

# Planetesimal Capture by an Evolving Giant Gaseous Protoplanet

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**Abstract.** Both the core-accretion and disk-instability models suggest that at the last stage of the formation of a gas-giant, the core of this object is surrounded by an extended gaseous envelope. At this stage, while the envelope is contracting, planetesimals from the protoplanetary disk may be scattered into the protoplanets atmosphere and deposit some or all of their materials as they interact with the gas. We have carried out extensive simulations of approximately  $10^4$  planetesimals interacting with a envelope of a Jupiter-mass protoplanet including effects of gas drag, heating, and the effect of the protoplanets extended mass distribution. Simulations have been carried out for different radii and compositions of planetesimals so that all three processes occur to different degrees. We present the results of our simulations and discuss their implications for the enrichment of ices in giant planets. We also present statistics for the probability of capture (i.e. total mass-deposition) of planetesimals as a function of their size, composition, and closest approach to the center of the protoplanetary body.

**Keywords.** planets and satellites: formation, solar system: formation, methods: n-body simulations.

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## 1. Introduction

Recently Helled *et al.* (2006, 2008) followed the evolution of a giant gaseous protoplanet formed according to the disk instability scenario (Boss 2000a). They showed that planetesimal capture during the early contraction stage could supply a proto-Jupiter with the super-solar high-Z materials observed (Young *et al.* 2003), while grain sedimentation of the accreted material could supply a core consistent with that expected from theoretical models (Saumon & Guillot 2004)

However, Helled *et al.* (2006, 2008) made a number of simplifying assumptions. Planetesimals were assumed to have a uniform size and composition, and were evenly distributed over a feeding zone of 3 Hill sphere radii on either side of the planet. For Jupiter, this is a region some 3 AU in width, and the number density of planetesimals over such a large region would be expected to decrease with distance from the Sun. In addition we might expect different compositions for planetesimals originating in different parts of this large region.

It was also assumed that the random velocities of these objects with respect to Jupiter were identical, and constant with time. As these objects were accreted, the background density was immediately re-adjusted to allow for the lowered planetesimal abundance, but the spatial distribution was kept uniform. Finally, planetesimal trajectories were computed neglecting the tidal effect of the Sun. Since the protoplanetary radius is originally of the order of the Hill sphere radius, 3-body effects must be taken into account.

Here we present a more careful calculation of planetesimal capture, which includes the effects mentioned above. As a proof of concept, we have carried out numerical simulations of the interactions of planetesimals with the proto-atmosphere of a growing giant planet in the DI model. Results obtained for the first stage of envelope contraction ( $\sim 3500$  years), point to ranges of planetesimal size, bulk density, and velocity for which the process of mass-deposition is efficient.

## 2. Effect of Gaseous Envelope

The presence of a gaseous envelope can affect the planetesimal trajectory in three different ways: 1. The gas drag exerts a force on the planetesimal which changes its trajectory. 2. Heating of the planetesimal both by the ambient gas and by drag heating. 3. The change in gravitational potential due to the mass being distributed over a finite volume.

We have run simulations to investigate the importance of these effects. The calculation was carried out in three distinct steps. First, an N-body integrator (MERCURY) is used to integrate the planetesimal's motion while still in the protoplanetary disk and subject to the gravitational forces of the Sun and the proto-giant planet. When the planetesimal enters the giant planet envelope, a second integration starts which, in addition to gravitational forces, includes the interaction of the planetesimal with the envelope through gas-drag using a special code designed for this purpose (Podolak *et al.* 1988). Once the planetesimal exits the envelope (if it is not captured) the N-body integrator continues computing the orbit until the next encounter or until the planetesimal leaves the system. In what follows we use the same set of model envelopes that were used by Helled *et al.* (2006, 2008).

