MATHEMATICAL ASPECTS OF EARTH-MOVING LINKED TO HYDRAULIC WORKS IN UR III UMMA

By Sergio Alivernini

This paper studies mathematical aspects of earthwork projects in the Ur III city of Umma, *c*.2053–2032 B.C. The main purpose of this paper is to describe the practical procedures involved in moving earth for hydraulic works around Umma. It also shows how Old Babylonian pedagogical "mathematical texts" about earthworks, from the early second millennium B.C., are indebted to the practical procedures adopted by Ur III officials.

Introduction

The role of irrigation management in the development of the first urban societies of Mesopotamia is still an important topic of discussion among philologists, archaeologists and historians.[1] Since the mid-twentieth century, the development of Mesopotamian civilization has been seen as being intrinsically linked to the building of water-related infrastructure such as dams and canals in order to tame the environment (see Postgate 1988). However, the importance of wetlands in the development of prehistoric communities in the Ubaid period has been emphasized by Oates (1960) and above all, more recently, by Pournelle (2003; 2007). Their research has revealed that the life of these communities was closely linked to the "wet environment" and that many sites were probably islands within the marshes. From the third millennium B.C. the region of Sumer, just to the north, became an important area for the production of cereals and crops, irrigated by artificial canals, had probably developed more rapidly than the wetlands. This may have been due to the fact that, although the marshes represented a wealth of natural resources such as reeds, fish and shellfish, they could be over-exploited to the point of exhaustion, while the cultivation of cereals represented the economic foundation of the growing Sumerian civilization (Wilkinson 2013: 38 with previous bibliography).

Earthwork projects and the building of reservoirs, dams and canals are closely related to the management of irrigation; several Ur III texts describe the building and repair of canals or their components. These works required the use of different materials, depending on the facilities to be built, but the most important was definitely earth. Civil (1994: 115) distinguishes two different categories of earthwork activity: earth moving (kin-sahar) and the reinforcement of levees (eg$_2$-e u$_2$-sag$_{11}$-a).[2] Activities such as digging canals and piling up earth to build dams and dykes belong to the first category of works, which is the subject of this article. More specifically, as noted by Civil (1994: 115), works involving earth-moving can be divided into sahar si-ga, "to pile up earth"; sahar zi-ga, "to remove earth"; sahar šu-ti, "to collect earth"; and sahar šu bal-a, "to transport earth". We must also add ba-al-la, "to dig", and šu / šu$_2$ luh… ak, "to clean" (the bed of a watercourse). These activities were necessary to build irrigation canals (pa$_4$/pa$_5$ or i$_7$), banks (eg$_2$), tanks and reservoirs for water control (kab$_2$-ku$_5$ or a-ga-am), barrages for waterways (a-igi), dams (giš-keš$_2$) and mud walls (im-du$_8$-a).

Several Old Babylonian mathematical problems highlight all the variables that had to be taken into account in order to carry out such works. Here, for example, is an excerpt from the text YBC 5037

[2] Often the two activities were involved the same operation; many texts record this combined procedure through the expressions kin u$_2$ sahar-ba, sahar u$_2$-sag$_{11}$ or u$_2$ sahar (Civil 1994: 121).

(Neugebauer and Sachs 1945: 59, Text F), a catalogue of problems that includes the excavation of earth as a common topic (Sumerian ki-la$_2$, Akkadian *kalakkum*).[3] Obverse, 1-2 reads:

1.   ki-la$_2$ 3½ nindan 3 kuš$_3$-ta-am$_3$ ib$_2$-si$_8$ 2½ kuš$_3$ bur$_3$-bi 7½ gin$_2$ eš$_2$-gar$_3$
2.   6 še a$_2$-bi a-ša$_3$ sahar erin$_2$-ḫa$_2$ u$_3$ ku$_3$-babbar en-nam

1.   An excavation: 3½ nindan 3 kuš$_3$ (*c*.22.5 m) is the length of each square-side; 2½ kuš$_3$ (*c*.1.25 m) is its depth; 7½ gin$_2$ (*c*.2.25 m^3) is the (daily) work-load (for each worker);
2.   6 še (c.0.27 g) (of silver) is its wage. What are the area, the volume, the (number of) workers and the (total expenses in) silver?

As we can see from this excerpt, the variables to be taken into account are:[4]

1. The length (in the example cited above 3½ nindan and 3 kuš$_3$), the width (3½ nindan and 3 kuš$_3$ – the text records that the length and the width are identical), and the depth of the earth to be excavated (2½ kuš$_3$);
2. The daily quantity of earth to be excavated by each worker (7 ½ gin$_2$);
3. The daily cost in silver (6 še);
4. The area of the base of excavation (to be found);
5. The total volume (to be found);
6. The total number of work-days (to be found);
7. The cost in silver (to be found).

The solution to the problem is therefore as follows:

The total area: 14 sar 3 ⅔ gin$_2$ 15 še (*c*.506 m^2);
The total volume: 35 sar 9 ⅓ gin$_2$ 7 ½ še (*c*.633 m^3);
The total number of work-days: 281 ¼;
The total cost in silver: 9 ⅔ gin$_2$ 7 ½ še (*c*.9.4 g).

It is clear from this typology of texts that there were challenges not only in the task itself but also in the practical procedure through which the measurements were obtained and calculated. From the point of view of the history of science and administration, it is fortunate that it is possible to identify, amongst the administrative texts of the Third Dynasty of Ur, documents that show how the Sumerians gathered this information.

All of the variables recorded in the Old Babylonian problem text shown above were necessary to complete the work; and they were the same as those used in daily practice by the officials of the Third Dynasty of Ur.[5] However, they never appear all together in a single document and so we need to study several different types of administrative text in order to throw light on the organization of such work.

The documentation
Calculations of the total amount of earth to be excavated. About twenty documents[6] from Ur III Umma specifically measure the amount of earth to be excavated. As discussed further below, we should understand them as surveys and notes, probably made directly in the field, in order to estimate the amount of work required. We can divide them into two main types. In Type 1, all three measurements (length, width and depth) are recorded, while in Type 2 the scribe recorded only the length and the volume of earth to be removed per unit of length (Civil 1994: 116).

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[3] For the expression ki-la$_2$/*kalakkum*, see Robson (1999: 101) with previous bibliography.
[4] This excerpt has been used as it represents, to the best of my knowledge, the clearest example for the discussion that follows. Many other mathematical texts deal with earthworks: see for example YBC 4657 (Neugebauer and Sachs 1945: 66 Text G), another catalogue of problems; and YBC 4663, which records mathematical problems with procedures (Neugebauer and Sachs 1945: 66 Text H). Moreover, we also have to add the coefficients for earthworks; for a complete list see Robson (1999: 93–110).

[5] Note, however, that the Ur III scribes never recorded their calculation of the area (though they presumably made this calculation as in intermediate step to finding the volume of excavation).
[6] AAICAB 1/2, Ashm. 1971-379 (Š 17/-/-), SAT 2 210 (Š 39/-/-), AnOr 1 33 (Š 45/-/-), Syracuse 459 (Š 46/-/-), SNAT 398 (AS 8/-/-), MVN 21 215 (ŠS 3/xii/-), CDLJ 2002: 1 15 (ŠS 4/-/-), MVN 16 961 (ŠS 6/-/-), CDLJ 2012: 1 3.04 (IS 2/-/-), AAICAB 1/3, Bod. S 157 (-/-/-), BPOA 7 2445 (-/-/-), CDLB 2017: 1, no. 8 (-/-/-), OrSP 47-49 511 (-/-/-), SACT 2 140 (-/-/-), Torino 2 666 (-/-/-), CUSAS 3 1513 (-/-/-).

Type 1 surveys, such as the undated text BPOA 7 2445, contain the following data:

- the measurements of the length (gid$_2$), width (dagal) and depth (buru$_3$);
- the calculation of the total volume (sahar-bi/kin-bi x sar), which results from the product of the length, width and depth recorded in the preceding line;[7]
- the total sum of the lengths (šu-nigin$_2$ x gid$_2$);[8]
- the total sum of the volumes (sahar-bi x sar);
- other administrative information, such as the kind of work, the place, the supervisors, etc.

Meanwhile Type 2 surveys, like the undated text AAICAB 1/3, Bod. S 157, give:

- The measurement of the length (gid$_2$) and the volume of the earth to be excavated for every unit of length (x sar-ta);
- The calculation of the total volume of earth to be excavated (sahar-bi/kin-bi x sar), which results from the product of the unit volume and the length recorded in the preceding line;
- The total sum of the lengths (šu-nigin$_2$ x gid$_2$);[9]
- The total sum of the volumes (sahar-bi x sar);
- Other administrative information, such as the kind of work, the place, the supervisors, etc.

