

# A survey of formation control and motion planning of multiple unmanned vehicles

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## SUMMARY

The increasing deployment of multiple unmanned vehicles systems has generated large research interest in recent decades. This paper therefore provides a detailed survey to review a range of techniques related to the operation of multi-vehicle systems in different environmental domains, including land based, aerospace and marine with the specific focuses placed on formation control and cooperative motion planning. Differing from other related papers, this paper pays a special attention to the collision avoidance problem and specifically discusses and reviews those methods that adopt flexible formation shape to achieve collision avoidance for multi-vehicle systems. In the conclusions, some open research areas with suggested technologies have been proposed to facilitate the future research development.

**KEYWORDS:** Collision avoidance; Cooperative path planning; Formation control; Trajectory optimisation; Unmanned vehicle formation.

## 1. Introduction

Over the past decade, there has been an increasing trend towards the development and deployment of unmanned vehicles. Due to an improved degree of autonomy and personnel risk minimisation through the reduction of the human operator interaction, unmanned vehicles are being used in a number of specialist fields, ranging from military operations to search and rescue, to carry out high risk tasks. In particular, for military operations, the importance of increasing the use of unmanned vehicles in the future battlefield has been addressed by the U.S. Department of Defense (DoD). In their report, published in 2011, it was noted that an increasing percentage of the defense budget had already been allocated to the studying, developing and improving of unmanned vehicle systems, with the vision of creating seamless integration of unmanned vehicle systems with conventional military assets.<sup>1</sup> In addition, UK Ministry of Defence has also cast its vision on unmanned vehicles and subsequently proposed and developed the ‘Unmanned Warriors’ project. This project has first involved more than 50 autonomous vehicles operating in different environmental domains to demonstrate the cooperation between these vehicles and their prospective benefits to the future battlefield.<sup>2</sup>

Depending on its designed mode of operation, an unmanned vehicle can be categorised as an Unmanned Aerial Vehicle (UAV), an Unmanned Ground Vehicle (UGV), an Unmanned Surface Vehicle (USV) or an Autonomous Underwater Vehicle (AUV).<sup>3</sup> For further clarity, UGV refers to a vehicle operating while in contact with the ground<sup>4</sup> while USV refers to an autonomous marine vehicle that navigates on the water surface.<sup>5</sup> A number of practical platforms for each of these autonomous vehicle types have already been built and deployed. When comparing the applications of these platforms, one of the common limitations that has been noted is that they are typically small in size and low in capacity, hence only capable of conducting relatively simple missions. In addition, most of the present unmanned vehicle platforms have low levels of autonomy while some are remote-controlled or are only semi-autonomous. To help overcome or mitigate these problems, it is often more effective to deploy these relatively small vehicles as a fleet in formation (or multi-vehicle formation

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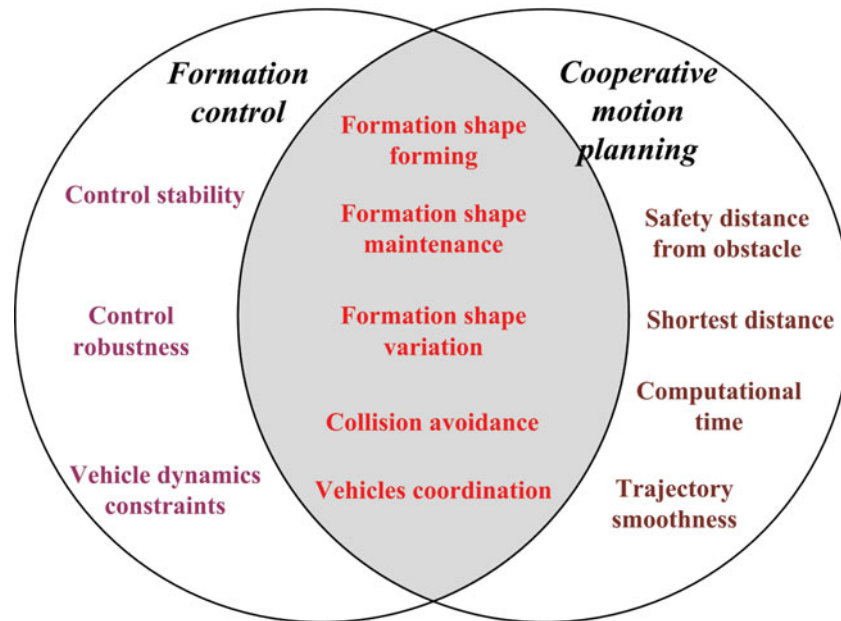


Fig. 1. Comparison of formation control and cooperative motion planning.

system) to carry out tasks since, when compared to a larger individual vehicle, a fleet is able to cover a wider mission area with improved system robustness, coordination and fault-tolerant capabilities.

To better deploy multi-vehicle formation systems, extensive formation related studies have been carried out in recent decades with formation control being the most actively investigated area. The aim of formation control is to generate appropriate control commands to drive multiple vehicles to achieve the prescribed constraints on their own states,<sup>6</sup> and a large body of the research has focused on consensus-based formation control, which utilises the inter-vehicle distance information to allow the formation to retain a certain shape while navigating. More recently, the concept of using flexible formation shapes for collision avoidance purposes has been proposed and studied in a number of different papers. However, the focus of these research efforts remains on generating the commands for low-level controllers with the absence of high-level decision-making capability.

In order to overcome this deficiency and thereby promote the utilisation of multi-vehicle formation systems in complex missions, another research area, i.e., cooperative motion planning, has become dominant in parallel with formation control. By taking into account information such as the mission start and end points and the environmental constraints, the aim of cooperative motion planning is to provide practical guidance information such as the optimised trajectories for the formation to benefit the coordination of multiple vehicles.<sup>7</sup> In addition, when performing the planning, apart from the costs that are routinely considered in conventional planning, such as the shortest distance cost, constraints specifically related to the formation itself, such as the required formation shape, also need to be considered to facilitate the formation control.<sup>8</sup>

Figure 1 provides a comparison of formation control and cooperative motion planning by listing the key factors that need to be considered when designing algorithms. For formation control, in addition to control stability and robustness, vehicle dynamic constraints are important when designing the controller;<sup>9</sup> whereas for cooperative motion planning, safety distance from obstacles, total distance cost, computational time and trajectory smoothness are key costs when planning the path.<sup>10,11</sup> It should also be noted that, as presented in Fig. 1, the large overlap between these two research topics clearly indicates that formation control and cooperative motion planning share a number of key concepts, and hence they should be working interactively when being implemented in multi-vehicle formation systems. For example, when performing cooperative motion planning, the trajectory for each vehicle should be generated with consideration for the required formation shape so that the shape can be attained efficiently. At the same time, the formation control strategy should also be capable of evaluating the features of the generated trajectories and decides whether or not to rigorously follow each individual path or modify them sufficiently it to avoid collisions.

Based upon the above discussions, in order to intelligently and securely operate a multi-vehicle formation system, the importance of the formation control and cooperative motion planning is evident. In fact, a large number of high quality survey papers<sup>12–14</sup> have investigated the formation control problem and pointed out several feasible control approaches including the leader–follower, virtual structure (VS) and behaviour-based methods. However, most of these papers only review mobile robots platforms and do not discuss the related technologies applied to unmanned vehicles, which have more complex motion constraints. Also, the absence of the reviewing of cooperative motion planning algorithms has also prevented these papers from providing a thorough vision on the development of multi-vehicle platforms.

Therefore, the purpose of this paper is to bridge this gap and to provide a review of the different approaches to formation control and cooperative motion planning used by unmanned vehicles over recent decades and to provide a comparison. The key focus is placed upon the analysis of how the different approaches are used to achieve various formation behaviours, such as the formation forming, maintenance and variation. The advantages and disadvantages of each method are analysed to determine common shortcomings and to consider the development trends for future research.

In addition, since this paper considers unmanned vehicles, which are usually deployed in practical environments and are required to avoid obstacles, collision avoidance has become an important criterion when evaluating different methodologies. Specific attention has been given to those studies that have developed evasive strategies that implement flexible and varying formation shapes. Such a strategy is able to provide efficient and effective collision avoidance performance and is therefore generally preferred for practical applications.

At this juncture it needs to emphasise that motion planning is at times referred to as path planning, and these two topics are closely related. The subtle difference between them is that path planning focuses on a collision-free or safe path from start to goal configuration, disregarding dynamic properties, i.e., velocity and acceleration; whereas, motion planning is the superset of path planning, with additional dynamic properties taken into consideration.<sup>15</sup> As a result, path planning typically refers to the computation of robot position and orientation geometric specifications only while motion planning involves evaluation of linear and angular velocities, taking robot or vehicle dynamics into account. However, because the difference is relatively minor, in many review papers (such as Tam *et al.*,<sup>15</sup> Campell *et al.*<sup>16</sup> and Elbanhawi *et al.*<sup>17</sup>), both terms have been used and share the same meaning. In this paper, a similar convention has been followed and both motion and path planning have not been particularly distinguished or compared. However, for reader seeking more in depth differentiation and comparison a paper that specifically discusses the motion planning problem, readers are referred to Goerzen *et al.*<sup>18</sup>

The organisation of this article is as follows. In Section 2, a general overview of the unmanned vehicles formation is presented. The historical development as well as the system architecture of unmanned vehicles formations are discussed. Sections 3 and 4 review the formation control strategies and formation path planning algorithms respectively with comprehensive comparison and analysis. Section 5 gives the conclusion remarks.

## 2. Unmanned Vehicles Formation

The concept of formation is inspired by natural animal behaviours such as birds flocking or fish schooling, where a number of animals adopt certain formations to enhance the survival of the individuals within a group strategy. By mimicking animal formation behaviour, groups of unmanned vehicles can be deployed in formation to accomplish complex tasks and improve the level of system autonomy.<sup>19</sup> In the 1980s, multi-robot formation systems had become a pioneering research field. Typical work included Fukuda's reconfigurable robot system, where the shape of a robot formation can be adjusted depending on task requirement,<sup>20</sup> and the ACTor-based robots and equipments synthetic system (ACTRESS), which is a system architecture allowing multiple robots to cooperatively accomplish tasks, developed by The Institute of Physical and Chemical Research, Japan.<sup>21</sup>

Then, as the technology became more mature, the concept, developed from the multi-robot systems, paved the way to the utilisation of multiple unmanned vehicle platforms in real-world applications. One of the crucial applications is the rescue missions carried out by UGV formations in disaster areas to minimise exploration time and reduce the risk of further casualties.<sup>22–24</sup> Similarly, a number of efforts have been put into the deployments including the area mapping<sup>25,26</sup> and border patrol

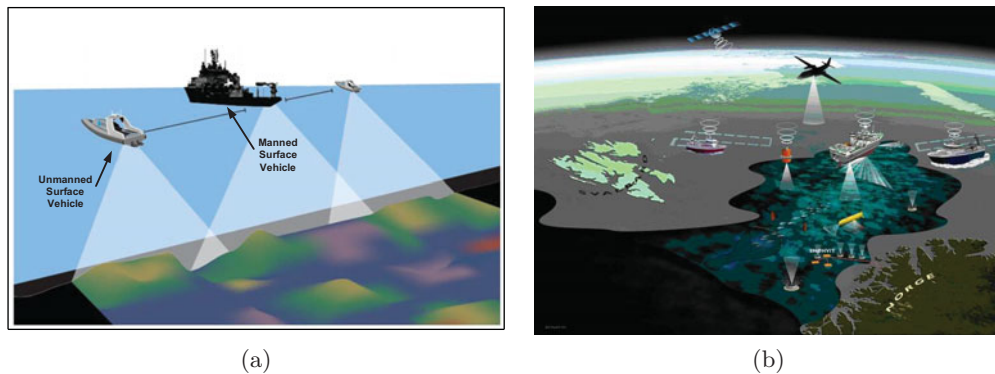


Fig. 2. USV formation applications. (a) The sea mapping operation accomplished by formation consisting of USV and manned vehicles. (b) The cooperation between USV, AUV and UAV. Both are taken from.<sup>29</sup>

and surveillance.<sup>27</sup> In addition, some highly task-oriented missions make use of multiple unmanned vehicles in special cases such as the lunar polar crater exploration missions conducted using a wheeled UGV, a legged scout and several immobile payload items.<sup>28</sup>

It is important to mention that an equivalent scale of the deployment of USV formations has not been seen in recent decades. However, it does not affect the impact brought by using the multi-USVs in accomplishing maritime activities in the future. In the report published by the DoD, the importance of the collaboration between multiple manned vessels and USVs has been addressed. The primary aim is to extend the hydrographic area where human operations cannot reach.<sup>1</sup> Figure 2 shows an example of how manned and unmanned vessels perform a sea mapping operation. The manned surface vehicle in the middle is acting as the leader vehicle to guide the two USVs to conduct the mission. Compared with single vessel operation, the dimensions of the area being explored are significantly increased.

