THEORY, OBSERVATION AND EXPERIMENT: STELLAR HYDRODYNAMICS

A Different Perspective

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Abstract. Computer technology now allows two dimensional (2D) simulations, with complex microphysics, of stellar hydrodynamics and evolutionary sequences, and holds the promise for 3D. Careful validation of astrophysical methods, by laboratory experiment, by critical comparison of numerical and analytical methods, and by observation are necessary for the development of simulation methods with reliable predictive capability. Recent and surprising results from isotopic patterns in pre-solar grains, 2D hydrodynamic simulations of stellar evolution, and laser tests and computer simulations of Richtmeyer-Meshkov and Rayleigh-Taylor instabilities will be discussed, and related to stellar evolution and supernovae.

1. Introduction

It is a personal pleasure to speak at this meeting in honor of Prof. Hanbury-Brown; this discussion may be seen as my colored reflection of his approach to making astrophysics a quantitative science.

A fundamental property of stars is their composition, and its possible variation throughout their structure. Stars are thermonuclear reactors, so that the change of abundances both drives the evolution and provides a diagnostic of that process. With the wealth of new quantitative data becoming available, theorists have an obligation to take a critical look at the conceptual framework within which predictions of nuclear abundance anomalies are made.

The investigation of rapidly evolving late stages of the lives of stars—by use of direct multidimensional hydrodynamic simulations—brings into question assumptions generally used in the theory of stellar evolution.

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Quantitative predictions of nucleosynthesis yields and pre-supernova characteristics will require attention to these new physical effects.

For at least the last three decades there has been a steady and dramatic increase in computer performance. This has been accompanied by a comparable (probably greater) increase in the quality and efficiency of algorithms. How may we validate these increasingly complex simulations, to convince ourselves that the results are real and not just virtual reality?

2. Toward a Predictive Theory: Tests with the NOVA Laser

Ultimately simulations must be well resolved in three spatial dimensions. One of the great assets of computers is their ability to represent complex geometries. If we can implement realistic representations of the *essential* physics, then simulations should become tools to predict—not "postdict"—phenomena. An essential step toward that goal is the testing of computer simulations against reality in the form of experiment(Remington, Weber, Marinak, et al., 1995). This is a venue in which we can alter conditions (unlike astronomical phenomena), and thereby understand the reasons for particular results. Experiments are intrinsically three dimensional, with two dimensional symmetry available with some effort, so that they provide a convenient way to assess the effects of dimensionality.

For Rayleigh-Taylor instabilities, the NOVA experiments not only sample temperatures similar to those in the helium layer of a supernova, but hydrodynamically scale to the supernova as well(Kane, Arnett, & Remington, 1997). The NOVA laser is physically imposing. The building is larger in area than an American football field; the lasers concentrate their beams on a target about the size a BB (or a small ball bearing). This enormous change in scale brings home just how high these energy densities are. Preliminary results show that the astrophysics code (PROMETHEUS) and the standard inertial confinement fusion code (CALE) both give qualitative agreement with the experiment. For example, the velocities of the spikes and bubbles are both in agreement with experiment, and analytic theory which is applicable in this experimental configuration(Kane, Arnett, & Remington, 1997). The two codes give similar, but not identical results. These differences will require new, more precise experiments to determine which is most nearly correct.

3. Applications to Stellar Hydrodynamics

In discussion of stellar evolution, one encounters the topics of rotation, convection, pulsation, mass loss, microturbulence, sound waves, shocks, and instabilities—to name a few—which are all just hydrodynamics. However, direct simulation of stellar hydrodynamics is limited by causality. In analogy to light cones in relativity, in hydrodynamics one may define space-time regions in which communication can occur by the motion of sound waves. To correctly simulate a wave traveling through a grid, the size of the time step must be small enough so that sound waves cannot "jump" zones. Thus the simulation is restricted to short time steps—an awkward problem if stellar evolution is desired. While simulations of the solar convection zone are feasible, the simulation time would be of order hours instead of the billions of years required for hydrogen burning. For the latter, a stellar evolution code is used, which damps out the hydrodynamic motion, obviating the need for the time step restriction. Any presumed hydrodynamic motion is then replaced by an algorithm (such as adiabatic structure and complete mixing in formally convective regions). Thus, *stellar evolution* deals with the long, slow phenomena, and *stellar hydrodynamics* has dealt with the short term.

