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Generation of magnetic macrostructures by electromagnetic drift turbulence

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Abstract. A system of equations for modulational interaction of the electromagnetic branch of drift waves with magnetic cells is constructed. The solution of these equations describes formation of a set of magnetic cells with oppositely directed magnetic fields at their boundaries.

Introduction

Magnetic convective cells.

An important property of drift turbulence is its ability for self-organization—creation of macroconvective cells by the Reynolds stresses imposed on a plasma by microturbulence. In the pioneering paper [1], generation of the convective cells by drift turbulence has been considered. The turbulence of the convective cells results in anomalous high plasma diffusion across the magnetic field. In the present paper we shall consider generation of magnetic structures by the electromagnetic part of the drift spectrum.

Such mechanisms of the magnetic field generation are important for the problem of the magnetic dynamo (see e.g. [2]).

The vector potential in the magnetic cell has a component along the ambient magnetic field which corresponds to the transverse magnetic structure

$$b_y(x) = -\frac{\partial a}{\partial x}, \tag{1}$$

where x is the direction of plasma inhomogeneity.

Plasma velocity components along the ambient magnetic field in the magnetic cell can be written as

$$v_{z\mathrm{i}} = -\frac{ea}{m_\mathrm{i}c}, \quad v_{z\mathrm{e}} = \frac{ea}{m_\mathrm{e}c}. \tag{2}$$

If we assume periodicity of the vector potential $a(x) \sim \cos k_0 x$, the described structure corresponds to plasma motion up and down along the magnetic field in the form of a convective cell closed by oppositely directed transverse components of the magnetic field (1).

Now let us consider the electromagnetic drift wave in detail. There are both electrostatic $\varphi(y)$ and vector $A_y(y)$ potentials in the drift wave. For the plane wave

$$\varphi(y,z) = \varphi \exp i(k_y y + ik_z z - \omega t),$$

the electric field can be written as

$$E_y = -ik_y\varphi + \frac{i\omega}{c}A_y$$

and the magnetic field is $B_x = -ik_z A_y$.

Plasma velocities across the magnetic field are to be calculated from the following equations:

$$\begin{aligned} -i\omega v_x - \omega_{\mathrm{ci}} v_y &= 0, \\ -i\omega v_y + \omega_{\mathrm{ci}} v_x &= \frac{e}{m_{\mathrm{i}}}E_y + \omega_{\mathrm{ci}}\frac{B_x}{B_0}v_z. \end{aligned} \tag{3}$$

On the right-hand side of (3), in addition to the electric force, the nonlinear term $\frac{\omega_{\mathrm{ci}}}{B_0}B_x v_z$ is also taken into account; that term describes coupling of the drift wave with the magnetic cell. ω_{ci} is the ion gyrofrequency. Solving (3) in the drift approximation ($\omega \ll \omega_{\mathrm{ci}}$), we have

$$\begin{aligned} v_x &= \frac{c}{B_0}E_y + v_z\frac{B_x}{B_0}, \\ v_y &= -i\frac{\omega}{\omega_{\mathrm{ci}}}\frac{c}{B_0}E_y - i\frac{\omega}{\omega_{\mathrm{ci}}}\left(1 + \frac{\omega^2}{\omega_{\mathrm{ci}}^2}\right)\frac{B_x}{B_0}v_z. \end{aligned} \tag{4}$$

Using the first of the equations (4), from the continuity equation we have the following relationship for the perturbation of ion density in the drift wave:

$$n_{\mathrm{i}}' = \frac{c}{B_0}\frac{dn_0}{dx}\left(-\frac{k_y}{\omega}\varphi + \frac{A}{c} - \frac{k_z v_z}{\omega}\frac{A}{c}\right). \tag{5}$$

As usual, for the low-frequency waves we can assume that electrons in the drift wave are Boltzmann distributed:

$$n_{\mathrm{e}}' = n_0\frac{e\varphi}{T_{\mathrm{e}}}$$

and, from the condition of quasineutrality $n_{\mathrm{e}}' = n_{\mathrm{i}}'$, we have

$$\varphi\left(1 + \frac{\omega^*}{\omega}\right) = \frac{\omega^*}{ck_y}\left(1 - \frac{k_z v_z}{\omega}\right)A_y, \tag{6}$$

where the following notation is used.

$$\omega^* = k_y\frac{cT_{\mathrm{e}}}{eB_0}\frac{1}{n_0}\frac{dn_0}{dx}$$

is the drift frequency.