In fact, due to the nature of surface operations of USVs, more important roles are going to be played by the USV in large-scale cross-platform cooperation acting across different unmanned vehicles. One of the potential utilisations is the cooperation of USVs with other unmanned vehicles to form an unmanned system network (shown in Fig. 2). The USV is unique in the sense that it is able to communicate with both above and under water vehicles. In the cooperative formation deployment of multiple unmanned vehicles, the USV can work as an interchange station such that the real-time information is gathered by one USV and distributed to other vehicles to improve communication efficiency.<sup>29</sup>

### 2.1. System architecture of multi-vehicle formation

A generic hierarchical architecture formation system has been proposed by Liu and Bucknall<sup>30</sup> as displayed in Fig. 3. The structure consists of three layers, i.e., the Task Management Layer, the Path Planning Layer and the Task Execution Layer.<sup>31</sup> The Task Management Layer allocates missions to individual vehicles based upon the criteria of maximum overall performance and minimum mission time.<sup>32</sup> A mission can be generally defined as a set of waypoints including mission start point and end point. In Gerkey *et al.*<sup>33</sup> and Khamis *et al.*,<sup>34</sup> comprehensive reviews regarding the multi-robot task allocation have been provided with dominant methodologies being listed. It also should be noted that due to the popularity of the utilisation of neural networks in solving robotics related problems, in recent years there has been a large amount of work using artificial neural network such as the self-organising map (SOM) to address multi-task allocation for unmanned vehicles. For example, Zhu *et al.*<sup>35</sup> proposed to use the SOM to plan tasks for multi-AUV systems and develop a velocity synthesis method for path planning according to the assigned tasks. Faigl and Hollinger<sup>36</sup> also applied the SOM for AUV systems and have specifically investigated SOM application in data collection missions. Liu and Bucknall<sup>37</sup> expanded utilisation of the SOM to USV platforms and have integrated the potential field into the SOM to achieve collision avoidance functionality.

According to mission requirements, the second layer, i.e., the Path Planning Layer, plans feasible trajectories for the formation. This layer is comprised of three sub modules: the real-time trajectory modification module, the data acquisition module and the cooperative path planning module. Among

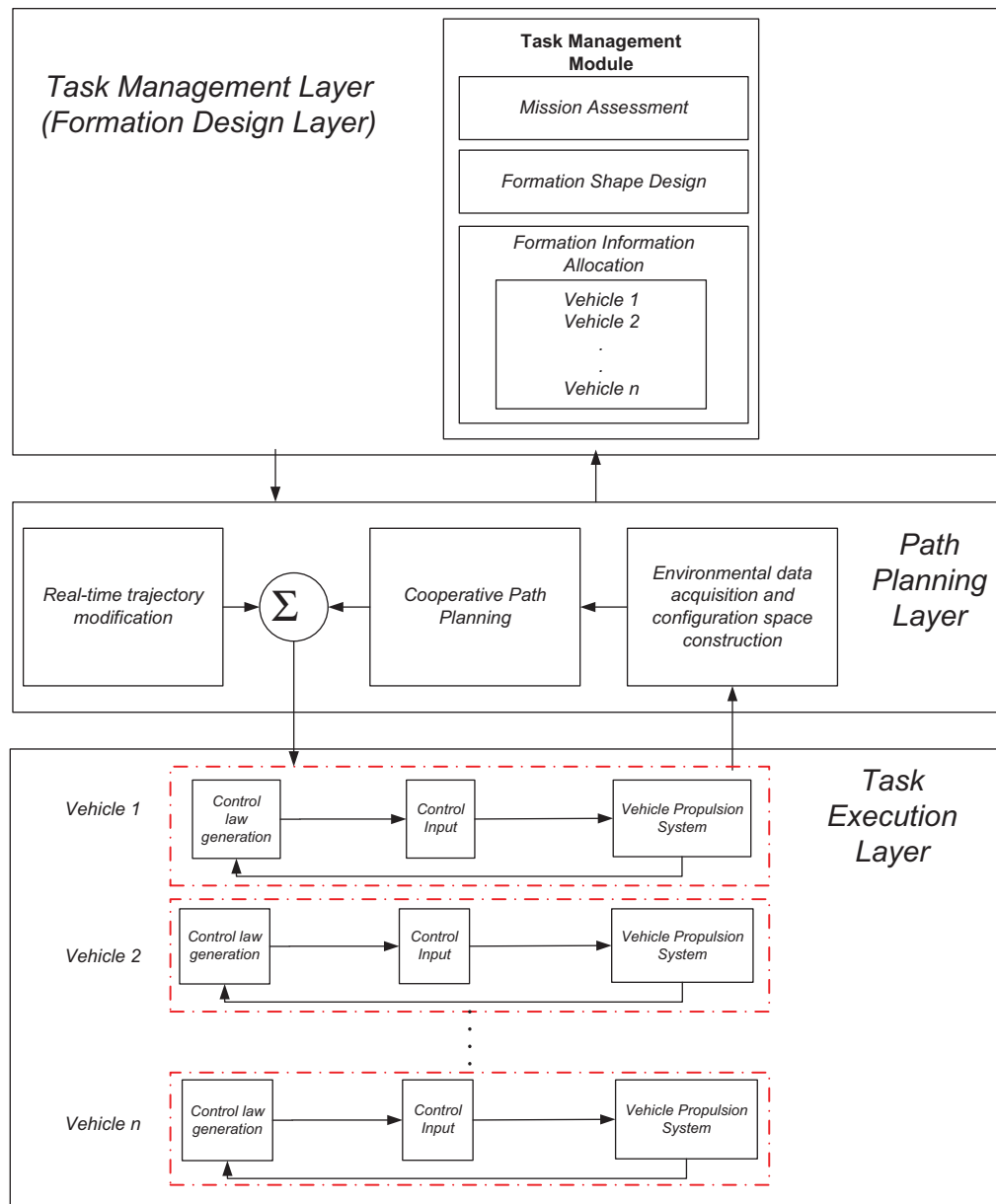


Fig. 3. Hierarchical structure of multiple unmanned vehicles system. There are three layers, i.e., task management layer, path planning layer and task execution layer from top to bottom.<sup>30</sup>

them, the cooperative path planning module is the core of the system and determines the overall optimised path for each vehicle. However, since a number of uncertainties may occur along the trajectory in practical applications, the real-time trajectory modification module is added to the system such that the formation is able to deal with emergency situations such as a suddenly emerged obstacle. A good example of integrating path planning capability with the task-planning requirement can be found at Munoz *et al.*,<sup>38</sup> where a unified framework has been proposed for exploration missions. Also, in Mahmoudzadeh *et al.*,<sup>39</sup> a novel combinatorial conflict-free task assignment and path planning strategy has been proposed for large-scale underwater missions and based upon such a strategy, Zhu *et al.*<sup>40</sup> incorporated a biologically inspired neural network into the task-allocation algorithm to address the dynamics constraints of the vehicles when generating the path.

Generated paths will then be passed down to the Task Execution Layer. This layer has the direct connection with the propulsion system of the unmanned vehicle and generates the control laws. In

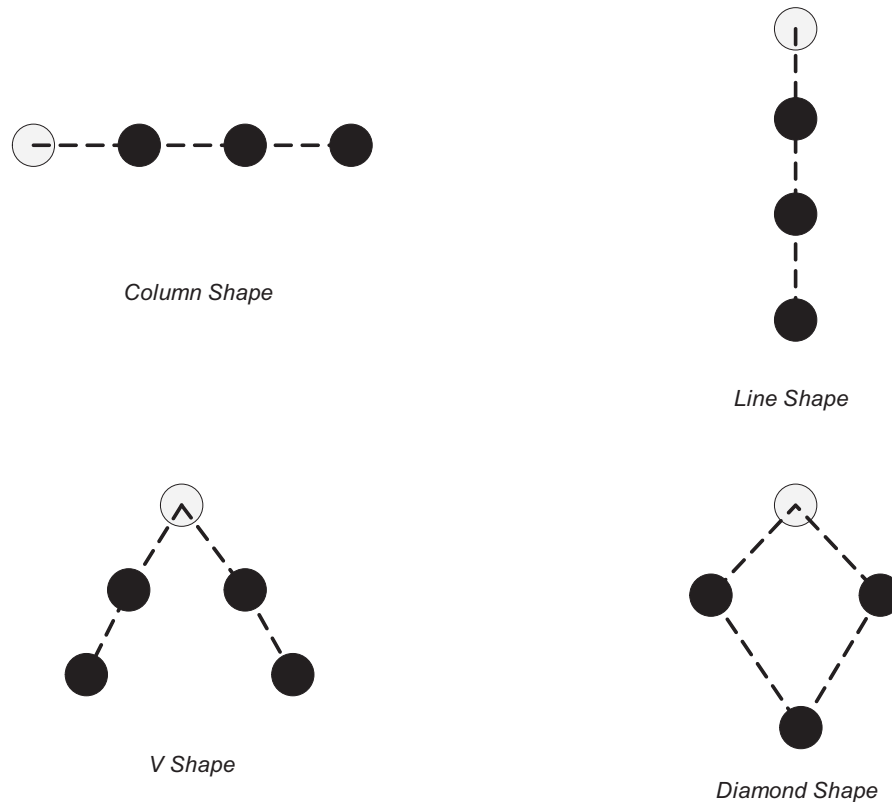


Fig. 4. Four different formation shapes.<sup>16</sup>

order to improve system performance, real-time information, i.e., vehicle velocity and position, is fed back to the upper layer to modify the trajectory in the near future, which generates a closed control loop. Present dominant control strategies include that Dong *et al.*<sup>41</sup> developed an approach adopting a switching interaction topologies to solve the time-varying formation control problem for UAVs. Yamchi *et al.*<sup>42</sup> proposed a distributed predictive controller that helps improve system stability as well as avoid collisions *en route* for mobile robots. Li *et al.*<sup>43</sup> improved the common receding horizon formation control to achieve stabilised tracking performance for AUVs.

