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MONEY, CREDIT, AND LIMITED PARTICIPATION

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An asset market segmentation model is constructed to study the distributional effects of monetary policy when economic individuals can choose means of payment among alternatives. In equilibrium, monetary policy has two distributional effects: a direct effect and an indirect effect through the choice of means of payment. When the government injects money, some purchase a greater variety of goods with cash whereas others purchase a greater variety of goods with credit. Credit can dampen fluctuations in consumption arising from monetary policy. The optimal money growth rate can be positive or negative. The Friedman rule is not optimal in general.

Keywords: Money, Credit, Limited Participation, Nonneutrality of Money, Distributional Effects

1. INTRODUCTION

This paper studies the distributional effects of monetary policy on the choice of multiple means of payment when an asset market is segmented. In the model, economic individuals choose either credit or cash to purchase goods, as there are circumstances where one is more advantageous than the other. In equilibrium, monetary policy has a redistributional effect and affects the choice of means of payment. The effect of monetary policy on the choice of credit or cash differs between *traders*, who participate in the asset market, and *nontraders*, who do not, and credit can be used as a buffer against monetary policy shocks.

Several models, including those of Lucas and Stokey (1987), Prescott (1987), Ireland (1994), Lacker and Schreft (1996), and Aiyagari et al. (1998), have been developed to discuss the coexistence of multiple means of payment. In Lucas and Stokey (1987), the choice of credit or cash is exogenously given and the model does not show the individual's choice of means of payment when purchasing consumption goods. Prescott (1987), Ireland (1994), Lacker and Schreft (1996), and Aiyagari et al. (1998), however, built models in which there are multiple means of payment and the choice is endogenous. In general, individuals use cash for smaller purchases, whereas they use alternative means of payment for larger ones. They

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substitute other means of payment for money if the nominal interest rate increases, but they prefer using cash if transactions costs of alternative means of payment increase. Further, in Ireland (1994) and Lacker and Schreft (1996), monetary policy affects a variety of goods purchased with cash and with other means of payment. However, these are representative agent models. Monetary policy delivers the same effects across individuals who make the same choice of means of payment.

A market segmentation model is useful for studying the distributional effects of monetary policy, because money is nonneutral. When the asset market is segmented, only a fraction of economic individuals receive the money injection from the government in the first round. Money becomes nonneutral in equilibrium and monetary policy can redistribute consumption goods among individuals. Limited participation models were initially developed by Grossman and Weiss (1983) and Rotemberg (1984) and extended by Lucas (1990) and Fuerst (1992). Recently, Alvarez and Atkeson (1997) and Alvarez et al. (2001, 2002) have made important contributions. Several features are in common among them, including Choi (2009). There are two types of households: traders and nontraders. In the asset market, traders exchange government nominal bonds and money and the government controls the money supply through open market purchases of interest-bearing government bonds. Thus, only traders receive the money injection in the asset market. In equilibrium, monetary policy has a redistributional effect between traders and nontraders. A positive money injection increases traders' consumption and decreases nontraders' consumption. The nominal interest rate decreases as the money stock increases; i.e., the liquidity effect arises. However, most limited participation models are built to explain the behavior of asset prices and the exchange rate in the short run instead of the choice of multiple means of payment.

Williamson (2008, 2009) has developed a new market segmentation model where both the goods market and the asset markets are segmented and cash and credit are introduced. However, credit is the sole medium of exchange for consumption goods in the goods market and outside money plays a role in clearing and settlement. Thus, his model does not show the choice of means of payment and the distributional effects of monetary policy on the choice of means of payment.

This paper extends the existing asset market segmentation model of Alvarez et al. (2001) using the approach of Ireland (1994). Some households are traders and the others are nontraders. Traders participate in the asset market, whereas nontraders do not. In the goods market each household can use either cash or credit to purchase consumption goods.

In equilibrium, monetary policy has distributional effects on the choice of means of payment and on consumption between traders and nontraders. In contrast to Alvarez et al. (2001), consumption for the household may increase or decrease with the money growth rate because there are two distributional effects of monetary policy: a direct effect and an indirect effect via the choice of means of payment.

Suppose the government injects money. Then traders can increase their consumption with cash because they receive the money injection in the asset market, whereas nontraders cannot—that is, the direct effect.

The indirect effect comes through the choice of means of payment, which may be different between traders and nontraders. When the money growth rate is constant, both traders and nontraders spend more on credit if the money growth rate increases. On the other hand, suppose the money growth rate is stochastic. Then, if the money growth rate increases, traders prefer using cash, whereas nontraders prefer using credit. Because traders hold more cash, they purchase a greater variety of consumption goods with cash. However, nontraders do not hold more cash and they purchase a larger variety of goods with credit.

Given the direct and indirect effects, consumption by traders and nontraders may increase or decrease. Thus, credit may be used to dampen fluctuations in consumption arising from monetary policy. The optimal money growth rate is negative with constant money growth, whereas it may be either positive or negative with stochastic money growth. The Friedman rule is not optimal in general.

The remainder of the paper is organized as follows. Section 2 describes the basic environment and Section 3 shows the equilibrium dynamics. Sections 4 and 5 study monetary policy implications for the nominal interest rate, the choice of means of payment, and consumption when the money growth rate is constant or stochastic. In Section 6, a creditless economy is introduced and the effect of monetary policy on the economies with credit and without credit will be discussed. Section 7 concludes.

2. THE ENVIRONMENT AND TIMING

Time is discrete and indexed by $t = 0, 1, 2, \ldots$. There is a continuum of infinitely lived households with unit mass indexed by $i \in [0, 1]$. Each household consists of a shopper and a worker. A fraction α of the households are traders and the rest, $1-\alpha$, are nontraders. There is a continuum of spatially separated markets indexed by $i \in [0,1]$ in each period. The household has preferences given by

$$U\left(\{c_t, x_t\}_{t=0}^{\infty}\right) = E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \int_0^1 \ln[c_t(i)]di - x_t \right\},$$

where E_0 is the expectation operator conditional on information in period 0, β is the discount factor, $c_t(i)$ represents consumption goods purchased at market i in period t and they are distinct and perishable, and x_t represents transaction costs.

At the beginning of each period t, traders enter the period with $M_{r,t}$ units of currency and B_t units of one-period nominal bonds. Nontraders enter the period with $M_{n,t}$ units of currency. Both traders and nontraders receive the same endowments, y, and they do not consume their own endowments.

Whereas nontraders stay at home, traders go to the asset market and exchange one-period government nominal bonds and money. Each bond sells for q_t units of money in period t and is a claim to one unit of money in period $t+1$. In the asset market, the government controls the money supply, M_t^s, through open market operations, where nominal bonds that are issued in period $t-1$ and mature in period t are denoted by $\bar{B}_t$. Thus, the government budget constraint is

$$\bar{B}_t - q_t \bar{B}_{t+1} = M^s_{t+1} - M^s_t, \tag{1}$$

$$M^s_{t+1} = (1 + \mu_t) M^s_t,$$

where $\bar{B}_{t+1}$ is newly issued nominal bonds with price q_t that mature at period $t+1$ and $\mu_t > -1$ is the net money growth rate.

Once the asset market is closed, traders and nontraders go to the goods market. Workers sell consumption goods to shoppers. Assume that every worker has same technology to convert endowment into good i at any market i. Shoppers travel from market to market to purchase them. There are two ways of acquiring goods for shoppers in each market i. One is to use non–interest bearing currency, which has the gross nominal interest rate as an opportunity cost. The other is to use credit, incurring transaction costs, $\gamma(i) > 0$. The transaction costs, $\gamma(i)$, are increasing and differentiable on i, $\gamma(0) = 0$, and $\lim_{i\to 1} \gamma(i) = \infty$. Credit becomes infinitely costly if shoppers use it in every market i. For example, if shoppers keep using credit from market to market, then markets farther away from a shopper's home are less willing to receive credit. This may create a technical difficulty of verifying the user's credit history. Transactions costs take the form of effort,

$$x_t(i) = \int_0^1 \xi_t(i)\gamma(i)\, di,$$

where $\xi_t(i)$ is an indicator variable: $\xi_t(i) = 1$ if shoppers use credit to buy good i at period t and $\xi_t(i) = 0$ if shoppers use currency. The cash-in-advance constraints in the goods markets of traders and nontraders are

$$\int_0^1 P_t(i)[1 - \xi_{r,t}(i)]c_{r,t}(i)di \le M_{r,t} + B_t - q_t B_{t+1},$$

$$\int_0^1 P_t(i)[1 - \xi_{n,t}(i)]c_{n,t}(i)di \le M_{n,t},$$

where $c_{r,t}(i)$ is consumption of good i purchased by a trader in period t, $c_{n,t}(i)$ is consumption good i purchased by a nontrader in period t, and $P_t(i)$ is the price of consumption good i at period t.

At the end of each period, all agents return home. Workers receive the revenue from sales, $P_t y$. No further trade or barter is allowed. The budget constraints of traders and nontraders are

$$\int_0^1 P_t(i)c_{r,t}(i)\, di + M_{r,t+1} = M_{r,t} + B_t - q_t B_{t+1} + P_t y,$$

$$\int_0^1 P_t(i)c_{n,t}(i)\, di + M_{n,t+1} = M_{n,t} + P_t y,$$

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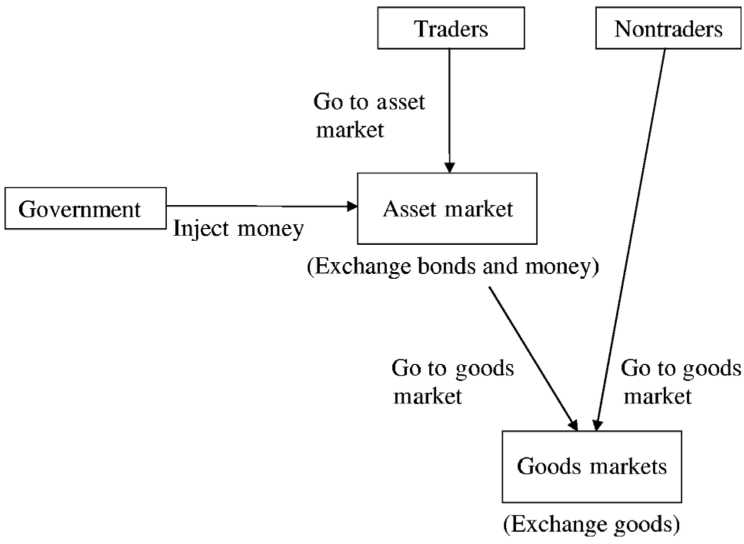

FIGURE 1. Timeline in period t.

where P_t is the average price level of consumption goods. Figure 1 summarizes the timeline of events within a period.

3. EQUILIBRIUM DYNAMICS

3.1. Optimization

Traders solve the optimization problem

$$\max_{c_{r,t}, x_{r,t}, \xi_{r,t}, M_{r,t+1}, B_{t+1}} E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \int_0^1 \ln[c_{r,t}(i)]\, di - x_{r,t} \right\}$$

subject to

$$\int_0^1 P_t(i)[1 - \xi_{r,t}(i)] c_{r,t}(i)\, di \leq M_{r,t} + B_t - q_t B_{t+1}, \tag{2}$$

$$\int_0^1 P_t(i) c_{r,t}(i)\, di + M_{r,t+1} = M_{r,t} + B_t - q_t B_{t+1} + P_t y, \tag{3}$$

$$x_{r,t} = \int_0^1 \xi_{r,t}(i) \gamma(i)\, di,$$

$$M_{r,t+1} \geq 0, \quad B_{t+1} \geq \bar{b}, \quad \text{where } \bar{b} \leq 0, \tag{4}$$

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where inequalities (4) are the nonnegativity constraints and the no–Ponzi scheme constraint. Now, nontraders solve

$$\max_{c_{n,t},x_{n,t},\xi_{n,t},M_{n,t+1}} E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \int_0^1 \ln[c_{n,t}(i)]di - x_{n,t} \right\}$$

subject to

$$\int_0^1 P_t(i)[1-\xi_{n,t}(i)]c_{n,t}(i)di \leq M_{n,t}, \tag{5}$$

$$\int_0^1 P_t(i)c_{n,t}(i)di + M_{n,t+1} = M_{n,t} + P_t y, \tag{6}$$

$$x_{n,t} = \int_0^1 \xi_{n,t}(i)\gamma(i)di,$$

$$M_{n,t+1} \geq 0, \tag{7}$$

where inequalities (7) are the nonnegativity constraints.

DEFINITION. *A* symmetric competitive equilibrium *consists of the sequences* $\{c_{j,t}(i), \xi_{j,t}, M_{j,t+1}, B_{t+1}, M_t^s, P_t(i), q_t\}_{t=0}^{\infty}$, *where* $i \in [0,1]$ *and* $j \in \{r,n\}$ *such that*

(1) $\{c_{j,t}(i), \xi_{j,t}, M_{j,t+1}, B_{t+1}\}_{t=0}^{\infty}$ *solves the household problems of traders and nontraders given* $\{M_t^s, P_t(i), q_t\}_{t=0}^{\infty}$ *for all market* i.