At each point of the planetesimal's trajectory the gas drag force is determined by three parameters: (1) The Knudsen number  $Kn = l/a$  where  $l$  is the mean free path of a gas molecule and  $a$  is the radius of the planetesimal, (2) the Reynolds number  $Re = \rho v a / \eta$  where  $\rho$  is the (envelope) gas density,  $v$  is the velocity of the planetesimal through the gas, and  $\eta$  is the gas viscosity, and (3) the Mach number  $Ma = v/c_s$  where  $c_s$  is the local sound speed. For  $Re < 1$ , the drag force is given by the Cunningham relation  $F_{\text{drag}} = 6\pi a \eta v / \psi$  (Öpik 1958) where  $\psi$  is an interpolation function usually written as  $\psi = 1 + Kn[A + Be^{-C/Kn}]$ . This formula interpolates between the Stokes formula when  $Kn \ll 1$  and the Epstein formula when  $Kn \gg 1$ . The constants  $A, B$ , and  $C$  are chosen to fit experimental data.

For  $Re > 1$ , the drag force is given by  $F = C_d \pi a^2 \rho v^2 / 2$  (Öpik 1958). The constant  $C_d$  depends on the respective values of the Knudsen, Reynolds, and Mach numbers. The detailed relations are given in Podolak *et al.* (1988). Although these relations were derived independently to fit experimental data, they agree well with the formulas given by Whipple (1972) and Weidenschilling (1977b). A more descriptive version of these equations can also be found in Haghighipour & Boss (2003).

As a rule of thumb, a planetesimal will be captured if it encounters a mass of gas equal to its own mass. A 1 km ice planetesimal will have a mass of  $1.33 \times 10^{15}$  g. Such a planetesimal crossing a diameter of the protoplanet in its initial configuration will encounter  $1.09 \times 10^{15}$  g of gas. This will cause a substantial change in the planetesimal velocity, and will almost certainly cause it to be captured, but it is an extreme case. Most trajectories do not pass through the center of the protoplanet and therefore encounter a much smaller mass of gas. For the initial planetary envelope, the densities are so low that gas drag has only a small effect, although smaller velocity changes can also result in capture under special circumstances.

It turns out that a 100 km planetesimal can pass through the entire planet with a minimal loss of velocity due to gas drag. 10 km rocky planetesimals also encounter less than 1% of their mass in gas, even if they pass through the center of the protoplanet. However, 10 km icy planetesimals encounter 10% of their mass in gas if they come within around  $2 \times 10^{12}$  cm of the center. 1 km icy planetesimals need to penetrate to within about  $4 \times 10^{12}$  cm = 0.27 AU from the center before encountering 10% of their mass in gas. For capture to occur the planetesimal needs to encounter somewhat more gas than this and, as we shall see below, 1 km mixed ice-rock planetesimals do not get captured unless they come within about 0.23 AU from the center of the protoplanet.

A second effect of the gas is to heat the planetesimal. For rocky planetesimals this is not important, at least in the early stages of evolution, where the temperature in the protoplanet never exceeds 400 K. Even gas drag heating due to the planetesimal's motion through the envelope will not affect these bodies significantly at this stage of the protoplanet's evolution. Rock will not undergo significant vaporization until the temperature reaches  $T \sim 1300$  K. For an encounter velocity of  $5 \text{ km s}^{-1}$  this requires a density of  $\rho = 4 \times 10^{-8} \text{ g cm}^{-3}$ . The density in the protoplanetary center is only  $2.6 \times 10^{-8} \text{ g cm}^{-3}$ , so rocky planetesimals lose almost no mass through gas drag heating.

Icy planetesimals, on the other hand, can lose substantial amounts of mass via ablation, and this is an important mechanism for planetesimal capture. For a 1 km ice planetesimal to lose 1% of its mass after a typical encounter time (one year), we require a surface temperature of  $T \sim 200$  K, while for a 100 km ice planetesimal we require  $T \sim 230$  K. The former temperature occurs at about  $2.7 \times 10^{12}$  cm = 0.18 AU from the center, while the latter occurs at about  $2.3 \times 10^{12}$  cm = 0.15 AU from the center. Thus for shallow encounters mass loss by gas heating can be neglected.