## 2.2. System patterns of multi-vehicle formation

When deploying the formation to efficiently accomplish tasks the choice of tactical formation shape can be essential. Based upon Campbell *et al.*,<sup>16</sup> four commonly used shapes are summarised as (see Fig. 4) follows:

- *Column shape*: The column shape gives a comparatively wide mission area, which is of special usage in mine sweeping and area mapping missions.
- *Line shape*: The line shape forms a small mission area but can be more useful in a highly constrained environment.
- *V shape*: The V shape is more suitable in normal operations as it offers a good view of the neighbouring situation. In the meantime, easy and direct communication can be established within the formation.
- *Diamond shape*: The diamond shape is a variation to the V shape, and it is also frequently used in normal operations.

It should be noted that there are no particular restrictions on the choice of formation shape. As shown in Fig. 3, the shape of formation is determined in the task management layer and should be selected adaptively according to specific mission requirements or environments, thus making the formation

'deformable'. For example, when a formation with a wide V shape is about to enter a narrower area, to avoid collisions with the obstacles the V shape could be modified towards a more linear shape. In addition, when the formation shape change is taking place, compared with the conventional approach, the design of the formation control strategy should consider additional constraints. These include<sup>16</sup>

- *Inter-vehicle collision avoidance*: Vehicles in the formation need to consider each other as additional moving obstacles and take appropriate evasive actions.
- *Coordinating of multiple vehicles*: The designed controller should avoid the situation that waiting or coming to full stop occurs due to one or more vehicles in the formation lagging behind.
- *Avoiding deadlock situation*: the movement of the vehicles should be controlled in such a manner that the scenario where one or more vehicles block the paths of others does not occur.

### 3. Review of Formation Control Strategies

While in operation, the control strategy is important for the formation to maintain the formation shape and behaviour. Three main types of formation maintenance are summarised here by the author according to different travel scenarios:

- *Formation generation and maintenance (Type 1)*: The formation shape has to be formed from a condition where the unmanned vehicles are located at random positions with arbitrary headings. Once attained the shape also needs to be maintained to continue the mission (shown in Fig. 5(a)).
- *Formation maintenance during trajectory tracking (Type 2)*: The formation shape needs to be rigorously maintained while the formation is in operation following a predefined trajectory (shown in Fig. 5(b)).
- *Formation shape variation and re-generation (Type 3)*: The formation shape needs to be maintained as defined in Type 2; however, shape also requires adjustment and re-generation while the formation is avoiding an obstacle (shown in Fig. 5(c)).

To achieve formation maintenance, a number of methods have been proposed including leader–follower, VS and behaviour-based methods. The first two methods address the formation maintenance problem better than behaviour-based approach; whereas, a formation controlled by the behaviour-based approach has a more flexible formation shape.<sup>44</sup> The detailed explanations of each method will be provided in the forthcoming sections with critical assessments. However, differing from the conventional way of reviewing the formation control, where the main focus has been placed on the analysis of the controller design as well as the controller's performances, in the following section, not only the design methods will be specifically reviewed and discussed, each method will also be compared against to the three formation maintenance types. This is in fact a new approach to categorise the formation control methods, specifically to assist with the evaluation of each method's suitability for immediate practical applications. For example, if one method can achieve all three types of formation maintenance at the same time, it is evident that in addition to the simple environments containing sparse obstacles (mainly using the Type 1 and Type 2 maintenance), the controller is also capable of satisfying the formation shape maintenance requirement for highly complex environment involving cluttered obstacles or multiple moving obstacles (mainly Type 3 maintenance), and therefore such a method will be better suited for addressing practical application needs.

#### 3.1. Leader–follower formation control

In the leader–follower control approach, one vehicle is regarded as the group leader with full access to the overall navigation information and works as the reference vehicle in the formation. In some cases where system robustness is critical, a *virtual leader* can be assigned to replace the actual vehicle in the formation.<sup>44</sup>

Apart from the leader vehicle, other vehicles in the formation are viewed as followers. Followers operate under the guidance of the leader with the primary aim being retention of the formation shape by maintaining the desired distance from and pose angle to the leader.

Figure 6 illustrates the leader–follower scheme designed by Wang.<sup>45</sup>  $L_{ij}$  and  $\Psi_{ij}$  are the actual distance and angle between leader and follower vehicle while  $L_{ij}^d$  and  $\Psi_{ij}^d$  are the desired distance and angle. The control task is to determine the linear velocity and angular velocity for follower vehicle to

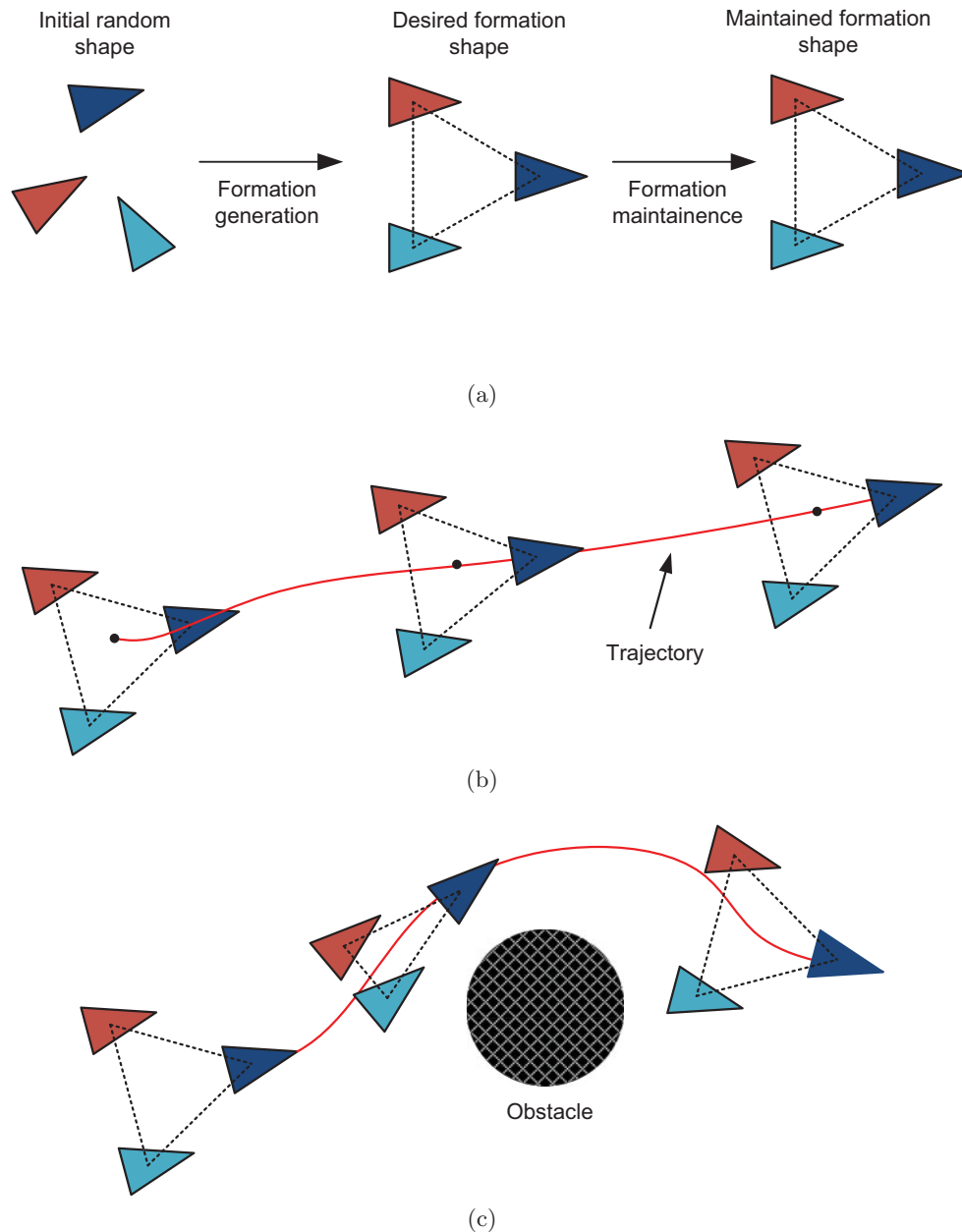


Fig. 5. Three different types of formation shape maintenance. (a) Formation generation and maintenance. (b) Formation maintenance while tracking trajectory. (c) Formation shape variation and re-generation.

eliminate the error value of distance and angle between leader and follower such that

$$\lim_{t \rightarrow \infty} (L_{ij} - L_{ij}^d) = 0 \quad (1)$$

$$\lim_{t \rightarrow \infty} (\Psi_{ij} - \Psi_{ij}^d) = 0 \quad (2)$$

Normally, two types of controllers are employed to design the control law: (1)  $l-l$  controller and (2)  $l-\phi$  controller. The first controller focuses on the relative positions between each vehicle in the formation; while the second one deals with the distance and angle between leader and follower.<sup>46</sup>

The leader–follower approach described here is only feasible for formation control in open space, as it only provides a solution to the *Type 1* and *Type 2* formation maintenance problems. Desai *et al.*<sup>48</sup> improved the leader–follower approach by adding collision avoidance capability to enhance control of



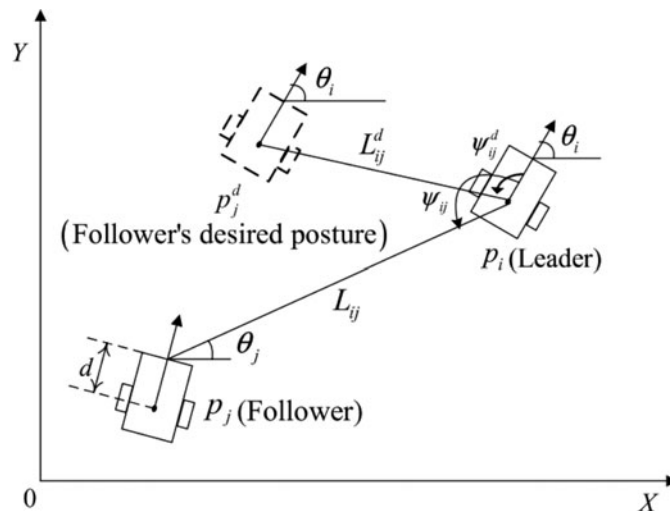


Fig. 6. Leader–follower formation control approach. Formation is maintained by keeping the desired distance and angle between the leader and each follower.<sup>47</sup>

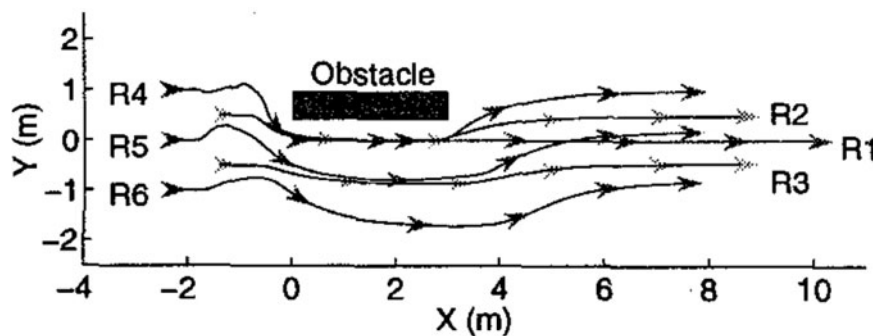


Fig. 7. Leader–follower formation control when the formation is encountering with an obstacle. Formation shape is changed to avoid the obstacle<sup>48</sup>

the formation in a cluttered environment (by solving the *Type 3* maintenance problem). The obstacle was avoided by letting the vehicle maintain a new desired distance, which is the distance between the vehicle and the obstacle. When the formation was avoiding the obstacle, the formation shape could be adaptively changed as shown in Fig. 7, and returned to the desired shape after the risk of collision was averted.

