

# TALENT UTILIZATION AND SEARCH FOR THE APPROPRIATE TECHNOLOGY

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This paper presents a model of development that is driven by matching between talents and technologies. Differences in productivity across countries are amplified by three dimensions of talent utilization: the range of talents utilized, the density of a specific talent utilized, and the average match quality in the economy. In our model, higher productivity increases the number of technologies available, enhancing the opportunities for individuals to match their talents to specific technologies and increasing the returns to search. More intensive search further contributes to talent utilization.

**Keywords:** Income Level, Total Factor Productivity, Technological Density, Appropriate Technology, Talent Utilization, Search

## 1. INTRODUCTION

Total factor productivity (TFP) is an important determinant of development. However, as measured by the Solow residual, it is no more than “the measure of our ignorance” [Abramovitz (1956)]. This research adds a new theoretical explanation for differences in TFP and, thus, income differences across countries. We build on evidence that the quality of the match between workers’ skills and production processes makes a large contribution to productivity [Neal (1995), Parent (2000)] and show how search and matching amplify small differences in productivity across countries.

The paper provides a mechanism by which the structure of production amplifies differences in productivity through better talent utilization. The premise of the paper is that each sector uses a distinct technology requiring a specific set of talents. As productivity increases, so does the number of sectors. With more sectors, individuals’ talents can be better matched to the talent requirements of sectors. Hence, economies that have different productivity will vary in their degree of talent utilization while efficiently allocating their resources. With additional information frictions, individuals need to search for the sector that best fits their talents. The incentives to search are determined by profitability, which in turn

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increases with the specialization created by higher search effort. This feedback between profitability and specialization lies at the core of the model.

These ideas are presented in a model of economic development where the final output is a composite of many intermediate goods. Each intermediate good is produced in a specific sector by raw labor and by entrepreneurs who have heterogeneous talents. The extent to which an entrepreneur's talent matches the sector's talent requirements determines the efficiency units of labor that an entrepreneur supplies. With decreasing returns to accumulated factors, the number of intermediate varieties is determined so that a fixed setup cost is recovered.

Productivity increases talent utilization through a few channels. Higher productivity increases entrepreneurial profits. As a result, a smaller continuum of entrepreneurs is needed to cover the fixed cost, yielding a larger number of varieties and a higher match quality on average. Both outcomes generate stronger incentives to search for the sector that best matches individuals' talents. Thus, investment in search increases with development, acting as another source of amplification by increasing the extent to which talents are utilized.

This paper belongs to a strand of literature that tries to explain why some countries are so much richer than others. The answers the empirical literature provides range from factor accumulation to the efficiency with which these factors are used.<sup>1</sup> Nonetheless, this literature identifies an important role for TFP in explaining cross-country differences in income.<sup>2</sup>

The importance of TFP in explaining large cross-country differences in income leaves us needing to understand the underlying technological differences across countries.<sup>3</sup> Acemoglu et al. (2007) present a model in which contractual incompleteness leads to the adoption of less advanced technologies. Basu and Weil (1998), Zeira (1998), and Acemoglu and Zilibotti (2001) are theoretical contributions that emphasize the role of appropriate technologies in explaining TFP differences. Zeira (1998) focuses on the range of technologies adopted due to differences in capital–labor ratios. Basu and Weil (1998) highlight the role of learning by doing. Acemoglu and Zilibotti (2001) emphasize skill supply for utilizing advanced technologies. In these papers, differences in factor distribution across countries drive the adoption or invention of the appropriate technology. In our model, appropriateness is at the micro level.<sup>4</sup> Each individual can be appropriately matched to a technology or not. Thus, countries may have the same factor distribution, yet differ in how appropriately these factors are allocated across production technologies.

The idea that better matching increases productivity can be traced back to Salop (1979). The literature that followed identified different mechanisms generating a mismatch or affecting the match quality between workers and their modes of employment. In Albrecht and Vroman (2002) the match quality is determined when firms choose their skill requirements in response to the distribution of skills.<sup>5</sup> Davidson et al. (2008) use a similar framework to show how trade liberalization can influence matching and therefore TFP. More recently, in Grossman and Helpman (2005), thick markets induce better matching; in Ahlin (2010), differences in risks

affect matching in micro-lending groups; and in Alder (2010), “crony capitalism” is responsible for the mismatch between firms and managers.<sup>6</sup> Closest to our structure is Kim (1989), in which a scale effect determines the assignment of individuals’ talents to firms’ requirements through a partial equilibrium model of the labor market. In contrast, our model has a general equilibrium setting, in which initial productivity differences are amplified as the number of varieties and the match quality adjust endogenously.<sup>7</sup> In addition, in our model information frictions further augment initial differences in productivity. Finally, our model can shed light on differences in the wage distribution and the size distribution of firms across countries.

Our work is also related to the burgeoning literature on search and matching between heterogeneous workers and firms. Sharing our concern regarding the mismatch between workers and jobs, this literature investigates the search and assignment process, focusing at times on the wage offer dynamics [Crawford and Knoer (1981)] or the search process and its direction [Gautier and Teulings (2004), Shimer (2005), Decreuse (2008)]. In our model, wage heterogeneity and the level of mismatch in the economy are determined by the number of sectors in equilibrium and the optimal search effort of individuals. Because search is directed toward the nearest match, the extent of mismatch diminishes with the number of varieties.<sup>8</sup>

The rest of the paper is organized as follows. Section 2 formalizes the arguments, Section 3 solves for the equilibrium, Section 4 provides a cross-country analysis, Section 5 presents some concluding remarks, and proofs appear in the Appendix.