(2) *Markets clear in every period:*

(a) *Bond market: each trader exchanges money and bonds such as*

$$B_t - q_t B_{t+1} = \frac{\mu_t M_t}{\alpha}.$$

(b) *Money market: for all* t,

$$M_{t+1}^s = M_{t+1} = \alpha M_{r,t+1} + (1-\alpha)M_{n,t+1}.$$

(c) *Goods market: for each market* i,

$$\alpha c_{r,t}(i) + (1-\alpha)c_{n,t}(i) = y,$$

and in the aggregate,

$$\alpha \int_0^1 c_{r,t}(i)di + (1-\alpha)\int_0^1 c_{n,t}(i)\,di = y.$$

3.2. Equilibrium

Suppose $\lambda_{j,t}^1$ and $\lambda_{j,t}^2$, where $j \in \{r,n\}$ denote the Lagrange multipliers associated with the cash-in-advance constraint and the budget constraint, respectively, for

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traders and nontraders at period t. Then, in equilibrium, the choices of traders and nontraders for $c_{j,t}(i)$, $\xi_{j,t}$, $M_{j,t+1}$, and B_{t+1} are as follows:

$$\frac{1}{c_{j,t}(i)} - \lambda^1_{j,t}[1 - \xi_{j,t}(i)]P_t - \lambda^2_{j,t}P_t = 0, \tag{8}$$

$$\frac{1}{c^1_{j,t}(i)} - \lambda^2_{j,t}P_t = 0, \qquad \text{if } \xi_{j,t}(i) = 1, \tag{9}$$

$$\frac{1}{c^0_{j,t}(i)} - \left(\lambda^1_{j,t} + \lambda^2_{j,t}\right)P_t = 0, \quad \text{if } \xi_{j,t}(i) = 0, \tag{10}$$

where $c^1_{j,t}(i)$ is consumption with credit by traders or nontraders at market i and $c^0_{j,t}(i)$ is consumption with cash by traders or nontraders at market i;

$$\xi_{j,t}(i) = \begin{cases} 1, & \text{if } \ln\left[c^1_{j,t}(i)\right] - \gamma(i_{j,t}) - \lambda^2_{j,t}c^1_{j,t}(i)P_t > \ln\left[c^0_{j,t}(i)\right] - c^0_{j,t}(i)\left(\lambda^1_{j,t} + \lambda^2_{j,t}\right)P_t, \\ 0, & \text{if } \ln\left[c^1_{j,t}(i)\right] - \gamma(i_{j,t}) - \lambda^2_{j,t}c^1_{j,t}(i)P_t < \ln\left[c^0_{j,t}(i)\right] - c^0_{j,t}(i)\left(\lambda^1_{j,t} + \lambda^2_{j,t}\right)P_t; \end{cases} \tag{11}$$

$$\beta E_t\left[\left(\lambda^1_{j,t+1} + \lambda^2_{j,t+1}\right) \mid \mu_t\right] = \lambda^2_{j,t}; \tag{12}$$

$$\beta E_t\left[\left(\lambda^1_{r,t+1} + \lambda^2_{r,t+1}\right) \mid \mu_t\right] = q_t\left(\lambda^1_{r,t} + \lambda^2_{r,t}\right). \tag{13}$$

As in Ireland (1994), the assumption that every worker has same technology to convert endowment into good i at any market i, along with the assumption that perfect competition prevails in all goods markets,[1] implies that in equilibrium, for each market i, $P_t(i) = P_t$ holds.[2] In equation (8), the marginal utility of consumption is identical, $c_{j,t}(i)P_t(i) = c_{j,t}(k)P_t(k)$, for any market $i, k \in [0, 1]$. Thus, in order to consume y, that is, $c_{j,t}(i) = y$ for all i, each market sells consumption goods at the same price, $P_t(i) = P_t$.

In equation (9), consumption with credit is equal among traders, $c^1_{r,t} = c^1_{r,t}(i)$, and nontraders, $c^1_{n,t} = c^1_{n,t}(i)$, because the marginal value of wealth is the same across markets. Similarly, in equation (10), consumption with cash is equal among traders, $c^0_{r,t} = c^0_{r,t}(i)$, and nontraders, $c^0_{n,t} = c^0_{n,t}(i)$, because the marginal value of cash is the same across markets.

In equations (10) and (12), the cash-in-advance constraints of traders and nontraders bind, i.e., $\lambda^1_{r,t} > 0$ and $\lambda^1_{n,t} > 0$, if for $j \in \{r, n\}$,

$$\beta E_t\left[\frac{c^0_{j,t}}{c^0_{j,t+1}}\frac{P_t}{P_{t+1}}\right] < 1, \tag{14}$$

where in equation (13), traders face a positive nominal interest rate, $q_t < 1$, in the asset market.

Assuming binding cash-in-advance constraints, equations (9) and (10) imply that both traders and nontraders use credit for larger purchases and use cash for smaller purchases:

$$\frac{c^0_{j,t}}{c^1_{j,t}} = \frac{\lambda^2_{j,t}}{\lambda^1_{j,t} + \lambda^2_{j,t}} < 1, \tag{15}$$

where $\lambda^1_{j,t} > 0$ and $\lambda^2_{j,t} > 0$.

In equation (11), the choice of credit or cash depends on the trade-off between the opportunity cost of money, the nominal interest rate, and the transaction costs of credit. In other words, the choice of credit or cash is determined if transactions costs are equal to the marginal rate of substitution of $c^0_{j,t}$ for $c^1_{j,t}$,

$$\gamma(i^*_{j,t}) = \ln\left(\frac{c^1_{j,t}}{c^0_{j,t}}\right) \in (0, \infty). \tag{16}$$

Given inequality (15) and equation (16), the cutoffs of the credit–cash choices of traders and nontraders,

$$i^*_{j,t} \in (0, 1),$$

imply the coexistence of money and credit in the economy. Shoppers use credit to acquire good i for $i < i^*_{j,t}$ and they use cash to acquire good i for $i > i^*_{j,t}$. The aggregate resource constraint is

$$\alpha\left\{i^*_{r,t}c^1_{r,t} + (1 - i^*_{r,t})c^0_{r,t}\right\} + (1 - \alpha)\left\{i^*_{n,t}c^1_{n,t} + (1 - i^*_{n,t})c^0_{n,t}\right\} = y, \tag{17}$$

where $i^*_{r,t}c^1_{r,t}$ is the aggregate consumption purchased with credit by traders and $(1 - i^*_{r,t})c^0_{r,t}$ is the aggregate consumption purchased with cash by traders. Similarly, $i^*_{n,t}c^1_{n,t}$ is the aggregate consumption purchased with credit by nontraders and $(1 - i^*_{n,t})c^0_{n,t}$ is the aggregate consumption purchased with cash by nontraders.

The binding cash-in-advance constraint and the budget constraint of traders in equations (2) and (3) are as follows:

$$P_t(1 - i^*_{r,t})c^0_{r,t} = M_{r,t} + \frac{\mu_t M_t}{\alpha}, \tag{18}$$

$$P_t i^*_{r,t}c^1_{r,t} + M_{r,t+1} = P_t y. \tag{19}$$

Similarly, in equations (5) and (6), those of nontraders are

$$P_t(1 - i^*_{n,t})c^0_{n,t} = M_{n,t}, \tag{20}$$

$$P_t i^*_{n,t}c^1_{n,t} + M_{n,t+1} = P_t y. \tag{21}$$

The term $\mu_t M_t/\alpha$ in equation (18) implies that money is nonneutral and a change in the money stock affects consumption between traders and nontraders in equilibrium. Traders initially receive the government money injection in the asset market, but nontraders do not. Thus, consumption for traders and nontraders change with the money injection.

At the end of the goods market, the distributional effect induced by the money injection disappears because every household receives the same revenue from sales, $P_t y$. Owing to credit payoffs, however, traders and nontraders will bring only a fraction of their revenue into the next period in the form of cash and the fraction would be different between traders and nontraders.

Suppose $\phi_{r,t} \in (0, 1)$ denotes the fraction of the revenue that traders bring to the next period in the form of cash and $\phi_{n,t} \in (0, 1)$ denotes the fraction of the revenue that nontraders bring to the next period. Then the real money holdings of traders and nontraders are

$$\frac{M_{j,t+1}}{P_t} = \phi_{j,t} y, \tag{22}$$

where $j \in \{r, n\}$ and in equilibrium, $\phi_{j,t}$ is determined by equation (12). Aggregate money demand is

$$\frac{M_{t+1}}{P_t} = \Phi_t y, \tag{23}$$

where

$$\Phi_t = \alpha\phi_{r,t} + (1-\alpha)\phi_{n,t} \in (0, 1),$$

and the inflation rate is

$$\frac{P_t}{P_{t-1}} = (1+\mu_t)\frac{\Phi_{t-1}}{\Phi_t}. \tag{24}$$

Given equations (18)–(24), consumption for traders and nontraders can be affected by two ways. One is through the redistributional effects of monetary policy, which transfers consumption goods between traders and nontraders as in Alvarez et al. (2001). The redistribution of consumption will be considered as a direct effect. Due to the asset market segmentation, if the government injects money, then traders receive extra cash from the asset market, whereas nontraders do not. Thus, consumption with cash for traders would increase and consumption with cash for nontraders would decrease as in Alvarez et al. (2001) and Choi (2009).

The other is through the choice of credit or cash, $i^*_{j,t}$. The existence of multiple means of payment allows shoppers to choose the one that is more beneficial, and it affects consumption with credit and with cash. The effect of a change in the cash–credit choice on consumption will be considered an indirect effect that does not show up in the economy of Alvarez et al. (2001) and Choi (2009).

4. CONSTANT MONEY GROWTH

This section will study the distributional effects of monetary policy when the money growth rate is constant. Suppose that $\mu_t = \mu$ for all t. Then the economy is deterministic and the variables become constant for all t. In equations (12), (16), and (18)–(24), traders' choices are

$$(1 - i_r^*)c_r^0 = \left\{ \frac{\phi_r + (\mu\Phi/\alpha)}{1+\mu} \right\} y, \tag{25}$$

$$i_r^* c_r^1 = (1 - \phi_r)y,$$

$$\frac{i_r^*}{(1 - i_r^*)} e^{\gamma(i_r^*)} = \frac{(1 - \phi_r)(1+\mu)}{\phi_r + (\mu\Phi/\alpha)}, \tag{26}$$

$$1 = \frac{\beta}{1+\mu}\left(\frac{c_r^1}{c_r^0}\right), \tag{27}$$

where in equation (25)

$$\mu \geq -\frac{\alpha\phi_r}{\Phi}$$

and

$$\Phi = \alpha\phi_r + (1-\alpha)\phi_n.$$

Nontraders' choices are

$$(1 - i_n^*)c_n^0 = \frac{\phi_n}{1+\mu} y, \tag{28}$$

$$i_n^* c_n^1 = (1 - \phi_n)y,$$

$$\frac{i_n^*}{(1 - i_n^*)} e^{\gamma(i_{n,t}^*)} = \frac{(1 - \phi_n)(1+\mu)}{\phi_n}, \tag{29}$$

$$1 = \frac{\beta}{1+\mu}\left(\frac{c_n^1}{c_n^0}\right). \tag{30}$$

When the money growth rate is constant, the liquidity effect disappears and only the Fisherian effect remains from equation (13),

$$q = \frac{\beta}{1+\mu}, \tag{31}$$

where the nominal interest rate is positive if $\mu > \beta - 1$.

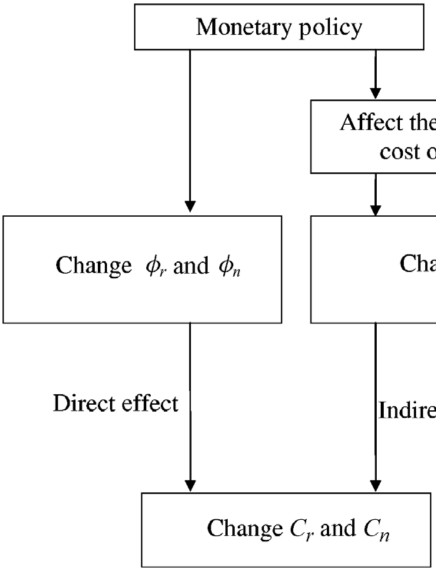

FIGURE 2. The transmission path of the constant monetary policy.

In equations (16), (27), and (30), the choice of cash or credit is identical between traders and nontraders,

$$i^* = i_r^* = i_n^*.$$

Traders and nontraders face the same trade-off between the opportunity cost of money, $1/q$, in equation (31) and the transaction costs of credit, where the intertemporal marginal rate of substitution is

$$\gamma(i^*) = \ln\left(\frac{1+\mu}{\beta}\right). \tag{32}$$

In equations (26), (29), and (32), the real money balances of traders, ϕ_r, and nontraders, ϕ_n, are determined by the following equations:

$$\gamma\left[\frac{1}{\frac{\phi_r + (\mu\Phi/\alpha)}{\beta(1-\phi_r)} + 1}\right] = \ln\left(\frac{1+\mu}{\beta}\right), \tag{33}$$

$$\gamma\left[\frac{1}{\frac{\phi_n}{\beta(1-\phi_n)} + 1}\right] = \ln\left(\frac{1+\mu}{\beta}\right), \tag{34}$$

where traders and nontraders hold different amounts of money in general.

Figure 2 describes the transmission path of monetary policy, that is, a change of the money growth rate, when the money growth rate is constant.

Monetary policy affects consumption for traders and nontraders in two ways. One is through redistribution of consumption goods, which is the direct effect. The other is through the choice of credit or cash in equation (32), which is the indirect effect. The change of the choice of credit or cash is driven by the trade-off between the opportunity cost of cash and the transactions costs of credit.

4.1. Distributional Effects on the Credit–Cash Choice

When the government injects money, in equation (31) and (32), both traders and nontraders prefer using credit over money for a greater variety of consumption goods:

$$\frac{\partial i^*}{\partial \mu} = \frac{1}{(1+\mu)\gamma'(i^*)} > 0. \tag{35}$$

In equations (32)–(35), traders' money holding, ϕ_r, may increase or decrease with μ and nontraders' money holding, ϕ_n, decrease:[3]

$$\frac{\partial \phi_r}{\partial \mu} > 0 \quad \text{or} \quad < 0, \tag{36}$$

$$\frac{\partial \phi_n}{\partial \mu} < 0, \tag{37}$$

which implies the effect of μ on aggregate money holding, $\partial \Phi / \partial \mu$, is ambiguous.

The money injection initially goes to traders and in equation (31) the nominal interest rate increases with the money growth rate. Traders hold more cash and use credit for a greater variety of goods in order to acquire nominal interest; this is the direct effect. Because traders tend to use credit more often, it may lead traders to reduce money holding, that is, the indirect effect. The direct effect and the indirect effect go to opposite directions and in inequality (36) traders may increase or decrease their money holding to the next period.

On the other hand, nontraders do not receive the money injection that increases the price of goods. They tend to hold more cash within the period and use credit for a larger variety of goods in order to compensate for the loss of consumption. Thus, in inequality (37), the net effect of monetary policy simply reduces the money holding of nontraders to the next period.

4.2. Distributional Effects on Trader's Choices

Because the effect of monetary policy on the traders' money balance is ambiguous, first, the net effect on consumption with cash, c_r^0, may increase or decrease:

$$\frac{\partial c_r^0}{\partial \mu} = \frac{\partial \left[\frac{(1-i^*)c_r^0}{1-i^*}\right]}{\partial \mu} = \frac{1}{[1-i^*]^2}\left\{\underbrace{\frac{\partial\left[(1-i^*)c_r^0\right]}{\partial \mu}(1-i^*)}_{\text{direct effect}} \underbrace{+(1-i^*)c_r^0\frac{\partial i^*}{\partial \mu}}_{\text{indirect effect}}\right\}. \tag{38}$$

Given inequalities (36) and (37), the effect on aggregate consumption with cash,

$$\frac{\partial\,(1-i^*)\,c_r^0}{\partial \mu} = \left[\frac{\partial \phi_r}{\partial \mu} + \left(\frac{1-\alpha}{\alpha}\right)\left\{\frac{\partial \phi_n}{\partial \mu}\left(\frac{\mu}{1+\mu}\right) + \phi_n\right\}\right] y,$$

implies that the direct effect is ambiguous, whereas the indirect effect via a change in the choice of credit or cash is positive.