However, the stages of evolution prior to a supernova explosion are fast and eventful. Here direct simulation is feasible(Bazan & Arnett, 1997). A key region for nucleosynthesis is the oxygen burning shell in a pre-supernova star. Besides producing nuclei from Si through Fe prior to and during the explosive event, it is the site at which the radioactive ^{56}Ni is made and is mixed. The conventional picture of this region relies upon the notion of thermal balance between nuclear heating and neutrino cooling in the context of complete microscopic mixing by convective motions.

This is usually treated by the mixing length scenario for convection, which assumes statistical (well developed) turbulence, random walk of convective blobs approximated by diffusion, subsonic motions, and almost adiabatic flow. These approximations are further constrained by a simplistic treatment of the boundaries of the convective region.

The timescales for the oxygen burning shell are unusual. The evolutionary time is $\tau_{evol} \approx 4 \times 10^3$ s. The convective "turnover" time is $\tau_{conv} \approx \Delta r/v_{conv} \approx \tau_{evol}/10$, while the sound travel time across the convective region is $\tau_{sound} \approx \Delta r/v_{sound} \approx \tau_{conv}/100$. The burning time is $\tau_{burn} = E/\varepsilon \leq \tau_{conv}$. Obviously the approximations of subsonic flow, well developed turbulence, complete microscopic mixing, and almost adiabatic flow are suspect.

These time scales are rapid enough to make the oxygen shell a feasible target for direct numerical simulations, and an extensive discussion is about to appear(Bazan & Arnett, 1997). The two dimensional simulations show qualitative differences from the previous one dimensional ones. The oxygen shell is not well mixed, but heterogeneous in coordinates θ and ϕ as well as r. The burning is episodic, localized in time and space, occurring in flashes rather than as a steady flame. The burning is strongly coupled to hydrodynamic motion of individual blobs, but the blobs are more loosely coupled to each other.

Acoustic and kinetic luminosity are not negligible, contrary to the assumptions of mixing length theory. The flow is only mildly subsonic, with Mach numbers of tens of percent. This gives non-spherical perturbations in density and temperature of several percent, especially at the boundaries of the convective region.

At the edges of convective regions, the convective motions couple to gravity waves, giving a slow mixing beyond the formally unstable region. The convective regions are not so well separated as in the one dimensional simulations; "rogue blobs" cross formally stable regions. A carbon rich blob became entrained in the oxygen convective shell, and underwent a violent flash, briefly outshining the oxygen shell itself by a factor of 100. Significant variations in neutron excess occur throughout the oxygen shell. Because of the localized and episodic burning, the typical burning conditions are systematically hotter than in one dimensional simulations, sufficiently so that details of the nucleosynthesis yields will be affected.

The two dimensional simulations are computationally demanding. Our radial zoning is comparable to that used in one dimensional simulations, to which we add several hundred angular zones, giving a computational demand several hundred times higher. This has limited us to about a quarter of the final oxygen shell burning in a SN1987A progenitor model. Given the dramatic differences from one dimensional simulations, it is important to pursue the evolutionary effects to see exactly how nucleosynthesis yields, pre-supernova structures, collapsing core masses, entropies, and neutron excesses will be changed. It may be that hydrostatic and thermal equilibrium on average, and the temperature sensitivity of the different burning stages, taken together, tend to give a rough layering in composition, even if the details of how this happens are quite different.