## 2. THE MODEL

Consider a small open economy in a world with one final good, which is used for consumption only. This final good is produced using a continuum of intermediate goods. For simplicity, the model assumes no physical capital and, therefore, intermediate goods are produced using labor only. All markets are assumed to be perfectly competitive. The final good, as well as each intermediate good, is assumed to be perfectly tradable, but labor is not tradable, and its market is domestic. For simplicity, there is no population growth and population size is normalized to one.

### 2.1. Production

*Production of the final good.* The final good is produced by the continuous log-linear production function

$$\log Y = \int_0^1 \log x(j) dj, \quad (1)$$

where  $Y$  is the total output produced in an economy and  $x(j)$  is the input of intermediate good  $j$ .

*Production of intermediate goods.* Each country produces a discrete variety of intermediate goods out of a potential continuum, which is located on the circumference of the circle of unit length. Each point on this unit circle represents a different type of intermediate good that requires a specific talent to operate the technology by which it is produced. This specific talent,  $j$ , will henceforth be called the “job requirements.”

Individuals are indexed on a unit circle with uniform density. The index  $i$  of each individual represents her talent. As job requirements represent the location of an entrepreneur whose talent accurately matches these requirements, individuals and intermediate goods are both indexed on the same unit circle without ambiguity.

Intermediate good  $j$  is produced within sector  $j$  by a continuum of entrepreneurs, each endowed with a specific talent that matches to some extent the job requirements of the technology used. The extent to which an entrepreneur’s talent matches the job requirements determines the number of efficiency units of labor this entrepreneur supplies, according to the function

$$h(j, i) = h_0 - bd(j, i), \tag{2}$$

where  $h(j, i)$  is the efficiency units of labor that entrepreneur  $i$  supplies for producing intermediate good  $j$ ,  $h_0$  is the maximum efficiency units of labor that an entrepreneur can have, and  $d(j, i)$  is the distance along the circle between the location of intermediate good  $j$ , which reflects its job requirements, and that of entrepreneur  $i$ , which reflects her entrepreneurship talent. This distance expresses the match quality. The greater the distance, the lower the match quality.

Each individual is a potential entrepreneur, who can produce an intermediate good  $j$  according to the production function

$$x(j, i) = A [l(j, i)]^\alpha [h(j, i)]^{(1-\alpha)}, \tag{3}$$

where  $\alpha \in (0, 1)$ ,  $x(j, i)$  is the output of intermediate good  $j$  produced by entrepreneur  $i$ ,  $l(j, i)$  is the number of workers she employs, and  $A$  is a country-specific productivity parameter. This coefficient may reflect geography (e.g., land quality, climate, access to the sea), resource endowments (e.g., land abundance or natural resources), or even infrastructure, and therefore differs across countries. In this formulation, labor productivity is affected not only by  $A$  but also by the match quality,  $[h(j, i)]^{(1-\alpha)}$ .

Each intermediate good is produced by a continuum of entrepreneurs taking prices as given. Namely, each entrepreneur  $i$  takes the equilibrium wage  $w$ , the cost  $r(j)$  of technology  $j$ ’s blueprint, and the price  $P(j)$  of a unit of intermediate good  $j$  and maximizes

$$\pi(j, i) = P(j)A [l(j, i)]^\alpha [h(j, i)]^{(1-\alpha)} - w[l(j, i)] - r(j). \tag{4}$$

*The monopolistic market for technologies.* Each intermediate good requires knowledge of a specific technology. This knowledge is owned by a monopoly. Because intermediate goods are substitutes in the production of the final good, competition arises among monopolies.

The market for technologies operates as follows. A monopolistic owner of a technology incurs a setup cost,  $C$ . This cost, which is measured in terms of the final good, can be interpreted as the cost of importing off-the-shelf technology for producing intermediate good  $j$ . The revenues are  $R(j)$ , which consist of total payments collected from all entrepreneurs using technology  $j$ . An owner who does not observe entrepreneurs' talents cannot discriminate and thus charges a uniform price,  $r(j)$ . Therefore, profit generated by monopolistic owner  $j$  is

$$\Pi(j) = R(j) - C, \quad (5)$$

where  $R(j) = \int_{i \in G(j)} r(j) di$  and  $G(j)$  is the set of entrepreneurs using technology  $j$ .

## 2.2. Individuals

Each individual derives utility from consuming the final good and, thus, individuals' maximization problems collapse to an income maximization problem. An individual can either work as an entrepreneur, utilizing her talent and earning some profits, or be employed as a simple worker, earning the equilibrium wage  $w$ .

For a nontrivial number of intermediate goods to be produced in equilibrium, an entrepreneur must earn at least as much as a simple worker. However, to be an entrepreneur, an individual must search and find an appropriate technology. The information friction is such that each individual does not know how well her talent matches existing technologies. This could be either because she does not know her own talent or because she does not know the technological requirements of  $j$ .

*Assumption 1.* The probability that entrepreneur  $i$  finds the nearest technology  $j$  is independent of  $i$ 's distance from technology  $j$ .

This assumption captures the symmetry in individuals' ignorance regarding technological requirements. Individuals are as likely to find the most appropriate technology for their talents whether they are very close to it or far away.<sup>9</sup> This assumption implies that investment in search is equal across individuals.

Individuals search for the most appropriate technology. This search effort determines the probability  $s$  of finding the nearest technology. The search procedure involves a cost  $g(s)$ , where  $g(s)$  is increasing and convex. An individual chooses the probability  $s$  that maximizes her expected income:

$$I = (1 - s)w + sE\{\max[\pi(j, i), w]\} - g(s). \quad (6)$$

The individual incurs a cost  $g(s)$ . With probability  $1 - s$  she does not succeed in finding the location of the nearest technology and earns the equilibrium wage  $w$ . With probability  $s$  she does succeed, and her income then depends on whether she works as an entrepreneur earning  $\pi(j, i)$  or as a simple worker earning  $w$ . Because workers do not know ex ante their mode of employment, they maximize the expected income.