The work of Wang<sup>45</sup> and Desai *et al.*<sup>48</sup> has become the standard approach when the leader–follower approach is applied to unmanned vehicle formation platforms. Adequate modifications are made according to specific needs provided by different platforms.

One of the problems of the implementation is the vehicle's bounded control inputs or constrained inputs. The inputs are normally subjected to a control boundary meaning the control system requires more reaction time, so the stability of the system could also be affected.<sup>49</sup> transformed the physical constraints on the velocity into a geometrical representation. As shown in Fig. 8, the follower's stable point was expanded to an arc instead of a point, which increases the system stability margin. Peng *et al.*<sup>47</sup> observed that impractically large control torque inputs could occur in conventional leader–follower controllers, which could lead to unstable performance. A bio-inspired neuro-dynamics based controller was developed to specifically reduce the required linear and angular velocities in initial state and subsequently reduced the force and torque inputs.

Another issue is system communication. When the leader–follower is implemented, robust communication throughout the formation needs to be assured such that leader and followers are able to exchange their pose information accurately. Unfortunately, such a communication channel is hardly available in practical applications. Edwards *et al.*<sup>50</sup> studied the malfunction problem brought

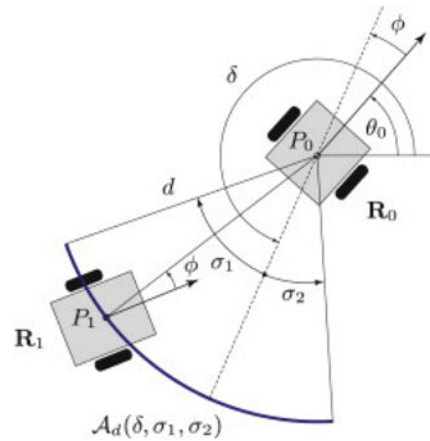


Fig. 8. The transformation of system stable point to a arc of circle as  $\mathcal{A}_d(\delta, \sigma_1, \sigma_2)$ .<sup>49</sup>

about by loss of communication. Orqueda *et al.*<sup>51</sup> proposed a monocular vision system to assist with recording the relative motion between leader and follower. A high gain observer is used to estimate the derivative of leader to follower distance and bearing angle. Peng *et al.*<sup>52</sup> investigated uncertainties associated with marine surface vehicles such as un-modelled hydrodynamics and disturbances from the environment when controlling the formation. An adaptive control law based upon neural networks and backstepping techniques was designed to compensate for uncertainties through an online learning scheme.

### 3.2. Virtual structure formation control

Another important formation control approach is the VS method proposed by Tan *et al.*<sup>53</sup> The VS as defined in this context is a collection of elements (unmanned vehicles), which maintain a rigid geometric relationship to each other and to a frame of reference.<sup>54</sup> The main concept behind the VS is that by treating the formation shape as a VS or a rigid body, the formation is maintained by minimising the position error between the VS and actual formation position. To achieve this, a bi-directional control scheme is proposed in an interacting way that the vehicles are controlled by the virtual force applied to the VS, while the positions of VS is determined by the positions of formation.

The specific control strategy of the VS method mainly involves three stages (see Fig. 9):

- VS position alignment (*stage 1*): Before moving the formation to the next point, a position error based upon the projection of the point-to-point error in  $x$ - $y$  coordinate may occur between the actual positions of the formation and the corresponding positions in the VS. Hence, at this stage, pre-defined one-to-one mapping is used to minimise such errors by following the equation:

$$f(X) = \sum_{i=1}^N d(r_i^W, I_R^W(X) \cdot p_i^R) \quad (3)$$

where  $N$  is the total number of vehicles in the formation,  $d(\bullet)$  is the function to calculate the distance,  $r_i^W$  is the position of vehicle in the world coordinate, whereas  $p_i^R$  is the corresponding position of the vehicle in the VS coordinate, and  $I_R^W(X)$  is the coordinate transforming function between world coordinate and VS coordinate.

- VS movement (*stage 2*): After the VS adjusts itself to the optimal position, a virtual force is applied at the VS to move the VS to the next point. It should be noted that the displacement of the VS is determined not only by the mission requirement but also the dynamic characteristics of the vehicles. The displacement needs to be appropriately calculated such that the vehicle can reach it in the next time step.
- Formation movement (*stage 3*): based upon the new position of the VS, each vehicle in the formation can now move towards its new position by referring to its corresponding point in the VS. A control input is generated for each vehicle, and to achieve more precise tracking performance, the vehicle

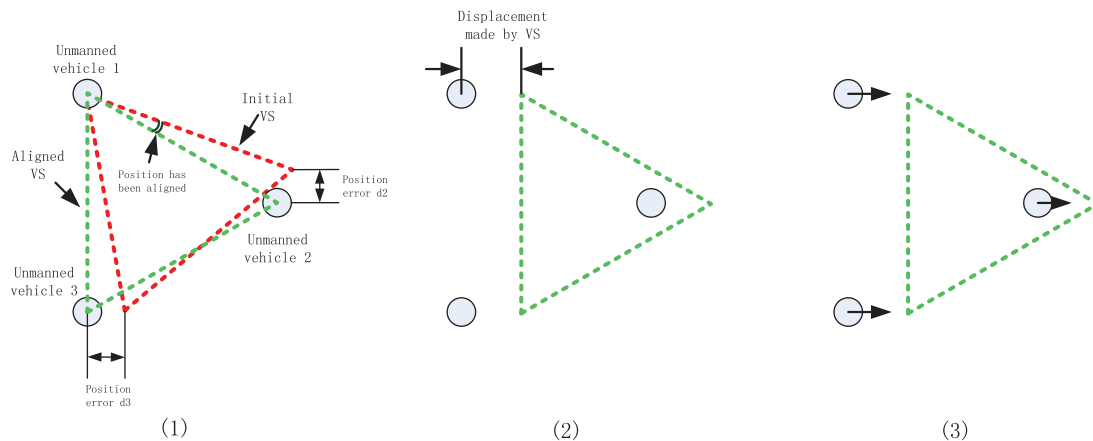


Fig. 9. Steps of virtual structure formation control. (1) VS position alignment. The triangle in red is the initial position of VS, and the triangle in green is the aligned VS by minimising the position error. (2) Move the VS to next position. (3) Move the formation according to the position of VS.

is first controlled to alter its heading to the desired orientation and then transits towards the target point.

Compared with the leader–follower approach, one of the most appealing advantages of the VS method is an increase in fault-tolerant capability. In leader–follower control, due to the lack of feedback of positions of each vehicle in the formation, a faulty vehicle will not be detected by other vehicles, causing the formation to disintegrate. However, such a drawback can be overcome by using the VS approach. It has been proven in Lewis and Tan<sup>54</sup> that the tracking error caused by the faulty robots can be compensated for by other robots in the *VS alignment stage* so that the formation can be retained (see Fig. 10). It should be noted that such formation maintenance is only a temporary solution as the faulty vehicle has not been repaired. To achieve comprehensive fault-tolerance, some high-level decision processes are needed to either change the formation shape or call up a new vehicle to replace the faulty unit.

Like the leader–follower approach, a robust communication channel is vital for the VS method as each vehicle is highly dependent on the information being exchanged to obtain real-time navigation data. In the work of Do and Pan,<sup>55</sup> such a problem has been addressed by introducing the communication limitation through a potential function. Suppose the designed controller for the *i*th vehicle is  $u_i$ , which was calculated not only related to its own position and velocity, but also the communication range, which was described as a potential function  $\beta_{ij}$  as follows:

$$\beta_{ij} = \begin{cases} = 0 & \text{if } d_{ij} \geq d_{com}, \\ > 0 & \text{if } 0 < d_{ij} < d_{com}, \\ = \infty & \text{if } d_{ij} = 0. \end{cases} \quad (4)$$

where  $d_{ij}$  is the distance between *i*th and *j*th formation agent, and  $d_{com}$  is the pre-defined communication range. It can be observed that if the distance between two vehicles is larger than the communication range, the potential value  $\beta_{ij}$  is zero and thereby the designed  $u_i$  is not dependent on *j*th agent information. Based on this, further work has been carried out by Do<sup>56</sup> to solve the USV formation control problem. An elliptical shape was adopted to simulate the dimension of the vessel with a circular area centred on the mid-point of the vessel representing the communication range.

### 3.3. Behaviour-based formation control

Behaviour-based formation control was first proposed by Balch and Arkin.<sup>57</sup> It solves the formation control problem by using a hybrid vector-weighted control function, which is able to generate the control command based upon various kinds of formation missions. For example, according to the general mission requirements, four different control schemes (‘‘behaviours’’) were developed as *move-to-goal* ( $u_{MG}$ ), *avoid-static-obstacle* ( $u_{AO}$ ), *avoid-robot* ( $u_{AR}$ ) and *maintain-formation* ( $u_{MF}$ )

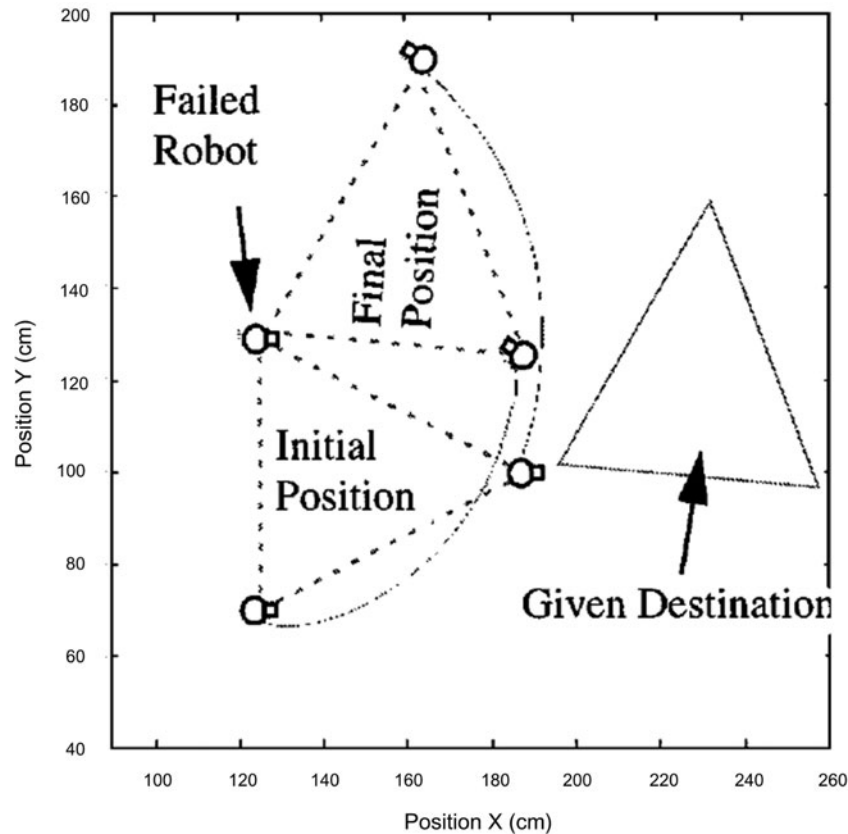


Fig. 10. Fault-tolerant formation control by using the virtual structure method. During a robot failure, the other robots adjust their paths to maintain formation. The new formation has the correct formation shape as well as the desired orientation.<sup>54</sup>

schemes. Each scheme was assigned with a gain value according to the specific mission or traffic environment, and the final control scheme was determined as the weighted combination of these gains by

$$u = a_1 \cdot u_{MG} + a_2 \cdot u_{AO} + a_3 \cdot u_{AR} + a_4 \cdot u_{MF} \quad (5)$$

where  $a_1$ ,  $a_2$ ,  $a_3$ ,  $a_4$  are the weighting gains for controllers with high gain value representing high importance for the corresponding behaviour. By implementing behaviour-based formation control, not only the formation generation and keeping, but also the collision avoidance can be simultaneously solved. It makes such a control approach superior to the other approaches in terms of practical application. However, in essence, the designed controller is not based upon kinematic/dynamic characteristics of the vehicles, thus the mathematical proof of system stability is highly complex, which makes it hard to theoretically justify the performance of this approach.<sup>58</sup> Despite this, the behaviour-based formation control is still of great importance, and a number of studies have adopted such an approach.