Records of the daily quantity of earth to be excavated by each worker. The amount of earth to be excavated by each individual worker is noted by the expression guruš-e x gin$_2$-ta. It is explicitly recorded in some tablets from the city of Umma. See, for example, BPOA 1 1147 (ŠS 2/-/-):

Obverse
1. 4 ⅔ sar sahar   4 ⅔ sar of earth:
2. guruš-e 7 ½ gin$_2$-ta   the worker's work is 7 ½ gin$_2$ each
3. eg$_2$ a-ša$_3$ ša-ra-hu-um-ma   for the levee of the field Šarahumma;
4. ugula šeš-kal-la   the supervisor is Šeškala;
Reverse
1. kišib ab-ba-ge-na   Seal of Abba-gena
2. mu ma$_2$ ^{d}en-ki ba-ab-du$_8$ Year: Šu-Sin 2
Seal
1. lu$_2$-^{dr}šara$_2$$^{\rceil}$ Lu-Šara
2. dub-sar   scribe
3. dumu ur-sa$_6$-[ga]   son of Ur-saga

Table 1 shows all the quantities of work-rates recorded, with texts given in chronological order.[10] As we can see, the two most frequently used values are 7 ½ and 10 gin$_2$.[11] Indeed, 10 gin$_2$ seems to have been the default value in the minds of the scribes, as in six texts it is not explicitly recorded on the tablet.[12] See, for example, Princeton 2466 (Š 47/-/-):

Obverse
1. 1 35 sar kin sahar   95 sar of earthwork
2. i$_7$-sal-la^{ki} ba-al-la   to dig the Sala canal;
3. a$_2$ lu$_2$ hun-ga$_2$-bi 6 sila$_3$-ta hired man's work is paid at a rate of 6 sila$_3$ each.
4. še-bi 11.2.0 gur   The total amount of barley is 11 gur, 2 ban$_2$.
5. kišib lugal-e$_2$-mah-e   Seal of Lugal-emahe.

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[7] In three texts belonging to this type (SAT 2 210 [Š 39/-/-], MVN 16 961 [ŠS 6/-/-], and CUSAS 3 1513 [unknown date]) the width is recorded only in the first set of measurements as it is constant throughout the text (2 nindan in SAT 2 210; ½ nindan 2 kuš$_3$ in MVN 16 961; and 1 ½ nindan in CUSAS 3 1513).
[8] Sometimes this information is omitted. See, for example, Syracuse 459 (Š 46/-/-).
[9] Sometimes this information is omitted. See, for example, SNAT 398 (AS 8/-/-).
[10] Data in this table and in the following ones are taken not only from texts about single earthwork projects but also from

long balanced accounts or others typologies of Ur III texts that also record works not related to this study. As one can see from the table, some texts record more than one daily work-rate, whether the same (e.g., BIN 5 727) or not MVN 20 151, where two different daily work-rates are recorded).
[11] It is probable that the differences in the daily quantity of the earth to be excavated depended on the quality of land to be dug.
[12] Princeton 2 466 (Š 47/-/-), MVN 13 308 (AS 3/-/-), JCS 16 14 1 (AS 3/-/-), SACT 2 126 (AS 7/-/-), YOS 4 209 (ŠS 7/-/-) and Georgica 2.11 ([-/-/-]).

TABLE 1 Daily work-rates for excavating earth attested in Ur III Umma administrative records

Daily work-rate	Texts and dates
5 gin₂	MVN 16 1337 (Š 42/xiii/-), BPOA 7 1711 (Š 46/-/-)
6 gin₂	BPOA 6 1538 (AS 5/-/-)
7 ½ gin₂	Syracuse 28 (Š 48/-/-), Princeton 1 501 (AS 3/v/-), UTI 4 2925 (AS 6/xiii/-), Syracuse 4 (AS 6/-/-), AUCT 3 311 (AS 6/-/-), SAT 2 308 (AS 6/-/-), AAICAB 1/1, Ashm. 1924-665 (AS 6/-/-), BPOA 1 1301 (AS 6/-/-), BPOA 2 2434 (AS 6/-/-), SACT 2 129 (AS 8/-/-), JCS 24 172 94 (AS 8/-/-), BPOA 1 1159 (AS 9/[x]/-), MVN 16 1324 (ŠS 2/v/-), UTI 3 1652 (ŠS 2/-/-), Princeton 2 385 (ŠS 2/-/-), UTI 4 2743 (ŠS 2/-/-), BPOA 1 1147 (ŠS 2/-/-), Nisaba 23 150 (ŠS 2/-/-), BPOA 7 1885 (ŠS 2/-/-), BPOA 7 2377 (ŠS 2/-/-), MVN 16 1158 (ŠS 3/-/-), UTI 4 2469 (ŠS 3/-/-), UTI 4 2766 (ŠS 3/-/-), BPOA 1 709 (ŠS 3/-/-), BPOA 6 1283 (ŠS 3/-/-), SAT 3 1537 (ŠS 4/-/-), BPOA 7 1889 (ŠS 4/-/-), BPOA 1 779 (ŠS 6/-/-)
7 ½ gin₂/10 gin₂	MVN 20 151 (Š 46/-/-), TCL 5 5676 (ŠS 2/-/-)
10 gin₂	BRM 3 119 (Š 46/-/-), BPOA 6 1313 (Š 46/-/-), OrSP 47-49 263 (Š 46/-/-), BPOA 6 1183 (Š 46/-/-), Ontario 2 139 (Š 47/-/-), TCL 5 5668 (Š 48/xii/-), Essen in Sumer, p. 66 (AS 1/ii/-), Syracuse 144 (AS 2/-/-), Aegyptus 8 264 11 (AS 3/-/-), SAT 2 755 (AS 3/-/-), OrSP 47-49 151 (AS 6/-/-), UTI 4 2869 (AS 8/-/-), BPOA 1 948 (AS 9/-/-), SNAT 477 (ŠS 3/-/-), MVN 21 150 (ŠS 5/viii/-), UTI 3 1906 (ŠS 5ʔ/-/-)
10 gin₂ / 10 gin₂	BIN 5 727 (AS 3/-/-)
10 gin₂ / 10 gin₂ / 10 gin₂	ASJ 8 68 (ŠS 4/-/-)
13 gin₂	OLP 4 17-70 n. 52 (AS 1/-/-), BPOA 1 625 (ŠS 3/-/-)

Reverse
1. inim-^{d}šara₂ i₃-dab₅   Inim-Šara took charge.
2. mu us₂-sa ki-maš^{ki} ba-hul   Year: Šulgi 47.
Seal
1. lugal-e₂-mah-e   Lugal-emahe,
2. dub-sar   scribe,
3. dumu lugal-ku₃-ga-ni   son of Lugal-kugani.

If we divide the total amount of barley (11 gur and 2 ban₂, i.e., 3,420 *sila₃*) by the daily expenditure on barley (6 sila₃) we arrive at the number of workers: 3,420 ÷ 6 = 570 workers. If we now divide the total quantity of earth to be excavated (95 sar, i.e., 5,700 gin₂, *c*.1,710 m³) by the total number of workers, we arrive at the quantity of earth to be excavated by each worker, namely 5700 ÷ 570 = 10 gin₂, *c*.3 m³.

Eleanor Robson (1999: 158) has already identified this quantity as the value of an Old Babylonian mathematical "coefficient" (logogram IGI.GUB, Akkadian *igigubbûm*). Coefficients are numbers that represent a mathematical relationship or constant, and are used for solving particular types of mathematical problem. In Babylonian mathematics, Robson (1999: 14) distinguishes between "absolute constants", such as, for example, that which links the diameter of a circle and its area; "metrological constants" between units of measure; and "empirical constants", which represent a relationship fixed by convention, such as, for example, the work-rate for making bricks. Robson (1999: xiii) worked with twelve coefficient lists: eight Old Babylonian, two dating from some time in the first first millennium B.C., and two from the Late Babylonian period. They are clearly recognizable in both formal features and content and represent a distinct genre of mathematical text. Every number written on the tablet has a name or a brief description, which often helps us to understand the context in which a coefficient was used. Robson (1999: 158) identifies some coefficients recorded in the Old Babylonian lists that were already in use during the Ur III period. The volumes 20, 15, 10 and 7 ½ gin₂ are recorded in the Old Babylonian coefficient lists and mathematical word problems, values which depend on the depth of the excavation to be made. Of these, Robson (1999: 139) remarks that 10 gin₂ "is exactly the most commonly recorded work-rate for earth moving in both Ur III administrative records and OB school mathematics". It is attested for instance in the coefficient list YBC 5022 (List F) obv. 43 as 10 GÍN ÉŠ.GÀR LÚ 1, "10 shekels, the work-load of 1 man", as well as in multiple problem texts and calculations (Robson 1999: 96).

