J. Fluid Mech. (2001), *vol.* 430, *pp.* 407–410. Printed in the United Kingdom

CORRIGENDUM

Radial pulsations of a fluid sphere in a sound wave

By S. Temkin

Journal of Fluid Mechanics, vol. 380 (1999), pp. 1–38

I am indebted to Professor Prosperetti and to Dr Ren for finding a significant error in my detailed analysis of the temperature and pressure fluctuations in a pulsating particle. As they point out, the error invalidates the results of that analysis for the temperature fluctuations in gas bubbles. Those results are withdrawn. The error also affects, although in a less significant manner, the results for the temperature fluctuations in pulsating liquid drops with specific heat ratio significantly different from 1. This includes the droplets chosen to illustrate the results, namely toluene droplets. However, correct results for both bubbles and droplets may be easily obtained and this is done here.

The error found by Prosperetti and Ren (see Prosperetti & Ren 2001) exists in both the internal and external solutions, but because the external fluid is in both cases a liquid having a very small value of $(\gamma_f - 1)$, the error has no effect there. Thus, fortuitously, the external solution is valid, provided the correct values of the interfacial temperature and pressure fluctuations are used. In addition, the simple solution presented in Appendix B, as well as the general expressions for derived quantities, such as those for the energy dissipation rates and the attenuation coefficients, are not affected by the error, provided, again, that the values of the temperature and pressure ratios appearing in them are properly evaluated. This is done below for both drops and bubbles by complementing the analysis of Appendix B with the polytropic approximation. The required steps are straightforward. First we note that when the external pressure fluctuations are retained, the derivative of the external temperature evaluated at the interface can be obtained from (5.1), which gives, instead of (B8),

$$\left(\frac{\partial \tau_f}{\partial y}\right)_1 = (1 - T_s)(1 - \mathrm{i}q) + (1 - \Pi_s)(1 - \mathrm{i}b). \tag{1}$$

Following the procedure used in Appendix B, (1) can be used to obtain the average and surface temperature. The results are, with $\Pi_s = \Pi$,

$$T = \xi\Pi - 3\frac{k_f}{k_p}\frac{\mathrm{i}}{q_i^2 F}\{\Pi(1-\xi) - (\Pi - 1)[1 + (1 - \mathrm{i}b)/(1 - \mathrm{i}q)]\}, \tag{2}$$

$$T_s = \xi\Pi + \frac{k_f}{k_p}\frac{G(q_i)}{F}\{\Pi(1-\xi) - (\Pi - 1)[1 + (1 - \mathrm{i}b)/(1 - \mathrm{i}q)]\}. \tag{3}$$

The second term inside the curly brackets may be regarded as the correction to the internal temperature that arises owing to the pressure disturbance in the external fluid. For bubbles in liquids, the correction term is negligible because of the very large value of ξ applicable to them. For droplets in liquids, where ξ can be order one, the magnitude of the second term can be evaluated by the use of a pressure disturbance in the external fluid that is based on isothermal conditions there. That calculation shows that the correction term in (2) and (3) can also be neglected for droplets. This means that, so far as the temperature is concerned, the pressure disturbance is

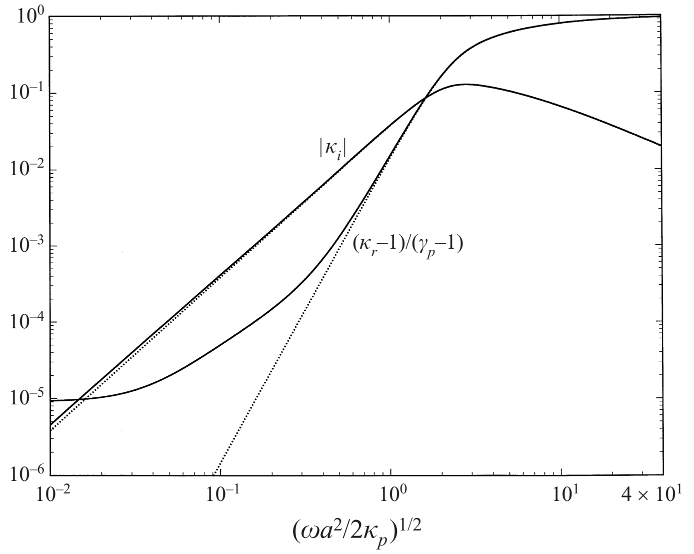

FIGURE 1. Values of $(\kappa_r/\zeta-1)/(\gamma_p-1)$ and of $|\kappa_i|/\zeta$ for a 100 micron diameter air bubble ($\zeta=1$) in water. ———, Present results; $\cdots\cdots$, Prosperetti & Ren (2001).