4. Mixing

We sometimes forget that stars are really very large. Let us make a order of magnitude estimate of diffusion time scales in a dense stellar plasma. It is the nuclei, not the electrons which define the composition. The coulomb cross section for pulling ions past each other is of order $\sigma \approx 10^{-16}$ cm^2. For a number density $N \approx 10^{24}$ cm^{-3}, this implies a mean free path $\lambda = 1/\sigma N \approx 10^{-8}$ cm. For a particle velocity $v_d \approx 10^8$ cm/s, this gives a diffusion time $\tau_d = (\Delta r)^2/\lambda v_d \approx (\Delta r)^2$ s cm^{-2}. For a linear dimension of stellar size, $\Delta r = 10^{11}$ cm, $\tau_d = 3 \times 10^{14}$ y, or 3,000 Hubble times! While one may quibble about the exact numbers used, it is clear that pure diffusion is ineffective for mixing stars.

Actually, we all know from common experience—such as stirring cream into coffee (tea)—that this is incomplete. To diffusion must be added *advection*, or stirring. Stars may be stirred too. For example, rotation may induce currents, as may accretion, and perturbations from a binary companion. However, the prime mechanism for stirring that is used in stellar evolutionary calculations is thermally induced convection. The idea is that convective motions will stir the heterogeneous matter, reducing the typical length scale Δr to a value small enough that diffusion can insure microscopic mixing. For our stellar example above, this would require a reduction in scale of $(\lambda/\Delta r)^{1/2} \approx 10^{-8}$. Convection is not perfectly efficient, so that the actual mixing time would still be finite. Given that such a limit exists, we must examine rapid evolutionary stages to see if microscopic mixing is a valid approximation. For pre-supernovae, the approximation is almost certainly not correct, so that these stars are not layered in uniform spherical shells as conventionally assumed, but heterogeneous in angle as well as radius.

5. Pre-solar Grains

This new view of the nucleosynthesis process in a supernova, both prior to and during the actual explosion, has implications for any attempt to identify pre-solar grains with such events. For example, on the basis of their enrichments in ^{28}Si and the presence of ^{44}Ti at the time of grain formation, type II supernovae have been suggested as the most likely sources of the X grains(Amari, et al., 1996). Using the one dimensional view of nucleosynthesis gives serious difficulties in detail(Hoppe, et al., 1996). The higher burning temperatures in the two dimensional simulations shifts the detailed constraints on yields. The heterogeneity in neutron excess, and the possibility of rogue blobs, further relaxes these constraints. The probability of macroscopic mixing prior to explosion, but with ashes of high temperature burning by various processes, gives still other options.

This leads us to suggest two different views of what might be occurring.

1. **The Layered Mixing Model.** This view has been explored in some detail(Hoppe, et al., 1996) for the X grains, and found to have problems. Inner layers, containing ^{44}Ti must be brought out to coincide with outer layers, containing ^{26}Al. This must occur without any microscopic mixing with intermediate layers which are oxygen rich, and could prevent SiC grain formation. However, just before grain formation, these layers must be microscopically mixed.
2. **The Path Integral Model.** It may be more accurate to think of the anomalies in the X grains as the result of a "path integral" over the trajectory of a blob, sampling a variety of conditions. The blob would be affected by burning prior to explosion as well as after. In general,

the blob might have all the anomalies produced with no mixing, or it might be moved out to the region with which it should mix, prior to the explosion, to be microscopically mixed with adjacent matter upon expansion to lower density.

The anomalies in the X grains have also been identified with explosive helium burning in ^{14}N-rich matter(Clayton, et al., 1997) in type I supernovae. He-rich blobs in type II supernovae might be possible sources as well.

6. Conclusions

- Multidimensional simulations provide new perspectives of direct relevance to the understanding of stellar evolution, and the interaction between theory and experiment promises to become even more exciting.
- Experimental tests of astrophysical simulation methods are feasible, and preliminary results from the NOVA laser are promising.
- The slow stirring outside the formally unstable regions, if it extends to more slowly evolving stages, may imply important and needed modifications for stellar evolutionary theory.
- The qualitative nature of the pre-supernova evolutionary stages is changed when direct hydrodynamic simulation techniques are used. This promises to modify our understanding of supernovae and nucleosynthesis. It is particularly applicable to the interpretation of presolar grains in meteorites.

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Discussion of this paper appears at the end of these Proceedings.