The total and daily cost in barley. The daily expenditure on barley is defined in the Ur III texts by the expression a_2 lu_2 hun-ga_2(-bi) x-$sila_3$-ta, "x $sila_3$ for the wages of hired men". See, for example, BPOA 1 971 (AS 2/-/-):

Obverse
1.	5 sar kin sahar	5 sar of earthwork:
2.	a_2 lu_2 hun-ga_2 6 $sila_3$-ta	the hired-man work is paid 6 $sila_3$ each (day)
3.	a-$ša_3$ a-geštin	in the field A-geštin.
4.	ugula lugal-e_2-mah-e	The supervisor is Lugal-emahe.

Reverse
1.	kišib lugal-e_2-mah-e	Seal of Lugal-emahe,
2.	šeš lugal-^{giš}kiri$_6$	brother of Lugal-kiri.
3.	mu ^{d}amar-^{d}suen lugal-e / ur-bi_2-lum^{ki} mu-hul	Year: Amar-Sin 2.

Seal
1.	lugal-e_2-mah-e	Lugal-emahe,
2.	dub-sar	scribe,
3.	dumu lugal-ku_3-ga-ni	son of Lugal-kugani.

The total cost is reported in the texts less frequently. An example is given in Princeton 2 497 (Š 47/-/-):

Obverse
1.	6 1/2 sar kin sahar-ra	6 ½ sar of earthwork:
2.	a_2-bi u_4 39-kam	its work is for 39 days;
3.	a_2 lu_2 hun-ga_2-bi 6 $sila_3$-ta	the hired-man work is paid 6 $sila_3$ each (day);
4.	še-bi 0.3.5 4 $sila_3$	its barley is 3 bariga 5 ban_2 4 $sila_3$

Reverse
1.	<a>-$ša_3$ a-geštin-ka	in the field A-geštin.
2.	ugula inim-^{d}šara$_2$ dumu Ur-nigar$_x$^{gar}	The supervisor is Inim-Šara son Ur-nigar.
3.	kišib nam-$ša_3$-tam a-kal-la nu-banda$_3$	Seal of administrative office of Akala the overseer.
4.	mu us_2-sa ki-maš^{ki} ba-hul	Year: Šulgi 47.

Seal
1.	a-kal-la	Akala,
2.	dub-sar	scribe,
3.	dumu ur-nigar$_x$^{gar} šuš$_3$	son of Ur-nigar the high official.

Table 2 shows the daily and the total expenses for earthwork activities. As one can see, the value is almost always 6 $sila_3$. It is worth noting that, unlike the "coefficients" seen in the previous section, the values of the labour costs per day are almost always given explicitly in the Ur III administrative texts. If this value is not recorded it cannot be established, since the tablet does not contain all the data necessary to calculate it, starting from the total expenditure on barley.

The total number of workers and work-days. As one can see in Table 3, the total number of workers is not always recorded in the texts. Some documents record only the work-days. The challenge for the administration was thus to calculate the total amount of work. It did not matter whether the work was done, for example, by a single worker employed for ten days or ten workers employed for one day: the expenditure on barley would have been the same.

The combination of the information. As already stated above, there are no texts which record all the information needed to complete an earthwork project. More specifically, the combinations of the information recorded in the texts are shown in the Appendix.

To sum up, we can make the following observations:

- there are about twenty texts which measure only the quantity of earth to be excavated, based on linear measurements;

TABLE 2 Daily and total expenses for earthwork activities in texts from Ur III Umma

Daily expenses	Total expenses	Texts and dates
—	106.4.0 še gur	AAICAB 1/3, Bod. S 207 (Š 45/-/-)
—	10.2.5 gur	Nisaba 24 13 (AS 6/-/-)
4 sila$_3$	—	Hirose 372 (AS 8/-/-)
5 sila$_3$	1.0.0 gur	BPOA 6 384 (AS 1/-/-)
5 sila$_3$	—	BPOA 6 850 (AS 1/-/-), JCS 16 14 1 (AS 3/-/-), MVN 18 490 (AS 9/-/-), BPOA 2 2590 (AS 9/-/-)
6 sila$_3$	—	Princeton 2 403 (Š 47/-/-), MVN 21 135 (Š 48/xii/-), MVN 21 136 (Š 48/-/-), SAT 2 607 (Š 48/-/-), BPOA 1 971 (AS 2/-/-), BPOA 1 948 (AS 9/-/-), BPOA 1 1451 (ŠS 1/-/-), BPOA 2 2547 (ŠS 2/-/-), BPOA 2 2568 (ŠS 2/-/-), MVN 16 802 (ŠS 4/vi/-)
6 sila$_3$	0.3.5 4 sila$_3$	Princeton 2 497 (Š 47/-/-)
6 sila$_3$	0.4.0	OrSP 47-49 151 (AS 6/-/-)
6 sila$_3$	11.2.0	Princeton 2 466 (Š 47/-/-)
[6 sila$_3$][1]	2.0.0 gur	Farmer's Instructions 2.11 ([-/-/-])
6 sila$_3$	21.4.1 2 sila$_3$ gur	OrSP 47-49 263 (Š 46/-/-)
6 sila$_3$	45.3.0 gur	MVN 10 105 ([-/-/-])
6 sila$_3$	67.1.0 ⌈gur⌉	UTI 3 1906 (ŠS 5?/-/-)
6 sila$_3$	276.1.4 8 sila$_3$ gur	YOS 4 209 (ŠS 7/-/-)
0.0.3	—	Nisaba 23 19 (ŠS 2/-/-)

[1] It is possible to calculate this value as the text also records the total number of work-days (100).

- there are about two hundred known documents which record, together with the volume of the earth already calculated, information on the quantity of work requested: the number of workers, work-days, the amount of earth to be excavated daily (Tables 1 and 3). These texts may also contain administrative information, such as the place where the work is to be carried out or the names of the officials;
- there are thirty identified texts that, together with the volume of earth already calculated, record the expenses incurred when undertaking the work (Table 2). As before, these texts may also contain administrative information, such as the place where the work is to be carried out or the names of the officials;
- finally, we have about twenty texts which record only the volume of the earth plus administrative information about the place where the work is to be carried out and the names of the officials in charge.[13]

The procedure
We do not know who had the authority to commission a hydraulic project. As Stefanie Rost (2011: 123) has already pointed out, the dominant theory that there was a centralized irrigation management plan during the Third Dynasty of Ur has not been established with absolute certainty:[14]

> It is also evident that irrigation can be organized in many different ways and thus centralized management is just one possibility. In addition, many ethnographic case studies provide evidence for varying degrees of centralization as is seen when the state government only organizes a certain aspect of irrigation such as water distribution.

[13] See, e.g., MVN 14 413 (AS 9/ix/-), SNAT 427 (AS 9/-/-) or BPOA 2 2217 (ŠS 1/-/-). Moreover, we should note that there are only a small number of texts that record "mixed" information: for example, information about the work requested, plus the expenses for doing the work (about ten texts, e.g., OrSP 47-49 263 [Š 46/-/-], OrSP 47-49 151 [AS 6/-/-] or BPOA 1 948 [AS 9/-/-]).

[14] In any case, the interrelationship between irrigation management and political power in Mesopotamia is thought to have been so important that the development of centralized polities must have been accompanied by comprehensive control of the resource of water (De Maaijer 1998: 57; Hruška 1988: 27; Renger 1990: 38–9).

TABLE 3 Earthwork texts from Ur III Umma showing total number of workers and work-days