In the work of Cao *et al.*,<sup>59</sup> the genetic algorithm (GA) was integrated with the behaviour-based formation control to assist the determination of weighting gain values of each behaviour. The simulation results show that besides improved control performance, the formation also presented a certain adaptability in an unknown environment by optimising the gain values detailed in Eq. (5).

Later, Cao *et al.*<sup>60</sup> investigated formation control in an unknown environment with moving obstacles. A prediction model based upon the recurrence least square algorithm was used to estimate the position of a moving obstacle, and a new behaviour named the *random behaviour* was established to operate in conjunction with the conventional four behaviours, to handle the unstable state occurring in a cluttered environment.

Since it is hard to mathematically analyse the formation stability by using the behaviour-based method, a hybrid control scheme that includes both the leader–follower and the behaviour-based methods was proposed by Yang *et al.*<sup>61</sup> The formation was generated and maintained by the leader–follower while the behaviour-based scheme specifically focused on the motion planning of individual vehicles. A supervision mechanism has been built between the leader and followers such that the formation integrity can be ensured when the number of controlled vehicles changes. The supervision is achieved in a way that an inter connection between the leader and each follower is established so that the leader can have a full-time monitoring of the status of followers.

### 3.4. Discussion on formation control strategies

In Table I, three main formation control strategies are summarised and compared. From the deployment platforms' perspectives, it shows that the most widely adopted strategy is the leader–follower approach, which has been applied not only on the mobile robot platforms, but every kind of unmanned vehicle platform. The primary reason for such wide scale deployment is probably because the leader–follower approach is relatively simple to design and implement. Such an approach is developed based upon the common concept when managing a group, i.e., a leader is selected from the group to supervise the group while other group members follow the behaviour of the leader.<sup>62</sup> Therefore, by using the leader–follower approach, the formation relationship is more explicit than with other approaches. Also, as mentioned in Section 2.2.1, the leader–follower approach adopts a centralised communication strategy, which requires vehicles in the formation to only establish connections with the leader. The overall amount of exchanged information is much less than with a decentralised approach, and as a consequence the communication efficiency is much higher. However, the primary disadvantage of the leader–follower is its high dependence on the leader vehicle's performance. If the leader malfunctions or the communication between the leader and the follower is disrupted, the formation is hard to control and maintain.

The VS strategy provides better performance in terms of formation maintenance as the formation is designed to follow the rigid body VS. However, such good performance in formation keeping is not beneficial for formation modification. The change of the formation requires re-design of the VS, which has the potential to increase the computational burden of the formation. The inflexibility in the formation eventually leads to limited capability for dealing with collision avoidance with obstacles, making the VS an unsuitable option for *Type 3* formation maintenance (shown in the 'Formation maintenance type' column in Table I).

The behaviour-based control methodology seems to be the most adoptable approach as it is able to accomplish a number of different mission requirements through one control command. But the lack of system stability analysis makes it unsuitable for large-scale utilisation.

As regards future development, a hybrid control strategy appears to be the trend. No single solution exists that is appropriate for all scenarios. A hybrid approach can be developed such that in the open space, where stabilisation of the system is the priority, the leader–follower and/or the virtual-leader method could be used. When the formation is navigating in a complex environment, the behaviour-based method takes over the control.

Another important development for formation control will be the integration with fault-tolerant control. One of the benefits gained from the deployment of unmanned vehicles as a formation is the improved system robustness that comes to the fore if and when vehicles in the formation fail. However, this aspect is generally ignored by much of the research work accomplished thus far. Fortunately, there have already been in-depth publications from Tousei *et al.*,<sup>63</sup> Yang *et al.*<sup>64</sup> and Tousei *et al.*,<sup>65</sup> who have studied the fault tolerance control from a mathematical perspective. There is no doubt that the seamless merging of fault-tolerance control and formation control would dramatically improve the utility of the research.

## 4. Review of Cooperative Formation Path Planning

In this section, algorithms developed for formation path planning will be grouped, reviewed and analysed accordingly with some typical work listed. It should be noted that alongside formation path planning, there has been research into another emerging path planning method in recent years, the *multi-vehicle cooperative path planning*. It can be viewed as a weaker challenge than the formation path planning with less conditions; however, solutions for the cooperative path planning are also

Table I. Comparison of formation control strategies.

Methods	Advantages	Disadvantages	Formation maintenance types	Platforms
Leader–follower <sup>45,47,48</sup>	<ol style="list-style-type: none"> <li>1. Easy to be designed and implemented</li> <li>2. Efficient communication within the system</li> </ol>	<ol style="list-style-type: none"> <li>1. Highly dependent on the leader vehicle</li> <li>2. Lack of the feedback from the follower to the leader</li> </ol>	Type 1, Type 2 and Type 3	Widely adopted across various platforms
Virtual structure <sup>54–56</sup>	<ol style="list-style-type: none"> <li>1. Good performance in shape keeping</li> <li>2. Good representation of the relationship and the coordination between each vehicle in the formation</li> </ol>	<ol style="list-style-type: none"> <li>1. Not flexible for shape deformation</li> <li>2. Not easy for collision avoidance</li> </ol>	Type 1 and Type 2	Most applications seen on mobile robots. Less application on unmanned vehicles
Behaviour-based <sup>57,59,61</sup>	<ol style="list-style-type: none"> <li>1. Capable of dealing with multi-task mission</li> </ol>	<ol style="list-style-type: none"> <li>1. Not easy to mathematically express the system behaviour</li> <li>2. Difficult to prove and guarantee the system stability.</li> </ol>	Type 1, Type 2 and Type 3	Mobile robots and UGVs are two popular platforms

beneficial with some core algorithms able to assist the formation with minor modifications to the algorithms. In this section, methods for both the formation path planning and the multi-vehicle cooperative path planning are going to be reviewed.

The path planning problem is to find a feasible route connecting the start and end points in a collision free space while satisfying a set of constraint conditions. The problem itself can be expressed as an optimisation process subjected to several costs, and for a 2-D path planning problem, it can be mathematically written as follows:<sup>31</sup>

$$P_s(x_s, y_s, \varphi_s) \xrightarrow[s.t. \prod_{\text{single}}]{\tau(t)} P_e(x_e, y_e, \varphi_e), \quad (6)$$

where  $P_s(x_s, y_s, \varphi_s)$  and  $P_e(x_e, y_e, \varphi_e)$  denote the start and end point configuration, respectively, which include start and end point coordinates and orientation.  $\tau(t)$  represents the trajectory which is subjected to the cost  $\prod_{\text{single}}$ . When extending the problem to the multi-vehicle formation path planning, the formulation can be written as follows:

$$P_{s,i}(x_s, y_s, \varphi_s) \xrightarrow[s.t. \prod_{\text{multiple}}]{\tau(t)} P_{e,i}(x_e, y_e, \varphi_e) \quad i = 1, 2, \dots, N \quad (7)$$

where  $N$  is the total number of vehicles in the formation and  $\prod_{\text{multiple}}$  is the cost for multiple vehicles paths.

In single vehicle path planning, to obtain the most effective and efficient path,  $\prod_{\text{single}}$  normally contains the least distance, the highest safety, the minimum energy consumption and so on. However, in contrast, costs for multiple vehicles path planning ( $\prod_{\text{multiple}}$ ) are more complicated, and a comparison between single vehicle and multiple vehicles costs has been provided in Liu and Bucknall.<sup>30</sup> As shown in Fig. 11, additional costs are explained as follows:

- *Internal collision avoidance*: As multiple vehicles are simultaneously and cooperatively working, each vehicle becomes a potential collision risk to other vehicles in the same group. To ensure the safety of the group, the internal collision avoidance needs to be addressed.
- *Formation behaviour*: If multiple vehicles are travelling in a formation, the formation behaviours, such as shape keeping and shape changing, are required.
- *Cooperation behaviour*: The cooperation behaviour is the most important factor, which can be expressed in two different forms as the *time cooperative behaviour* and the *time and position cooperative behaviour*. Illustrations of these two different forms are displayed in Fig. 12. The first one only imposes time requirements on the final trajectories, i.e., by following planned trajectories, each vehicle within the group should leave and arrive at each mission point simultaneously or in order. Since no formation behaviour is represented *en route* except the start and end points, the path planning problem involving such behaviour is known as the *multi-vehicle cooperative path planning*.  
In contrast, the second form not only places the requirement on time but also on instantaneous position of each vehicle. Generated trajectories should, to the most extent, maintain the pre-defined distances between each other thereby solving the formation path planning problem.
- *Total distance*: To achieve the most efficient outcome, the total distance of all trajectories should be optimised.

#### 4.1. Path planning algorithms and the categorising

A number of different path planning algorithms have been proposed, and according to Tam *et al.*<sup>66</sup> these algorithms can be categorised into two different general approaches based upon its searching characteristics (as shown in Fig. 13):

- Deterministic path planning.
- Heuristic path planning.

