}^{\perp} \mid \mathscr{M}\wp\mathscr{M} \mid \mathscr{M}\otimes\mathscr{M}$.

Given a formula F in $\mathscr{M}$, the notation $F^{\perp}$ is a shorthand for the unique formula F' of $\mathscr{M}$ such that $F' \equiv F^{\perp}$ where $F^{\perp}$ is a formula of $\mathscr{F}$.

The cut-free multiplicative sequent calculus is defined by the following rules:

$$\frac{}{\vdash a, a^{\perp}}ax \qquad \frac{\vdash \Gamma, A, B}{\vdash \Gamma, B, A}exch \qquad \frac{\vdash \Gamma \quad \vdash \Delta}{\vdash \Gamma, \Delta}mix \qquad \frac{\vdash \Gamma, A, B}{\vdash \Gamma, A\wp B}par \qquad \frac{\vdash \Gamma, A \quad \vdash \Delta, B}{\vdash \Gamma, \Delta, A\otimes B}ts$$

Notice that the axiom is restricted to the formulae in $\mathscr{P}$, as the η-expansion property allows.

2.2. *Cut-free and constant-free multiplicative proof structures and nets with* mix

We deal here with cut-free constant-free multiplicative proof nets (Girard 1987) where the logic is enriched with the *mix* rule (Fleury and Retoré 1994). We simply call them *proof structures and proof nets.*

We use a characterisation *à la* Danos–Regnier (Danos and Regnier 1989; Troelstra 1992; Girard 1995), where proof structures are graphs. More precisely, we use the following definition.

Definition 1. A cut-free *proof structure* with conclusions $C_1, \ldots, C_n$ is a graph whose vertices are occurrences of formulae that consist of:

— the subformula trees of $C_1, \ldots, C_n$ – the syntactic forest of the sequent $\vdash C_1, \ldots, C_n$

— a set of pairwise disjoint edges $a \overline{\quad} a^{\perp}$ called axioms covering the pendant vertices (or leaves) of the syntactic forest.

A '`par`-link' of a proof structure is a full subgraph of it consisting of three vertices A, B and $A\wp B$, where $A\wp B$ is a subformula of some C_i. The vertices A and B are said to be the premises of the `par`-*link*, and the vertex $A\wp B$ is said to be the conclusion of the `par`-link.

The definition of a '`tensor`-link' is obtained by replacing `par` with `tensor` and $\wp$ with $\otimes$ in the definition of a `par`-link,

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A link is said to be *final* whenever $A * B$ is some C_i, that is, a root of the syntactic forest. A *splitting* `tensor`-*link* is a final `tensor`-link, each edge of which is an isthmus of the proof structure.

Following Fleury and Retoré (1994), we use the following definition.

Definition 2. A *feasible* path of a proof structure is a path that does not contain the two edges of the same `par`-link. A proof structure is a *proof net* whenever there is no feasible cycle – we equivalently say, in this case, that the proof structure is *correct*.

As shown in Fleury and Retoré (1994), these proof nets correspond exactly to the proofs of the sequent calculus that we gave. This entails the following useful proposition – directly established in Retoré (1996).

Proposition 3. A proof net always contains a final `par`-link or a splitting `tensor`-link, unless it consists of a family of axioms.

2.3. *Coherence semantics*

We recall here a few definitions – more details can be found in Girard (1995), Girard *et al.* (1988), Troelstra (1992) and Retoré (1994).

Definition 4. A *coherence space* U is a non-directed graph:

— the set of vertices or *tokens* is called the *web* and is written $|U|$;
— adjacency, called *strict coherence* is an anti-reflexive and symmetrical relation written $x \frown y[U]$.

The following shorthand is convenient:

$x \stackrel{\frown}{\smile} y[U] : x = y$ or $x \frown y[U]$ – coherent,

$x \stackrel{\smile}{\frown} y[U]$: not $x \frown y[U]$ – incoherent,

$x \smile y[U] : x \neq y$ and not $(x \frown y[U])$ – strictly incoherent.