Number of days or work-days[1]	Number and type of workers	Text and date
—	[...] + 6 UN-ga$_6$	MVN 14 261 (IS 2/i/16)
	[x] guruš	AnOr 1 73 (Š 48/vi/-)
	8 guruš ša$_3$-gu$_4$	Princeton 1 408 (ŠS 4/xi/-)
	10 geme$_2$ uš-bar	MCS 3 54 BM 105547 (ŠS 2/-/-)
	10 guruš	UTI 4 3454 (AS 7/-/-)
	14 geme$_2$ uš-bar	BPOA 6 1420 (ŠS 2/-/-)
	19 guruš / 51 guruš / 24 guruš	UTI 4 2892 (ŠS 2/-/-)
	20 guruš hun-ga$_2$	SAT 2 501 (Š 46/-/-)
	27 guruš / 25 guruš / 26 guruš / 26 guruš / 103 guruš	UTI 4 2884 (ŠS 2/-/-)
	27 UN-ga$_6$	UTI 3 1854 (IS 2/1/20)
	30 geme$_2$ uš-bar	BPOA 7 2099 (ŠS 2/-/-)
	30 UN-ga$_6$	UTI 5 3208 (IS 2/i/1), UTI 3 1874 (IS 2/i/9), MVN 14 549 (IS 2/i/10), SAT 3 1954 (IS 2/i/16), UTI 5 3202 (IS 2/i/18)
	31 UN-ga$_6$	MVN 16 657 (IS 2/i/4)
	32 UN-ga$_6$	UTI 3 2056 (IS 2/i/8)
	33 UN-ga$_6$	UTI 3 2070 (IS 2/i/6)
	34 UN-ga$_6$ / 31 UN-ga$_6$	UTI 3 2114 (IS 2/ii/27)
	34 UN-ga$_6$ / 33 UN-ga$_6$	MVN 16 648 (IS 2/ii/23)
	35 UN-ga$_6$	MVN 16 706 IS 2/i/17)
	35 UN-ga$_6$ / 28 UN-ga$_6$	BPOA 6 1238 (IS 2/ii/5)
	35 UN-ga$_6$ / 36 UN-ga$_6$	OrSP 47-49 458 IS (2/iii/12)
	36 UN-ga$_6$	UTI 6 3818 (IS 2/i/23), UTI 5 3307 (IS 2/i/24)
	37 UN-ga$_6$ / 30 UN-ga$_6$	UTI 3 1862 (IS 2/ii/1)
1	[...]+3 guruš	BPOA 6 767 (ŠS 1/i/-)
1	3 guruš	SACT 2 4 (ŠS 5/-/-)
1	4 guruš	BPOA 1 1168 (ŠS 1/ix/-)
1	5	BPOA 7 1891 (AS 9/ix/-), AnOr 1 144 (AS 9/ix/-)
1	5 guruš	BPOA 6 1349 (ŠS 1/ix/-)
1	6	MVN 16 1449 (AS 9/ix/-)
1	7 ½ UN-ga-tur-tur	Aleppo 197 (Š 40/xiii/-)
1	8 geme$_2$	SAT 3 1286 (ŠS 2/-/-)
1	8 guruš	SACT 2 3 (ŠS 5/-/-)
1	9 ½ guruš	SACT 2 5 (ŠS 9/-/-)
1	10 geme$_2$	UCP 9-2-2 21 (ŠS 2/-/-)
1	10 guruš	Nik. 2 160 (Š 45/xii/-), SAT 3 1387 (ŠS 3/-/-), BCT 2 36 (ŠS 4/xi/-), UTI 3 1636 (ŠS 4/-/-)
1	11	UTI 4 2687 (AS 9/ix/-)
1	11 guruš	OrSP 47-49 349 (Š 42/-/-), MVN 14 38 (Š 45/xi/-), OrSP 47-49 325 (AS 2/-/-)
1	12 guruš	ASJ 6 138 4 (ŠS 5/-/-)
1	13 geme$_2$	BPOA 6 1416 (ŠS 2/xi/-)
1	13 guruš	UTI 5 3166 (ŠS 2/-/-)
1	14 guruš	Princeton 1 524 (ŠS 4/-/-)
1	15 guruš	UCP 9-2-1 8 (ŠS 5/-/-), BPOA 7 2062 (AS 3/-/-), BPOA 1 398 (AS 8/i/-)
	16 guruš	MVN 1 88 (AS 4/-/-)
	17 guruš	BPOA 1 388 (ŠS 2/-/-), MVN 16 991 (ŠS 3/-/-), BPOA 7 1765 (ŠS 1/-/-)
	18 geme$_2$	MCS 3 54 BM 112942 (ŠS 8/-/-)
	20 geme$_2$	OrSP 47-49 230 (Š 43/vii/-)
	20 guruš	SAT 2 782 (AS 4/-/-)
	20 guruš ša$_3$-gu$_4$	SAT 3 1348 (ŠS 3/-/-)
	23 guruš	Aleppo 203 (AS 3/xii/-)
	25 ½ guruš	BPOA 7 2224 (AS 7/-/-)
	29 geme$_2$	Nisaba 23 19 (ŠS 2/-/-)
	30	Ontario 2 139 (Š 47/-/-)

TABLE 3 (*Continued*)

Number of days or work-days[1]	Number and type of workers	Text and date
	30 guruš	AOS 32 D19 (AS 4/-/-)
	34 geme$_2$	BBVO 317 (ŠS 2/-/-)
	35 guruš	TCL 5 5676 (ŠS 2/-/-)
	38 guruš	BPOA 6 1178 (AS 3/-/-)
	40 geme$_2$	BPOA 1 1186 (AS 8/x/-)
	40 guruš	BPOA 1 1348 (Š 46/-/-)
	44 guruš	Princeton 2 429 (AS 7/-/-)
	44 ša$_3$-sahar	Princeton 1 483 (AS 1/-/-)
	60 guruš	BPOA 7 1711 (Š 46/-/-)
	84 guruš	SACT 2 31 (ŠS 2/-/-)
	195 guruš	MVN 16 1567 (ŠS 2/-/-)
	3045 ½ guruš	BPOA 7 2318 (ŠS 3/-/-)
	3120 UN-ga$_6$	BPOA 7 2391 (Š 41/-/-)
1 / 1	[x] + 45 guruš / 107	MVN 18 397 (ŠS 2/-/-)
1 / 1	107 guruš / 195 guruš	CDLJ 2003 1: 1 (ŠS 2/-/-)
1 / 1	16 guruš / 36 guruš	SACT 2 2 (ŠS 2/-/-)
1 / 1 / 1	15 guruš / 32 guruš / 10 guruš	UTI 5 3402 (ŠS 3/-/-)
1 / 1 / 1 / 1	13 guruš / 10 guruš / 10 guruš / 12 / 10 guruš	BPOA 7 2444 (AS 2/-/-)
1 / 1 / 1 / 90 ⅔	21 guruš / 67 guruš / 22 guruš	JCS 24 172 94 (AS 8/-/-)
1 / 1/ 1	20 ½ guruš/ 10 guruš / 24 guruš	MVN 21 82 (ŠS 3/-/-)
1 / 1/ 1/ 1	135 guruš / 45 guruš / 45 guruš / 235 guruš	BPOA 2 2545 (ŠS 1/-/-)
1 / 6	17 guruš / 15 guruš	UTI 2 1789 (ŠS 2/-/-)
2	10 guruš	Aleppo 201 (Š 48/vi/-), MVN 16 903 (ŠS 2/-/-), SACT 2 9 (ŠS 2/-/-)
2	11 guruš	BPOA 6 1384 (ŠS 1/ix/-)
2	13 guruš	BPOA 6 545 (ŠS 1/-/-)
2	15 guruš	BPOA 6 1466 (ŠS 2/-/-), Syracuse 276 (AS 8/-/-)
2	15 lu$_2$ hun-ga$_2$	OLP 4 17-70 n. 52 (AS 1/-/-)
2	16 guruš	MVN 14 24 (Š 44/-/-)
2	70 guruš	BPOA 2 2445 (Š 42/ix/-)
2 / 2	17 guruš / 17 guruš	BPOA 6 892 (ŠS 2/-/-)
3	3 guruš, 3 a$_2$ ⅔	BRM 3 126 (Š 46/iii/-)
3	7 guruš	BPOA 2 2335 (Š 46/-/-)
3	10 guruš	MVN 21 53 (AS 3/-/-)
3	12 guruš	BPOA 7 2215 (ŠS 2/iii/-)
3	15 ⅓ guruš	MVN 16 1475 (ŠS 1/ix/-)
3	15 guruš	BPOA 1 748 (ŠS 1/ix/-)
3	18 guruš	Syracuse 116 (Š 46/-/-)
3 / 2 / 3	15 guruš / 14 guruš / 10 guruš	SAT 2 715 (AS 2/-/-)
4	10 guruš	BIN 5 263 (AS 2/-/-), UTI 3 1737 (ŠS 1/ix/-), UTI 4 2463 (ŠS 4/xii/-)
5	10 guruš	MVN 14 355 (AS 7/-/-)
5	16 UN-ga$_6$	Nisaba 9 156 (Š 41/-/-)
5	19 guruš	UTI 3 1809 (AS 9/-/-)
5	8 guruš	Princeton 1 432 (AS 5/-/-)
5 / 4	18 guruš / 16 ½ guruš	BPOA 1 639 (ŠS 2/-/-)
5 / 5	16 guruš / 15 guruš	UTI 5 3380 (ŠS 1/-/-)
6	—	BPOA 7 1889 (ŠS 4/-/-)
6	[…] + 5 guruš	Syracuse 069 (AS 9/-/-)
6	18 guruš	Nik. 2 136 (ŠS 3/-/-)
6 / 4	[…] + 2 guruš / 16 guruš	MVN 16 1387 (AS 9/-/-)
8	11 guruš	MVN 14 329 (AS 7/-/-)
9	6 guruš	BPOA 7 1853 (AS 3/-/-)
15	16 guruš	BPOA 1 513 (ŠS 2/-/-)

Continued

TABLE 3 (*Continued*)