Gas drag heating can also be important. For a planetesimal velocity of  $5 \text{ km s}^{-1}$  relative to the gas, a temperature of 200 K can be achieved by drag heating at a density of  $2 \times 10^{-10} \text{ g cm}^{-3}$ . In the atmospheric model we are considering, this occurs at a radius of around  $5.6 \times 10^{12}$  cm.

A planetesimal composed of a mixture of ice and rock can lose even more mass. The reason is that we model the rock as being composed of small grains embedded in an ice matrix. The rate of ice loss is the same as for the pure ice case, but as the ice is lost, the embedded grains are lost as well, and the result is a higher total mass loss. For the cases we investigated, typical values of mass loss are no more than a few percent, so this effect is usually small.

If the planetesimal penetrates more deeply into the envelope, it will encounter higher temperatures, and will lose more mass. The mass loss depends on the vapor pressure of the evaporating material, which is very sensitive to the surface temperature of the planetesimal. Thus unless the planetesimal is moving with velocities of more than  $\sim 5 \text{ km s}^{-1}$ , it will need to penetrate to within  $2.6 \times 10^{12}$  cm of the center before mass loss will become important.

The third important parameter that affects the capture of planetesimals is the fact that the protoplanetary mass is distributed over a finite volume, rather than in a point mass. The difference in the gravitational potential between the two cases can lead to significant changes in the trajectory of a planetesimal inside the envelope, and has consequences for the probability of capturing the planetesimal.

### 3. Numerical Simulations

Using the same envelope models as Helled *et al.* (2006, 2008), we computed the planetesimal capture rate using our simulations. The planetesimal disk was populated by

$\sim 10^4$  objects with sizes of 1, 10, and 100 km at two regions; 3.7-4.0 AU and 6.2-6.6 AU. The radial profile of the disk surface density was set to  $r^{-3/2}$  (Hayashi 1981, Weidenschilling 1977a) and the orbital eccentricities and inclinations of the planetesimals, and their angular orientations were chosen randomly. The mass of the protoplanet was set equal to the mass of Jupiter, and the radius of the envelope was taken to be 0.47 AU.

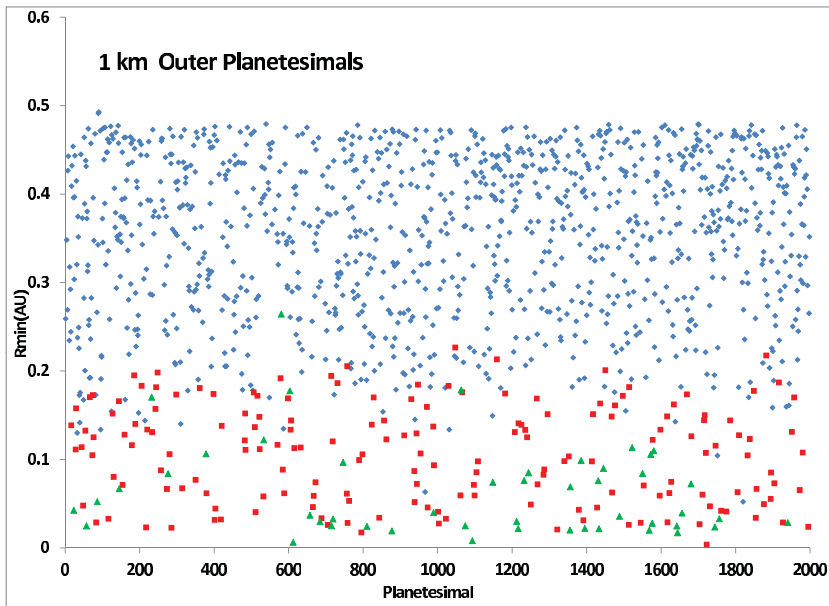
The planetesimals were assumed to be composed of either pure rock, pure ice, or a mixture of rock + ice. Other relevant parameters, such as density, material strength, melting temperature, etc., are provided for each chosen planetesimal material.