Next, the net effect on consumption with credit, c_r^1, is ambiguous as well:

$$\frac{\partial c_r^1}{\partial \mu} = \frac{\partial\left[\frac{i^* c_r^1}{i^*}\right]}{\partial \mu} = \frac{1}{(i^*)^2}\left\{\underbrace{\frac{\partial\left(i^* c_r^1\right)}{\partial \mu} i^*}_{\text{direct effect}} \underbrace{-(1-\phi_r) y \frac{\partial i^*}{\partial \mu}}_{\text{indirect effect}}\right\}.$$

Given inequalities (36), the effect on aggregate consumption with credit implies that the direct effect is ambiguous, whereas the indirect effect via a change in the choice of credit or cash is positive.

4.3. Distributional Effects on Nontrader's Choices

Similarly, the net effect on consumption with cash, $c_{n,t}^0$, is ambiguous:

$$\frac{\partial c_n^0}{\partial \mu} = \frac{\partial \left[\frac{(1-i^*)c_n^0}{1-i^*}\right]}{\partial \mu} = \frac{1}{(1-i^*)^2}\left\{\underbrace{\frac{\partial\left[(1-i^*)c_n^0\right]}{\partial \mu}(1-i^*)}_{\text{direct effect}} \underbrace{+(1-i^*)c_n^0\frac{\partial i^*}{\partial \mu}}_{\text{indirect effect}}\right\}. \tag{39}$$

Given inequality (37), the effect on aggregate consumption with cash,

$$\frac{\partial\,(1-i^*)\,c_n^0}{\partial \mu} = \frac{y}{(1+\mu)^2}\left\{\frac{\partial \phi_n}{\partial \mu}(1+\mu) - \phi_n\right\} < 0,$$

implies that the direct effect is negative, whereas the indirect effect via the choice of credit or cash is positive.

Now, the net effect on consumption with credit by nontraders, c_n^1, is ambiguous as well:

$$\frac{\partial c_n^1}{\partial \mu} = \frac{\partial \left[\frac{i^* c_n^1}{i^*}\right]}{\partial \mu} = \frac{1}{(i^*)^2}\left\{\underbrace{\frac{\partial \left(i^* c_n^1\right)}{\partial \mu} i^*}_{\text{direct effect}} \underbrace{-(1-\phi_n) y \frac{\partial i^*}{\partial \mu}}_{\text{indirect effect}}\right\}.$$

Given inequality (37), the effect on aggregate consumption with credit implies that the direct effect is positive, but the indirect effect via a change in the choice of credit or cash is negative.

4.4. Welfare and Optimal Money Growth

Given the effects of monetary policy on traders' and nontraders' choices, the effects of monetary policy on welfare are as follows. Traders' welfare is

$$\begin{aligned} W_r &= i^* \ln\left(c_r^1\right) + (1-i^*)\ln\left(c_r^0\right) - \int_0^{i^*} \gamma\,(i)\, di \\ &= \ln\left(c_r^0\right) + i^*\gamma(i^*) - \int_0^{i^*} \gamma\,(i)\, di. \end{aligned}$$

The effect of monetary policy at $\mu = 0$ on traders' welfare[4] is

$$\left.\frac{\partial W_r}{\partial \mu}\right|_{\mu=0} = \phi\left(\frac{(\beta-1)\{\phi+\beta(1-\phi)\}}{\phi\beta\gamma'(i^*)} - \frac{1}{\phi+\beta(1-\phi)} + \frac{1}{\alpha}\right), \qquad \textbf{(40)}$$

where $\phi_r = \phi_n = \phi$ and $i_r^* = i_n^* = i^*$. The welfare may reach its peak with a positive money growth rate. The traders' participation in the asset market can have an ambiguous effect on traders' welfare. First, a money injection redistributes consumption goods from nontraders and traders. However, traders spend on credit for a greater variety of goods. Thus, consumption with cash, c_r^0, and with credit, c_r^1, may increase or decrease. Thus, the optimal money growth rate for traders can be either positive or negative and the Friedman rule is not optimal.

Nontraders' welfare is

$$\begin{aligned} W_n &= i^* \ln\left(c_n^1\right) + (1-i^*)\ln\left(c_n^0\right) - \int_0^{i^*} \gamma\,(i)\, di \\ &= \ln\left(c_n^0\right) + i^*\gamma(i^*) - \int_0^{i^*} \gamma\,(i)\, di. \end{aligned}$$

The effect of monetary policy at $\mu = 0$ on nontraders' welfare[5] is

$$\left.\frac{\partial W_n}{\partial \mu}\right|_{\mu=0} = \frac{\phi\,(\beta-1)\{\phi+\beta(1-\phi)\}}{\phi\beta\gamma'(i^*)} - \frac{\phi}{\phi+\beta(1-\phi)} < 0, \qquad \textbf{(41)}$$

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where $\phi_r = \phi_n = \phi$ and $i_r^* = i_n^* = i^*$. The welfare is maximized when the money growth rate is negative. Unlike traders, a negative money growth rate redistributes consumption goods from traders to nontraders and decreases transactions costs because nontraders use cash for a greater variety of goods. Thus, nontraders benefit from the negative money growth rate, which is optimal. The Friedman rule is optimal.

Finally, welfare of the economy is defined as

$$W = \alpha W_r + (1-\alpha) W_n.$$

From equations (40) and (41) the effect of monetary policy at $\mu = 0$ on welfare[6] is

$$\begin{aligned}\left.\frac{\partial W}{\partial \mu}\right|_{\mu=0} &= \alpha \left.\frac{\partial W_r}{\partial \mu}\right|_{\mu=0} + (1-\alpha)\left.\frac{\partial W_n}{\partial \mu}\right|_{\mu=0} \\ &= \frac{1}{\phi}\left(\alpha \left.\frac{\partial \phi_r}{\partial \mu}\right|_{\mu=0} + (1-\alpha)\left.\frac{\partial \phi_n}{\partial \mu}\right|_{\mu=0}\right) + \frac{1}{(1-i^*)\,\gamma'(i^*)} + i^* \\ &= \phi(\beta-1)\left\{\frac{\phi+\beta(1-\phi)}{\phi\beta\gamma'(i^*)} + \frac{1-\phi}{\phi+\beta(1-\phi)}\right\} < 0. \end{aligned} \quad \textbf{(42)}$$

Welfare is maximized with a negative money growth rate, although traders' welfare may not be maximized in equation (40). When the money growth rate is negative, given inequality (35), credit is used for a smaller variety of goods and the transaction costs of credit decreases. Thus, when credit and cash exist and the money growth rate is constant, in inequalities (35)–(37) an increase in nontraders' money holding and a decrease in the credit–cash choice improve welfare although a change in traders' money holding is ambiguous. The optimal money growth rate is negative and the Friedman rule is optimal.

5. STOCHASTIC MONEY GROWTH

Now, assume the money growth rate changes in a random manner. The comparison of monetary policy between Sections 4 and 5 will have interesting implications. Suppose that the money growth rate, μ_t, is independent and identically distributed. Then, from equations (18)–(23), traders' aggregate consumption with cash and with credit is

$$(1-i_{r,t}^*)c_{r,t}^0 = \frac{\phi_{r,t-1} + (\mu_t \Phi_{t-1}/\alpha)}{1+\mu_t}\left(\frac{\Phi_t}{\Phi_{t-1}}\right) y, \quad \textbf{(43)}$$

$$i_{r,t}^* c_{r,t}^1 = (1-\phi_{r,t})y, \quad \textbf{(44)}$$

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where

$$\mu_t > -\frac{\alpha\phi_{r,t-1}}{\Phi_{t-1}}.$$

Given equations (12), (43), and (44), the choice of credit or cash, $i^*_{r,t}$ or $\phi_{r,t}$, are determined. First, equations (43) and (44) imply that

$$\left(\frac{i^*_{r,t}}{1-i^*_{r,t}}\right)e^{\gamma(i^*_{r,t})} = \frac{(1-\phi_{r,t})(1+\mu_t)}{\phi_{r,t-1}+(\mu_t\Phi_{t-1}/\alpha)}\left(\frac{\Phi_{t-1}}{\Phi_t}\right). \quad \textbf{(45)}$$

Next, equations (12),

$$1 = \beta E_t\left[\frac{c^1_{r,t}}{c^0_{r,t+1}}\left(\frac{\Phi_{t+1}}{\Phi_t}\frac{1}{1+\mu_{t+1}}\right)\right],$$

and (44) imply that

$$i^*_{r,t} = \beta\left(\frac{1-\phi_{r,t}}{\Phi_t}\right)y\Psi_r, \quad \textbf{(46)}$$

where Ψ_r is constant,

$$\Psi_r = E_t\left[\frac{\Phi_{t+1}}{c^0_{r,t+1}(1+\mu_{t+1})}\right].$$

Similarly, nontraders' aggregate consumption with cash and with credit is

$$(1-i^*_{n,t})c^0_{n,t} = \frac{\phi_{n,t-1}}{1+\mu_t}\left(\frac{\Phi_t}{\Phi_{t-1}}\right)y, \quad \textbf{(47)}$$

$$i^*_{n,t}c^1_{n,t} = (1-\phi_{n,t})y. \quad \textbf{(48)}$$

The choices of credit or cash, $i^*_{n,t}$ and $\phi_{n,t}$, are determined given equations (12),

$$1 = \beta E_t\left[\frac{c^1_{n,t}}{c^0_{n,t+1}}\left(\frac{\Phi_{t+1}}{\Phi_t}\frac{1}{1+\mu_{t+1}}\right)\right],$$

(47), and (48) such as

$$\left(\frac{i^*_{n,t}}{1-i^*_{n,t}}\right)e^{\gamma(i^*_{n,t})} = \frac{(1-\phi_{n,t})(1+\mu_t)}{\phi_{n,t-1}}\left(\frac{\Phi_{t-1}}{\Phi_t}\right), \quad \textbf{(49)}$$

$$i^*_{n,t} = \beta\left(\frac{1-\phi_{n,t}}{\Phi_t}\right)y\Psi_n, \quad \textbf{(50)}$$

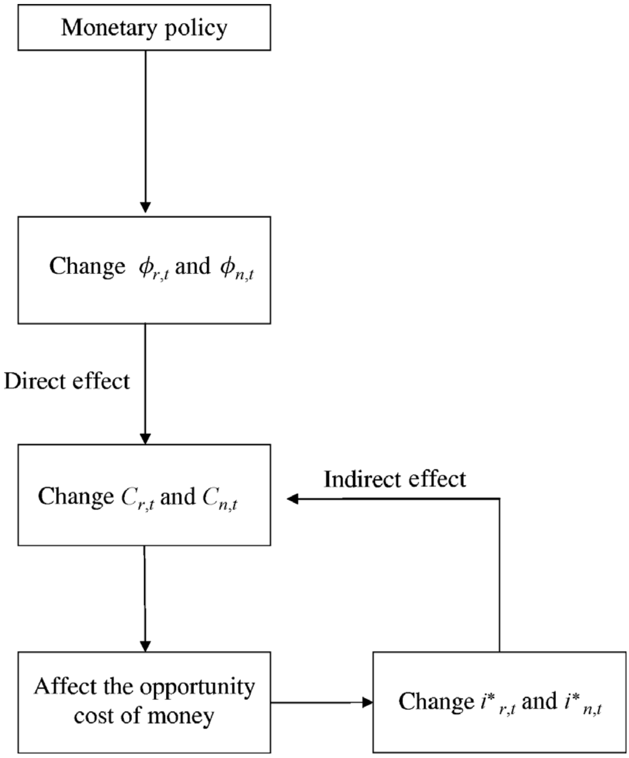

FIGURE 3. The transmission path of the stochastic monetary policy.

where Ψ_n is constant,

$$\Psi_n = E_t\left[\frac{\Phi_{t+1}}{c^0_{n,t+1}(1+\mu_{t+1})}\right].$$

When the money growth rate is stochastic, the transmission path is as in Figure 3. Monetary policy affects consumption for traders and nontraders in two ways: through the direct redistribution of consumption goods, as in Figure 2, and through the choice of credit or cash.

However, unlike Figure 2, the change in choice of credit or cash is driven by redistribution of consumption goods between traders and nontraders. The change in consumption with cash has an effect on the opportunity cost of money, because money is nonneutral. In equations (12) and (14), the opportunity costs of money for traders and nontraders consist of the intertemporal marginal rate of substitution between current consumption with cash and future consumption with cash: for traders,

$$q_t = \beta E_t \left[\frac{c^0_{r,t}}{c^0_{r,t+1}} \left(\frac{1}{1+\mu_{t+1}} \right) \left(\frac{\Phi_{t+1}}{\Phi_t} \right) \right] < 1, \tag{51}$$

where q_t is the nominal interest rate, and for nontraders,

$$\beta E_t \left[\frac{c^0_{n,t}}{c^0_{n,t+1}} \left(\frac{1}{1+\mu_{t+1}} \right) \left(\frac{\Phi_{t+1}}{\Phi_t} \right) \right] < 1. \tag{52}$$

Monetary policy has a distributional effect on $c^0_{r,t}$ and $c^0_{n,t}$ in equations (43) and (47) and it also affects the intertemporal marginal rate of substitution and the opportunity cost of money.

5.1. Distributional Effects on the Credit–Cash Choice

The choices of credit or cash for traders and nontraders can be solved by inserting equation (46) into equation (45),[7]

$$\frac{e^{\gamma \left[\beta \left(\frac{1-\phi_{r,t}}{\Phi_t} \right) y\Psi_r \right]}}{\frac{1}{\beta y \Psi_r} - \frac{1-\phi_{r,t}}{\Phi_t}} = \frac{1+\mu_t}{\frac{\phi_{r,t-1}}{\Phi_{t-1}} + \frac{\mu_t}{\alpha}} \tag{53}$$

and inserting equations (50) into (49):[8]

$$\frac{e^{\gamma \left[\beta \left(\frac{1-\phi_{n,t}}{\Phi_t} \right) y\Psi_n \right]}}{\frac{1}{\beta y \Psi_n} - \frac{1-\phi_{n,t}}{\Phi_t}} = \frac{(1+\mu_t)\Phi_{t-1}}{\phi_{n,t-1}}. \tag{54}$$

Given equations (53) and (54), the effects of monetary policy on $\phi_{r,t}$ and $\phi_{n,t}$ are[9]

$$\frac{\partial \phi_{r,t}}{\partial \mu_t} > 0, \tag{55}$$

$$\frac{\partial \phi_{n,t}}{\partial \mu_t} < 0. \tag{56}$$

When the government injects money, traders receive it in the asset market, but nontraders do not. Traders can increase the real money holding, $\phi_{r,t}$, in the next period, whereas nontraders reduce the real money holding, $\phi_{n,t}$. Thus, the effect of monetary policy on aggregate money holding, $\partial \Phi_t / \partial \mu_t$, is ambiguous.