Number of days or work-days[1]	Number and type of workers	Text and date
158 ⅓	—	BPOA 6 1538 (AS 5/-/-)
16	—	TCL 5 5675 (AS 4/-/-), SAT 2 308 (AS 6/-/-)
18	—	TCL 5 5674 (AS 3/xii/-), SNAT 477 (ŠS 3/-/-)
18 / 12 / 9	—	SACT 2 126 (AS 7/-/-)
30	—	MVN 13 666 (Š 46/-/-)
36	—	BPOA 7 1838 (Š 46/-/-), BPOA 7 2377 (ŠS 2/-/-)
39	—	Princeton 2 497 (Š 47/-/-)
40	—	MVN 16 1158 (ŠS 3/-/-)
40 ⅔	—	BPOA 2 2434 (AS 6/-/-)
48	—	Aleppo 263 (Š 48/-/-)
49	—	Nisaba 23 150 (ŠS 2/-/-)
52	—	Syracuse 4 (AS 6/-/-)
54	—	BPOA 7 2116 (AS 6/-/-)
69 / 13 / 15 / 13	—	SNAT 295 (Š 46/-/-)
78	—	AAICAB 1/1, Ashm. 1924-0665 (AS 6/-/-)
80	—	BPOA 7 1885 (ŠS 2/-/-)
100	—	Farmer's Instructions 2.11 ([-/-/-])
132 / 27 / 99	—	MVN 21 199 (Š 47?/-/-)
142	—	BPOA 1 1062 (Š 46/-/-)
192	—	BPOA 7 1811 (Š 46/-/-)
210	—	BRM 3 119 (Š 46/-/-)
270	—	TCL 5 5668 (Š 48/xii/-), Essen in Sumer, p. 66 (AS 1/ii/-)
324	—	Aegyptus 8 264 11 (AS 3/-/-)
3360	—	UTI 3 1906 (ŠS 5?/-/-)

[1] When the number of workers is provided, this column refers to the number of days. When the number of workers is not provided, the data in this column refers to the number of work-days (number of work-days = number of workers × number of days).

The most important information seems to be that which is related to the total volume of earth—which, in fact, is recorded in almost all the documents. Once this information had been obtained, the scribe could calculate the number of people needed to complete the work and therefore the total expense in barley. The calculations for the amount of work were based on two different coefficients, which represented the amount of earth that a typical worker was expected to dig in a day (10 and 7 ½ gin_2);[15] the difference between them is probably due to the physical characteristics of the soil. The total expense mainly derived the daily cost of labour, which was commonly 6 $sila_3$ of barley per worker. Using this data, the administration could quickly calculate expenditure of labour and wages. However, the scribe did not have to record all the information on the tablet: all that was needed was information about the total volume of earth to be excavated in order to calculate, first, the number of workers and then, the cost of the work.

Obviously, the first step in the process was to conduct a survey to gather the basic information needed. This information was recorded on a tablet (such as the texts in Table 1). It should be emphasised again that these texts never record the name of the person authorised to order the work to be done. The second step was to calculate how many workers were needed to complete the work (see the texts in Table 3). It was simple enough to do this by dividing the total volume by the coefficient representing the daily amount of earth to be excavated by one worker. At this point, a scribe had to simply multiply the number of workers by the amount of daily barley that each worker received in order to calculate the total cost of a particular project (texts in Table 2). We can summarise this sequence as follows:

[15] As mentioned above, while the value of 10 gin_2 is not explicitly recorded but clearly implied in at least six texts, this situation does not occur in relation to the value 7 ½ gin_2. However, we must also consider it to be a coefficient, both because this value is widely attested in the Ur III texts and because it is recorded in the Old Babylonian coefficients lists, as discussed.

TABLE 4 The structure of an Old Babylonian mathematical problem text compared to those of Ur III administrative texts about earthworks

	Old Babylonian problem text		Ur III Texts
Phase of work	Given information	Information to be found	
Survey	ki-la$_2$ 3½ nindan 3 kuš$_3$-ta-am$_3$ ib$_2$-si$_8$ 2½ kuš$_3$ bur$_3$-bi	sahar	Type 1 or Type 2 Texts
Calculation of the total work	7½ gin$_2$ eš$_3$-gar$_3$	erin$_2$-ḫa$_2$	Texts Table 1 and Table 3
Calculation of the cost of the work	6 še a$_2$-bi	ku$_3$-babbar	Texts Table 2

Initial decision (?) → Survey to collect raw data → Calculation of volume to be excavated → Calculation of the total work → Calculation of the cost of the work.

It must be stressed that the sequence described above is inferred from administrative texts from Umma which show just the data and the results of the calculations: we do not have texts showing how the actual calculations were performed or the measurements taken. The linear measurements were probably taken by using a rope provided with knots at fixed lengths, whilst the calculations were probably performed with account tokens. After the intermediate results of the calculations were jotted down, either on tablets or on writing boards, they were probably erased by the scribe. As Robson (2008: 78) notes, "Good scribes never showed their working".[16] What the tablets mostly show us are just the final results of all the operations.

Let us now compare the Old Babylonian problem texts introduced at the beginning of this paper with the Ur III documentation (Table 4). The relationship between Ur III administrative accounting practices and Old Babylonian mathematics was studied by Robson (1999: 165), who concluded that:

> During the early Ur III period a sophisticated accounting procedure was developed for some sectors of the state economy, whereby the output of work-gangs could be monitored and compared to a theoretical maximum productivity. This overly bureaucratic system did not survive the end of Ur III empire, and records of production from the OB period reveal a truer picture of variable efficiency. Nevertheless, the rates of output deducible from the OB documents appear to cluster around the Ur III ideals. A few of the Ur III quotas appear in contemporary school practice texts, and many more seem to be the precursors to the OB quantity surveying coefficients.[17]

To sum up, as evidenced by the mathematical text presented at the beginning of this paper, and others like it, Old Babylonian mathematical problems combined with a study of Ur III administrative documentation about earthworks allow us to reconstruct the practical procedure, used by Ur III officials, to prepare and complete earthworks. Moreover, it confirms the supposition that Old Babylonian mathematics about earthworks probably reflect Ur III procedures, while presenting the whole process in the form of a problem to be solved.

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[16] For the purposes of calculation, the measurements were probably converted into sexagesimal place value notation (Powell 1976; Robson 2008: 16, 77–80; Alivernini and Greco 2014).

[17] Robson also found other work-rates which are the same in Ur III and Old Babylonian periods, such as, "carrying distance", "demolishing walls" and "mixing mortar".

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Sergio Alivernini
Oriental Institute of the Czech Academy of Sciences
Prague
alivernini@orient.cas.cz

رياضيات الاعمال الترابية المرتبطة بالمشاريع الإروائية في مدينة أُوما Umma سلالة أور الثالثة
بقلم : سيرجيو أليفرنيني

يركز هذا البحث على رياضيات الاعمال الترابية المرتبطة بالمشاريع الإروائية في مدينة أُوما Umma خلال سلالة أور الثالثة 2053–2032 قبل الميلاد. الغرض الرئيسي من البحث هو شرح الإجراءات العملية خلال الاعمال الترابية الخاصة بالمشاريع الإروائية حول مدينة أُوما. كما تبين تأثير الإجراءات العملية المتبناة من قبل المسؤولين في سلالة أور الثالثة على النصوص الرياضية المدرسية البابلية القديمة من اوائل الألف الثانية قبل الميلاد والخاصة بالاعمال الترابية.

APPENDIX: Summary of the information included in the Ur III Umma texts about earthworks

Text and date	Total amount of earth to be excavated	Quantity of earth to be excavated by each worker	Daily cost in barley	Total cost in barley	Total number and type of workers	Total number of days or work-days
AAICAB 1/2, Ashm. 1971-379 (Š 34/-/-)	155 sar					
AAICAB 1/1, Ashm. 1911-486 (Š 34/-/-)	485 ⅚ sar					
Nisaba 9 47 (Š 35/-/-)	108 ⅓ sar					
Nisaba 26 37 (Š 36/i/-)	22 sar / 32 sar / 46 sar / 38 sar / 46 sar / 22 sar					
BIN 5 265 (Š 39/6/-)	6 ½ sar					
Aleppo 197 (Š 40/xiii/-)					7 ½ guruš	1
BPOA 7 2391 (Š 41/-/-)					3120 UN-ga$_6$-tur-tur	1
Nisaba 9 156 (Š 41/-/-)					16 UN-ga$_6$	5
BPOA 2 2445 (Š 42/ix/-)	180 ⅓ sar				70 guruš	2
SAT 2 950 (Š 42/x/-)	15 sar					
OrSP 47-49 349 (Š 42/-/-)					11 guruš	1
MVN 16 1337 (Š 42/xiii/-)	⅚ sar 6 gin$_2$	5 gin$_2$				
OrSP 47-49 230 (Š 43/vii/-)					20 geme$_2$	1
MVN 14 24 (Š 44/-/-)					16 guruš	2
MVN 14 38 (Š 45/xi/-)					11 guruš	1
Nik. 2 160 (Š 45/xi/-)	1 ⅔ sar				10 guruš	1
AAICAB 1/3, Bod. S 207 (Š 45/-/-)				106.4.0 še gur		
BPOA 2 2563 (Š 45/-/-)	1572 sar					
BRM 3 126 (Š 46/iii/-)					3 guruš, 3 a$_2$ ⅔	3
Vicino Oriente 8/1 11 (Š 46/vi/-)	1 sar					
SAT 2 486 (Š 46/vi/-)	9 ⅔ sar					
BRM 3 119 (Š 46/-/-)	35 sar	10 gin$_2$				210
OrSP 47-49 263 (Š 46/-/-)	182 sar	10 gin$_2$	6 sila$_3$	21.4.1 2 še gur		
Orient 17 19 (Š 46/-/-)					6 guruš	1
Syracuse 116 (Š 46/-/-)					18 guruš	3
Syracuse 459 (Š 46/-/-)	545 ½ sar					
MVN 13 666 (Š 46/-/-)	5 sar					30
MVN 14 215 (Š 46/-/-)	1 sar					
SNAT 295 (Š 46/-/-)	11 ½ sar / 2 sar 15 gin$_2$ / 2 ½ sar 7 ½ gin$_2$ / 2 sar 15 gin$_2$					69 / 13 / 15 / 13