From the Maxwell equation

$$\frac{\partial B_x}{\partial z} = \frac{1}{c}\frac{\partial E_y}{\partial t} + \frac{4\pi}{c}j_y,$$

we have the following equation for the vector potential:

$$\frac{\partial^2 A}{\partial z^2} = \frac{i\omega}{c}E_y - \frac{4\pi}{c}j_y. \tag{7}$$

Evaluating the y components of the electron velocities similarly to what was done in (4) for ions, it is easy to see that the highest in $\omega/\omega_{\mathrm{ci}}$ order nonlinear terms in j_y for electrons and ions are cancelled, and therefore the main contribution to the

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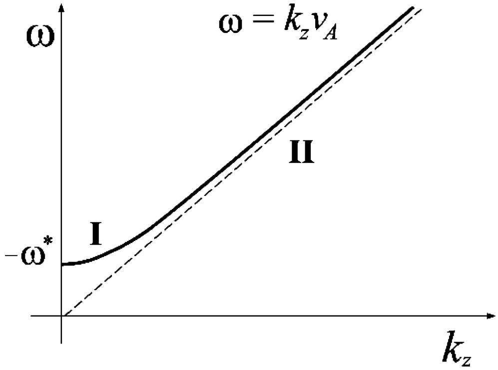

Figure 1. Branches of the drift waves in the dispersion equation (10).

nonlinear current is from ions in the next order ($\sim\omega^2$) over the parameter $\omega/\omega_{\mathrm{ci}}$. The final expression for the j_y current is given by

$$j_y = en_0 \frac{-i\omega}{\omega_{\mathrm{ci}}}\left(\frac{cE_y}{B_0} + \frac{\omega^2}{\omega_{\mathrm{ci}}^2} v_z \frac{B_x}{B_0}\right). \tag{8}$$

Omitting in the expression for j_y the nonlinear term $\sim v_z$, we can write the following equation for the vector potential:

$$\left(k_z^2 - \frac{\omega^2}{v_{\mathrm{A}}^2} - \frac{\omega^2}{c^2}\right) A_y = -\frac{\omega}{c} k_y \varphi \left(1 + \frac{c^2}{v_{\mathrm{A}}^2}\right), \tag{9}$$

where ω is the drift wave frequency determined by the dispersion relationship (10), see below, and

$$v_{\mathrm{A}}^2 = \frac{B_0^2}{4\pi n_0 m_{\mathrm{i}}} \text{ -- square of Alfvén velocity.}$$

Combining this equation with (6), in which we omit the nonlinear term $\sim v_z$, we obtain a linear dispersion relation for the drift wave:

$$\omega^2 = k_z^2 \widehat{v}_{\mathrm{A}}^2 \left(1 + \frac{\omega^*}{\omega}\right), \tag{10}$$

where $\widehat{v}_{\mathrm{A}}^2 = (c^2 v_{\mathrm{A}}^2)/(c^2 + v_{\mathrm{A}}^2)$.

In the limit $\omega \ll k_z v_{\mathrm{A}}$ electromagnetic effects are not important and (10) reduces to the dispersion relation of the electrostatic drift wave:

$$\omega = -\omega^*; \tag{10a}$$

in the limit $\omega \gg \omega^*$ electromagnetic effects start to be important and (10) reduces to the dispersion relationship of the Alfvén wave:

$$\omega^2 = k_z^2 \widehat{v}_{\mathrm{A}}^2; \tag{10b}$$

see Fig. 1. Modulational interaction of magnetic cells and electromagnetic drift waves analysis of the nonlinear equations.

Substituting in (7) the vector potential of the drift wave in the form

$$A(t) e^{i(k_y y + k_z z - \omega t)},$$

expressing j_y from (8) and φ from (6) and saving nonlinear terms $\sim v_z B_x$ coupling the drift wave with the magnetic cell, we obtain the following final equation for the

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vector potential in the drift wave:

$$i(2\omega+3\omega^*)\frac{dA}{dt}+k_z v_{z_e}\frac{c^2}{c^2+v_A^2}\left(\omega^*-\frac{\omega^2}{\omega_{ci}^2}(\omega+\omega^*)\right)A=0. \tag{11}$$