We followed the evolution of each object and identified those that entered the giant planet's envelope. Once the planetesimal entered the envelope we computed its trajectory twice. The first time we simply used the MERCURY code as before, and treated the protoplanet as a point mass. Gas drag effects were neglected. We refer to such trajectories as *point-mass trajectories*. The second time we integrated the trajectory allowing for both gravitational and drag forces as described above in section 2. We refer to such trajectories as *extended-mass trajectories*.

To do a full history for each planetesimal using extended-mass trajectories would be very time consuming, because a planetesimal can have many encounters with a planet in a relatively short time. To shorten the computation we followed only the first encounter of each planetesimal with both trajectories and developed statistics for the expected outcome. We then computed full histories for the planetesimals using point-mass trajectories and used the statistics we developed to evaluate the outcome of these additional encounters that the planetesimal later experienced. Fig. 1 shows the results for 2005 1 km planetesimals originating in the region exterior to Jupiter (6.2-6.6 AU). Each planetesimal is represented by a point in the figure. The abscissa is the identification number of the planetesimal, while the ordinate is the closest distance, in AU, that the planetesimal approaches to the center of the planet during its first encounter assuming a point-mass trajectory. The outcome of the encounter, as computed with the extended-mass trajectory is given by the color of the corresponding point in the figure. The blue squares represent planetesimals that passed through the planet with essentially no mass loss. Red triangles represent mixed ice-rock planetesimals that were captured. Green triangles represent rock planetesimals that were captured.

Note that the mixed planetesimals fall into three distinct regions. If the planetesimal never gets closer than 0.25 AU it does not get captured. Of the 111 planetesimals that approached to within between 0.24 and 0.28 AU from the center of the planet, only one, PLAN0580, was captured, both for the case of a mixed ice-rock planetesimal and for a pure rock planetesimal. As noted above, at these distances the temperature in the protoplanet is not high enough to cause significant ablation, nor is the gas drag alone sufficient to cause significant energy loss and trapping. Rather it is a result of a particularly long temporary capture. For most encounters the planetesimal stays in the Hill sphere on the order of a year, but this particular case the planetesimal remained in the Hill sphere for over five years. This allowed it to lose energy through the other mechanisms and eventually to be captured.

For distances less than 0.12 AU essentially all the mixed ice-rock planetesimals are captured. Again there are a very few exceptions. PLAN1820 comes to within 0.052 AU from the center of the planet and is not captured. This is a special case because the orbit is very close to Jupiter's and as a result the relative velocity is very low. A very slight mismatch between the many-body code and the gas-drag code can result in large differences in the end result. We have included the outcome in our presentation of the results, but its effect on the statistics is negligible.