In equation (53) and inequality (55), traders spend cash for a larger variety of goods if the money growth rate increases,[10]

$$\frac{\partial i^*_{r,t}}{\partial \mu_t} = \frac{-\beta y \Psi_r}{(\Phi_t)^2}\left\{\frac{\partial \phi_{r,t}}{\partial \mu_t}\Phi_t + (1-\phi_{r,t})\frac{\partial \Phi_t}{\partial \mu_t}\right\} < 0. \qquad \textbf{(57)}$$

For traders, cash is more advantageous than credit because they receive the money injection in the asset market. For nontraders, in equation (54) and inequality (56), if the money growth rate goes up, then nontraders use credit for a wider variety of goods, [11]

$$\frac{\partial i^*_{n,t}}{\partial \mu_t} = \frac{-\beta y \Psi_n}{(\Phi_t)^2}\left\{\frac{\partial \phi_{n,t}}{\partial \mu_t}\Phi_t + (1-\phi_{n,t})\frac{\partial \Phi_t}{\partial \mu_t}\right\} > 0. \qquad \textbf{(58)}$$

Credit is more advantageous because they do not receive the money injection. In other words, the effect of monetary policy on the choice of credit or cash for traders and nontraders goes in opposite directions.

5.2. Distributional Effects on Trader's Choices

Given inequality (57), the net effect on consumption with cash, $c^0_{r,t}$, is ambiguous,[12]

$$\frac{\partial c^0_{r,t}}{\partial \mu_t} = \frac{1}{(1-i^*_{r,t})^2}\left\{\underbrace{\frac{\partial\left[(1-i^*_{r,t})c^0_{r,t}\right]}{\partial \mu_t}(1-i^*_{r,t})}_{\text{direct effect}} \underbrace{+(1-i^*_{r,t})c^0_{r,t}\frac{\partial i^*_{r,t}}{\partial \mu_t}}_{\text{indirect effect}}\right\}, \qquad \textbf{(59)}$$

where in equation (43) the effect on aggregate consumption with cash[13] is

$$\frac{\partial\left[(1-i^*_{r,t})c^0_{r,t}\right]}{\partial \mu_t} = \frac{\left(\frac{1-\alpha}{\alpha}\right)\phi_{n,t-1}y}{(1+\mu_t)^2}\left(\frac{\Phi_t}{\Phi_{t-1}}\right) + \left(\frac{y}{\Phi_{t-1}}\right) \times \left[\frac{\phi_{r,t-1} + (\mu_t \Phi_{t-1}/\alpha)}{1+\mu_t}\right]\frac{\partial \Phi_t}{\partial \mu_t}.$$

In equation (43) and inequalities (55) and (56), the direct effect is ambiguous. Although the money injection increases traders' money holding, aggregate consumption with cash may decrease if aggregate money holding, Φ_t, decreases. In inequality (57) the indirect effect via a change in the choice of credit or cash is negative. Further, given equations (51) and (59), the liquidity effect may arise if the effect of monetary policy on $c^0_{r,t}$ is positive.

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Now, the net effect[14] on consumption with credit, $c^1_{r,t}$, is also ambiguous and it depends on the change of the aggregate money balance,[15]

$$\frac{\partial c^1_{r,t}}{\partial \mu_t} = \frac{1}{(i^*_{r,t})^2}\left\{\underbrace{\frac{\partial\left(i^*_{r,t}c^1_{r,t}\right)}{\partial \mu_t}i^*_{r,t}}_{\text{direct effect}}\underbrace{-(1-\phi_{r,t})y\frac{\partial i^*_{r,t}}{\partial \mu_t}}_{\text{indirect effect}}\right\} = \frac{y}{i^*_{r,t}}\left(\frac{1-\phi_{r,t}}{\Phi_t}\right)\frac{\partial \Phi_t}{\partial \mu_t}. \tag{60}$$

The effect on aggregate consumption with credit in equation (44) and inequality (55) implies that the direct effect is negative. However, in inequality (57) the indirect effect via a change in the choice of credit or cash is positive.

5.3. Distributional Effects on Nontrader's Choices

For nontraders, in inequality (58) the net effect on consumption with cash, $c^0_{n,t}$, is ambiguous,[16]

$$\frac{\partial c^0_{n,t}}{\partial \mu_t} = \frac{1}{(1-i^*_{n,t})^2}\left\{\underbrace{\frac{\partial\left[(1-i^*_{n,t})c^0_{n,t}\right]}{\partial \mu_t}(1-i^*_{n,t})}_{\text{direct effect}}\underbrace{+(1-i^*_{n,t})c^0_{n,t}\frac{\partial i^*_{n,t}}{\partial \mu_t}}_{\text{indirect effect}}\right\}, \tag{61}$$

where in equation (47) the effect on aggregate consumption with cash[17] is

$$\frac{\partial\left[(1-i^*_{n,t})c^0_{n,t}\right]}{\partial \mu_t} = \frac{-\phi_{n,t-1}y}{(1+\mu_t)^2}\left(\frac{\Phi_t}{\Phi_{t-1}}\right) + \left(\frac{y}{\Phi_{t-1}}\right)\left(\frac{\phi_{n,t-1}}{1+\mu_t}\right)\frac{\partial \Phi_t}{\partial \mu_t}.$$

In equation (47) and inequality (56) the effect on aggregate consumption with cash implies that the direct effect is ambiguous. Although the money injection decreases nontraders' money holding, aggregate consumption with cash may increase if aggregate money holding, Φ_t, increases. In inequality (58) the indirect effect via a change in the choice of credit or cash is positive.

The net effect[18] on consumption with credit by nontraders, $c^1_{n,t}$, is ambiguous and it depends on the change of the aggregate money balance,[19]

$$\frac{\partial c^1_{n,t}}{\partial \mu_t} = \frac{1}{(i^*_{r,t})^2}\left\{\underbrace{\frac{\partial\left(i^*_{n,t}c^1_{n,t}\right)}{\partial \mu_t}i^*_{n,t}}_{\text{direct effect}}\underbrace{-(1-\phi_{n,t})y\frac{\partial i^*_{n,t}}{\partial \mu_t}}_{\text{indirect effect}}\right\} = \frac{y}{i^*_{n,t}}\left(\frac{1-\phi_{n,t}}{\Phi_t}\right)\frac{\partial \Phi_t}{\partial \mu_t}. \tag{62}$$

In equation (48) and inequality (56) the effect on aggregate consumption with credit implies that the direct effect is positive. However, in inequality (58) the indirect effect via a change in the choice of credit or cash is negative.

5.4. Welfare and Optimal Money Growth

The welfare of the economy is defined as

$$W_t = \alpha W_{r,t} + (1-\alpha) W_{n,t},$$

where

$$W_{r,t} = \ln\left(c^0_{r,t}\right) + i^*_{r,t}\gamma(i^*_{r,t}) - \int_0^{i^*_{r,t}} \gamma\,(i)\,di$$

and

$$W_{n,t} = \ln\left(c^0_{n,t}\right) + i^*_{n,t}\gamma(i^*_{n,t}) - \int_0^{i^*_{n,t}} \gamma\,(i)\,di.$$

When the money growth rate is stochastic, however, it is not easy to get analytical welfare implications of monetary policy. Thus, introducing an additional assumption on the value of the expected money growth rate would be helpful. Suppose μ_t is independent and identically distributed with zero mean, $E_t[\mu_{t+1}] = 0$. Then no monetary policy is expected by traders and nontraders in the future. In other words, no redistributional effect caused by the segmented asset market would be expected in the future. Thus, the following holds:

$$\Psi_r = \Psi_n = \Psi. \tag{63}$$

If the money growth rate is

$$\widehat{\mu}_t = \frac{\alpha}{\Phi_{t-1}}\left\{\left(\frac{1-\phi_{r,t}}{1-\phi_{n,t}}\right)\phi_{n,t-1} - \phi_{r,t-1}\right\}, \tag{64}$$

then, in equations (45) and (49), the choices of credit or cash of traders and nontraders are the same,[20]

$$i^*_{r,t} = i^*_{n,t} = \widehat{i^*_t},$$

where equations (46) and (50) imply that

$$\frac{\Psi_r}{\Psi_n} = \frac{1-\phi_{n,t}}{1-\phi_{r,t}}. \tag{65}$$

In equations (63) and (65), the real money holdings of traders and nontraders become constant,

$$\phi_{r,t} = \phi_{n,t} = \widehat{\phi}_t,$$

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and the effect of monetary policy on aggregate money holding is zero,

$$\frac{1}{\Phi_t}\frac{\partial \Phi_t}{\partial \mu_t}\bigg|_{\mu_t=\widehat{\mu}_t} = 0.$$

The effect of monetary policy at $\mu_t = \widehat{\mu}_t$ on traders' welfare[21] is

$$\frac{\partial W_{r,t}}{\partial \mu_t}\bigg|_{\mu_t=\widehat{\mu}_t} = \left(\frac{1-\alpha}{\alpha}\right)\frac{(1-\widehat{i}_t^*)\Phi_{t-1}}{\phi_{n,t-1}}\left[1 - \frac{(1-\widehat{i}_t^*)}{\beta y \widehat{\Psi}\left\{\gamma'(\widehat{i}_t^*)(1-\widehat{i}_t^*)+1\right\}e^{\gamma(\widehat{i}_t^*)}}\right]$$

and the effect of monetary policy at $\mu_t = \widehat{\mu}_t$ on nontraders' welfare[22] at $\widehat{\mu}_t$ is

$$\frac{\partial W_{n,t}}{\partial \mu_t}\bigg|_{\mu_t=\widehat{\mu}_t} = \frac{(1-\widehat{i}_t^*)\Phi_{t-1}}{\phi_{n,t-1}}\left[-1 + \frac{(1-\widehat{i}_t^*)}{\beta y \widehat{\Psi}\left\{\gamma'(\widehat{i}_t^*)(1-\widehat{i}_t^*)+1\right\}e^{\gamma(\widehat{i}_t^*)}}\right].$$

The money growth rate where the choice of credit or cash is the same between traders and nontraders does not maximize either the welfare of traders or that of nontraders in general.

However, aggregate welfare is maximized at $\mu_t = \widehat{\mu}_t$ because the effect of monetary policy at $\mu_t = \widehat{\mu}_t$ on aggregate welfare is zero,[23]

$$\frac{\partial W_t}{\partial \mu_t}\bigg|_{\mu_t=\widehat{\mu}_t} = \alpha\frac{\partial W_{r,t}}{\partial \mu_t}\bigg|_{\mu_t=\widehat{\mu}_t} + (1-\alpha)\frac{\partial W_{n,t}}{\partial \mu_t}\bigg|_{\mu_t=\widehat{\mu}_t} = 0. \qquad \textbf{(66)}$$

By making traders and nontraders choose the same means of payment for each market i, $\widehat{\mu}_t$ minimizes the asymmetric redistributional effect on consumption and the choice of credit or cash between traders and nontraders driven by the segmented asset market. The optimal money growth rate, $\mu_t^* = \widehat{\mu}_t$, can be either positive or negative depending on the value of $\phi_{r,t-1}$ and $\phi_{n,t-1}$ in equation (64), and the Friedman rule is not optimal in general.

6. A CREDITLESS ECONOMY

In this paper, in a segmented asset market economy with multiple means of payment, a money injection results in two different distributional effects on consumption for traders and nontraders. To understand the net effect of the direct and indirect effects better, a discussion of a creditless economy will be useful.

6.1. Distributional Effects and Welfare

Suppose cash is the only means of payment when the asset market is segmented. Then the real money balance for credit purchases is zero and there is no choice of credit. In other worlds, $\phi_{r,t} = 1$, $\phi_{n,t} = 1$, $\Phi_t = 1$, $i_{r,t}^* = 0$, $i_{n,t}^* = 0$, and $c_{r,t}^1 = 0 = c_{n,t}^1$. In equations (43) and (47), consumption for traders and

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nontraders becomes as in Alvarez et al. (2001) with unit velocity:

$$c_{r,t}^0 = \left(\frac{1+\mu_t/\alpha}{1+\mu_t}\right) y$$

and

$$c_{n,t}^0 = \left(\frac{1}{1+\mu_t}\right) y.$$

If the government injects money, then only the direct distributional effect remains. The indirect effect disappears because there is no effect from a change in the choice of credit or cash. Traders simply increase consumption and nontraders decrease it. For nontraders, inflation taxes their consumption. Thus, without credit, consumption for traders and nontraders are highly affected by monetary policy because they do not have a device, that is, credit, to alleviate the monetary policy shock.

Welfare is defined as

$$W_t = \alpha \ln\left(c_{r,t}^0\right) + (1-\alpha)\ln\left(c_{n,t}^0\right)$$

and the effect of monetary policy on welfare in the creditless economy is

$$\frac{\partial W_t}{\partial \mu_t} = \frac{-\left(\frac{1-\alpha}{\alpha}\right)}{1+\mu_t}\left(\frac{\mu_t}{1+\frac{\mu_t}{\alpha}}\right).$$

Welfare is maximized if the money growth rate is zero,

$$\left.\frac{\partial W_t}{\partial \mu_t}\right|_{\mu_t=0} = 0.$$

Without monetary policy, there is no redistributional effect on traders and nontraders and they consume equally. Besides, without credit, there is no transactions costs arising from credit. Thus, the government can maximize welfare and smooth out consumption with $\mu_t = 0$. The optimal money growth rate is zero, $\mu_t^* = 0$, and the Friedman rule is not optimal.

6.2. Discussion

When the asset market is segmented, introducing an alternative means of payment, credit, enables traders and nontraders to ease the effect of monetary policy on consumption. Monetary policy, that is, a change in the money growth rate, directly redistributes consumption goods between nontraders and traders. However, the indirect effect on consumption via the change in the choice of credit or cash, $i_{r,t}^*$ and $i_{n,t}^*$, may alleviate the direct effect. By switching from one means of payment to another, traders and nontraders can control their consumption more effectively against monetary policy shock, which is infeasible without credit. Therefore, the net effects of monetary policy on consumption for traders and nontraders become ambiguous, unlike those in the creditless economy. This result is very interesting in the sense that the household carries multiple means of payment not simply to reduce its opportunity costs but to partially compensate for fluctuations of consumption when monetary policy moves against it.

Further, the existence of credit in the segmented asset market implies that the optimal money growth rate is not zero in general. The government has to consider not only the redistributional effect of monetary policy but also the effect on the choice of credit or cash to implement the optimal monetary policy. For example, in Section 4, where the money growth rate is constant, the optimal money growth rate is negative. Both traders and nontraders use credit for a smaller variety of goods if the money growth rate decreases. Thus, by decreasing the money growth rate, the government can redistribute consumption goods from traders to nontraders and reduce the transaction costs of credit. On the other hand, if the money growth rate is stochastic, as in Section 5, then the effects of monetary policy on the choice of credit or cash go to the opposite, unlike Section 4. The optimal money growth rate maximizing welfare can be either positive or negative in general.