**2.3. Labor Market**

The labor market consists of entrepreneurs and simple workers. Entrepreneurs produce intermediate goods by employing simple workers using technology  $j$ . The continuum of entrepreneurs using technology  $j$  is referred to as sector  $j$ . Let  $J$  denote the equilibrium number of sectors and  $\phi(j, i)$  the density of entrepreneurs of talent  $i$  within sector  $j$ . Each entrepreneur  $i$  producing within sector  $j$  demands  $l(j, i)$  workers. Let  $\{j_1, \dots, j_J\}$  be the set of technologies arising in equilibrium. Aggregate demand for labor is

$$\sum_{j \in \{j_1, \dots, j_J\}} \int_{i \in G(j)} \phi(j, i) l(j, i) \, di,$$

and aggregate supply of labor is

$$1 - \sum_{j \in \{j_1, \dots, j_J\}} \int_{i \in G(j)} \phi(j, i) \, di.$$

**3. EQUILIBRIUM**

An equilibrium is a vector  $\{s, r(j), G(j), P(j), l(j, i), w, J\}$  of search effort, price of technology, set of entrepreneurs utilizing each technology, prices of the intermediate goods, employment of workers by entrepreneurs, wage of workers, and number of technologies, which is a solution to (i) individuals’ maximization of income; (ii) the monopolistic owners of technologies’ profit maximization and (iii) zero profits condition; (iv) the final good maximization problem; (v) the intermediate goods maximization problem; (vi) a threshold condition on individuals’ choice of employment; and (vii) the labor market clearing condition. In the remainder of the section we solve for the equilibrium.

**3.1. Final Good Market**

Let the final good serve as the numeraire. Profit maximization by final good producers leads to the following first-order condition:

$$P(j) = \frac{\partial Y}{\partial x(j)} = \frac{Y}{x(j)}. \tag{7}$$

Substituting equation (7) into (1), we get that the condition  $\int_0^1 \log P(j) dj = 0$  must hold at the optimum. Because of symmetry and the world competition in markets for intermediate goods, all prices must be equal. Hence  $P(j) = P = 1$ .

### 3.2. Intermediate Goods Market

Profit maximization by entrepreneur  $i$ , who produces intermediate good  $j$ , leads to the following demand for labor:

$$l(j, i) = \left(\frac{\alpha A}{w}\right)^{\frac{1}{1-\alpha}} [h_0 - bd(j, i)]. \tag{8}$$

An entrepreneur would like to set up a firm producing intermediate good  $j$  as long as it is profitable. However, the profits of entrepreneur  $i$  decreases with the distance to technology  $j$ . Hence, the threshold condition for the marginal entrepreneur in sector  $j$ , who is indifferent between being an entrepreneur or working as an employee in any firm, is

$$\pi[j \pm \bar{d}(j)] = (1 - \alpha)A[\bar{l}(j)]^\alpha [\bar{h}(j)]^{(1-\alpha)} - r(j) = w, \tag{9}$$

where  $\bar{h}(j)$  is the number of efficiency units of labor that the marginal entrepreneur has and  $\bar{l}(j)$  is the number of workers she employs. Recall from equation (2) that  $\bar{h}(j) = h_0 - b\bar{d}(j)$ , where  $\bar{d}(j)$  is the maximal distance between the requirements of sector  $j$  and the talent of the marginal entrepreneur.

### 3.3. The Monopolistic Market for Technologies

In equilibrium, sectors are symmetric. To see this, note that (4), (8), and the result that the prices of all intermediate goods are equal imply that monopolies face the same demand. Given the symmetry in their cost, they charge the same price,  $r(j) = r$ . Because, in addition, the marginal individual earns the same competitive wage  $w$ , his or her distance from the technology is equal across sectors; that is,  $\bar{d}_j = \bar{d}$ .

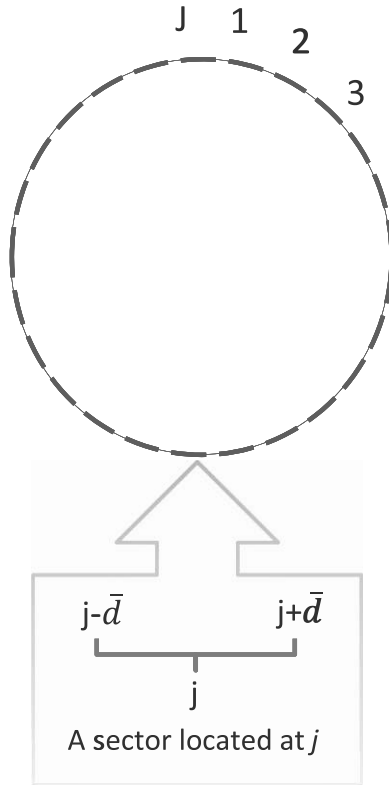
Because entrepreneurs join sector  $j$  from both sides, the size of sector  $j$  is represented by the width of that sector, which is the interval  $[j - \bar{d}, j + \bar{d}]$ . The size of each sector, which is  $2\bar{d}$ , represents the continuum of firms that produces the same intermediate good  $j$  (Figure 1).

An additional intensive margin is given by the density of entrepreneurs of talent  $i$  working in sector  $j$ ,  $\phi(j, i)$ .

LEMMA 1. *At the macro level,  $\phi(j, i) = s_{ij}$ .*

Proof. Follows directly from the law of large numbers. ■

COROLLARY 1. *The density of entrepreneurs of talent  $i$  is independent of their distance to the nearest technology  $j$ ; i.e.,  $\forall i, j$  s.t.  $i \in G(j)$ ,  $\phi(j, i) = s$ .*



**FIGURE 1.** The dashed thick curves describe the continuum of entrepreneurs out of the unit circle and the thin curves describe the simple workers. Each thick curve is a sector that produces a specific intermediate good using the technology located at the center of the curve. The size of each sector, which is described by the length of each thick curve, is  $2\bar{d}$ .