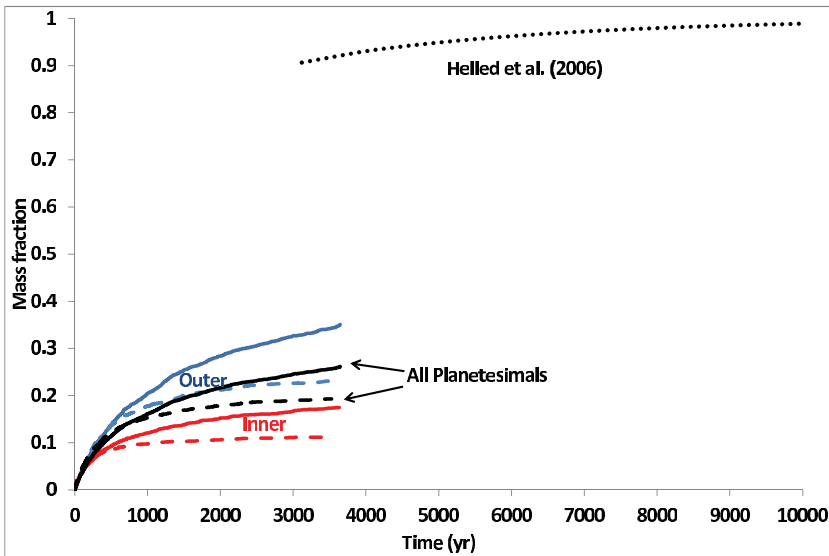


**Figure 1.** Outcome of extended-mass first encounter with the protoplanet as a function of closest approach distance for a point-mass trajectory (see text). Blue diamonds are for negligible interaction, red squares signify capture of a 1 km mixed ice + rock planetesimals, green triangles are for 1 km rock planetesimals.

Finally, there is a transition region between 0.12 and 0.24 AU where the outcome is sensitive to additional parameters such as the encounter speed, although here too the probability of capture increases with decreasing encounter distance. The mixed ice-rock planetesimals have a capture probability going from zero to essentially one with an approximately linear rise for minimum encounter distances between 0.26 and 0.1 AU.

Fig. 1 also shows that 1 km rock planetesimals do not exhibit any clear correlation between minimum encounter distance and capture probability. The higher density of rock requires more gas is to slow it and in the initial configuration of the protoplanet this is more difficult. In addition, the mass loss due to ablation is much lower for a rock planetesimal. The only way these planetesimals can be captured is if the orbital parameters allow a temporary capture so that the planetesimal spends enough time in the inner envelope for gas drag to be effective. Because additional orbital parameters are important, there is no clear correlation with the minimum encounter distance of the point-mass trajectory. The planetesimals originating inside of Jupiter's orbit show similar trends.

We performed two different calculations of mass capture by the protoplanet. In the first calculation (*full calculation*) we followed the planetesimals through their first encounter with the protoplanet, using the full gas drag code. We included gas drag, ablation, and the extended mass distribution in computing the trajectory. For this case we computed the mass deposited in the protoplanet either by ablation or by complete capture of the planetesimal. However, because of computer limitations we followed only the first encounter for each planetesimal. In the second calculation (*statistical calculation*) we assumed that any encounter where the point-mass trajectory of the planetesimal never took it closer than 0.25 AU would have no noticeable interaction with the gas. We therefore continued integrating the trajectory with the MERCURY code until it came within 0.25 AU



**Figure 2.** Fraction of planetesimal mass captured as a function of time. Red curves are for 1 km mixed ice+rock planetesimals originating in the region inside of Jupiter's orbit, blue curves are for those originating outside of Jupiter's orbit. The black curves are for all planetesimals. Dashed curves show detailed results for the first encounter, solid curves show statistical results for the first close encounter (see text). The dotted black curve near the top is from Helled *et al.* (2008).

or less from the center of the protoplanet. We used this minimum distance in the statistical model we described above to compute the capture probability. In this case we assumed that each encounter resulted in either a complete capture of the planetesimal or no capture at all.

The results are shown in Fig. 2. The red curves show the fraction of planetesimals captured which originated inside of Jupiter's orbit, and the blue curves show the fraction captured from among those which originated outside of Jupiter's orbit. The dashed curves are for the *full calculation* and the solid curves are for the *statistical calculation*. The solid and dashed black curves are the corresponding curves for the combination of both inner and outer planetesimals. One important result is that the rate of capture of planetesimals in our model for both of the cases differs substantially from that found by Helled *et al.* (2008). This is given by the dotted curve in the upper part of the figure. The current calculation, although only covering  $\sim 3500$  yr (red curve) shows that a considerably smaller fraction of the available mass, roughly one third, is captured than Helled *et al.* (2008) found.

In addition, we find that the contribution to the accreted mass by planetesimals from outside Jupiter's orbit is noticeably higher than the contribution by planetesimals from inside Jupiter's orbit.

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