7. CONCLUSION

An asset market segmentation model has been constructed to study the distributional effects of monetary policy when there are multiple means of payment. There are traders, who participate in the goods market, and nontraders, who do not. In the asset market, when the government injects money through open market operations, traders initially receive the money injection and nontraders do not. In the goods market, traders and nontraders can use either credit or cash to purchase consumption goods. By using cash, they forego nominal interest, and by using credit, they bear transactions costs.

In equilibrium, money is nonneutral and monetary policy has distributional effects on the choice of means of payment and consumption. When the money growth rate is constant, both traders and nontraders use credit for a greater variety of goods if the money growth rate increases. However, when the money growth rate is stochastic, traders use cash for a greater variety of goods, whereas nontraders prefer to use credit if the money growth rate increases. Next, unlike Alvarez et al. (2001), the money injection may increase or decrease consumption for traders and nontraders because of a direct effect and an indirect effect via the choice of credit or cash. The existence of credit allows traders and nontraders to dampen fluctuations in consumption arising from monetary policy, which can be tested in future research. Liquidity effects disappear when the money growth rate is constant, and they may appear when the money growth rate is stochastic. The optimal money growth rate is negative with a constant money growth and it may be either positive or negative with a stochastic money growth. The Friedman rule is not optimal in general.

NOTES

1. Other than the assumption of perfect competition, a Walrasian auctioneer setup can also imply that $P_t(i) = P_t$ for all i.
2. When a shopper chooses either cash or credit to purchase goods, he or she bears the opportunity costs of cash and credit. In other words, a worker is indifferent between accepting cash and credit, because a worker does not take any of the opportunity costs of cash or credit. Thus, in this paper, the no-surcharge rule does not apply, because a shopper bears any costs incurred by credit. However, as a future extension, it would be very interesting to study the effect of the no-surcharge rule on the choice of credit or cash when a worker is unable or partially able to transfer the transaction costs of credit to a shopper.
3. The derivation is in Appendix A.
4. The derivation is in equations (B.1) and (B.8) in Appendix B.
5. The derivation is in equations (B.2) and (B.9) in Appendix B.
6. The derivation is in Appendix B.
7. The derivation is in Appendix C.
8. The derivation is in Appendix D.
9. The derivation is in Appendix E.
10. The derivation is in Appendix F.
11. The derivations is in Appendix H.
12. Proposition 1 in Appendix J implies it.
13. The derivation is at K.1 in Appendix K.
14. The derivation is in Appendix G.
15. Proposition 1 in Appendix J implies it.
16. Proposition 1 in Appendix J implies it.
17. The derivation is at K.2 in Appendix K.
18. The derivation is in Appendix I.
19. Proposition 1 in Appendix J implies it.
20. The assumption of $E_t[\mu_{t+1}] = 0$ where $\Psi_r = \Psi_n = \Psi$ and Proposition 1 in Appendix J result in them.
21. The derivation is in equations (K.1) and (K.3) in Appendix K.
22. The derivation is in equations (K.2) and (K.4) in Appendix K.
23. The derivation is at K.3 in Appendix K.
24. The derivation is in Appendix A.
25. The total derivative of equation (45) with respect to μ_t provides the equation.
26. The total derivative of equation (49) with respect to μ_t provides the equation.

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APPENDIX A: DERIVATION OF INEQUALITIES (36) AND (37))

In equation (32),

$$\gamma(i^*) = \ln\left(\frac{1+\mu}{\beta}\right).$$

First, in equation (32) and inequality (35),

$$\frac{\partial i^*}{\partial \mu} > 0.$$

Now, insert equation (32) into equations (26) and (29). Then, after they are arranged, equations (33) and (34) hold as

$$i^* = \frac{1}{\dfrac{\phi_n}{\beta(1-\phi_n)} + 1} \tag{A.1}$$

and

$$i^* = \frac{1}{\dfrac{\phi_r + (\mu\Phi/\alpha)}{\beta(1-\phi_r)} + 1}. \tag{A.2}$$

Now, equation (A.1) has the following relation to μ:

$$\frac{\partial \phi_n}{\partial \mu} < 0, \tag{A.3}$$

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where

$$\frac{\partial i^*}{\partial \mu} = \frac{-(i^*)^2}{\beta(1-\phi_n)^2}\left(\frac{\partial \phi_n}{\partial \mu}\right) > 0. \quad \textbf{(A.4)}$$

Next, equation (A.2) with inequality (A.3) implies that ϕ_r may increase or decrease with μ_t,

$$\frac{\partial \phi_r}{\partial \mu}\left[1+\mu+\frac{(1-\alpha)\phi_n\mu}{\alpha}\right]+\left(\frac{1-\alpha}{\alpha}\right)\frac{\partial \phi_n}{\partial \mu}\frac{\mu}{\alpha}(1-\phi_r) < -\frac{\Phi}{\alpha}(1-\phi_r),$$

from

$$0 < \frac{\partial i^*}{\partial \mu} = \frac{-(i^*)^2}{\beta(1-\phi_r)^2}\left\{\frac{\partial \phi_r}{\partial \mu}\left[1+\mu+\frac{(1-\alpha)\phi_n\mu}{\alpha}\right]+\frac{\Phi}{\alpha}(1-\phi_r)\right.$$
$$\left.+\left(\frac{1-\alpha}{\alpha}\right)\frac{\partial \phi_n}{\partial \mu}\frac{\mu}{\alpha}(1-\phi_r)\right\}. \quad \textbf{(A.5)}$$

APPENDIX B: DERIVATION OF EQUATIONS (40), (41), AND (42)

B.1. EFFECTS OF MONETARY POLICY ON W_r AND W_n

The effects of monetary policy on traders' and nontraders' welfare are

$$\begin{aligned}\frac{\partial W_r}{\partial \mu} &= \frac{1}{c_r^0}\frac{\partial c_r^0}{\partial \mu} + i^*\gamma'(i^*)\frac{\partial i^*}{\partial \mu}\\
&= \frac{y}{(1-i^*)c_r^0}\left[\frac{\partial \phi_r}{\partial \mu}+\left(\frac{1-\alpha}{\alpha}\right)\left\{\frac{\partial \phi_n}{\partial \mu}\left(\frac{\mu}{1+\mu}\right)+\phi_n\right\}\right]\\
&\quad + \frac{1}{(1+\mu)(1-i^*)\gamma'(i^*)}+\frac{i^*}{1+\mu}\\
&= \frac{1+\mu}{\phi_r+(\mu\Phi/\alpha)}\left[\frac{\partial \phi_r}{\partial \mu}+\left(\frac{1-\alpha}{\alpha}\right)\left\{\frac{\partial \phi_n}{\partial \mu}\left(\frac{\mu}{1+\mu}\right)+\phi_n\right\}\right]\\
&\quad + \frac{1}{(1+\mu)(1-i^*)\gamma'(i^*)}+\frac{i^*}{1+\mu}\end{aligned} \quad \textbf{(B.1)}$$

and

$$\begin{aligned}\frac{\partial W_n}{\partial \mu} &= \frac{1}{c_n^0}\frac{\partial c_n^0}{\partial \mu} + i^*\gamma'(i^*)\frac{\partial i^*}{\partial \mu}\\
&= \frac{y}{(1-i^*)c_n^0}\left\{\frac{\partial \phi_n}{\partial \mu}\left(\frac{1}{1+\mu}\right)-\frac{\phi_n}{(1+\mu)^2}\right\}+\frac{1}{(1+\mu)(1-i^*)\gamma'(i^*)}+\frac{i^*}{1+\mu}\\
&= \frac{1+\mu}{\phi_n}\left\{\frac{\partial \phi_n}{\partial \mu}\left(\frac{1}{1+\mu}\right)-\frac{\phi_n}{(1+\mu)^2}\right\}+\frac{1}{(1+\mu)(1-i^*)\gamma'(i^*)}+\frac{i^*}{1+\mu},\end{aligned} \quad \textbf{(B.2)}$$

where

$$\frac{\partial \int_0^{i^*} \gamma(i)di}{\partial \mu} = \left[\frac{\partial \int_0^{i^*} \gamma(i)di}{\partial i^*} \right] \frac{\partial i^*}{\partial \mu} = \gamma'(i^*) \frac{\partial i^*}{\partial \mu}$$

and the effect of monetary policy on the choice of credit or cash in equation (35) is

$$\frac{\partial i^*}{\partial \mu} = \frac{1}{(1+\mu)\gamma'(i^*)}.$$

B.2. EFFECTS OF MONETARY POLICY AT $\mu = 0$ ON WELFARE

Now, suppose the money growth rate is set to zero. Then, first, in equations (A.4) and (A.5) in Appendix A, the effect of monetary policy on money holding is

$$\left. \frac{\partial \phi_n}{\partial \mu} \right|_{\mu=0} = \frac{-\beta(1-\phi)^2}{(i^*)^2 \gamma'(i^*)}, \tag{B.3}$$

$$\left. \frac{\partial \phi_r}{\partial \mu} \right|_{\mu=0} = \frac{-\beta(1-\phi)^2}{(i^*)^2 \gamma'(i^*)} - \frac{\phi(1-\phi)}{\alpha}, \tag{B.4}$$

$$\left. \left[\frac{1}{(1+\mu)(1-i^*)\gamma'(i^*)} + \frac{i^*}{1+\mu} \right] \right|_{\mu=0} = \frac{\phi + \beta(1-\phi)}{\phi\gamma'(i^*)} + \frac{\beta(1-\phi)}{\phi+\beta(1-\phi)} \tag{B.5}$$

with

$$\left. \frac{\partial i^*}{\partial \mu} \right|_{\mu=0} = \frac{1}{\gamma'(i^*)}, \tag{B.6}$$

where given $\mu = 0$, traders and nontraders will consume equally, $c_r = c_n = c$, and hold the same amount of money, $\phi_r = \phi_n = \phi$. The choice of credit or cash becomes[24]

$$i^* = \frac{1}{\dfrac{\phi}{\beta(1-\phi)} + 1}. \tag{B.7}$$

Thus, given equations (B.3)–(B.7), the effects of monetary policy on the welfare of traders, nontraders, and the economy in (40), (41), and (42) are as follows. First, from equation (B.1),

$$\begin{aligned}
\left. \frac{\partial W_r}{\partial \mu} \right|_{\mu=0} &= \frac{1}{\phi} \left[\left. \frac{\partial \phi_r}{\partial \mu} \right|_{\mu=0} + \left(\frac{1-\alpha}{\alpha} \right) \phi \right] + \frac{\phi + \beta(1-\phi)}{\phi\gamma'(i^*)} + \frac{\beta(1-\phi)}{\phi+\beta(1-\phi)} \\
&= \frac{1}{\phi} \left[\frac{-\beta(1-\phi)^2}{(i^*)^2 \gamma'(i^*)} - \frac{\phi(1-\phi)}{\alpha} \right] + \left(\frac{1-\alpha}{\alpha} \right) \\
&\quad + \frac{\phi + \beta(1-\phi)}{\phi\gamma'(i^*)} + \frac{\beta(1-\phi)}{\phi+\beta(1-\phi)}
\end{aligned}$$

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$$= \frac{-\beta(1-\phi)^2}{\phi\gamma'(i^*)}\left[\frac{\phi+\beta(1-\phi)}{\beta(1-\phi)}\right]^2 - \frac{(1-\phi)}{\alpha} + \left(\frac{1-\alpha}{\alpha}\right)$$

$$+ \frac{\phi+\beta(1-\phi)}{\phi\gamma'(i^*)} + \frac{\beta(1-\phi)}{\phi+\beta(1-\phi)}$$

$$= \frac{\phi+\beta(1-\phi)}{\phi\beta\gamma'(i^*)}[-\phi-\beta(1-\phi)+\beta] + \frac{\phi}{\alpha} + \frac{\beta(1-\phi)-\phi-\beta(1-\phi)}{\phi+\beta(1-\phi)}$$

$$= \phi\left(\frac{(\beta-1)\{\phi+\beta(1-\phi)\}}{\phi\beta\gamma'(i^*)} - \frac{1}{\phi+\beta(1-\phi)} + \frac{1}{\alpha}\right). \qquad \textbf{(B.8)}$$

Next, from equation (B.2),

$$\left.\frac{\partial W_n}{\partial\mu}\right|_{\mu=0} = \frac{1}{\phi}\left(\left.\frac{\partial\phi_n}{\partial\mu}\right|_{\mu=0} - \phi\right) + \frac{\phi+\beta(1-\phi)}{\phi\gamma'(i^*)} + \frac{\beta(1-\phi)}{\phi+\beta(1-\phi)}$$

$$= \frac{-\beta(1-\phi)^2}{(i^*)^2\phi\gamma'(i^*)} - 1 + \frac{\phi+\beta(1-\phi)}{\phi\gamma'(i^*)} + \frac{\beta(1-\phi)}{\phi+\beta(1-\phi)}$$

$$= \frac{-\beta(1-\phi)^2}{(i^*)^2\phi\gamma'(i^*)} - 1 + \frac{\phi+\beta(1-\phi)}{\phi\gamma'(i^*)} + \frac{\beta(1-\phi)}{\phi+\beta(1-\phi)}$$

$$= \frac{-\beta(1-\phi)^2}{\phi\gamma'(i^*)}\left[\frac{\phi+\beta(1-\phi)}{\beta(1-\phi)}\right]^2 - 1 + \frac{\phi+\beta(1-\phi)}{\phi\gamma'(i^*)} + \frac{\beta(1-\phi)}{\phi+\beta(1-\phi)}$$

$$= \frac{\phi+\beta(1-\phi)}{\phi\beta\gamma'(i^*)}(-\phi-\beta(1-\phi)+\beta) + \frac{\beta(1-\phi)-\phi-\beta(1-\phi)}{\phi+\beta(1-\phi)}$$

$$= \frac{\phi(\beta-1)\{\phi+\beta(1-\phi)\}}{\phi\beta\gamma'(i^*)} - \frac{\phi}{\phi+\beta(1-\phi)} < 0. \qquad \textbf{(B.9)}$$