In order to obtain a closed system of equations describing modulational interaction of the drift waves and magnetic cells, it is necessary to use an equation for the generation of magnetic cells by the nonlinear interaction of the drift waves. Since the main input into the current is from field-aligned electron motion, the equation for the vector potential in the magnetic cell $a(t,x)$ can be written as

$$\nabla^2 a=\frac{4\pi e n_0}{c}v_{\|e}. \tag{12}$$

Evaluating the equation for the magnetic structure, we shall follow the treatment of [3]. The equation of motion of electrons along the magnetic field can be written as

$$m_e\left(\frac{\partial}{\partial t}+\frac{c}{B_0}\frac{\partial\varphi}{\partial x}\frac{\partial}{\partial y}-\frac{c}{B_0}\frac{\partial\varphi}{\partial y}\frac{\partial}{\partial x}\right)v_{\|e}=-eE_{\|}, \tag{13}$$

where we have calculated electron velocities transverse to the magnetic field in the drift approximation the nonlinear term which serves as the driver for magnetic cells is created by the drift wave.

The parallel component of electron current in the drift wave $-env_{\|e}$ can be expressed as $\nabla^2 A$. The magnetic field aligned component of the electric field can be written as

$$E_{\|}=E_z+\frac{\mathbf{E}_\perp\mathbf{B}_\perp}{B_0}=-\frac{1}{c}\frac{\partial a}{\partial t}-\frac{1}{B_0}\left(\frac{\partial\varphi}{\partial x}\frac{\partial A}{\partial y}-\frac{\partial\varphi}{\partial y}\frac{\partial A}{\partial x}\right). \tag{14}$$

Here, z is the direction of the ambient magnetic field and $\bar{B}_\perp$ is the perturbation of the magnetic field created by the drift wave. Combining (13) and (14), we obtain the following final equation for the vector potential in the magnetic cell:

$$\begin{gathered}\frac{\partial}{\partial t}(1-\lambda_e^2\nabla^2)a=-\frac{c}{B_0}\left(\frac{\partial\varphi}{\partial x}(1-\lambda_e^2\nabla^2)\frac{\partial A}{\partial y}-\frac{\partial\varphi}{\partial y}(1-\lambda_e^2\nabla^2)\frac{\partial A}{\partial x}\right),\\ \lambda_e=\frac{c}{\omega_{pe}}.\end{gathered} \tag{15}$$

As usual, in modulational interaction we have two satellites of the test drift wave coupled with the pump wave by the mode of the magnetic cell:

$$\begin{aligned}&\varphi=\varphi_0 e^{i(k_{0y}y+k_{0z}z)}+\varphi_+(t)e^{i(k_{0y}y+k_{0z}z)}e^{i\kappa_0 x}+\varphi_-(t)e^{i(k_{0y}y+k_{0z}z)}e^{-i\kappa_0 x},\\ &a=\tfrac{1}{2}a(t)e^{i\kappa_0 x}+c.c.\end{aligned} \tag{16}$$

From (11), we have the following set for the amplitudes of the drift wave satellites:

$$\frac{\partial}{\partial t}\begin{pmatrix}\varphi_+\\ \varphi_-\end{pmatrix}=i\frac{k_z e}{m_e c}\begin{pmatrix}a\\ a^*\end{pmatrix}f(\omega)\varphi_0, \tag{17}$$

where the following notation is used:

$$f(\omega)=\frac{\omega^*}{2\omega+3\omega^*}\frac{\breve{v}_A^2}{v_A^2}-\frac{\omega^2}{\omega_{ci}^2}\frac{\omega+\omega^*}{2\omega+3\omega^*}.$$

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Using (15) for the vector potential of the magnetic cell and the relationship (9) connecting vector and scalar potentials in the drift wave, we have

$$\frac{\partial a}{\partial t} = \frac{c}{B_0}\frac{(\omega+\omega^*)^2 k_y^2 c^2}{\omega^2\omega^{*2}}\lambda_{\rm e}^2\left(\frac{\partial\varphi^*}{\partial y}\frac{\partial}{\partial x}\nabla^2\varphi - \frac{\partial\varphi^*}{\partial x}\frac{\partial}{\partial y}\nabla^2\varphi\right). \tag{18}$$

It follows from (18) that there is no coupling with the magnetic cell in a particular case of a drift electrostatic wave ($\omega = -\omega^*$).