The deterministic searching approach is accomplished through the following of a set of defined steps and has the advantage of completeness and consistency. A searching result can be guaranteed to be found as long as it exists; also, the output remains the same each time if there is no variation

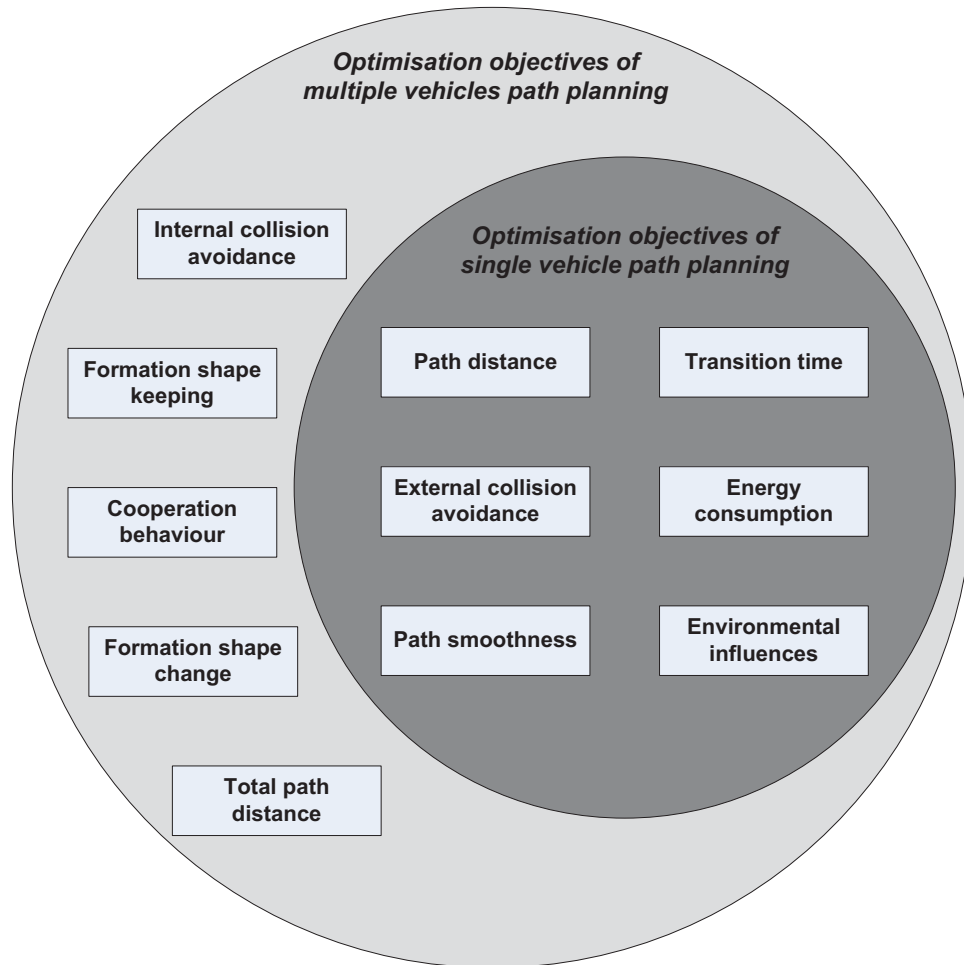


Fig. 11. Comparison of costs between single vehicle path planning and multiple vehicles path planning.<sup>30</sup>

of searching environment. Therefore, the deterministic algorithm can also be referred to as an exact algorithm. Popular deterministic approaches include the artificial potential field (APF), the roadmap-based algorithm and the optimisation method.

The heuristic searching approach is proposed to specifically solve the problem that can-not be efficiently addressed by a deterministic approach. Also, it is able to provide an approximate solution when exact solutions are hard to find<sup>67</sup> and thus the heuristic algorithm can also be referred to as an approximation algorithm. However, since it only searches the subspace of the search space, the global optimality of the results cannot be guaranteed, i.e., only near optimal result can be obtained. In addition, because the algorithm stochastically searches within the space, the consistency of the results is not as good as those delivered by the deterministic method.<sup>68</sup> Typical heuristic algorithms include the evolutionary algorithm (EA) such as the GA, the particle swarm optimisation (PSO) and the ant colony optimisation (ACO).<sup>66</sup>

Another promising classification strategy for path planning algorithms, as proposed in Sharma *et al.*,<sup>69</sup> is to evaluate the algorithm depending upon if it has been developed in a deliberative or reactive way. For example, when the environment is partially known to the vehicle, algorithms can only generate the trajectory within a certain area and therefore has to constantly and reactively update the trajectory as the vehicle is navigating; hence, such a strategy is regarded as a reactive approach. Conversely, when the environment is fully mapped a deliberative approach is adopted and in this case, the generated trajectory is able to provide full guidance information to the vehicle and is always used as the global reference path. In Fig. 14, favourable path planning algorithms have been re-grouped based upon reactive or deliberative approaches. It can be seen that compared with Fig. 13, EA and roadmap-/grid-based algorithms now belong to the deliberative approach; whereas, potential field



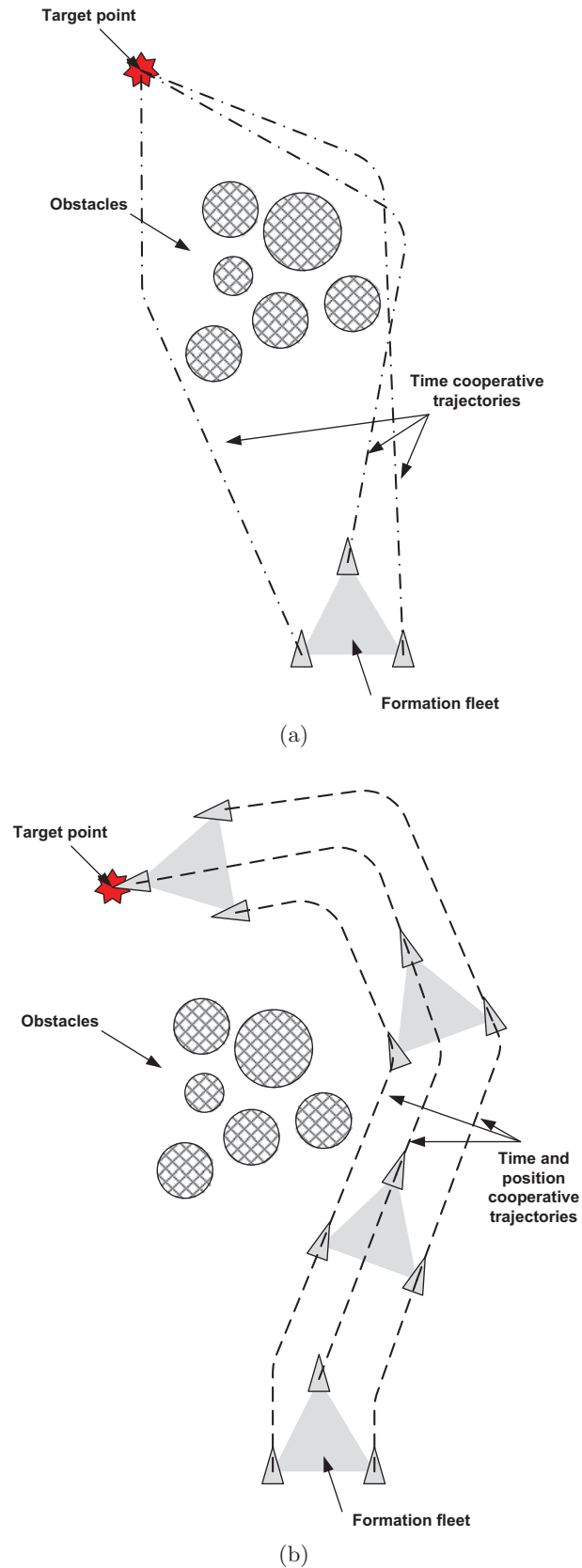


Fig. 12. Cooperative path planning. (a) Time cooperative trajectories: by following the generated paths, vehicles need depart from the start point and arrive at end point simultaneously but without the need to keep the relative position to each other. (b) Time and position cooperative trajectories: apart from the same departing and arriving time, vehicles also need to keep the relative distance *en route*, and they tend to move in a formation.<sup>31</sup>

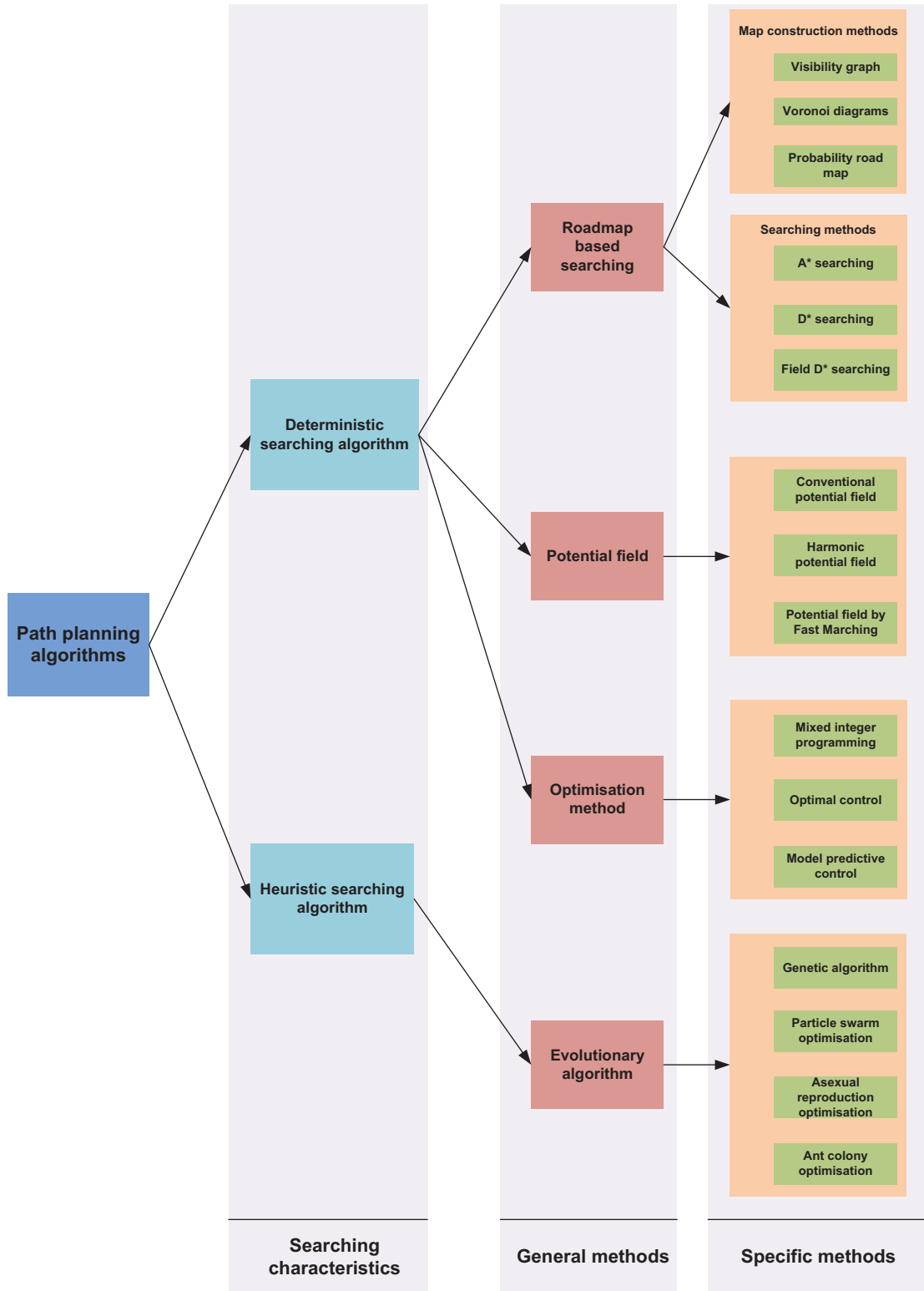


Fig. 13. The categorising of path planning algorithms based upon deterministic and heuristic approaches.

algorithms are grouped in the reactive category together with the optimisation method (especially the model predictive control).

In the following sections, literature in regard to multiple vehicles path planning is going to be reviewed based upon the adopted searching methodology.