The *dual* $U^{\perp}$ of a coherence space U is defined by its web $|U^{\perp}| = |U|$, and its strict coherence: $x \frown y[U^{\perp}]$ iff $x \smile y[U]$. In other words, $U^{\perp}$ is the complement graph of U. Given two coherence spaces U and V, the coherence spaces $U \wp V$ and $U \otimes V$ are defined by:

$|U \wp V| = |U \otimes V| = |U| \times |V|$

$(x, y) \stackrel{\frown}{\smile} (x', y')[U \otimes V]$ iff $x \stackrel{\frown}{\smile} x'[U]$ and $y \stackrel{\frown}{\smile} y'[V]$

$(x, y) \frown (x', y')[U \wp V]$ iff $x \frown x'[U]$ or $y \frown y'[V]$

Definition 5. An *interpretation* ∇ is the choice, for each propositional variable a of a coherence space a_{∇}. Thus, defining $(A \wp B)_{\nabla}$ as $A_{\nabla} \wp B_{\nabla}$, $(A \otimes B)_{\nabla}$ as $A_{\nabla} \otimes B_{\nabla}$ and $(A^{\perp})_{\nabla}$ as $(A_{\nabla})^{\perp}$, each formula F is associated with a coherence space F_{∇}. Here are the two dual coherence spaces '$\mathsf{N} = \mathsf{Z}^{\perp}$' and '$\mathsf{Z} = \mathsf{N}^{\perp}$':

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An ‘NZ-interpretation’ is an interpretation in which any atomic formula is interpreted as N or as Z.

2.4. *Experiments*

The starting point of this note is the so-called method of *experiments* of Girard (1987, §3.17–18) for computing the coherence semantics of a proof directly from the proof net. It can be harmlessly extended to proof structures, as in the following definition.

Definition 6. Let ∇ be an interpretation. Let Π be a proof structure with conclusions $F_1, \ldots, F_n$. A ∇-*experiment* of Π is a labelling of its vertices – *i.e.*, of the occurrences of the subformulae of the F_i. The label of a vertex A is a token, say u, of the web $|A_\nabla|$ of the coherence space A_∇, and we write $u : A$ for this. A ∇-experiment is obtained as follows:

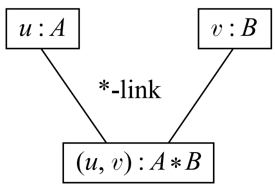
— for each axiom $a\overline{}a^{\perp}$ we arbitrarily choose a *single* token $x \in |a_\nabla| = |a^{\perp}_\nabla|$, which is their *common* label

$$x : a\overline{}x : a^{\perp},$$

and this completely determines the experiment;

— these labels are spread all over the proof net, from the premises of links to their conclusions as follows:

Let $* \in \{\wp, \otimes\}$. If the label of the left premise is $u \in |A_\nabla|$ and the label of the right premise is $v \in |B_\nabla|$, the label of the conclusion $A * B$ is (u, v), which belongs to $|(A * B)_\nabla| = |A_\nabla| \times |B_\nabla|$.



The *result* of an experiment $\mathscr{E}$ is the tuple $|\mathscr{E}| = (t_1, \ldots, t_n)$ of the tokens t_i labelling the conclusion vertices: $t_1 : F_1$, … , and $t_n : F_n$. Thus it is a token of $(F_1 \wp \cdots \wp F_n)_\nabla$. The semantics $\| \Pi \|_\nabla$ of a Π according to an interpretation ∇ is the set of results of the ∇-experiments[†].

From Girard (1987, 3.18), we have, among others, the following theorem.

Theorem 7. (‘soundness’) Let ∇ be any interpretation. Let Π be a proof structure with conclusions $F_1, \ldots, F_n$. If Π is a proof net, then $\| \Pi \|_\nabla$ is a clique of the coherence space $(F_1 \wp \cdots \wp F_n)_\nabla$.

For a proof in this setting, see Retoré (1994).