MVN 20 151 (Š 46/-/-)	39 sar 15 gin$_2$ / 56 ⅓ sar 5 gin$_2$	7 ½ gin$_2$ / 10 gin$_2$				
MVN 21 134 (Š 46/-/-)	148 sar 5 gin$_2$					
BPOA 1 1062 (Š 46/-/-)	22 sar					142
BPOA 1 1348 (Š 46/-/-)					40 guruš	1
BPOA 2 2313 (Š 46/-/-)	145 ½ sar					
BPOA 2 2335 (Š 46/-/-)					7 guruš	3
BPOA 6 1454 (Š 46/-/-)	15 ⅔ sar					
BPOA 7 1582 (Š 46/-/-)	12 sar					
BPOA 7 1711 (Š 46/-/-)	5 sar	5 gin$_2$			60 guruš	1
BPOA 7 1811 (Š 46/-/-)	31 sar 10 gin$_2$					192
BPOA 7 1838 (Š 46/-/-)	6 sar					36
SAT 2 501 (Š 46/-/-)					20 guruš	
BRM 3 162 (Š 47/-/-)	33 ⅔ sar		6 sila$_3$			
Ontario 2 139 (Š 47/-/-)	5 sar	10 gin$_2$			30 guruš	1
Princeton 2 403 (Š 47/-/-)	37 ⅓ sar		6 sila$_3$			
Princeton 2 466 (Š 47/-/-)	95 sar		6 sila$_3$	11.2.0 gur		
Princeton 2 497 (Š 47/-/-)	6 ½ sar		6 sila$_3$	0.3.5 4 sila$_3$		39
MVN 21 199 (Š 47$^?$/-/-)	22 sar / 4 ½ sar / 16 ½ sar 3 gin$_2$					132 / 27 / 99
BPOA 7 1728 (Š 48/v/-)	12 ½ sar					
AnOr 1 73 (Š 48/v/-)					[x] guruš	
Aleppo 201 (Š 48/x/-)					10 guruš	2
Syracuse 152 (Š 48/xi/-)	1 sar					
TCL 5 5668 (Š 48/xii/-)	22 ½ sar	10 gin$_2$				270
MVN 21 135 (Š 48/xii/-)	205 ⅓ sar 10 gin$_2$ / 13 ⅚ 7 ½ gin$_2$		6 sila$_3$			
Aleppo 230 (Š 48/xii/-)	200 sar					
TIM 6 1 (Š 48/-/-)	219 ⅚ sar 6 ½ gin$_2$ / 406 ½ sar 5 gin$_2$ / 1185 ⅚ sar 6 ⅔ gin$_2$					
Syracuse 28 (Š 48/-/-)	5 sar / 4 sar	7 ½ gin$_2$				
MVN 21 136 (Š 48/-/-)	3 ⅓ sar		6 sila$_3$			
SAT 2 602 (Š 48/-/-)	11 sar					
SAT 2 607 (Š 48/-/-)	77 ⅔ sar 7 gin$_2$ / 44 sar 15 gin$_2$		6 sila$_3$			
SAT 3 2113 (Š 48/-/-)	46 ½ sar 5 gin$_2$					
BPOA 1 925 (Š 48/-/-)	1 ⅚ sar					
BPOA 7 2417 (Š 48/-/-)	9 ½ sar					
Aleppo 200 (Š 48/-/-)	22 ½ sar					

Appendix: (*Continued*)

Text and date	Total amount of earth to be excavated	Quantity of earth to be excavated by each worker	Daily cost in barley	Total cost in barley	Total number and type of workers	Total number of days or work-days
Aleppo 263 (Š 48/-/-)	8 sar					48
Essen in Sumer, p. 66 (AS 1/ii/-)	22 ½ sar	10 gin$_2$				270
OrSP 47-49 313 (AS 1/-/-)	6 ⅔ sar					
SAKF 17 (AS 1/-/-)	28 ⅓ sar					
SAKF 21 (AS 1/-/-)	1 ⅔ sar					
OLP 4 17-70 n. 52 (AS 1/-/-)	7 ½ sar	13 gin$_2$			15 lu$_2$ hun-ga$_2$	2
Princeton 1 483 (AS 1/-/-)	7 ⅓ sar				44 ša$_3$-sahar (	1
BPOA 6 384 (AS 1/-/-)	10 sar		5 sila$_3$	1.0.0 gur		
BPOA 6 850 (AS 1/-/-)	168 sar		5 sila$_3$			
BPOA 6 1432 (AS 1/-/-)	10 sar					
BPOA 7 1835 (AS 1/-/-)	⅔ sar					
CST 576 (AS 2/i/-)	1 ⅚ sar					
YOS 4 223 (AS 2/-/-)	17 ½ sar					
OrSP 47-49 325 (AS 2/-/-)					11 guruš	1
AnOr 1 79 (AS 2/-/-)	2 ⅔ sar					
BIN 5 263 (AS 2/-/-)	4 ⅚ sar					
Syracuse 144 (AS 2/-/-)	4 sar	10 gin$_2$				
SAT 2 715 (AS 2/-/-)					15 guruš / 14 guruš / 10 guruš	3 / 2 / 3
AnOr 7 199 (AS 2/-/-)	3 sar 10 gin$_2$					
BPOA 1 971 (AS 2/-/-)	5 sar		6 sila$_3$			
BPOA 7 2444 (AS 2/-/-)					13 guruš / 10 guruš / 10 guruš / 12 / 10 guruš	1 / 1 / 1 / 1
NYPL 93 (AS 3/iii/-)	1 ½ sar					
Princeton 1 501 (AS 3/v/-)	5 ⅓ sar	7 ½ gin$_2$				
TCL 5 5674 (AS 3/xii/-)	3 sar					18
Aleppo 203 (AS 3/xii/-)					23 guruš	1
Aegyptus 8 264 11 (AS 3/-/-)	54 sar	10 gin$_2$				324
JCS 16 14 1 (AS 3/-/-)	50 sar / 10 sar		5 sila$_3$			
Farmer's Instructions 2.04 (AS 3/-/-)	10 sar					
MVN 21 53 (AS 3/-/-)					10 guruš	3
SAT 2 755 (AS 3/-/-)	2 sar	10 gin$_2$				
BPOA 1 1256 (AS 3/-/-)	1 ½ sar					
BPOA 2 2436 (AS 3/-/-)	20 sar					
RA 101 44 13 (AS 3/-/-)	1 ⅔ sar					