Finally, from equations (B.7) and (B.8),

$$\left.\frac{\partial W}{\partial\mu}\right|_{\mu=0} = \frac{1}{\phi}\left(\alpha\left.\frac{\partial\phi_r}{\partial\mu}\right|_{\mu=0} + (1-\alpha)\left.\frac{\partial\phi_n}{\partial\mu}\right|_{\mu=0}\right) + \frac{1}{(1-i^*)\gamma'(i^*)} + i^*$$

$$= \frac{-\beta(1-\phi)^2}{(i^*)^2\phi\gamma'(i^*)} - (1-\phi) + \frac{1}{(1-i^*)\gamma'(i^*)} + i^*$$

$$= \frac{-\beta(1-\phi)^2}{\phi\gamma' i^*}\left[\frac{\phi+\beta(1-\phi)}{\beta(1-\phi)}\right]^2 - (1-\phi) + \frac{\phi+\beta(1-\phi)}{\phi\gamma'(i^*)} + \frac{\beta(1-\phi)}{\phi+\beta(1-\phi)}$$

$$= \frac{\phi+\beta(1-\phi)}{\phi\beta\gamma'(i^*}[-\phi-\beta(1-\phi)+\beta] + \frac{1-\phi}{\phi+\beta(1-\phi)}[-\phi-\beta(1-\phi)+\beta]$$

$$= \phi(\beta-1)\left\{\frac{\phi+\beta(1-\phi)}{\phi\beta\gamma'(i^*)} + \frac{1-\phi}{\phi+\beta(1-\phi)}\right\} < 0.$$

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APPENDIX C: DERIVATION OF EQUATION (53)

Insert equation (46) into equation (45). Then

$$\left[\frac{\beta(1-\phi_{r,t})y\Psi_r}{\Phi_t-\beta(1-\phi_{r,t})y\Psi_r}\right]e^{\gamma\left[\beta\left(\frac{1-\phi_{r,t}}{\Phi_t}\right)y\Psi_r\right]}=\left(\frac{1-\phi_{r,t}}{\Phi_t}\right)\left(\frac{1+\mu_t}{\frac{\phi_{r,t-1}}{\Phi_{t-1}}+\frac{\mu_t}{\alpha}}\right),$$

where

$$\frac{i^*_{r,t}}{1-i^*_{r,t}}=\frac{\beta(1-\phi_{r,t})y\Psi_r}{\Phi_t-\beta(1-\phi_{r,t})y\Psi_r},$$

and they imply that

$$\frac{e^{\gamma\left[\beta\left(\frac{1-\phi_{r,t}}{\Phi_t}\right)y\Psi_r\right]}}{\dfrac{1}{\beta y\Psi_r}-\dfrac{1-\phi_{r,t}}{\Phi_t}}=\frac{1+\mu_t}{\dfrac{\phi_{r,t-1}}{\Phi_{t-1}}+\dfrac{\mu_t}{\alpha}}.$$

APPENDIX D: DERIVATION OF EQUATION (54)

Insert equation (50) into equation (49). Then

$$\left[\frac{\beta(1-\phi_{n,t})y\Psi_n}{\Phi_t-\beta(1-\phi_{n,t})y\Psi_n}\right]e^{\gamma\left[\beta\left(\frac{1-\phi_{n,t}}{\Phi_t}\right)y\Psi_n\right]}=\left(\frac{1-\phi_{n,t}}{\Phi_t}\right)\left(\frac{1+\mu_t}{\frac{\phi_{n,t-1}}{\Phi_{t-1}}}\right),$$

where

$$\frac{i^*_{n,t}}{1-i^*_{n,t}}=\frac{\beta(1-\phi_{n,t})y\Psi_n}{\Phi_t-\beta(1-\phi_{n,t})y\Psi_n},$$

and they imply that

$$\frac{e^{\gamma\left[\beta\left(\frac{1-\phi_{n,t}}{\Phi_t}\right)y\Psi_n\right]}}{\dfrac{1}{\beta y\Psi_n}-\dfrac{1-\phi_{n,t}}{\Phi_t}}=\frac{(1+\mu_t)\Phi_{t-1}}{\phi_{n,t-1}}.$$

APPENDIX E: DERIVATION OF INEQUALITIES (55) AND (56)

First, in equation (53), the right-hand side of equation (53) is

$$\text{RHS}=\frac{1+\mu_t}{\dfrac{\phi_{r,t-1}}{\Phi_{t-1}}+\dfrac{\mu_t}{\alpha}},$$

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and the effect of the money growth rate on RHS is negative,

$$\frac{\partial \text{RHS}}{\partial \mu_t} = \frac{\dfrac{\phi_{r,t-1}}{\Phi_{t-1}} - \dfrac{1}{\alpha}}{\left(\dfrac{\phi_{r,t-1}}{\Phi_{t-1}} + \dfrac{\mu_t}{\alpha}\right)^2} < 0, \tag{E.1}$$

where

$$\frac{1}{\alpha}\left(\frac{\alpha\phi_{r,t-1}}{\Phi_{t-1}} - 1\right) < 0.$$

Next, the left-hand side of equation (53) is

$$\text{LHS} = \frac{e^{\gamma(\beta A_t y \Psi_r)}}{\dfrac{1}{\beta y \Psi_r} - A_t},$$

where

$$A_t = \frac{1-\phi_{r,t}}{\Phi_t}$$

and

$$\Phi_t = \alpha\phi_{r,t} + (1-\alpha)\phi_{n,t}.$$

Because the RHS decreases with μ_t, the effect of the money growth rate on the LHS should be negative:

$$\frac{\partial \text{LHS}}{\partial \mu_t} = \frac{\{\gamma'(\beta A_t y \Psi_r)(1-\beta A_t y \Psi_r) + 1\}\, e^{\gamma(\beta A_t y \Psi_r)}}{\left(\dfrac{1}{\beta y \Psi_r} - A_t\right)^2}\left(\frac{\partial A_t}{\partial \mu_t}\right)$$

$$= \frac{(\beta y \Psi_r)^2\left\{\gamma'\left(i^*_{r,t}\right)\left(1-i^*_{r,t}\right)+1\right\} e^{\gamma\left(i^*_{r,t}\right)}}{\left(1-i^*_{r,t}\right)^2}\left(\frac{\partial A_t}{\partial \mu_t}\right) < 0, \tag{E.2}$$

where in equation (46), $i^*_{r,t} = \beta A_t y \Psi_r$ and

$$\frac{1}{\beta y \Psi_r} - A_t = \frac{1}{\beta y \Psi_r} - \frac{i^*_{r,t}}{\beta y \Psi_r}.$$

Therefore, the following should hold:

$$\frac{\partial A_t}{\partial \mu_t} = \frac{-1}{(\Phi_t)^2}\left\{\frac{\partial \phi_{r,t}}{\partial \mu_t}\Phi_t + (1-\phi_{r,t})\frac{\partial \Phi_t}{\partial \mu_t}\right\}$$

$$= \frac{-1}{(\Phi_t)^2}\left\{\frac{\partial \phi_{r,t}}{\partial \mu_t}[\Phi_t + \alpha(1-\phi_{r,t})] + (1-\alpha)(1-\phi_{r,t})\frac{\partial \phi_{n,t}}{\partial \mu_t}\right\} < 0. \tag{E.3}$$

Second, the right-hand side of equation (54) is

$$\text{RHS} = \frac{(1+\mu_t)\Phi_{t-1}}{\phi_{n,t-1}},$$

and the effect of the money growth rate on the RHS is negative:

$$\frac{\partial \text{RHS}}{\partial \mu_t} = \frac{\Phi_{t-1}}{\phi_{n,t-1}} > 0. \tag{E.4}$$

Now, the left-hand side of equation (54) is

$$\text{LHS} = \frac{e^{\gamma(\beta B_t y \Psi_n)}}{1/\beta y \Psi_n - B_t},$$

where

$$B_t = \frac{1-\phi_{n,t}}{\Phi_t}$$

and

$$\Phi_t = \alpha \phi_{r,t} + (1-\alpha)\phi_{n,t}.$$

Because the RHS increases with μ_t, the effect of the money growth rate should be positive:

$$\begin{aligned}\frac{\partial \text{LHS}}{\partial \mu_t} &= \frac{\{\gamma'(\beta B_t y \Psi_n)(1-\beta B_t y \Psi_n)+1\} e^{\gamma(\beta B_t y \Psi_n)}}{\left(\frac{1}{\beta y \Psi_n} - B_t\right)^2}\left(\frac{\partial B_t}{\partial \mu_t}\right) \\ &= \frac{(\beta y \Psi_n)^2 \left\{\gamma'\left(i^*_{n,t}\right)\left(1-i^*_{n,t}\right)+1\right\} e^{\gamma\left(i^*_{n,t}\right)}}{\left(1-i^*_{n,t}\right)^2}\left(\frac{\partial B_t}{\partial \mu_t}\right) > 0,\end{aligned} \tag{E.5}$$

where in equation (50), $i^*_{n,t} = \beta B_t y \Psi_n$ and

$$\frac{1}{\beta y \Psi_n} - B_t = \frac{1-i^*_{n,t}}{\beta y \Psi_n}.$$

Therefore, the following should hold:

$$\begin{aligned}\frac{\partial B_t}{\partial \mu_t} &= \frac{-1}{(\Phi_t)^2}\left\{\frac{\partial \phi_{n,t}}{\partial \mu_t}\Phi_t + (1-\phi_{n,t})\frac{\partial \Phi_t}{\partial \mu_t}\right\} \\ &= \frac{-1}{(\Phi_t)^2}\left\{\frac{\partial \phi_{n,t}}{\partial \mu_t}[\Phi_t + (1-\alpha)(1-\phi_{n,t})] + \alpha(1-\phi_{n,t})\frac{\partial \phi_{r,t}}{\partial \mu_t}\right\} > 0.\end{aligned} \tag{E.6}$$

Overall, inequalities (E.3) and (E.6) imply that

$$\frac{\partial \phi_{r,t}}{\partial \mu_t} > 0,$$

$$\frac{\partial \phi_{n,t}}{\partial \mu_t} < 0,$$

where

$$\frac{\partial \phi_{n,t}}{\partial \mu_t}\left(\frac{\Phi_t}{1-\phi_{n,t}}\right) < -\frac{\partial \Phi_t}{\partial \mu_t} < \frac{\partial \phi_{r,t}}{\partial \mu_t}\left(\frac{\Phi_t}{1-\phi_{r,t}}\right). \tag{E.7}$$

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APPENDIX F: DERIVATION OF INEQUALITY (57)

Equation (46) and inequality (E.3) in Appendix D,

$$\frac{\partial A_t}{\partial \mu_t} = \frac{-1}{(\Phi_t)^2}\left\{\frac{\partial \phi_{r,t}}{\partial \mu_t}\Phi_t + (1-\phi_{r,t})\frac{\partial \Phi_t}{\partial \mu_t}\right\} < 0,$$

imply that

$$\frac{\partial i^*_{r,t}}{\partial \mu_t} = \beta \frac{\partial A_t}{\partial \mu_t} y\Psi_r < 0. \tag{F.1}$$

APPENDIX G: DERIVATION OF INEQUALITY (60)

First, in equation (44) and inequality (55),

$$\frac{\partial\left(i^*_{r,t}c^1_{r,t}\right)}{\partial \mu_t} = -\frac{\partial \phi_{r,t}}{\partial \mu_t} y < 0. \tag{G.1}$$

Now, from equation (60) and inequalities (E.3), (F.1), and (G.1),

$$\begin{aligned}\frac{\partial c^1_{r,t}}{\partial \mu_t} &= \frac{1}{\left(i^*_{r,t}\right)^2}\left\{\frac{\partial\left(i^*_{r,t}c^1_{r,t}\right)}{\partial \mu_t} i^*_{r,t} - (1-\phi_{r,t})y\frac{\partial i^*_{r,t}}{\partial \mu_t}\right\}\\ &= \frac{1}{\left(i^*_{r,t}\right)^2}\left\{-\frac{\partial \phi_{r,t}}{\partial \mu_t} y i^*_{r,t} - (1-\phi_{r,t})y\beta\frac{\partial A_t}{\partial \mu_t}y\Psi_r\right\}\\ &= \frac{1}{\left(i^*_{r,t}\right)^2}\left\{-\frac{\partial \phi_{r,t}}{\partial \mu_t} y i^*_{r,t} + \left(\frac{y}{\Phi_t}\right)\beta\left(\frac{1-\phi_{r,t}}{\Phi_t}\right)y\Psi_r\left[\frac{\partial \phi_{r,t}}{\partial \mu_t}\Phi_t + (1-\phi_{r,t})\frac{\partial \Phi_t}{\partial \mu_t}\right]\right\}.\end{aligned}$$

Equation (46) implies that the indirect effect is greater than the direct effect:

$$\begin{aligned}\frac{\partial c^1_{r,t}}{\partial \mu_t} &= \frac{1}{\left(i^*_{r,t}\right)^2}\left\{-\frac{\partial \phi_{r,t}}{\partial \mu_t} y i^*_{r,t} + \left(\frac{y}{\Phi_t}\right) i^*_{r,t}\left\{\frac{\partial \phi_{r,t}}{\partial \mu_t}\Phi_t + (1-\phi_{r,t})\frac{\partial \Phi_t}{\partial \mu_t}\right\}\right\}\\ &= \frac{y}{i^*_{r,t}}\left(\frac{1-\phi_{r,t}}{\Phi_t}\right)\frac{\partial \Phi_t}{\partial \mu_t}.\end{aligned} \tag{G.2}$$

Finally, inequality (E.7),

$$-\frac{\partial \phi_{r,t}}{\partial \mu_t} < \left(\frac{1-\phi_{r,t}}{\Phi_t}\right)\frac{\partial \Phi_t}{\partial \mu_t},$$

implies that $c^1_{r,t}$ may increase or decrease with μ_t:

$$\frac{\partial c^1_{r,t}}{\partial \mu_t} > -\left(\frac{y}{i^*_{r,t}}\right)\frac{\partial \phi_{r,t}}{\partial \mu_t},$$

where

$$\frac{\partial \phi_{r,t}}{\partial \mu_t} > 0.$$

APPENDIX H: DERIVATION OF INEQUALITY (58)

Equation (50) and inequality (E.6) in Appendix D,

$$\frac{\partial B_t}{\partial \mu_t} = \frac{-1}{(\Phi_t)^2}\left\{\frac{\partial \phi_{n,t}}{\partial \mu_t}\Phi_t + (1-\phi_{n,t})\frac{\partial \Phi_t}{\partial \mu_t}\right\} > 0,$$

imply that

$$\frac{\partial i^*_{n,t}}{\partial \mu_t} = \beta \frac{\partial B_t}{\partial \mu_t} y \Psi_n > 0. \tag{H.1}$$

APPENDIX I: DERIVATION OF INEQUALITY (62)

First, in equation (48) and inequality (56), aggregate consumption with credit increases with inflation:

$$\frac{\partial \left(i^*_{n,t} c^1_{n,t}\right)}{\partial \mu_t} = -\frac{\partial \phi_{n,t}}{\partial \mu_t} y > 0. \tag{I.1}$$