Linearizing the right-hand side of (18) over the amplitude of the probe wave and using (17), we can write the following equation for the vector potential:

$$\frac{\partial^2 a}{\partial t^2} = \omega_{\rm ce}\frac{c}{2B_0^2}\frac{(1+(\omega/\omega^*))^3}{\omega^2\omega^{*2}}v_{\rm A}^2\frac{k_y^2 c}{(1+2(\omega^*/\omega))}k_z\lambda_{\rm e}^2|\varphi_0|^2 k_y\frac{\partial}{\partial x}\nabla^2 a$$
$$\times\left[\frac{\omega^4}{\omega_{ci}^2 v_{\rm A}^2} + \omega\omega^*\left(\frac{1}{c^2}+\frac{1}{v_{\rm A}^2}\right)\right]. \tag{19}$$

Now let us consider nonlinear solutions developing as the result of modulational interaction of magnetic cells and drift waves.

Assuming that the solution has a form of a traveling wave

$$a = a(x-ut), \tag{20}$$

we obtain for a the following equation:

$$u^2\frac{da}{dx} = \frac{c^2}{2B_0^2}\frac{(1+(\omega/\omega^*))^3}{\omega^2\omega^{*2}}v_{\rm A}^2\frac{k_y^3 k_z\lambda_{\rm e}^2|\varphi_0|^2}{1+2(\omega^*/\omega)}\left(\frac{\omega^4}{\omega_{\rm ci}^2 v_{\rm A}^2}+\omega\omega^*\left(\frac{1}{c^2}+\frac{1}{V_{\rm A}^2}\right)\right)\frac{d^2 a}{dx^2}. \tag{21}$$

We can rewrite this equation as

$$\beta_1\frac{da}{dx} = \beta_2\frac{d^2a}{dx^2},$$

where the following notation is used:

$$\beta_1 = 1\frac{u^2 B_0^2\omega^2\omega^{*2}}{1k_z|\varphi_0|^2(1+(\omega^*/\omega))^3}\frac{1+(2\omega^*/\omega)}{v_{\rm A}^2 k_y^2 k_z\lambda_{\rm e}^2|\varphi_0|^2},$$
$$\beta_2 = \frac{\omega^4}{\omega_{\rm ci}^2 v_{\rm A}^2} + \omega\omega^*\left(\frac{1}{c^2}+\frac{1}{v_{\rm A}^2}\right). \tag{22}$$

Substituting the function a as $a = w\exp(-(\beta_2/2\beta_1)x)$, we obtain for w the following equation:

$$\frac{d^2w}{dx^2} + \frac{\beta_2^2}{4\beta_1^2}w = 0. \tag{23}$$

We shall assume that the nonlinear velocity of traveling wave propagation in (20) is a function of amplitude $u = u_0 + \alpha a^2, \alpha > 0$. Then, if we assume that the condition $(\beta_2/\beta_1)x \ll 1$ is fulfilled, we are able to rewrite the last equation as

$$\frac{d^2w}{dx^2} + \left(A - A\frac{\alpha w^2}{4u_0^2}\right)w = 0,$$

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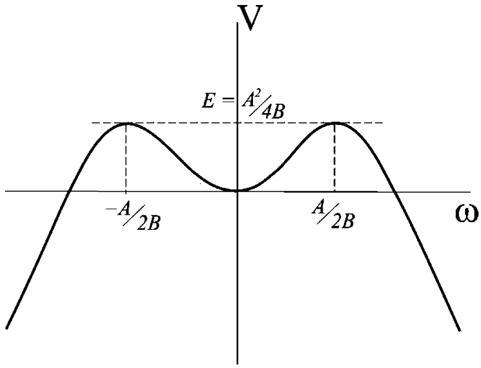

Figure 2. Potential well for quasiparticle oscillations in (23).

with the following notation:

$$A = \frac{\beta_2}{\beta_1^0},$$

$$\beta_1^0 = \beta_1 \frac{u_0^2}{u^2}.$$

The above equation has an integral $1/2(dw/dx)^2 + V(w) = E$, where E is a constant, corresponding to the nonlinear oscillations of a quasiparticle in the potential well $V(w) = Aw^2 - B(w^4/4), B = \alpha/4u_0^2$, as shown in Fig. 2. Those oscillations describe a set of the magnetic cells with magnetic field polarity changing oppositely in every cell; the value $E = A^2/4B$ corresponds to the quasiparticle oscillations between two turning points $\pm A/2B$ reached at infinity.

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