[†] There is no need to restrict the semantics to the experiments that *succeed*, because we only deal here with cut-free proof structures – see Retoré (1994) for more details.

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3. Result

The purpose of this note is to establish the following theorem.

Theorem 8. Let ∇ be a given arbitrary NZ-interpretation. Let Π be a proof structure with conclusions $F_1, \ldots, F_n$. Let $\mathscr{E}_1$ be a given arbitrary ∇-experiment of Π. If any ∇-experiment $\mathscr{E}_2$ satisfies $|\mathscr{E}_1| \frown\!\!\!\smile |\mathscr{E}_2|[(F_1 \wp \cdots \wp F_n)_\nabla]$, then Π is a proof net.

This result obviously entails the following corollary.

Corollary 9. Let ∇ be a given arbitrary NZ-interpretation. Let Π be a proof structure with conclusions $F_1, \ldots, F_n$. If $\| \Pi \|_\nabla$ is a clique of $(F_1 \wp \cdots \wp F_n)_\nabla$, then Π is a proof net.

This clearly entails Corollary 10, which is the converse of Theorem 7.

Corollary 10. ('completeness') Let Π be a proof structure with conclusions $F_1, \ldots, F_n$. If $\| \Pi \|_\nabla$ is a clique of $(F_1 \wp \cdots \wp F_n)_\nabla$ for any interpretation ∇, then Π is a proof net.

The respective converses of Theorem 8 and its two corollaries are immediate consequences of Theorem 7. Thus Theorem 8 and its corollaries are all semantic *characterisations* of the correctness of a proof structure.

An advantage of Theorem 8 or Corollary 9 with respect to Corollary 10 is that it provides an algorithm for asserting or refuting the correctness of a proof structure. This algorithm simply consists of:

1 Choose an arbitrary NZ-interpretation ∇.
2 Choose an arbitrary ∇-experiment $\mathscr{E}_1$.
3 Check that any ∇-experiment $\mathscr{E}_2$ satisfies $|\mathscr{E}_1| \frown\!\!\!\smile |\mathscr{E}_2|[(F_1 \wp \cdots \wp F_2)_\nabla]$ – because $|\mathsf{N}|$ is finite, there are finitely many $\mathscr{E}_2$.

Unfortunately, it is not an efficient algorithm: if the proof structure has N `axiom`-links, there are 4^N ∇-experiments to be checked (polynomial algorithms are known: for example, Danos (1990), Fleury and Retoré (1994) and Retoré (1996)). However, this is the only semantic characterisation, and is directly applicable to the proof expressions of Abramsky (1993), without considering the corresponding proof structures.

4. Proof

Notation 11. In this section:

— ∇ denotes a given but arbitrary NZ-interpretation,
— Π denotes a proof structure with conclusions $F_1, \ldots, F_n$,
— $\mathscr{E}_1, \mathscr{E}_2$ are ∇-experiment of Π,
— $A\!:\!\frown, A\!:\!\smile, A\!:\!\asymp, A\!:\!\frown\!\!\!\smile, A\!:=$ Given a vertex A of Π, and two ∇-experiments $\mathscr{E}_1$ and $\mathscr{E}_2$, the expression $A\!:\!\frown$ means: the two tokens t_1 and t_2 labelling the vertex A according to $\mathscr{E}_1$ and $\mathscr{E}_2$ satisfy $t_1 \frown t_2[A_\nabla]$ – the other similar expressions are defined in the same way.

Proposition 12. Let $\mathscr{E}_1$ be a given but arbitrary ∇-experiment of Π. Let $a_1 \overline{\quad} a_1^\perp$, $a_2 \overline{\quad} a_2^\perp$ … and $a_p \overline{\quad} a_p{}^\perp$ be a family of axioms of Π (hence all the a_i are atomic), and let ϕ and ψ be two functions from $[1, p]$ to $\{a_1, a_1^\perp, a_2, a_2^\perp, \ldots, a_p, a_p{}^\perp\}$ such that $\{\psi(i), \phi(i)\} = \{a_i, a_i{}^\perp\}$ for $i \in [1, p]$.