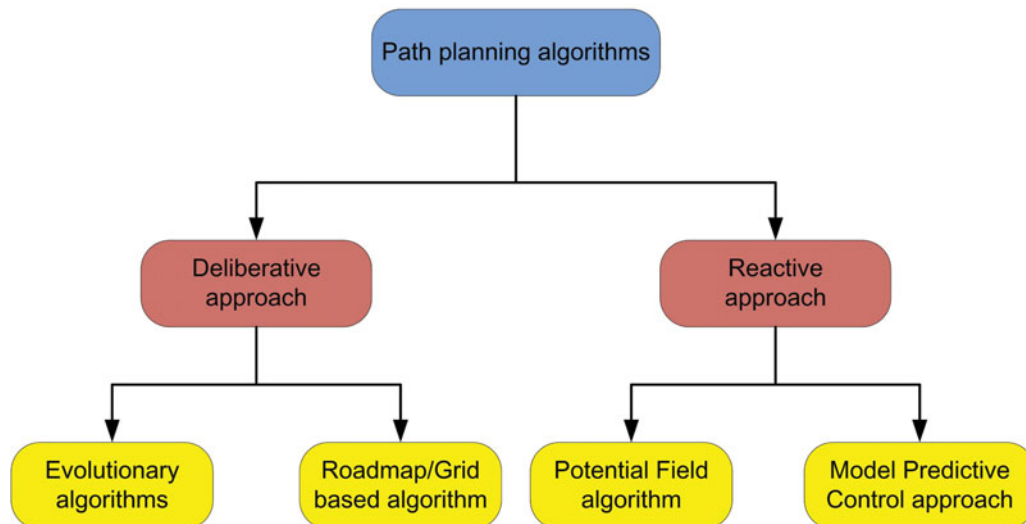


Fig. 14. The categorising of path planning algorithms based upon reactive and deliberative approaches.<sup>69</sup>

#### 4.2. The potential field method

The APF method was first proposed by Khatib<sup>70</sup> to control a robot manipulator. The method converts the configuration space into the potential field, which consists of an attractive field ( $U_{att}$ ) around the target point and repulsive fields ( $U_{rep}$ ) around obstacles. The attractive field is proportional to the distance to the target point and is influential over the whole space; whereas the repulsive fields are inversely proportional to the distance to the obstacles and are only effective in certain areas around obstacles. The path is calculated by following the total force at each location, which is the gradient of the sum of fields as follows:

$$F_{total} = F_{att} + F_{rep} = \nabla(U_{att}) + (-\nabla(U_{rep})) \quad (8)$$

In terms of the implementation of the APF in formation path planning, in addition to the attractive and repulsive fields, new fields are needed to represent cooperative formation behaviours. An internal attractive potential field first needs to be constructed to maintain the formation shape (shown as the red line in Fig. 15) such that when a vehicle is away from its formation position, the force is capable of dragging it back to prevent destruction of the formation shape. In addition, internal repulsive fields are also needed to prevent two vehicles from moving too close and colliding with each other (blue line in Fig. 15).

Wang *et al.*<sup>72</sup> constructed such potential fields by referring to the concepts of electric fields. Each vehicle was treated as a point source in the electric field with varying electrical polarity. If the distance between vehicles was larger than the expected value, opposite charges were used to attract them to move towards each other; otherwise, like polarities were used to prevent them from colliding if the distance between vehicles was less than the expected value.

Paul *et al.*<sup>73</sup> also built the fields to solve the problem of UAV formation path planning. To increase control accuracy, an attractive potential field was a function of the error value between desired distance and actual distance, such that any deflection from the desired position can be quickly modified and corrected.

Yang *et al.*<sup>74</sup> published work on motion planning for an AUV formation in an environment with obstacles based upon the APF. The algorithm concentrated on overall mission requirements instead of the development of an individual vehicle's control law and treated the AUV formation as a multi-body system with each vehicle modelled as a point mass with full actuation. Potential fields for formation path planning were constructed for particular mission requirements, ocean environment and formation geometry.

It should be noted that the primary disadvantage of using the APF is the local minima problem. It is caused by the sum of total forces at certain point equalling zero, which results in the vehicle becoming 'trapped' at that point. Many researchers solved this problem by constructing new kinds

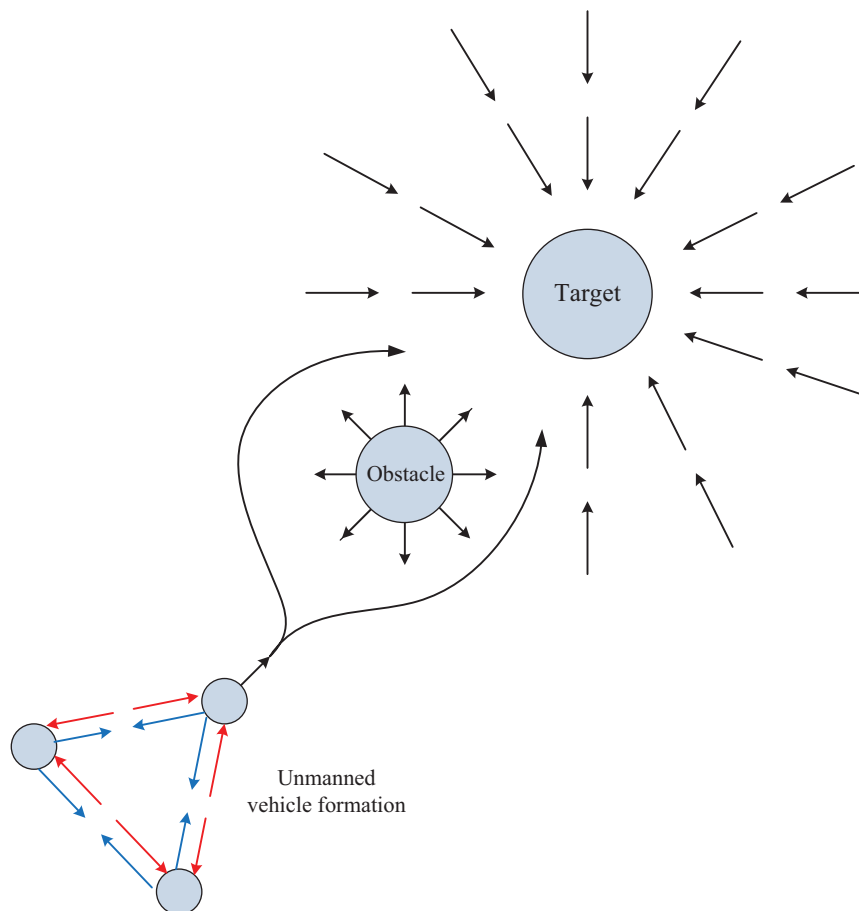


Fig. 15. The formation path planning using the APF. An internal attractive potential field first needs to be constructed to maintain the formation shape (shown as the red line). Internal repulsive fields are also needed to prevent two vehicles from moving too close and colliding with each other (shown as the blue line).<sup>71</sup>

of fields such as the Harmonic Potential Fields,<sup>75–78</sup> which is constructed by the harmonic function containing no local minima.

Recently, another effective way to deal with the local minima problem was reported in Garrido *et al.*<sup>10</sup> and Gomez *et al.*,<sup>11</sup> which employed the Fast Marching Method (FMM) to construct the potential field. Differing from the conventional way of combining all fields to generate the total potential field; the FMM produces the potential field by simulating the propagation of an electromagnetic wave. A propagation index ranging from 0 to 1 is first calculated at each point to indicate the speed of propagation of the wave, i.e., 0 value means the wave cannot pass and hence is given to obstacle area. The wave then emits from the start point by obeying the propagation index and stops when the target point is reached. The generated potential field represents the local arrival time of the wave and only has the minima potential at the start point.

#### 4.3. Evolutionary algorithm

The EA method including GA, PSO and ACO is a heuristic search algorithm based upon biological evolution process. When applied to the path planning problem, the EA mimics the natural selection process in the way that possible paths are treated as individuals and evolve themselves through mutation, reproduction and recombination by comparison against a fitness function. The function, as a weighted function consisting of several optimisation criteria, determines the quality of each individual and only the individual with the best fitness result will survive as the final path.

In terms of using the EA for multiple vehicles cooperative path planning, a two-layer evolution process is normally used. In Fig. 16, each vehicle has its own EA process, which generates an optimal trajectory subject to each individual vehicle's planning conditions. Then, all the individual paths are

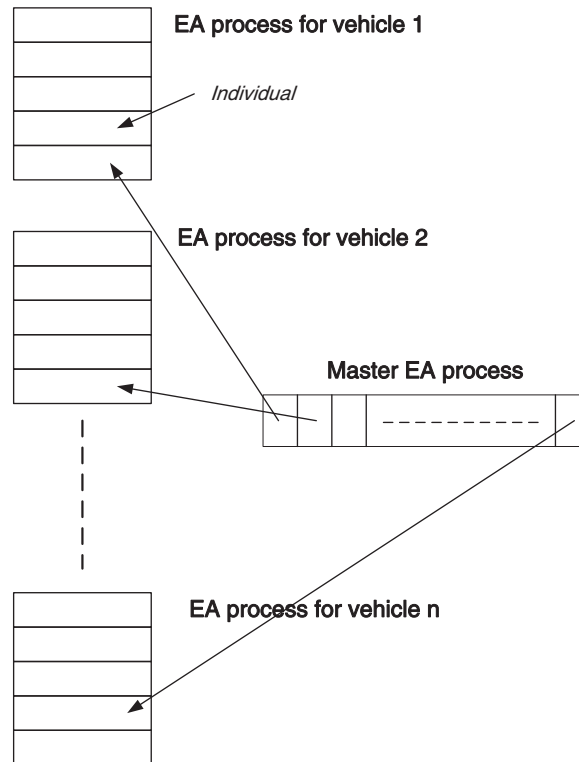


Fig. 16. The two-layered EA process for multiple vehicles cooperative path planning.

compared against a master fitness function in the Master EA process to achieve cooperative behaviour. The master fitness function takes costs such as the internal collisions, the target point arrival time and the distance between each vehicle into consideration, and re-evolves individuals to make them suitable for multiple vehicle cooperation. Note that the cooperative behaviour addressed by the EA method normally belongs to time cooperative behaviour because of the characteristic of the randomness of EA search, which makes it hard to follow the rigorous condition of formation shape maintenance.

Among the applications of EA in multiple vehicle path planning, Zheng *et al.*<sup>79</sup> proposed a co-evolving and cooperating path planner for multiple UAVs based upon the GA. In order to make the generated path practical, dynamic characteristics constraints such as the minimum path led length, the minimum flying height, and the maximum climbing angle were incorporated into the algorithm. However, the computation speed was not fast enough to make the algorithm applicable for real-time planning. Hence, Kala<sup>80</sup> and Qu *et al.*<sup>81</sup> improved it by introducing new evolution operators to increase the convergence speed of the algorithm.

4.4. Optimal control method

Using the optimal control method is another main approach for multiple vehicle cooperative path planning. This approach considers the path planning problem as a numerical optimisation problem by following a set of constraints.<sup>82</sup> It breaks down the multiple vehicle path planning into several single vehicle path planning processes, and the multi-vehicle cooperation is achieved by satisfying a set of pre-defined ‘cooperation constraints’.