Now, from equation (62) and inequalities (E.6), (H.1), and (I.1),

$$\frac{\partial c^1_{n,t}}{\partial \mu_t} = \frac{1}{\left(i^*_{n,t}\right)^2}\left\{\frac{\partial \left(i^*_{n,t} c^1_{n,t}\right)}{\partial \mu_t} i^*_{n,t} - (1-\phi_{n,t}) y \frac{\partial i^*_{n,t}}{\partial \mu_t}\right\}$$

$$= \frac{1}{\left(i^*_{n,t}\right)^2}\left\{-\frac{\partial \phi_{n,t}}{\partial \mu_t} y i^*_{n,t} - (1-\phi_{n,t}) y \beta \frac{\partial B_t}{\partial \mu_t} y \Psi_n\right\}$$

$$= \frac{1}{\left(i^*_{n,t}\right)^2}\left\{-\frac{\partial \phi_{n,t}}{\partial \mu_t} y i^*_{n,t} + \left(\frac{y}{\Phi_t}\right)\beta\left(\frac{1-\phi_{n,t}}{\Phi_t}\right) y \Psi_n \left[\frac{\partial \phi_{n,t}}{\partial \mu_t}\Phi_t + (1-\phi_{n,t})\frac{\partial \Phi_t}{\partial \mu_t}\right]\right\}.$$

Equation (50) implies that

$$\frac{\partial c^1_{n,t}}{\partial \mu_t} = \frac{1}{\left(i^*_{n,t}\right)^2}\left\{-\frac{\partial \phi_{n,t}}{\partial \mu_t} y i^*_{n,t} + \left(\frac{y}{\Phi_t}\right) i^*_{n,t}\left\{\frac{\partial \phi_{n,t}}{\partial \mu_t}\Phi_t + (1-\phi_{n,t})\frac{\partial \Phi_t}{\partial \mu_t}\right\}\right\}$$

$$= \frac{y}{i^*_{n,t}}\left(\frac{1-\phi_{n,t}}{\Phi_t}\right)\frac{\partial \Phi_t}{\partial \mu_t}. \tag{I.2}$$

Finally, inequality (E.7),

$$-\frac{\partial \phi_{n,t}}{\partial \mu_t} > \left(\frac{1-\phi_{n,t}}{\Phi_t}\right)\frac{\partial \Phi_t}{\partial \mu_t},$$

implies that $c^1_{n,t}$ may increase or decrease with μ_t:

$$\frac{\partial c^1_{n,t}}{\partial \mu_t} < -\left(\frac{y}{i^*_{n,t}}\right)\frac{\partial \phi_{n,t}}{\partial \mu_t},$$

where

$$\frac{\partial \phi_{n,t}}{\partial \mu_t} < 0.$$

APPENDIX J: FIND OUT $\partial \Phi_t / \partial \mu_t$

In Appendix D, the effect of μ_t on equation (53) follows from equations (E.1) and (E.2):

$$\frac{(\beta y \Psi_r)^2 \left\{\gamma'\left(i_{r,t}^*\right)\left(1 - i_{r,t}^*\right) + 1\right\} e^{\gamma\left(i_{r,t}^*\right)}}{\left(1 - i_{r,t}^*\right)^2} \left(\frac{\partial A_t}{\partial \mu_t}\right) = \frac{\dfrac{\phi_{r,t-1}}{\Phi_{t-1}} - \dfrac{1}{\alpha}}{\left(\dfrac{\phi_{r,t-1}}{\Phi_{t-1}} + \dfrac{\mu_t}{\alpha}\right)^2},$$

where in equation (E.3)

$$\frac{\partial A_t}{\partial \mu_t} = \frac{-1}{(\Phi_t)^2} \left\{\frac{\partial \phi_{r,t}}{\partial \mu_t} \Phi_t + (1 - \phi_{r,t}) \frac{\partial \Phi_t}{\partial \mu_t}\right\}.$$

Therefore, they imply that

$$\begin{aligned} &\alpha \frac{\partial \phi_{r,t}}{\partial \mu_t} + \alpha \left(\frac{1 - \phi_{r,t}}{\Phi_t}\right) \frac{\partial \Phi_t}{\partial \mu_t} \\ &= \frac{-\alpha \Phi_t \left(1 - i_{r,t}^*\right)^2}{(\beta y \Psi_r)^2 \left\{\gamma'\left(i_{r,t}^*\right)\left(1 - i_{r,t}^*\right) + 1\right\} e^{\gamma\left(i_{r,t}^*\right)}} \left\{\frac{\dfrac{\phi_{r,t-1}}{\Phi_{t-1}} - \dfrac{1}{\alpha}}{\left(\dfrac{\phi_{r,t-1}}{\Phi_{t-1}} + \dfrac{\mu_t}{\alpha}\right)^2}\right\}. \end{aligned} \tag{J.1}$$

Next, in Appendix E, the effect of μ_t on equation (54) follows from equations (E.4) and (E.5),

$$\frac{(\beta y \Psi_n)^2 \left\{\gamma'\left(i_{n,t}^*\right)\left(1 - i_{n,t}^*\right) + 1\right\} e^{\gamma\left(i_{n,t}^*\right)}}{\left(1 - i_{n,t}^*\right)^2} \left(\frac{\partial B_t}{\partial \mu_t}\right) = \frac{\Phi_{t-1}}{\phi_{n,t-1}},$$

where in equation (E.3)

$$\frac{\partial B_t}{\partial \mu_t} = \frac{-1}{(\Phi_t)^2} \left\{\frac{\partial \phi_{n,t}}{\partial \mu_t} \Phi_t + (1 - \phi_{n,t}) \frac{\partial \Phi_t}{\partial \mu_t}\right\}.$$

Therefore, they imply that

$$\begin{aligned} &(1 - \alpha) \frac{\partial \phi_{n,t}}{\partial \mu_t} + (1 - \alpha) \left(\frac{1 - \phi_{n,t}}{\Phi_t}\right) \frac{\partial \Phi_t}{\partial \mu_t} \\ &= \frac{-(1 - \alpha) \Phi_t \left(1 - i_{n,t}^*\right)^2}{(\beta y \Psi_n)^2 \left\{\gamma'\left(i_{n,t}^*\right)\left(1 - i_{n,t}^*\right) + 1\right\} e^{\gamma\left(i_{n,t}^*\right)}} \left(\frac{\Phi_{t-1}}{\phi_{n,t-1}}\right). \end{aligned} \tag{J.2}$$

Now, add equations (J.1) and (J.2):

$$\frac{1}{\Phi_t}\frac{\partial \Phi_t}{\partial \mu_t} = \frac{-\alpha\Phi_t\left(1-i^*_{r,t}\right)^2}{(\beta y \Psi_r)^2\left\{\gamma'\left(i^*_{r,t}\right)\left(1-i^*_{r,t}\right)+1\right\}e^{\gamma\left(i^*_{r,t}\right)}}\left\{\frac{\dfrac{\phi_{r,t-1}}{\Phi_{t-1}}-\dfrac{1}{\alpha}}{\left(\dfrac{\phi_{r,t-1}}{\Phi_{t-1}}+\dfrac{\mu_t}{\alpha}\right)^2}\right\}$$
$$-\frac{(1-\alpha)\Phi_t\left(1-i^*_{n,t}\right)^2}{(\beta y \Psi_n)^2\left\{\gamma'\left(i^*_{n,t}\right)\left(1-i^*_{n,t}\right)+1\right\}e^{\gamma\left(i^*_{n,t}\right)}}\left(\frac{\Phi_{t-1}}{\phi_{n,t-1}}\right)$$
$$=\frac{(1-\alpha)\Phi_t\left(1-i^*_{r,t}\right)^2}{(\beta y \Psi_r)^2\left\{\gamma'\left(i^*_{r,t}\right)\left(1-i^*_{r,t}\right)+1\right\}e^{\gamma\left(i^*_{r,t}\right)}}\left\{\frac{\dfrac{\phi_{n,t-1}}{\Phi_{t-1}}}{\left(\dfrac{\phi_{r,t-1}}{\Phi_{t-1}}+\dfrac{\mu_t}{\alpha}\right)^2}\right\}$$
$$-\frac{(1-\alpha)\Phi_t\left(1-i^*_{n,t}\right)^2}{(\beta y \Psi_n)^2\left\{\gamma'\left(i^*_{n,t}\right)\left(1-i^*_{n,t}\right)+1\right\}e^{\gamma\left(i^*_{n,t}\right)}}\left(\frac{\Phi_{t-1}}{\phi_{n,t-1}}\right). \qquad \textbf{(J.3)}$$

PROPOSITION 1. *Given $\phi_{r,t-1}$ and $\phi_{n,t-1}$, equation (J.3) satisfies*

$$\frac{1}{\Phi_t}\frac{\partial \Phi_t}{\partial \mu_t} = \begin{cases} = 0 & \text{if } \mu_t = \widehat{\mu}_t, \\ < 0 & \text{if } \mu_t < \widehat{\mu}_t, \\ > 0 & \text{if } \mu_t > \widehat{\mu}_t, \end{cases}$$

where

$$\widehat{\mu}_t = \frac{\alpha}{\Phi_{t-1}}\left\{\left(\frac{1-\phi_{r,t}}{1-\phi_{n,t}}\right)\phi_{n,t-1}-\phi_{r,t-1}\right\}. \qquad \textbf{(J.4)}$$

Proof. In equations (45), (46), (49), and (50), if $\mu_t = \widehat{\mu}_t$, then

$$i^*_{r,t} = i^*_{n,t} = \widehat{i^*_t},$$

where

$$\frac{1-\phi_{r,t}}{1-\phi_{n,t}} = \frac{\Psi_n}{\Psi_r}.$$

Given equation (J.4), equation (J.3) shows that

$$\left.\frac{1}{\Phi_t}\frac{\partial \Phi_t}{\partial \mu_t}\right|_{\mu_t=\widehat{\mu}_t} = 0 = \frac{(1-\alpha)\Phi_t\left(1-\widehat{i^*_t}\right)^2\left\{\left(\frac{\Phi_{t-1}}{\phi_{n,t-1}}\right)\left(\frac{1}{\Psi_n}\right)^2-\left(\frac{\Phi_{t-1}}{\phi_{n,t-1}}\right)\left(\frac{1}{\Psi_n}\right)^2\right\}}{(\beta y)^2\left\{\gamma'\left(\widehat{i^*_t}\right)\left(1-\widehat{i^*_t}\right)+1\right\}e^{\gamma\left(\widehat{i^*_t}\right)}}.$$

Next, if μ_t decreases below $\widehat{\mu}_t$, then inequalities (57) and (58) imply that $i^*_{r,t}$ increases and $i^*_{n,t}$ decreases. Thus, $i^*_{r,t} > \widehat{i^*_t} > i^*_{n,t}$ holds if $\mu_t < \widehat{\mu}_t$. In equation (J.3), the effect of μ_t is negative because the first term decreases and the second term increases given $\phi_{r,t-1}$ and

$\phi_{n,t-1}$:

$$\frac{1}{\Phi_t}\frac{\partial \Phi_t}{\partial \mu_t} = \frac{(1-\alpha)\Phi_t}{(\beta y \Psi_r)^2 \left\{ \frac{\gamma'\left(i^*_{r,t}\right)}{1-i^*_{r,t}} + \frac{1}{\left(1-i^*_{r,t}\right)^2} \right\} e^{\gamma\left(i^*_{r,t}\right)}} \left\{ \frac{\frac{\phi_{n,t-1}}{\Phi_{t-1}}}{\left(\frac{\phi_{r,t-1}}{\Phi_{t-1}} + \frac{\mu_t}{\alpha}\right)^2} \right\}$$

$$- \frac{(1-\alpha)\Phi_t}{(\beta y \Psi_n)^2 \left\{ \frac{\gamma'\left(i^*_{n,t}\right)}{1-i^*_{n,t}} + \frac{1}{\left(1-i^*_{n,t}\right)^2} \right\} e^{\gamma\left(i^*_{n,t}\right)}} \left(\frac{\Phi_{t-1}}{\phi_{n,t-1}}\right) < 0.$$

Last, if μ_t increases above $\widehat{\mu}_t$, then inequalities (57) and (58) imply that $i^*_{r,t}$ decreases and $i^*_{n,t}$ increases. Thus, $i^*_{r,t} < \widehat{i}^*_t < i^*_{n,t}$ holds if $\mu_t > \widehat{\mu}_t$. In equation (J.3), the effect of μ_t is positive because the first term increases and the second term decreases given $\phi_{r,t-1}$ and $\phi_{n,t-1}$:

$$\frac{1}{\Phi_t}\frac{\partial \Phi_t}{\partial \mu_t} = \frac{(1-\alpha)\Phi_t}{(\beta y \Psi_r)^2 \left\{ \frac{\gamma'\left(i^*_{r,t}\right)}{1-i^*_{r,t}} + \frac{1}{\left(1-i^*_{r,t}\right)^2} \right\} e^{\gamma\left(i^*_{r,t}\right)}} \left\{ \frac{\frac{\phi_{n,t-1}}{\Phi_{t-1}}}{\left(\frac{\phi_{r,t-1}}{\Phi_{t-1}} + \frac{\mu_t}{\alpha}\right)^2} \right\}$$

$$- \frac{(1-\alpha)\Phi_t}{(\beta y \Psi_n)^2 \left\{ \frac{\gamma'\left(i^*_{n,t}\right)}{1-i^*_{n,t}} + \frac{1}{\left(1-i^*_{n,t}\right)^2} \right\} e^{\gamma\left(i^*_{n,t}\right)}} \left(\frac{\Phi_{t-1}}{\phi_{n,t-1}}\right) > 0.$$

■

APPENDIX K: DERIVATION OF EQUALITY (66)

K.1. EFFECTS OF MONETARY POLICY ON $W_{r,t}$

The effect of monetary policy on traders' welfare is

$$\frac{\partial W_{r,t}}{\partial \mu_t} = \frac{1}{c^0_{r,t}}\frac{\partial c^0_{r,t}}{\partial \mu_t} + i^*_{r,t}\gamma'\left(i^*_{r,t}\right)\frac{\partial i^*_{r,t}}{\partial \mu_t}$$

$$= \frac{1}{(1-i^*_{r,t})c^0_{r,t}}\frac{\partial\left[\left(1-i^*_{r,t}\right)c^0_{r,t}\right]}{\partial \mu_t} + \left[\frac{1}{1-i^*_{r,t}} + i^*_{r,t}\gamma'\left(i^*_{r,t}\right)\right]\frac{\partial i^*_{r,t}}{\partial \mu_t}$$

$$= \frac{\left(\frac{1-\alpha}{\alpha}\right)\frac{\phi_{n,t-1}}{1+\mu_t}}{\phi_{r,t-1} + (\mu_t\Phi_{t-1}/\alpha)} + \frac{1}{\Phi_t}\frac{\partial \Phi_t}{\partial \mu_t} - \frac{\left(\frac{1-\alpha}{\alpha}\right)\frac{\phi_{n,t-1}}{1+\mu_t}}{\phi_{r,t-1} + \left(\mu_t\Phi_{t-1}/\alpha\right)}\left[\frac{\beta(1-\phi_{r,t})y\Psi_r}{\Phi_t}\right]$$