Text						
BPOA 6 1178 (AS 3/-/-)					38 guruš	1
BPOA 7 1853 (AS 3/-/-)					6 guruš	9
BPOA 7 2062 (AS 3/-/-)	2 ½ sar				15 guruš	1
BPOA 7 2303 (AS 3/-/-)	1 ⅓ sar					
BPOA 7 2472 (AS 3/-/-)	8 sar					
BIN 5 272 (AS 3/-/-)	26 sar 10 gin$_2$ / 50 ⅚ sar	10 gin$_2$ / 10 gin$_2$				
BPOA 1169 (AS 4/vi/-)	13 ⅓ sar					
AOS 32 D19 (AS 4/-/-)					30 guruš	1
MVN 1 88 (AS 4/-/-)					16 guruš	1
SAT 2 782 (AS 4/-/-)	2 sar				20 guruš	1
MVN 18 475 (AS 4/-/-)	3 ½ sar					
BPOA 7 1741 (AS 4/-/-)	6 sar […] gin$_2$					
PPAC 4 158 (AS 4/-/-)	5 sar 5 ⅔ gin$_2$					
TCL 5 5675 (AS 4/-/-)	2 ⅔ sar					16
BPOA 1 1264 (AS 5/iii/-)	2 sar					
MVN 13 362 (AS 5/vi/-)	2 ⅔ sar					
Princeton 1 432 (AS 5/-/-)					8 guruš	5
BPOA 6 1538 (AS 5/-/-)	15 ⅚ sar	6 gin$_2$				158 ⅓
UTI 4 2925 (AS 6/xiii/-)	1 ½ sar	7 ½ gin$_2$				
OrSP 47-49 151 (AS 6/-/-)	6 ⅔ sar	10 gin$_2$	6 sila$_3$	0.4.0		
Syracuse 4 (AS 6/-/-)	[x] ½ sar	7 ½ gin$_2$				52
AUCT 3 311 (AS 6/-/-)	[…] ⌜x⌝ sar	7 ½ gin$_2$				
SAT 2 308 (AS 6/-/-)	2 sar	7 ½ gin$_2$				16
AAICAB 1/1, Ashm. 1924-0665 (AS 6/-/-)	9 ⅓ sar 5 gin$_2$	7 ½ gin$_2$				78
BPOA 1 1301 (AS 6/-/-)	3 sar	7 ½ gin$_2$				
BPOA 2 2434 (AS 6/-/-)	5 sar 6 gin$_2$	7 ½ gin$_2$				40 ⅔
BPOA 7 2116 (AS 6/-/-)	9 sar					54
Nisaba 24 13 (AS 6/-/-)				10.2.5 gur		
SACT 2 124 (AS 7/xii/-)	13 gin$_2$					
Nisaba 23 118 (AS 7/xii/-)	60 sar					
SACT 2 126 (AS 7/-/-)	3 sar / 2 sar / 1 ½ sar					18 / 12 / 9
MVN 14 329 (AS 7/-/-)					11 guruš	8
MVN 14 355 (AS 7/-/-)					10 guruš	5
UTI 4 2460 (AS 7/-/-)	2 sar					
UTI 4 2554 (AS 7/-/-)	39 sar					
UTI 4 3454 (AS 7/-/-)					10 guruš	
Princeton 2 429 (AS 7/-/-)	3 ½ sar				44 guruš	1
BPOA 7 2224 (AS 7/-/-)					25 ½ guruš	1
BPOA 1 398 (AS 8/i/-)					15 guruš	1
BPOA 1 1186 (AS 8/x/-					40 geme$_2$	1

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APPENDIX: (*Continued*)

Text and date	Total amount of earth to be excavated	Quantity of earth to be excavated by each worker	Daily cost in barley	Total cost in barley	Total number and type of workers	Total number of days or work-days
JCS 24 172 94 (AS 8/-/-)	11 ⅓ sar	7 ½ gin$_2$			21 guruš / 67 guruš / 22 guruš	1 / 1 / 1 / 90 ⅔
SACT 2 129 (AS 8/-/-)	2 sar	7 ½ gin$_2$				
Syracuse 276 (AS 8/-/-)					15 guruš	2
Hirose 372 (AS 8/-/-)	50 sar		4 sila$_3$			
SNAT 398 (AS 8/-/-)	6100 sar					
UTI 3 1655 (AS 8/-/-)	1 ½ sar					
UTI 4 2781 (AS 8/-/-)	15 sar 12 ½ gin$_2$					
UTI 4 2791 (AS 8/-/-)	35 sar 15 gin$_2$ / 60 ⅓ sar					
UTI 4 2869 (AS 8/-/-)	6 ⅓ sar	10 gin$_2$				
BPOA 7 2247 (AS 8/-/-)	5 ⅓ sar					
MVN 14 413 (AS 9/ix/-)	[…] sar 6 gin$_2$					
MVN 16 1449 (AS 9/ix/-)	⅚ sar 5 gin$_2$				6 guruš	1
UTI 4 2687 (AS 9/ix/-)					11	1
BPOA 7 1891 (AS 9/ix/-	1 sar				5	1
AnOr 1 144 (AS 9/ix/-)					5	1
MVN 14 397 (AS 9/xii/-)	5 ⅓ sar					
MVN 16 1549 (AS 9/xii/-)	3 ⅔ sar					
BPOA 1 1159 (AS 9/[…]/-)	3 ⅓ sar	7 ½ gin$_2$				
Syracuse 69 (AS 9/-/-)					[…] + 5 guruš	6
SNAT 427 (AS 9/-/-)	6 ⅚ sar					
UTI 3 1808 (AS 9/-/-)	5 ⅔ sar					
UTI 3 1809 (AS 9/-/-)					19 guruš	5
MVN 16 1387 (AS 9/-/-)					[…] + 2 guruš / 16 guruš	6 / 4
UTI 5 3499 (AS 9 /-/-)	69 ½ sar / 82 ⅓ sar / 66 sar					
MVN 18 490 (AS 9/-/-)	[…] + 5 sar 15 gin$_2$		5 sila$_3$			
BPOA 1 511 (AS 9/-/-)	3 ⅚ sar 5 gin$_2$					
BPOA 1 948 (AS 9/-/-)	20 sar	10 gin$_2$	6 sila$_3$			
BPOA 2 2435 (AS 9/-/-)	20 sar					
BPOA 2 2590 (AS 9/-/-)	259 sar 10 gin$_2$		5 sila$_3$			
BPOA 6 24 (AS 9/-/-)	4 ½ sar					
BPOA 7 2059 (AS 9/-/-)	6 ½ sar					
BPOA 6 767 (ŠS 1/i/-)					[…] + 3 guruš	1
UTI 3 1737 (ŠS 1/ix/-)					10 guruš	4
MVN 16 1475 (ŠS 1/ix/-)					15 ⅓ guruš	3

BPOA 1 748 (ŠS 1/ix/-)				15 guruš	3
BPOA 1 1168 (ŠS 1/ix/-)				4 guruš	1
BPOA 6 1349 (ŠS 1/ix/-)				5 guruš	1
BPOA 6 1384 (ŠS 1/ix/-)				11 guruš	2
BPOA 1 1451 (ŠS 1/-/-)	70 sar		6 sila₃		
BPOA 2 2217 (ŠS 1/-/-)	1 sar 10 gin₂				
BPOA 2 2545 (ŠS 1/-/-)				135 guruš / 45 guruš / 45 guruš / 235 guruš	1 / 1/ 1/ 1
BPOA 6 545 (ŠS 1/-/-)				13 guruš	2
BPOA 6 1272 (ŠS 1/-/-)	7 sar 5 gin₂				
BPOA 7 1765 (ŠS 1/-/-)				17 guruš	1
UTI 5 3380 (ŠS 1/-/-)				16 guruš / 15 guruš	5 / 5
BPOA 7 2215 (ŠS 2/iii/-)				12 guruš	3
MVN 16 1324 (ŠS 2/v/-)	120 ⅚ sar	7 ½ gin₂			
BPOA 6 1416 (ŠS 2/xi/-)				13 geme₂	1
UCP 9-2-2 21 (ŠS 2/-/-)				10 geme₂	1
MCS 3 54 BM 105547 (ŠS 2/-/-)				10 geme₂ uš-bar	
SACT 2 2 (ŠS 2/-/-)				16 guruš / 36 guruš	1 / 1
SACT 2 9 (ŠS 2/-/-)				10 guruš	2
SACT 2 31 (ŠS 2/-/-)				84 guruš	1
SACT 2 135 (ŠS 2/-/-)	11 ⅓ sar 7 gin₂				
UTI 3 1652 (ŠS 2/-/-)	3 ⅚ sar	7 ½ gin₂			
UTI 2 1789 (ŠS 2/-/-)				17 guruš / 15 guruš	1 / 6
MVN 16 903 (ŠS 2/-/-)				10 guruš	2
MVN 16 1567 (ŠS 2/-/-)				195 guruš	1
UTI 4 2743 (ŠS 2/-/-)	5 ⅚ sar	7 ½ gin₂			
UTI 4 2884 (ŠS 2/-/-)				27 guruš / 25 guruš / 26 guruš / 26 guruš / 103 guruš	
UTI 4 2892 (ŠS 2/-/-)				19 guruš / 51 guruš / 24 guruš	
UTI 5 3166 (ŠS 2/-/-)				13 guruš	1
SAT 3 1286 (ŠS 2/-/-)				8 geme₂	1
CDLJ 2003 1: 1 (ŠS 2/-/-)				107 guruš / 195 guruš /	1 / 1
Princeton 2 385 (ŠS 2/-/-)	[…] sar 7 ½ gin₂ / 6 ⅔ sar 2 7 ½ gin₂	7 ½ gin₂			
TCL 5 5676 (ŠS 2/-/-)	25 sar / 6 sar	7 ½ gin₂ / 10 gin₂		35 guruš	1
AnOr 7 231 (ŠS 2/-/-)	5 ⅔ sar				
MVN 18 397 (ŠS 2/-/-)				[x] + 45 guruš / 107 guruš	1 / 1