A general form of using the optimal control method for multiple vehicle path planning was reported in Schouwenaars *et al.*<sup>83</sup> The group consisted of a number of *N* vehicles, and for the *p*th vehicle in the group, a fuel-optimal cost function was first defined as follows:

$$J_p = \sum_{i=1}^T q'_p |s_{pi} - s_{pf}| + \sum_{i=1}^T r'_p |u_{pi}| + p'_p |s_{pT} - s_{pf}| \tag{9}$$

where  $s_{pi}$ ,  $u_{pi}$  and  $s_{pf}$  denote the state, input and final state of the vehicle, respectively.  $q'_p$ ,  $r'_p$  and  $p'_p$  are the weighting factors. Constraints for single vehicle optimisation included boundary conditions for the vehicle's state and control inputs, and position constraints to avoid static and moving obstacles. Then, a 'cooperation constraint' was defined, in this case, to keep two vehicles away from each other by a certain distance to maintain the safety as

$$|x_{pi} - x_{qi}| \geq d_x \quad |y_{pi} - y_{qi}| \geq d_y \quad (10)$$

where  $(x_{pi}, y_{pi})$  and  $(x_{qi}, y_{qi})$  are the coordinates for  $p$ th and  $q$ th vehicle at time step  $i$ , and  $d_x$  and  $d_y$  are the two safety distances. By subjecting to all constraint conditions, mixed integer linear programming (MILP) was used to find the optimal control input  $u$  for each vehicle, which could finally generate a feasible path by substituting it into the system dynamic functions. Yilmaz *et al.*<sup>84</sup> expanded such a method to a larger scale multiple vehicle cooperation such as AUV-USV cooperation, AUV-Shore station cooperation and AUV-AUV cooperation. To achieve these cooperation, constraints were developed to ensure sufficient distances were maintained to keep robust communication.

However, even though the MILP is powerful enough to handle different constraints for the optimisation problem, high computation complexity is its main disadvantage and prevents its use for on-line planning. To improve MILP, Bemporad and Rocchi<sup>85</sup> applied receding horizon control (RHC) to solve optimisation problems for UAV formations. Unlike conventional methods that seek for the optimal result for the whole time period, an on-the-fly strategy is used by the RHC to only minimise the cost function for a relatively short horizon in each time step and compute the according control input, which could largely decrease the computation time. Based on such an online scheme, Chen *et al.*<sup>86</sup> designed a formation hybrid formation path planner by combining the RHC and the APF methods for UAVs. An additional control force generated by APF was added to the system control input to improve the collision avoidance capability of the formation.

#### 4.5. Discussion on formation path planning

Formation path planning, working as a command generator for the formation control system (referring to Fig. 3), takes the description of the environment as the input and produces sets of waypoints as trajectories. The APF method, the EA and the optimal control method are three mainstream approaches used for multi-vehicle path planning, and a comparison of these approaches is listed in Table II. Among them, the potential field and the evolutionary based methods are the most widely adopted approaches. These are significantly different from single vehicle path planning, where the grid based<sup>87-89</sup> or the road map based methods<sup>90-93</sup> are preferred.

A possible reason is the multiple-vehicle system needs a path planning algorithm to have fast computation speed as a number of vehicles are involved; however, both the road map and the grid-based methods need significant memory capacity to store the environment information, which has the potential to decrease the speed of the algorithm. More importantly, the trajectories generated by the potential field or the EA are more practical than other methods. The potential field method can produce a smooth and continuous path and the EA is able to optimise the trajectory's costs for different mission requirements.

However, the FMM-based potential field method may have more advantages than the EA. First, in terms of the algorithm completeness and consistency, the FMM performs well whereas the evolutionary method lacks consistency and the conventional potential field method is not complete.

Second, the FMM is able to achieve various cooperative behaviours. Generated trajectories can either be time cooperative, or time-and-position cooperative, and a 'deformable' formation shape can be easily established, which is difficult to achieve with the other methods. In Gomez *et al.*<sup>11</sup> and Garrido *et al.*,<sup>10</sup> a generic formation path planning algorithm based upon the FMM has been proposed and employed for indoor robot formations. From the simulation results, the formation is able to adjust its shape to avoid complex obstacles such as a narrow pathway.

Third, differing from the conventional potential field method of only constructing attractive and repulsive fields; some other fields representing different costs can also be used by the FMM. In Garrido *et al.*,<sup>94</sup> a weighting matrix that addresses different path constraints was used and blended with the potential field to generate the path. The final trajectory was optimised in terms of the least energy consumption, the shortest distance and the plainest terrain.

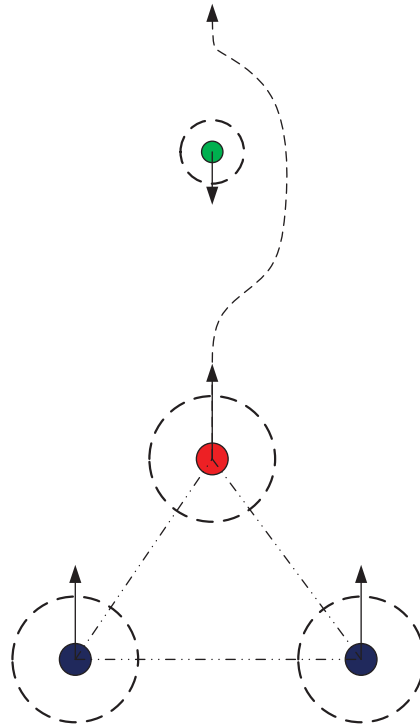


Fig. 17. The split-merge formation collision avoidance strategy. When the formation encounters a small-size obstacle, which only has the collision risk with the vehicle in red, the red vehicle needs to take manoeuvres while the others remain unaffected.

However, some limitations of current multi-vehicle path planning also need be taken into consideration:

- The collision avoidance strategies were not effective enough to deal with the complex environments. Most publications used either rigid formations or dynamic formations to avoid the obstacles. However, this may not be the best solution, and in some cases the formation could be partially maintained to seek a more optimised result. For example, as shown in Fig. 17, the split-merge strategy can be adopted when the formation encounters a small sized obstacle.
- Formation path planning in an environment with true dynamic obstacles has not been studied. For the purpose of simplicity, most of the dynamic obstacles moved only at slow or constant speed. In reality, such obstacles normally have unpredictable movement patterns, which requires a path planning algorithm to be integrated with advanced sensors and a prediction algorithm. In Yao *et al.*,<sup>95</sup> the Kalman filter was used to predict the path of a moving obstacle in the immediate future, and the path planning algorithm can accordingly adjust the path to avoid the obstacles more effectively.
- Environmental factors were not included in the problem. In real applications, environmental aspects such as wind for UAVs and sea currents for AUVs or USVs have immediate influences on the vehicle. Harsh environment conditions can severely degrade or even cut-off the communication, especially for the multi-vehicle systems. For example, when deploying multiple USVs to investigate a flooded area, the communication between vessels and the ground station can be affected by debris or line-of-sight obstructions. To address such issues, UAVs can be used as communication relay to retain the communication and support the mission of USVs.

## 5. Conclusion and Future Research Areas

A review of the multiple unmanned vehicles formation system has been presented in this paper. The principle structure of the multi-vehicle system as well as the critical development technologies

Table II. Comparison of cooperative multi-vehicle path planning algorithms.

Method	Deployment platforms	Algorithm completeness	Algorithm consistency	Cooperative behaviour type	Capability for multi-optimisation
Potential field method <sup>72-74</sup>	Mobile robots AUV UAV	Incomplete	Consistent	1. Time and position cooperative behaviour 2. Time cooperative behaviour	No
Fast marching method based potential field <sup>10,11</sup>	Mobile robots	Complete	Consistent	1. Time and position cooperative behaviour 2. Time cooperative behaviour	Yes
Evolutionary <sup>79,81</sup> algorithm <i>Genetic algorithm</i>	Mobile robots UAV	Probabilistic complete	Inconsistent	1. Time cooperative behaviour	Yes
<i>Partical swarm optimisation</i> Optimal control method <sup>83,86</sup> <i>Mixed integer linear programming</i> <i>Model predictive control</i>	Mobile robots UAV AUV	Complete	Consistent	1. Time and position cooperative behaviour 2. Time cooperative behaviour	Yes



have been reviewed. In terms of the key research involved in the multi-vehicle formation system, both the formation control and cooperative path planning are important. Even though they are two different research topics, a number of overlaps make them coherent. For example, the problem of collision avoidance, which primarily resides in the path planning problem, has many solutions in formation control literature, having designed and generated necessary control commands to manoeuvre the vehicle. In the meantime, the recent path planning trend is towards the *kinodynamic planning*, for which velocity, acceleration and force/torque limitations must be satisfied.<sup>96</sup> The paths generated by the *kinodynamic planning* algorithms are physically compliant with the vehicle's dynamics, which facilitates the controllers to track, and are also able to avoid obstacles in the environment.<sup>97,98</sup>

Compared with single platform deployment, a relatively small number of multi-vehicle system deployments have been seen in recent decades. However, there is considerable potential future development for such systems as the multi-vehicle system is more effective and able to undertake complex missions for which single vehicles are incapable. It is without doubt that by fully implementing a formation control and navigation system into current unmanned system platforms, the autonomy and efficiency of unmanned vehicles can be successfully enhanced.

To further push the boundary of the research of multi-vehicle systems, extensive work needs to be carried out from both control and path planning perspectives. First, as presented in this review, most of the work only focuses on single types of platforms, and the problem of formation control and path planning for multi-vehicle cross-platform system has not been rigorously addressed. In the future, the dominant approach to deploy multi-vehicle systems may be to use various types of vehicles to cooperatively work together to provide the persistent autonomy. For example, a cross-platform system consisting of UAVs, USVs and AUVs can be deployed for search and rescue missions in post-disaster scenarios, where the UAV provides long-range detection capability, the USV works as a communication relay station and the AUV is responsible for underwater search and detection. To effectively operate such a combined system, new considerations must be given to the development of the associated control algorithms. In terms of formation control, as each type of vehicle has its unique dynamic characteristics, such a system becomes highly heterogeneous and consequently its formation control become more challenging. In addition, when deploying such cross-platform systems to conduct persistent mission, the energy consumption will become a significant limitation and would need to be properly addressed by balancing the energy usage issue with other requirements. With respect to the path planning, computation efficiency is the major issue that needs to be specifically taken into account as a cross-platform system would normally be conducting missions in a 3-D environment. Path planning algorithms reviewed in this paper normally belong to grid-based path planning algorithm, which are powerful in dealing with 2-D environments but lack effectiveness in 3-D. Therefore, the sampling-based algorithm such as the rapidly exploring random tree<sup>97</sup> can be modified and improved for this application.

Another important research area is development of multi-vehicle systems towards the swarm concept. Because the number of vehicles involved in a swarm is far more than that in a formation,<sup>99</sup> the required algorithm for operating a swarm is different and more complex. This has therefore led to a phenomenon that a large number of bio-inspired control methods such as insect colonies and flocks of birds, have been adopted as they are capable of providing solutions to the complex problem that conventional approaches cannot address.<sup>100</sup> In fact, when developing the algorithm for a swarm, due to the large amount of vehicles, which provides a certain degree of redundancy, new functionality called the *obstacle enclosure* can be considered as a potential research area. This in fact will be a new way of dealing with moving obstacles. For example, for a conventional formation system, generating the safe evasive actions is always the priority when the formation encounters moving obstacles. However, for a swarm system, instead of avoiding the obstacles, part of the swarm can be used to enclose an obstacle to effectively block its trajectory and delay its movement, while the rest of the swarm can continue to transit towards the target point. It might still be viewed as having accomplished the mission even if not all the vehicles but only part of them arrive at the target point. To successfully implement such a strategy, the choice of the obstacle enclosure time would be critical and should be calculated according to the movement of the obstacle. Also, internal collision within the swarm when performing the enclosure might not be negligible and must be addressed in the algorithm design.

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