negligible for both bubbles and droplets. Thus,

$$\frac{T}{\Pi}=\xi+3\frac{k_f}{k_p}\frac{\mathrm{i}}{q_i^2F}(\xi-1),\qquad \frac{T_s}{\Pi}=\xi-\frac{k_f}{k_p}\frac{G(q_i)}{F}(\xi-1). \tag{4a,b}$$

These can be used to show that, contrary to what is displayed in my earlier figures 2 and 8(b), the average particle temperature is, at all frequencies but the smallest, significantly larger than the surface temperature. In fact, in the case of bubbles in water, the ratio $|T_s/T|$, as given by (4a,b) is nearly equal to that shown by Prosperetti & Ren (2001) in their figure 2.

Now, to obtain the particle's average temperature via (4a) we require the pressure ratio Π. This is done by means of the harmonic-oscillator model under the polytropic approximation. First, we define a complex polytropic index for pulsating particles, using (2.6) instead of the corresponding perfect-gas relation. Thus, instead of (4.9) we now have the more general result

$$\kappa=\zeta\left(1-\frac{(\gamma_p-1)}{\gamma_p\xi}\frac{T}{\Pi}\right)^{-1}, \tag{5}$$

where $\zeta=\rho_{p0}c_{sp}^2/\gamma_p p_0$, a quantity which reduces to unity for perfect gases. Figure 1 shows the quantities $|\kappa_i|/\zeta$ and $(\kappa_r/\zeta-1)/(\gamma_p-1)$ for an air bubble in water. The corresponding graph for toluene droplets in water is similar and is not shown to save space. Figure 1 also shows Properetti's results (Prosperetti 1977) for both quantities. It is seen that both sets of results agree closely for values of z_p of order one or larger. At lower frequencies, differences exist owing to the neglect, in Prosperetti's theory, of the temperature fluctuations in the external fluid. I should add that experimental confirmation of Prosperetti's result for the real part of the polytropic index had been reported some time ago by Crum (1982) in one of the papers I missed when writing mine. Other important papers I missed are listed in Prosperetti & Ren (2001).

The remaining steps in the calculation of the complex-pressure ratio, Π, are also straightforward, and only a sketch is presented here. Thus, the displacement of the

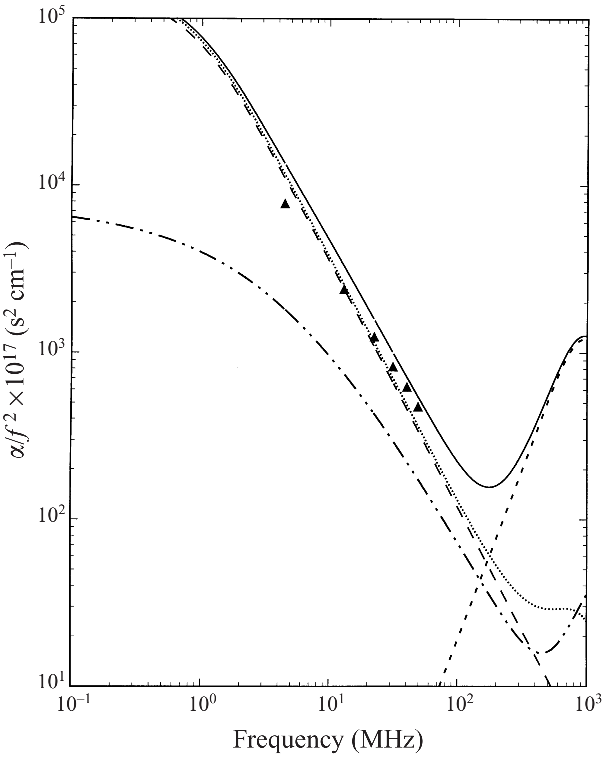

FIGURE 2. Scaled attenuation, $\alpha/f^2 \times 10^{17}\,\mathrm{s^2\,cm^{-1}}$, in an emulsion of toluene droplets. $D = 0.7\,\mu\mathrm{m}$, $\phi_v = 0.2$. ▲, Allegra & Hawley. The lines represent dilute attenuation theories: — ·· — ·· —, translational; – – – – – –, acoustic; ········, thermal, present results; — — — —, thermal, Epstein & Carhart; ——— total attenuation.