Continued

Appendix: (*Continued*)

Text and date	Total amount of earth to be excavated	Quantity of earth to be excavated by each worker	Daily cost in barley	Total cost in barley	Total number and type of workers	Total number of days or work-days
BBVO 18 317 (ŠS 2/-/-)					34 geme$_2$	1
BPOA 1 388 (ŠS 2/-/-)					17 guruš	1
BPOA 1 513 (ŠS 2/-/-)					16 guruš	15
BPOA 1 639 (ŠS 2/-/-)					18 guruš / 16 ½ guruš	5 / 4
BPOA 1 1147 (ŠS 2/-/-)	4 ⅔ sar	7 ½ gin$_2$				
BPOA 2 2547 (ŠS 2/-/-)	84226 ½ sar 4 ⅚ gin$_2$		6 sila$_3$			
BPOA 2 2568 (ŠS 2/-/-)	418 ⅓ sar 4 ⅓ gin$_2$ / 506 sar 7 gin$_2$ / 23 sar					
Nisaba 23 19 (ŠS 2/-/-)			0.0.3		29 geme$_2$	1
Nisaba 23 150 (ŠS 2/-/-)	6 sar 7 ½ gin$_2$	7 ½ gin$_2$				49
BPOA 6 892 (ŠS 2/-/-)					17 guruš / 17 guruš	2 / 2
BPOA 6 1420 (ŠS 2/-/-)					14 geme$_2$ uš-bar	
BPOA 6 1466 (ŠS 2/-/-)					15 guruš	2
BPOA 7 1885 (ŠS 2/-/-)	10 sar	7 ½ gin$_2$				80
BPOA 7 2099 (ŠS 2/-/-)					30 geme$_2$ uš-bar	
BPOA 7 2377 (ŠS 2/-/-)	4 ½ sar	7 ½ gin$_2$				36
Nik. 2 136 (ŠS 3/-/-)					18 guruš	6
HUCA 29 87 13 (ŠS 3/-/-)	5 sar					
SNAT 477 (ŠS 3/-/-)	3 sar	10 gin$_2$				18
MVN 16 991 (ŠS 3/-/-)					17 guruš	1
MVN 16 1158 (ŠS 3/-/-)	5 sar	7 ½ gin$_2$				40
UTI 4 2469 (ŠS 3/-/-)	4 ⅚ sar	7 ½ gin$_2$				
UTI 4 2766 (ŠS 3/-/-)	300 sar	7 ½ gin$_2$				
MVN 21 82 (ŠS 3/-/-)					20 ½ guruš/ 10 guruš / 24 guruš	1 / 1/ 1
UTI 5 3402 (ŠS 3/-/-)					15 guruš / 32 guruš / 10 guruš	1 / 1 / 1
SAT 3 1348 (ŠS 3/-/-)					20 guruš ša$_3$-gu$_4$	1
BPOA 1 625 (ŠS 3/-/-)	4 ⅚ sar	13 gin$_2$				
BPOA 1 709 (ŠS 3/-/-)	4 ⅚ sar	7 ½ gin$_2$				
BPOA 2 2495 (ŠS 3/-/-)	12 ⅓ sar					
BPOA 6 1283 (ŠS 3/-/-)	3 ⅚ sar	7 ½ gin$_2$				
BPOA 7 2318 (ŠS 3/-/-)					3045 ½ guruš	1
SAT 3 1387 (ŠS 3/-/-)					10 guruš	1
MVN 16 802 (ŠS 4/vi/-)	57 ⅓ sar		6 sila$_3$			
Princeton 1 408 (ŠS 4/xi/-)					8 guruš ša$_3$-gu$_4$	

BCT 2 36 (ŠS 4/xi/-)					10 guruš	1
UTI 4 2463 (ŠS 4/xi/-)					10 guruš	4
ASJ 8 68 (ŠS 4/-/-)	3 ⅓ sar / 3 ⅚ sar 4 gin_2 / 1 ⅓ sar	10 gin_2 / 10 gin_2 / 10 gin_2				
Princeton 1 524 (ŠS 4/-/-)					14 guruš	1
UTI 3 1636 (ŠS 4/-/-)					10 guruš	1
Farmer's Instructions 2.9 (ŠS 4/-/-)	6 sar					
JCS 46 22 9 (ŠS 4/-/-)	8 sar / 5 sar / 8 sar / 3 sar					
MVN 16 871 (ŠS 4/-/-)	5 ½ sar					
SAT 3 1537 (ŠS 4/-/-)	3 ⅔ sar / 4 ⅔ sar	7 ½ gin_2				
AnOr 7 240 (ŠS 4/-/-)	5 ½ sar / 5 gin_2					
BPOA 7 1889 (ŠS 4/-/-)	⅔ sar 5 gin_2	7 ½ gin_2				6
MVN 21 150 (ŠS 5/viii/-)	⅚ sar	10 gin_2				
UCP 9-2-1 8 (ŠS 5/-/-)					15 guruš	1
SACT 2 3 (ŠS 5/-/-)					8 guruš	1
SACT 2 4 (ŠS 5/-/-)					3 guruš	1
ASJ 6 138 4 (ŠS 5/-/-)					12 guruš	1
UTI 3 1906 (ŠS 5$^?$/-/-)	560 sar	10 gin_2	6 $sila_3$	67.1.0 gur		3360
MVN 16 961 (ŠS 6/-/-)	1486 ⅔ sar					
BPOA 1 779 (ŠS 6/-/-)	8 ½ sar	7 ½ gin_2				
BPOA 1 848 (ŠS 7/x/-)	2 ⅓ sar 3 ½ gin_2					
YOS 4 209 (ŠS 7/-/-)	2303 sar / 723 ⅔ sar		6 $sila_3$	276.1.4 8 $sila_3$ gur		
CST 574 (ŠS 7/-/-)	25 sar					
MCS 3 54 BM 112942 (ŠS 8/-/-)					18 $geme_2$	1
SACT 2 5 (ŠS 9/-/-)					9 ½ guruš	1
Ontario 2 141 (IS 1/-/-)	15 sar					
UTI 5 3208 (IS 2/i/1)					30 UN-ga_6	
MVN 16 657 (IS 2/i/4)					31 UN-ga_6	
UTI 3 2070 (IS 2/i/6)					33 UN-ga_6	
UTI 3 2056 (IS 2/i/8)					32 UN-ga_6	
UTI 3 1874 (IS 2/i/9)					30 UN-ga_6	
MVN 14 549 (IS 2/i/10)					30 UN-ga_6	
MVN 14 261 (IS 2/i/16)					[…] + 6 UN-ga_6	
SAT 3 1954 (IS 2/i/16)					30 UN-ga_6	
MVN 16 706 (IS 2/i/17)					35 UN-ga_6	
UTI 5 3202 (IS 2/i/18)					30 UN-ga_6	
UTI 3 1854 (IS 2/i/20)					27 UN-ga_6	
UTI 6 3818 (IS 2/i/23)					36 UN-ga_6	
UTI 5 3307 (IS 2/i/24)					36 UN-ga_6	

APPENDIX: (*Continued*)

Text and date	Total amount of earth to be excavated	Quantity of earth to be excavated by each worker	Daily cost in barley	Total cost in barley	Total number and type of workers	Total number of days or work-days
UTI 6 3606 (IS 2/i/26)					40 UN-ga$_6$	
UTI 3 1862 (IS 2/ii/1)					37 UN-ga$_6$ / 30 UN-ga$_6$	
BPOA 6 1238 (IS 2/ii/5)					35 UN-ga$_6$ / 28 UN-ga$_6$	
MVN 16 648 (IS 2/ii/23)					34 UN-ga$_6$ / 33UN-ga$_6$	
UTI 3 2114 (IS 2/ii/27)					34 UN-ga$_6$ / 31 UN-ga$_6$	
OrSP 47-49 458 (IS 2/iii/12)					35 UN-ga$_6$ / 36 UN-ga$_6$	
CDLJ 2012:1 3.04 (IS 2/-/-)	99 ⅚ sar					
AAICAB 1/3, Bod. S 157 (-/-/-)	467 sar					
OrSP 47-49 511 (-/-/-)	1.0.2 ½ ¼ GAN$_2$					
SACT 2 140 (-/-/-)	55 ½ sar					
UTI 5 3480 ([/-/-])	6 sar / 6 sar / 5 sar					
MVN 10 105 ([-/-/-])	380 sar		6 sila$_3$	45.3.0 gur		
Farmer's Instructions 2.11 ([-/-/-])	16 ⅔ sar		[6 sila$_3$]	2.0.0 gur		100