Proof. Follows from Assumption 1 and Lemma 1. ■

The density of entrepreneurs for a given talent is the probability that an entrepreneur matches with the nearest technology. This probability,  $s$ , is the same for all individuals. Hence, (5) becomes

$$\Pi = r \cdot 2 \left( \int_0^{\bar{d}} s \, dt \right) - C. \tag{10}$$

From equation (9) it follows that the price that an owner of a technology charges for selling it to other entrepreneurs,  $r$ , affects entrepreneurs' profits and therefore affects the size of sectors. The first-order condition with respect to the monopolistic price yields

$$\frac{\partial \bar{d}}{\partial r} r + \bar{d} = 0. \tag{11}$$



Substituting equation (8) into (9) and applying the implicit function theorem implies that

$$\frac{\partial \bar{d}}{\partial r} = \frac{-w^{\frac{\alpha}{1-\alpha}}}{\alpha^{\frac{\alpha}{1-\alpha}} (1-\alpha) b A^{\frac{1}{1-\alpha}}}. \tag{12}$$

Substituting equation (12) into equation (11) and isolating  $w$  yields

$$w = \alpha(1-\alpha)^{\frac{1-\alpha}{\alpha}} b^{\frac{1-\alpha}{\alpha}} A^{\frac{1}{\alpha}} \left[ \frac{\bar{d}(j)}{r(j)} \right]^{\frac{1-\alpha}{\alpha}}. \tag{13}$$

Substituting equation (13) and (8) into (9) and isolating  $r$  yields

$$r = \gamma \frac{b\bar{d}}{(h_0 - 2b\bar{d})^\alpha} A, \tag{14}$$

where  $\gamma = \alpha^\alpha (1-\alpha)^{(1-\alpha)}$ . Substituting (14) into (13) yields

$$w = \gamma (h_0 - 2b\bar{d})^{1-\alpha} A. \tag{15}$$

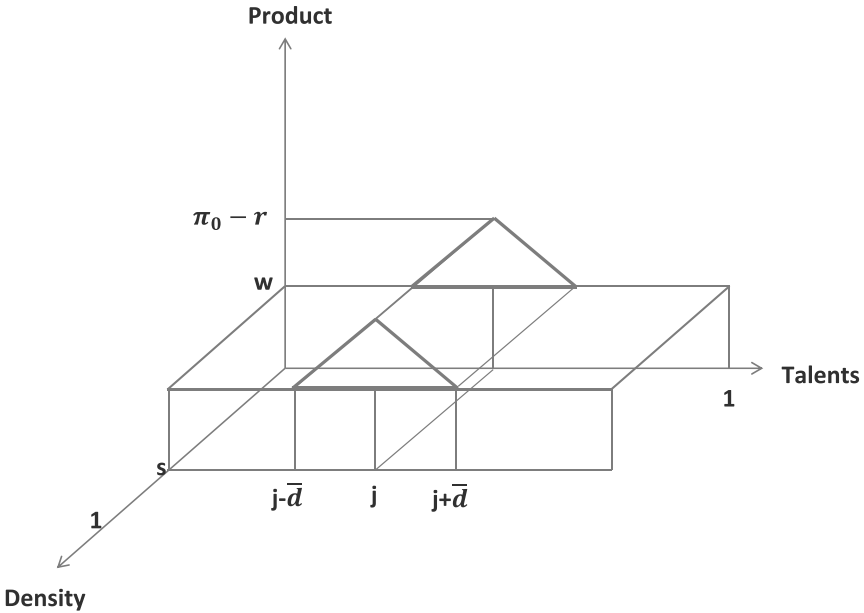
Another potential entrepreneur  $j'$ , located far from  $j$ , finds it profitable to initiate a new sector that produces a different intermediate good.  $j'$  incurs the setup cost  $C$ , and through the previously described market for technologies, sells the blueprint to other entrepreneurs close to her. Ultimately, many sectors are being established, where each sector produces a unique intermediate good by a continuum of firms. The larger the variety of intermediate goods, the smaller the profits for each monopolistic owner. This conclusion is driven by the assumption of substitution of the intermediates in producing the final good. As a result, at equilibrium, the variety of intermediate goods in an economy is determined by applying a zero profit condition on equation (10) and substituting (14), which gives

$$2s\gamma \frac{b(\bar{d})^2}{(h_0 - 2b\bar{d})^\alpha} A = C. \tag{16}$$

Figure 2 draws one descriptive sector located at  $j$  with size  $2\bar{d}$ . The surplus of each entrepreneur within the support  $[j - \bar{d}, j + \bar{d}]$  is measured on the vertical axis. The closer an entrepreneur is to  $j$ , the higher is her surplus. The surplus of an entrepreneur who perfectly matches the job requirement of the sector is  $\pi_0 - r$ , where  $\pi_0 = \gamma \frac{Ah_0}{(h_0 - 2b\bar{d})^\alpha}$ . The diagonal axis measures the density,  $s$ , of entrepreneurs who find the suitable sector.