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Then there always exists another ∇-experiment such that $\phi(i){:}\frown\!\!-\!\!\psi(i){:}\smile$, and, for any axiom not in the family, $b{:}{=}\!-\!b^{\perp}{:}{=}$.

Proof. Assume the token for the axiom i according to $\mathscr{E}_1$ is $x_i \in |\mathsf{N}| = |\mathsf{Z}|$. Then in the interpretation of $\phi(i)$ that is N or Z, there exists another token y_i such that $x_i \frown y_i[\phi(i)_\nabla]$, and since $\psi(i) = \phi(i)^\perp$, we also have $x_i \smile y_i[\psi(i)_\nabla]$. Indeed for any token x in $|\mathsf{N}| = |\mathsf{Z}|$ there exists another token y (respectively, another token z) of $|\mathsf{N}| = |\mathsf{Z}|$ such that $x \frown y[\mathsf{N}]$ (respectively, $x \frown z[\mathsf{Z}]$). □

4.1. *Experiments and feasible paths*

In this subsection, $\mathscr{E}_1$ denotes a given but arbitrary ∇-experiment of Π, and the proof structure Π is assumed to be a proof *net*†.

Let γ be a feasible path (*cf.* Definition 2) of Π from a conclusion X to a conclusion Y. Notice that γ necessarily uses some `axiom`-links, which are all distinct, because Π is a proof net.

Let $a_1\!-\!a_1^\perp, a_2\!-\!a_2^\perp \ldots, a_p\!-\!a_p^\perp$, be the sequence of `axiom`-links that γ uses and let $\phi(i)$ (respectively, $\psi(i)$) be the first (respectively, second) vertex of $a_i\!-\!a_i^\perp$ met by γ – thus $\{\phi(i), \psi(i)\} = \{a_i, a_i^\perp\}$.

Proposition 12 provides another ∇-experiment $\mathscr{E}_2$ such that: $\phi(i){:}\frown\!\!-\!\!\psi(i){:}\smile$, and, for any other axiom, $b{:}{=}\!-\!b^{\perp}{:}{=}$.

We then have the following lemma.

Lemma 13. $\mathscr{E}_1$ and $\mathscr{E}_2$ satisfy $X{:}\frown$ and $Y{:}\smile$, while $Z{:}\asymp$ for any other conclusion Z.

Proof. We proceed by induction, using Proposition 3.

1 If the proof net is a union of `axiom`-links, then, because of the existence of the (feasible) path γ, X and Y are the two conclusions of the same axiom, and the result is obvious.
2 If there is a final `par`-link, we arbitrarily choose one, and call Π' the proof net obtained by removing this final `par`-link.
   (a) If X is its conclusion. Then the path γ makes use of one of the edges of the `par`-link. We call the corresponding premise X', and use γ' for the restriction of γ to Π'. Therefore γ' uses the same `axiom`-links in the same order. We can apply the induction hypothesis to Π', X' and γ', and therefore we obtain $X'{:}\frown$ and $Y{:}\smile$ with $Z{:}\asymp$ for any other conclusion Z. From the coherence according to `par`, we obtain the result.
   (b) If Y is its conclusion, we proceed similarly, noticing that $a' \smile b'[Y'_\nabla]$ and $a'' \asymp b''[Y''_\nabla]$ implies $(a', a'') \smile (b', b'')[(Y' \wp Y'')_\nabla = Y_\nabla]$.
   (c) If neither of X, Y is its conclusion, then γ does not use this link. So we apply the induction hypothesis to Π', X, Y and γ, and the result immediately follows.

† This is in fact not needed, as can be seen in Retoré (1994), but it makes both the statement of the lemma and its proof easier.

3 If there is no final `par`-link, then there exists a splitting `tensor`-link, and we choose one. Arbitrarily putting any totally disconnected part of Π with one or the other premise of the chosen splitting `tensor`-link, we obtain a partition of Π – minus the two edges of the chosen splitting `tensor`-link – into three full subgraphs: two proof nets Π', Π'', respectively, containing one and the other premise of the splitting `tensor`-link, and a single vertex, which is the conclusion of the splitting `tensor`-link.