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$$- \frac{\beta y \Psi_r}{\Phi_t} \left[\frac{\partial \phi_{r,t}}{\partial \mu_t} + \frac{\left(1-\phi_{r,t}\right)}{\Phi_t} \frac{\partial \Phi_t}{\partial \mu_t} \right]$$

$$= \frac{\left(\frac{1-\alpha}{\alpha}\right) \frac{\phi_{n,t-1}}{1+\mu_t} \left(1-i^*_{r,t}\right)}{\phi_{r,t-1} + \left(\mu_t \Phi_{t-1}/\alpha\right)} + \frac{\left(1-i^*_{r,t}\right)}{\Phi_t} \frac{\partial \Phi_t}{\partial \mu_t} - \frac{i^*_{r,t}}{1-\phi_{r,t}} \frac{\partial \phi_{r,t}}{\partial \mu_t}, \tag{K.1}$$

where

$$\frac{\partial \int_0^{i^*_{r,t}} \gamma(i) di}{\partial \mu_t} = \left[\frac{\partial \int_0^{i^*_{r,t}} \gamma(i) di}{\partial i^*_{r,t}} \right] \frac{\partial i^*_{r,t}}{\partial \mu_t} = \gamma'(i^*_{r,t}) \frac{\partial i^*_{r,t}}{\partial \mu_t},$$

$$\frac{\partial c^0_{r,t}}{\partial \mu_t} = \frac{1}{\left(1-i^*_{r,t}\right)^2} \left\{ \frac{\partial \left[\left(1-i^*_{r,t}\right) c^0_{r,t}\right]}{\partial \mu_t} \left(1-i^*_{r,t}\right) + (1-i^*_{r,t}) c^0_{r,t} \frac{\partial i^*_{r,t}}{\partial \mu_t} \right\}$$

$$= \left(\frac{1}{1-i^*_{r,t}}\right) \frac{\partial \left[\left(1-i^*_{r,t}\right) c^0_{r,t}\right]}{\partial \mu_t} + \left(\frac{c^0_{r,t}}{1-i^*_{r,t}}\right) \frac{\partial i^*_{r,t}}{\partial \mu_t},$$

$$\frac{\partial \left(1-i^*_{r,t}\right) c^0_{r,t}}{\partial \mu_t} = \frac{y}{\left(1+\mu_t\right)^2} \left(\frac{\Phi_t}{\Phi_{t-1}}\right) \left[\frac{(1+\mu_t)\Phi_{t-1}}{\alpha} - \phi_{r,t-1} - \frac{\mu_t}{\alpha} \Phi_{t-1} \right]$$

$$+ \left(\frac{y}{\Phi_{t-1}}\right) \left[\frac{\phi_{r,t-1} + \left(\mu_t \Phi_{t-1}/\alpha\right)}{1+\mu_t} \right] \frac{\partial \Phi_t}{\partial \mu_t}$$

$$= \frac{\left(\frac{1-\alpha}{\alpha}\right) \phi_{n,t-1} y}{\left(1+\mu_t\right)^2} \left(\frac{\Phi_t}{\Phi_{t-1}}\right) + \left(\frac{y}{\Phi_{t-1}}\right) \left[\frac{\phi_{r,t-1} + \left(\mu_t \Phi_{t-1}/\alpha\right)}{1+\mu_t} \right] \frac{\partial \Phi_t}{\partial \mu_t},$$

and in equation (45), the effect[25] of monetary policy on $i^*_{r,t}$ is

$$\left(\frac{1}{1-i^*_{r,t}} + i^*_{r,t} \gamma'\left(i^*_{r,t}\right) \right) \frac{\partial i^*_{r,t}}{\partial \mu_t} = - \frac{\left(\frac{1-\alpha}{\alpha}\right) \frac{\phi_{n,t-1}}{1+\mu_t}}{\phi_{r,t-1} + \left(\mu_t \Phi_{t-1}/\alpha\right)} \left[\frac{\beta \left(1-\phi_{r,t}\right) y \Psi_r}{\Phi_t} \right]$$

$$- \frac{\beta y \Psi_r}{\Phi_t} \left[\frac{\partial \phi_{r,t}}{\partial \mu_t} + \frac{\left(1-\phi_{r,t}\right)}{\Phi_t} \frac{\partial \Phi_t}{\partial \mu_t} \right].$$

K.2. EFFECTS OF MONETARY POLICY ON $W_{n,t}$

The effect of monetary policy on nontraders' welfare is

$$\frac{\partial W_{n,t}}{\partial \mu_t} = \frac{1}{c^0_{n,t}} \frac{\partial c^0_{n,t}}{\partial \mu_t} + i^*_{n,t} \gamma'\left(i^*_{n,t}\right) \frac{\partial i^*_{n,t}}{\partial \mu_t}$$

$$= \frac{1}{\left(1-i^*_{n,t}\right) c^0_{n,t}} \frac{\partial \left[\left(1-i^*_{n,t}\right) c^0_{n,t}\right]}{\partial \mu_t} + \left[\frac{1}{1-i^*_{n,t}} + i^*_{n,t} \gamma'\left(i^*_{n,t}\right) \right] \frac{\partial i^*_{n,t}}{\partial \mu_t}$$

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$$= \frac{-1}{1+\mu_t} + \frac{1}{\Phi_t}\frac{\partial \Phi_t}{\partial \mu_t} + \frac{1}{1+\mu_t}\left[\frac{\beta(1-\phi_{n,t})y\Psi_n}{\Phi_t}\right]$$
$$- \frac{\beta y \Psi_n}{\Phi_t}\left[\frac{\partial \phi_{n,t}}{\partial \mu_t} + \frac{(1-\phi_{n,t})}{\Phi_t}\frac{\partial \Phi_t}{\partial \mu_t}\right]$$
$$= \frac{-\left(1-i^*_{n,t}\right)}{1+\mu_t} + \frac{\left(1-i^*_{n,t}\right)}{\Phi_t}\frac{\partial \Phi_t}{\partial \mu_t} - \frac{i^*_{n,t}}{1-\phi_{n,t}}\frac{\partial \phi_{n,t}}{\partial \mu_t}, \tag{K.2}$$

where

$$\frac{\partial c^0_{n,t}}{\partial \mu_t} = \frac{1}{\left(1-i^*_{n,t}\right)^2}\left\{\frac{\partial\left[(1-i^*_{n,t})c^0_{n,t}\right]}{\partial \mu_t}\left(1-i^*_{n,t}\right) + (1-i^*_{n,t})c^0_{n,t}\frac{\partial i^*_{n,t}}{\partial \mu_t}\right\}$$
$$= \left(\frac{1}{1-i^*_{n,t}}\right)\frac{\partial\left[(1-i^*_{n,t})c^0_{n,t}\right]}{\partial \mu_t} + \left(\frac{c^0_{n,t}}{1-i^*_{n,t}}\right)\frac{\partial i^*_{n,t}}{\partial \mu_t},$$

$$\frac{\partial\left(1-i^*_{n,t}\right)c^0_{n,t}}{\partial \mu_t} = \frac{-\phi_{n,t-1}y}{(1+\mu_t)^2}\left(\frac{\Phi_t}{\Phi_{t-1}}\right) + \left(\frac{y}{\Phi_{t-1}}\right)\left(\frac{\phi_{n,t-1}}{1+\mu_t}\right)\frac{\partial \Phi_t}{\partial \mu_t},$$

and in equation (49), the effect[26] of monetary policy on $i^*_{n,t}$ is

$$\left[\frac{1}{1-i^*_{n,t}} + i^*_{n,t}\gamma'\left(i^*_{n,t}\right)\right]\frac{\partial i^*_{n,t}}{\partial \mu_t} = \frac{1}{1+\mu_t}\left[\frac{\beta\left(1-\phi_{n,t}\right)y\Psi_n}{\Phi_t}\right]$$
$$- \frac{\beta y \Psi_n}{\Phi_t}\left[\frac{\partial \phi_{n,t}}{\partial \mu_t} + \frac{(1-\phi_{n,t})}{\Phi_t}\frac{\partial \Phi_t}{\partial \mu_t}\right].$$

K.3. EFFECT OF MONETARY POLICY AT $\widehat{\mu}_t$ ON WELFARE

Given the assumption of $\Psi_r = \Psi_n = \Psi$, $\widehat{\mu}_t$ in equation (64), $i^*_{r,t} = i^*_{n,t} = \widehat{i}^*_t$ implies $\phi_{r,t} = \phi_{n,t} = \widehat{\phi}_t$ in equations (44) and (48). First, given equations (K.1), the effect of monetary policy on traders' welfare at $\widehat{\mu}_t$ is

$$\left.\frac{\partial W_{r,t}}{\partial \mu_t}\right|_{\mu_t=\widehat{\mu}_t} = \frac{\left(\frac{1-\alpha}{\alpha}\right)\left(1-\widehat{i}^*_t\right)}{1+\alpha\left(\frac{\Psi_n}{\Psi_r}-1\right)}\left(\frac{\Phi_{t-1}}{\phi_{n,t-1}}\right)\left(\frac{\Psi_n}{\Psi_r}\right) - \left(\frac{\beta\Psi_r y}{\Phi_t}\right)\left.\frac{\partial \phi_{r,t}}{\partial \mu_t}\right|_{\mu_t=\widehat{\mu}_t}$$
$$= \left(\frac{1-\alpha}{\alpha}\right)\left(1-\widehat{i}^*_t\right)\left(\frac{\Phi_{t-1}}{\phi_{n,t-1}}\right) - \left(\frac{\beta\widehat{\Psi}y}{\widehat{\Phi}_t}\right)\left.\frac{\partial \phi_{r,t}}{\partial \mu_t}\right|_{\mu_t=\widehat{\mu}_t}$$
$$= \left(\frac{1-\alpha}{\alpha}\right)\left(1-\widehat{i}^*_t\right)\left(\frac{\Phi_{t-1}}{\phi_{n,t-1}}\right) - \frac{\left(\frac{1-\alpha}{\alpha}\right)\left(1-\widehat{i}^*_t\right)^2}{\left(\beta y\widehat{\Psi}\right)\left\{\gamma'\left(\widehat{i}^*_t\right)\left(1-\widehat{i}^*_t\right)+1\right\}e^{\gamma\left(\widehat{i}^*_t\right)}}\left(\frac{\Phi_{t-1}}{\phi_{n,t-1}}\right)$$
$$= \left(\frac{1-\alpha}{\alpha}\right)\left(1-\widehat{i}^*_t\right)\left(\frac{\Phi_{t-1}}{\phi_{n,t-1}}\right)\left[1 - \frac{\left(1-\widehat{i}^*_t\right)}{\left(\beta y\widehat{\Psi}\right)\left\{\gamma'\left(\widehat{i}^*_t\right)\left(1-\widehat{i}^*_t\right)+1\right\}e^{\gamma\left(\widehat{i}^*_t\right)}}\right], \tag{K.3}$$

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where from equation (J.1),

$$\left.\frac{\partial \phi_{r,t}}{\partial \mu_t}\right|_{\mu_t=\widehat{\mu}_t} = \frac{\left(\frac{1-\alpha}{\alpha}\right)\widehat{\phi}_t\left(1-\widehat{i}_t^*\right)^2}{\left(\beta y\widehat{\Psi}\right)^2\left\{\gamma'\left(\widehat{i}_t^*\right)\left(1-\widehat{i}_t^*\right)+1\right\}e^{\gamma\left(\widehat{i}_t^*\right)}}\left(\frac{\Phi_{t-1}}{\phi_{n,t-1}}\right).$$

Next, given equation (K.2), the effect of monetary policy on nontraders' welfare at $\widehat{\mu}_t$ is

$$\begin{aligned}
\left.\frac{\partial W_{n,t}}{\partial \mu_t}\right|_{\mu_t=\widehat{\mu}_t} &= \frac{-\left(1-\widehat{i}_t^*\right)}{1+\alpha\left(\frac{\Psi_n}{\Psi_r}-1\right)}\left(\frac{\Phi_{t-1}}{\phi_{n,t-1}}\right)-\left(\frac{\beta\Psi_n y}{\Phi_t}\right)\left.\frac{\partial \phi_{n,t}}{\partial \mu_t}\right|_{\mu=\widehat{\mu}_t}\\
&= \left(1-\widehat{i}_t^*\right)\left(\frac{\Phi_{t-1}}{\phi_{n,t-1}}\right)-\left(\frac{\beta\widehat{\Psi}y}{\widehat{\Phi}_t}\right)\left.\frac{\partial \phi_{r,t}}{\partial \mu_t}\right|_{\mu_t=\widehat{\mu}_t}\\
&= -\left(1-\widehat{i}_t^*\right)\left(\frac{\Phi_{t-1}}{\phi_{n,t-1}}\right)+\frac{\left(1-\widehat{i}_t^*\right)^2}{\left(\beta y\widehat{\Psi}\right)\left\{\gamma'\left(\widehat{i}_t^*\right)\left(1-\widehat{i}_t^*\right)+1\right\}e^{\gamma\left(\widehat{i}_t^*\right)}}\left(\frac{\Phi_{t-1}}{\phi_{n,t-1}}\right)\\
&= \left(1-\widehat{i}_t^*\right)\left(\frac{\Phi_{t-1}}{\phi_{n,t-1}}\right)\left[-1+\frac{\left(1-\widehat{i}_t^*\right)}{\left(\beta y\widehat{\Psi}\right)\left\{\gamma'\left(\widehat{i}_t^*\right)\left(1-\widehat{i}_t^*\right)+1\right\}e^{\gamma\left(\widehat{i}_t^*\right)}}\right], \qquad \textbf{(K.4)}
\end{aligned}$$

where from equation (J.2),

$$\left.\frac{\partial \phi_{n,t}}{\partial \mu_t}\right|_{\mu_t=\widehat{\mu}_t} = \frac{-\widehat{\phi}_t\left(1-\widehat{i}_t^*\right)^2}{\left(\beta y\widehat{\Psi}\right)^2\left\{\gamma'\left(\widehat{i}_t^*\right)\left(1-\widehat{i}_t^*\right)+1\right\}e^{\gamma\left(\widehat{i}_t^*\right)}}\left(\frac{\Phi_{t-1}}{\phi_{n,t-1}}\right).$$

Thus, the effect of monetary policy on welfare in the economy from equations (K.3) and (K.4) is

$$\left.\frac{\partial W_t}{\partial \mu_t}\right|_{\mu_t=\widehat{\mu}_t} = \alpha\left.\frac{\partial W_{r,t}}{\partial \mu_t}\right|_{\mu_t=\widehat{\mu}_t} + (1-\alpha)\left.\frac{\partial W_{n,t}}{\partial \mu_t}\right|_{\mu_t=\widehat{\mu}_t} = 0.$$