### 3.4. Labor Market Clearing

Let  $d_i$  be the distance between an entrepreneur and her technology and  $l(d_i)$  her labor demand. Given that  $J$  is the equilibrium number of sectors, labor



**FIGURE 2.** The 3-dimensional figure shows one descriptive sector located at  $j$  with size  $2\bar{d}$ . The surplus of each entrepreneur within the support  $[j - \bar{d}, j + \bar{d}]$  is measured on the vertical axis. The closer the distance of an entrepreneur to  $j$  the higher is her surplus. The surplus of an entrepreneur who perfectly matches the job requirement of the sector is  $\pi_0 - r$ , where  $\pi_0 = \gamma \frac{Ah_0}{(h_0 - 2bd)^\alpha}$ . The diagonal axis measures the density,  $s$ , of entrepreneurs who find the suitable sector. Notice that the surplus is drawn to be linear with distance only for simplicity.

market clearing implies that

$$2J \left[ \int_0^{\bar{d}} sl(t) dt \right] = 1 - 2J \left[ \int_0^{\bar{d}} s dt \right]. \tag{17}$$

The term  $2J \int_0^{\bar{d}} s dt$  represents the measure of entrepreneurs out of a normalized population. Therefore, the left-hand side of (17) represents the demand for labor and the right-hand side of (17) represents the supply for labor.

Substituting equation (15) into (8), and using the result on symmetry, equation (8) can be rewritten as a function of the distance of an entrepreneur from the nearest sector,  $d_i$ :

$$l(d_i) = \frac{\alpha}{1 - \alpha} \frac{h_0 - bd_i}{h_0 - 2b\bar{d}}. \tag{18}$$

**PROPOSITION 1.** *Firm size is positively affected by the match quality.*

**Proof.** Follows from equation (18). ■

Thus, Proposition 1 states that a firm’s size increases with entrepreneurial talent, a result that is consistent with the empirical evidence provided by Brown and Medoff (1989) and Idson and Oi (1999).

Substituting equations (18) into (17) yields

$$J = \frac{1}{s\bar{d} \left( \frac{\alpha}{1-\alpha} \frac{2h_0 - b\bar{d}}{h_0 - 2b\bar{d}} + 2 \right)}. \tag{19}$$

### 3.5. Individuals’ Optimization

As  $2\bar{d}J$  is the measure of entrepreneurs, (6) could be rewritten as

$$I = (1 - s)w + s[2\bar{d}JE(\pi) + (1 - 2\bar{d}J)w] - g(s), \tag{20}$$

where  $E(\pi)$  is the expected profits from being an entrepreneur, which are given by

$$E(\pi) = \int_0^{\bar{d}} \{ (1 - \alpha)A[l(t)]^\alpha [h(t)]^{1-\alpha} - r \} f(t) dt, \tag{21}$$

where  $f(t)$  is the density function of talent with distance  $t$ . Given that  $t$  is uniformly distributed on  $[0, \bar{d}]$ ,  $f(t) = (1/\bar{d})$ .

Substituting equation (18) and (14) into (21) yields

$$E(\pi) = \gamma \frac{h_0 - \frac{3}{2}b\bar{d}}{(h_0 - 2b\bar{d})^\alpha} A. \tag{22}$$

Maximizing equation (20) yields the first-order condition

$$2\bar{d}J[E(\pi) - w] = g'(s). \tag{23}$$

The intuition behind equation (23) is straightforward. The left-hand side of (23) is the gain from a marginal increase in  $s$  and the right-hand side is its cost.

Substituting (16), (15), (19), and (22) into (23) yields

$$\frac{1}{s^2 g'(s)} \frac{C}{2\bar{d} \left( \frac{\alpha}{1-\alpha} \frac{2h_0 - b\bar{d}}{h_0 - 2b\bar{d}} + 2 \right)} = 1. \tag{24}$$

Ultimately, (16) and (24) solve for the equilibrium values of  $s$  and  $\bar{d}$  and (19), in turn, solves for  $J$ .

## 4. TALENT UTILIZATION ACROSS COUNTRIES

This section examines cross-country differences in talent utilization. More specifically, we show how small differences in productivity are amplified through different channels. (i) Higher productivity yields more diversification, reflected by a wider variety of technologies. Such an environment potentially allows better

matching between individuals’ talents and technologies. (ii) Within this better environment, the range of different talents being utilized is wider. That is, more developed economies utilize some talents that are wasted in less developed economies. (iii) This better environment also increases the marginal cost of mismatch, yielding better matches by employing a narrower range of talents in each technology. (iv) Finally, this better environment, under reasonable assumptions, induces individuals to increase their search effort, resulting in a higher intensity of talent utilization, and therefore better match quality.

The two core mechanisms underlying these channels are diversification and search. Although these two mechanisms are interrelated, it will prove useful to isolate the role of diversification by holding search effort constant. Thus, initially, in Sections 4.1–4.3, we analyze an economy in which the density of entrepreneurs,  $s$ , and hence the intensity of talent utilization are constant.

### 4.1. The Quality of Matches

The match quality in the economy could be measured by the inverse of the average distance of entrepreneurs to their nearest technology,  $2/\bar{d}$ .

**PROPOSITION 2.** *With exogenous search, more developed economies are characterized by a higher average match quality. Formally,  $\partial \bar{d} / \partial A < 0$ .*

*Proof.* Follows directly from applying the implicit function theorem on (16), which shows that

$$\frac{\partial \bar{d}}{\partial A} = -\frac{\bar{d}}{2A \left(1 + \frac{\alpha b \bar{d}}{h_0 - 2b\bar{d}}\right)} < 0. \tag{25}$$

■

Hence, in a more developed country, a smaller continuum of talents are employed in each sector.

**COROLLARY 2.** *The monopolistic price for selling technologies increases with development. Formally,  $\partial r / \partial A > 0$ .*

*Proof.* Differentiating (14) with respect to  $A$  and substituting (25) yields

$$\frac{\partial r}{\partial A} = \frac{\gamma b \bar{d}}{(h_0 - 2b\bar{d})^\alpha} \left(1 - \frac{1 + \frac{2\alpha b \bar{d}}{h_0 - 2b\bar{d}}}{2 + \frac{2\alpha b \bar{d}}{h_0 - 2b\bar{d}}}\right) > 0. \tag{26}$$

■

The intuition for the result described in Proposition 2 is as follows. Entrepreneurs in more developed economies are more productive and thus are willing to pay higher prices, but their willingness to pay also declines more steeply with their distance  $d_i$ . The monopoly facing a steeper demand sets a higher price. In addition, as we shall see later, wages in this economy are higher. Thus, the marginal

entrepreneur at distance  $\bar{d}$  faces both higher alternative wages and higher prices, and thus must be more productive, i.e., better matched.