(a) If X is the conclusion of the splitting `tensor`-link, say Y is in Π', and call X' the premise of X in Π'. Then, necessarily, γ starts with the edge $X - X'$, and we use γ' for the rest of γ, which is necessarily included in Π'. We apply the induction hypothesis to Π', X', Y and γ'. Noticing that all conclusions in Π'' are $Z{:}{=}$, and the result is clear.

(b) If Y is the conclusion of the splitting `tensor`-link, we proceed similarly.

(c) If neither X nor Y is the conclusion of the splitting `tensor`-link, they either lie in the same part Π' or different parts, say $X \in \Pi'$ and $Y \in \Pi''$.

i If X, Y are in the same part, since the `tensor`-link is splitting, γ does not use it – otherwise there would exist a feasible cycle. So we apply the induction hypothesis to Π', X, Y and γ, and the result follows – all conclusions are $Z{:}{=}$ in Π''.

ii If X is in Π' and Y in Π'', then γ uses the splitting `tensor`-link. We use U' for its premise in Π', γ' for the part of γ from X to U' (included in Π'), U'' for its premise in Π'', and γ'' for the part of γ from U'' to Y (included in Π''). We apply the induction hypothesis to Π', X, U' with γ' and to Π'', U'' and Y with γ''. The result follows, since $U'{:}\smile$ and $U''{:}\frown$ implies $U'{\otimes}U''{:}\smile$.

□

4.2. *Experiments and proof structures*

We now prove the contrapositive of Theorem 8, i.e:

Theorem 14. Let ∇ be a given arbitrary NZ-interpretation
Let $\mathscr{E}_1$ be a given arbitrary ∇-experiment of the proof structure Π.
If the proof structure Π is not a proof net, there exists another ∇-experiment such that $|\mathscr{E}_1| \smile |\mathscr{E}_2| [(F_1 \wp \cdots \wp F_n)_\nabla]$.

Proof. Remember that the proof structure is not a proof net whenever it possesses a feasible cycle, while the two experiments are not coherent in the `par` of its conclusions whenever all conclusions are $Z{:}\asymp$, one of them being $W{:}\smile$.

Here too we proceed by induction on the number of links of the proof structure.

1 Π cannot be a union of `axiom`-links.

2 If Π has a final `par`-link, say $F_1 = X \wp Y$, then the proof structure Π' obtained by removing this final `par`-link is not a proof net either. We apply the induction hypothesis, and thus obtain an experiment $\mathscr{E}_2$ of Π', for which $|\mathscr{E}_1| \smile |\mathscr{E}_2| [(X \wp Y \wp F_2 \wp \cdots \wp F_n)_\nabla]$. So $\mathscr{E}_2$ viewed as an experiment of Π gives the result.

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3 Otherwise Π possesses a final `tensor`-link, say $F_1 = X \otimes Y$.

(a) If the proof structure obtained by removing this final `tensor`-link is not a proof net either, we apply the induction hypothesis, and we are done, as in 2: if $|\mathscr{E}_1| \smile |\mathscr{E}_2|[((X \wp Y) \wp F_2 \cdots \wp F_n)_\nabla]$, then $|\mathscr{E}_1| \smile |\mathscr{E}_2|[((X \otimes Y) \wp F_2 \wp \cdots \wp F_n)_\nabla]$.

(b) Otherwise, the proof structure obtained by removing this final `tensor`-link is a proof net. Therefore this proof net contains a feasible path γ between the two premises of this final `tensor`-link, say X and Y. We apply Lemma 13, and thus we obtain another experiment $\mathscr{E}_2$ such that $X : \frown$ and $Y : \smile$, the other conclusions being $Z : \asymp$. This obviously provides another experiment $\mathscr{E}_2$ of Π such that $X \otimes Y : \smile$ and $Z : \asymp$ for any other conclusion. □

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