interface under a harmonic force that includes the externally applied pressure P_f' as well as the fluid's reaction to the particle's pulsations, is found to be

$$\varepsilon = -(4\pi a^2/M_0\omega_0^2)\chi P_f', \tag{6}$$

where $\chi = [1 - (\omega/\omega_0)^2 - \mathrm{i}\hat{d}]^{-1}$ is the *displacement*-response function for the particle, $\hat{d} = \hat{d}_{th} + \hat{d}_{ac}$ is the total, non-dimensional damping coefficient when viscosity is neglected, $\hat{d}_{th} = |\kappa_i|/\kappa_r$ and $\hat{d}_{ac} = b(\omega/\omega_0^2)$ are, respectively, the thermal and acoustic damping coefficients, $M_0 = 4\pi\rho_{f0}a^2/(1+b^2)$ is the induced interfacial mass, which is due to the incompressible part of the fluid's reaction on the pulsating particle, and ω_0 is the dissipationless resonance frequency, which is given by

$$\omega_0^2 = \omega_{T0}^2(1+b^2)\kappa_r/\zeta. \tag{7}$$

Finally, the fluctuating pressure in the pulsating particle is obtained from the linearized polytropic relation $\bar{p}_p' = -3p_0\kappa\varepsilon/a$. This gives

$$\Pi = (1 - \mathrm{i}\hat{d}_{th})\chi. \tag{8}$$

This completes the solution for both bubbles and droplets. Because the harmonic model has not been used in the past to study the pulsational response, to sound waves, of droplets in liquids, it is of interest to assess its validity. To do this, we compute the contribution to the sound attenuation coefficient in a dilute emulsion arising from the pulsational motion, namely acoustic and thermal attenuations, using the above results. The working equations are still, respectively, (7.2) and (7.4) of my 1999 paper. (Incidentally, the front factor on the right-hand side of (7.2) should be $3/2b$.) As figure 2 shows, the present results provide a better agreement with the experimental data of Allegra & Hawley (1972) than the earlier results. Also shown in the figure is Epstein & Carhart's (1953) thermal attenuation. It is seen that thermal attenuation computed from the harmonic model theory agrees well with it, except at high frequencies where the effects of droplet compressibility play a more important role.

To conclude it might be useful to state that my detailed analysis in my 1999 paper applies only when both internal and external fluids have specific heat ratios very nearly equal to one. Thus that analysis does not apply to gas bubbles and to droplets that have a large specific heat ratio. For these the corrections given here may be used. I should also state that my incorrect results have been used in a more recent paper (Temkin 2000). Thus, the same corrections apply there.

REFERENCES

Allegra, J. R. & Hawley, S. A. 1972 Attenuation of sound in suspensions and emulsions: theory and experiments. *J. Acoust. Soc. Am.* **51**, 1545–1564.

Crum, L. A. 1983 The polytropic exponent of gas contained within air bubbles pulsating in a liquid. *J. Acoust. Soc. Am.* **73**, 116–120.

Epstein, P. S. & Carhart, R. R. 1953 The absorption of sound in suspensions and emulsions. I. Water fog in air. *J. Acoust. Soc. Am.* **25**, 553–565.

Prosperetti, A. 1977 Thermal effects and damping mechanisms in the radial oscillations of gas bubbles in liquids. *J. Acoust. Soc. Am.* **61**, 17–27.

Prosperetti, A. & Ren, M. 2001 Comments on 'Radial pulsations of a fluid sphere in a sound wave' by S. Temkin. *J. Fluid Mech.* **430**, 401–405.

Temkin, S. 2000 Attenuation and dispersion of sound in dilute suspensions of spherical particles. *J. Acoust. Soc. Am.* **108**, 121–147.