Next we show how development increases both the variety of technologies and the variety of talents utilized.

### 4.2. The Variety of Sectors

PROPOSITION 3. *With exogenous search, higher productivity induces more diversification, that is, a larger number of intermediate goods. Formally,  $\partial J / \partial A > 0$ .*

Proof. Follows directly from (25) and (19). ■

Intuitively, in more developed countries, both entrepreneurs and workers are more productive factors. Because at equilibrium the zero profit condition holds, it requires fewer factors to cover the same fixed costs,  $C$ . As each technology captures a smaller share of the factors of production, more technologies arise at equilibrium.

### 4.3. The Range of Talents

PROPOSITION 4. *With exogenous search, a higher level of development is associated with a larger range of talents utilized. Formally,  $\partial 2J\bar{d} / \partial A > 0$ .*

Proof. Rewriting (19) as

$$2J\bar{d} = \frac{2}{s \left( \frac{\alpha}{1-\alpha} \frac{2h_0 - b\bar{d}}{h_0 - 2b\bar{d}} + 2 \right)},$$

shows that  $2J\bar{d}$  decreases with  $\bar{d}$ . ■

Proposition 4 states that although the size of each sector is smaller in more developed economies, the increase in the variety of sectors dominates. Thus, higher productivity increases the share of entrepreneurs in the population. Because in this model talents play a role only through entrepreneurial activities, it turns out that in more developed countries a greater variety of talents are utilized, albeit the same ante distribution of talents is found in all countries.

Proposition 2 and Proposition 4 together imply that more individuals enjoy higher match quality in more developed countries. Another way to relate these two results is that individuals are more likely to receive returns to their talent, which implies less randomness in income.

### 4.4. Amplification through Search

In this section we would like to learn how individuals' search effort for the appropriate technology varies across economies. As described previously, different

economies foster different environments, which shapes the incentives individuals face when searching for the appropriate technology.

**COROLLARY 3.** *The size of each sector,  $2\bar{d}$ , and the investment in search,  $s$ , are negatively correlated. That is, in an economy where the average match quality is higher, individuals invest more in search for the appropriate technology. Formally,  $\partial s / \partial \bar{d} < 0$ .*

Proof. See the Appendix. ■

To elaborate the results of our model with endogenous search, we assume that the weakly convex cost function takes the general form  $g(s) = s^\delta$ , where  $\delta \in [1, \infty)$ .

**PROPOSITION 5.** *For any weakly convex cost function given in the preceding, a higher level of development is associated with a higher investment in search if  $\alpha(1 + \delta) > 1$ .*

Proof. See the Appendix. ■

Two remarks should be made concerning the condition in Proposition 5. First, this is a sufficient condition for search effort to increase with development for any weakly convex cost function from the form described previously. Second, the left-hand side of this condition increases with  $\delta$ . Thus, the range of  $\alpha$  for which this condition holds increases with the convexity of the cost function. The smallest range of  $\alpha$  for which this condition still holds is when we take  $\delta$  to the corner and assume a linear cost function,  $\delta = 1$ . In this case, the condition becomes  $\alpha > 1/2$ .

Turning to the variety of sectors,  $J$ , equation (19) implies that holding the average mismatch  $\bar{d}/2$  constant, an increase in search decreases  $J$ . However, as  $\bar{d}$  declines, the labor market clearing condition implies that  $J$  increases. To see the overall impact on  $J$ , we substitute the probability and cost functions in equation (24), isolating  $s$  and substituting it in equation (19), which yields

$$J = \frac{\left(\frac{2\delta}{C}\right)^{\frac{1}{1+\delta}}}{\left[\bar{d} \left(\frac{\alpha}{1-\alpha} \frac{2h_0 - b\bar{d}}{h_0 - 2b\bar{d}} + 2\right)\right]^{\frac{\delta}{1+\delta}}}. \tag{27}$$

**PROPOSITION 6.** *With endogenous search, higher productivity induces more diversification, that is, a larger number of intermediate goods. Formally,  $\partial J / \partial A > 0$ .*

Proof. Follows directly from Corollary 3, Proposition 5, and equation (27). ■

Finally, equation (27) and Proposition 6 reveal that, with endogenous search, the measure of entrepreneurs,  $2J\bar{d}s$ , increases with development. Thus, we have shown that for our general cost function with  $\alpha > 0.5$ , a higher level of development is associated with a greater variety of technologies,  $J$ , a stronger search effort,  $s$ , a larger measure of talents being utilized,  $2\bar{d}Js$ , and a higher average match quality in the economy,  $2/\bar{d}$ .

### 4.5. Total Effect of Development

Total output  $Y$  is produced by a continuum of entrepreneurs with the measure  $2\bar{d}$  in each of the  $J$  sectors. Thus,

$$Y = 2J \int_0^{\bar{d}} Al(t)^\alpha h(t)^{1-\alpha} dt. \tag{28}$$

Substituting (18) and (27) into (28) yields

$$Y = \frac{2 \left(\frac{2\delta}{C}\right)^{\frac{1}{1+\delta}} \left(\frac{\alpha}{1-\alpha}\right)^\alpha (h_0 - 2b\bar{d})^{-\alpha}}{\left[\bar{d} \left(\frac{\alpha}{1-\alpha} \frac{2h_0 - b\bar{d}}{h_0 - 2b\bar{d}} + 2\right)\right]^{\frac{\delta}{1+\delta}}} \int_0^{\bar{d}} (h_0 - bt) dt. \tag{29}$$

**PROPOSITION 7.** *Talent utilization amplifies the impact of differences in productivity on output for a wide range of the parameter space. Moreover, this amplification effect increases with the share of efficiency units of labor in the production of intermediate goods,  $1 - \alpha$ , as well as the convexity of the search cost function,  $\delta$ .*

*Proof.* Notice that the amplification effect exists when  $\partial \ln(Y)/\partial \ln(A) > 1$ . Thus, solving for the integral in (29) and applying  $\ln$  on both sides yields

$$\begin{aligned} \ln(Y) &= \ln(A) - \alpha \ln(h_0 - 2b\bar{d}) + \ln(2h_0 - b\bar{d}) \\ &\quad - \frac{\delta}{1 + \delta} \ln\left(\frac{\alpha}{1 - \alpha} \frac{2h_0 - b\bar{d}}{h_0 - 2b\bar{d}} + 2\right) + \ln\left(\frac{\alpha}{1 - \alpha}\right)^\alpha + \ln\left(\frac{2\delta}{C}\right)^{\frac{1}{1+\delta}}. \end{aligned} \tag{30}$$

Differentiating (30) with respect to  $\ln(A)$  gives

$$\frac{\partial \ln(Y)}{\partial \ln(A)} = 1 - \frac{\partial \bar{d}}{\partial \ln(A)} \left( \frac{\delta}{1 + \delta} \frac{\frac{\alpha}{1-\alpha} \frac{3bh_0}{(h_0-2b\bar{d})^2}}{\frac{\alpha}{1-\alpha} \frac{2h_0-b\bar{d}}{h_0-2b\bar{d}} + 2} - \frac{2\alpha b}{h_0 - 2b\bar{d}} + \frac{b}{2h_0 - b\bar{d}} \right).$$

An amplification effect exists if  $\partial \ln(Y)/\partial \ln(A) > 1$ . Because Corollary 3 and Proposition 5 imply that  $\partial \bar{d}/\partial \ln(A) < 0$ , an amplification effect exists if

$$\frac{\delta}{1 + \delta} \frac{\frac{\alpha}{1-\alpha} \frac{3bh_0}{(h_0-2b\bar{d})^2}}{\frac{\alpha}{1-\alpha} \frac{2h_0-b\bar{d}}{h_0-2b\bar{d}} + 2} - \frac{2\alpha b}{h_0 - 2b\bar{d}} + \frac{b}{2h_0 - b\bar{d}} > 0,$$

$\Leftrightarrow$

$$\frac{\delta}{1 + \delta} \frac{3bh_0}{(h_0 - 2b\bar{d}) \left[ (2h_0 - b\bar{d}) + 2 \frac{1-\alpha}{\alpha} (h_0 - 2b\bar{d}) \right]} > \frac{2\alpha b}{h_0 - 2b\bar{d}} - \frac{b}{2h_0 - b\bar{d}},$$

⇔

$$\frac{\delta}{1 + \delta} \frac{3bh_0}{[(2h_0 - b\bar{d}) + 2\frac{1-\alpha}{\alpha}(h_0 - 2b\bar{d})]} > \frac{2\alpha b(2h_0 - b\bar{d}) - b(h_0 - 2b\bar{d})}{2h_0 - b\bar{d}},$$

$$\frac{\delta}{1 + \delta} 3bh_0(2h_0 - b\bar{d}) > 2\alpha b(2h_0 - b\bar{d})^2 + 4(1 - \alpha)b(2h_0 - b\bar{d})(h_0 - 2b\bar{d})$$

$$- \underbrace{b(2h_0 - b\bar{d})(h_0 - 2b\bar{d}) - 2\frac{1-\alpha}{\alpha}b(h_0 - 2b\bar{d})^2}_{M(\bar{d}, \alpha)}. \tag{31}$$

Notice that the right-hand side of (31), denoted by  $M(\bar{d}, \alpha)$ , strictly increases with  $\alpha$ . Thus, taking  $\alpha$  to the corner and substituting  $\alpha = 1$  tightens the condition under which inequality (31) holds. As a result, the right-hand side of (31),  $M(\bar{d}, \alpha)$ , becomes

$$M(\bar{d}, 1) = 3bh_0(2h_0 - b\bar{d}).$$

Finally, as the left-hand side of (31) could then be written as  $\frac{\delta}{1+\delta}M(\bar{d}, 1)$ , the left-hand side of (31) is equal to its right-hand side when  $\delta$  approaches infinity. This implies that, given strong convexity of the cost function, the amplification effect exists for a wide range of  $\alpha$ . Alternatively, as  $\alpha$  declines, the right-hand side of (31) declines and the range of  $\delta$  for which inequality (31) holds increases. Specifically, taking  $\alpha$  to the other corner, that is, for  $\alpha = 0$ , (31) holds for the whole range of  $\delta$ . ■

### 5. CONCLUDING REMARKS

This paper argues that small differences in productivity are amplified by talent utilization. Talent utilization is a result of matching between technologies’ requirements and individuals’ talents. The amplification process works through three different channels: first, the variety of different talents utilized; second, the density of a specific talent utilized; third, the average match quality in the economy.

The model can be used to understand differences in economic structure across countries. Not only does it explain differences in the number of sectors, but also it predicts the size of each sector, the size distribution of firms, and wage inequality. Moreover, the model could be extended to deal with unemployment, exploring different search technologies, an interesting dimension that we leave for future research.

#### NOTES

1. For an updated survey of such development accounting literature, see Caselli (2005).
2. On the one hand, Mankiw et al. (1992), Parente et al. (2000), Seshadri and Manuelli (2005), Weil (2005), and Fuentes and Morales (2011) find that most of the cross-country differences in per capita output are induced by factor accumulation. On the other hand, Chari et al. (1996), Klenow and



Rodriguez-Clare (1997), Prescott (1998), Hall and Jones (1999), Bils and Klenow (2000), Parente and Prescott (2000), Hendricks (2002), and Jeong and Townsend (2007) find that most of the cross-country differences in per capita output are induced by TFP.

3. See Moro (2012) for the effect of biased technical change on TFP.

4. Our research is motivated by evidence on matching at the individual level [Baumgardner (1988a,b); Garicano and Hubbard (2009)].

5. See also Acemoglu (1998).

6. See also Yang and Shi (1992), Duranton and Puga (2004), and Zhou (2004).

7. For a model of endogenous variety see Grossman and Helpman (1991).

8. See also Lentz and Mortensen (2010) for an unemployment search and matching model with product variety.

9. As will be seen later, the average distance is shorter in more developed countries. Thus, relaxing this assumption and allowing higher success probability for shorter distances will strengthen the amplification result.

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### APPENDIX: PROOFS

**Proof of Corollary 3.** First, rewrite equations (16) and (24) as the following nonlinear system:

$$\begin{cases} F(s, \bar{d}, A) = 2s\gamma \frac{b\bar{d}^2}{(h_0 - 2b\bar{d})^\alpha} A - C = 0 \\ G(s, \bar{d}) = \frac{1}{s^2 g'(s)} \frac{C}{2\bar{d} \left( \frac{\alpha}{1-\alpha} \frac{2h_0 - b\bar{d}}{h_0 - 2b\bar{d}} + 2 \right)} - 1 = 0. \end{cases} \tag{32}$$

The derivatives of  $s$  and  $\bar{d}$  with respect to  $A$  are calculated as

$$\frac{\partial s}{\partial A} = - \frac{\det \begin{pmatrix} 2s\gamma \frac{b\bar{d}^2}{(h_0 - 2b\bar{d})^\alpha} A & 2s\gamma \frac{2b\bar{d} + \frac{2\alpha b^2 \bar{d}^2}{(h_0 - 2b\bar{d})}}{(h_0 - 2b\bar{d})^\alpha} A \\ 0 & \frac{1}{s^2 g'} D(\bar{d}) \end{pmatrix}}{\det \begin{pmatrix} 2\gamma \frac{b\bar{d}^2}{(h_0 - 2b\bar{d})^\alpha} A & 2s\gamma \frac{2b\bar{d} + \frac{2\alpha b^2 \bar{d}^2}{(h_0 - 2b\bar{d})}}{(h_0 - 2b\bar{d})^\alpha} A \\ Q(s) \frac{C}{2\bar{d} \left( \frac{\alpha}{1-\alpha} \frac{2h_0 - b\bar{d}}{h_0 - 2b\bar{d}} + 2 \right)} & \frac{1}{q^2 g'} D(\bar{d}) \end{pmatrix}} \tag{33}$$

and

$$\frac{\partial \bar{d}}{\partial A} = - \frac{\det \begin{pmatrix} 2\gamma \frac{b\bar{d}^2}{(h_0 - 2b\bar{d})^\alpha} A & 2s\gamma \frac{b\bar{d}^2}{(h_0 - 2b\bar{d})^\alpha} A \\ Q(s) \frac{C}{2\bar{d} \left( \frac{\alpha}{1-\alpha} \frac{2h_0 - b\bar{d}}{h_0 - 2b\bar{d}} + 2 \right)} & 0 \end{pmatrix}}{\det \begin{pmatrix} 2\gamma \frac{b\bar{d}^2}{(h_0 - 2b\bar{d})^\alpha} A & 2s\gamma \frac{2b\bar{d} + \frac{2\alpha b^2 \bar{d}^2}{(h_0 - 2b\bar{d})}}{(h_0 - 2b\bar{d})^\alpha} A \\ Q(s) \frac{C}{2\bar{d} \left( \frac{\alpha}{1-\alpha} \frac{2h_0 - b\bar{d}}{h_0 - 2b\bar{d}} + 2 \right)} & \frac{1}{s^2 g'} D(\bar{d}) \end{pmatrix}}, \tag{34}$$

where

$$Q(s) = \frac{-2s g' - s^2 g''}{s^4 g'^2}$$

and

$$D(\bar{d}) = - \frac{\left( \frac{\alpha}{1-\alpha} \frac{2h_0 - b\bar{d}}{h_0 - 2b\bar{d}} + 2 \right) + \left( \frac{\alpha}{1-\alpha} \frac{3b\bar{d}h_0}{(h_0 - 2b\bar{d})^2} \right)}{2\bar{d}^2 \left( \frac{\alpha}{1-\alpha} \frac{2h_0 - b\bar{d}}{h_0 - 2b\bar{d}} + 2 \right)^2} C.$$

Notice that  $D(\bar{d}) < 0$  always holds and that  $Q(s) < 0$  holds for any weakly convex cost function  $g(s)$ . Thus, the sign of the numerator of equation (33) is negative, whereas the sign of the numerator of equation (34) is positive, which yields the corollary. ■

**Proof of Proposition 5.** Isolating  $s$  from equation (16), substituting it along with  $g(s) = s^\delta$  into equation (24), and rearranging yields

$$\delta \left(\frac{C}{2}\right)^\delta \left(\frac{1}{\gamma b A}\right)^{\delta+1} = \frac{\bar{d}^{1+2\delta}}{\underbrace{\frac{\alpha}{1-\alpha} [(2h_0 - b\bar{d}) + 2\frac{1-\alpha}{\alpha} (h_0 - 2b\bar{d})] (h_0 - 2b\bar{d})^{\alpha(1+\delta)-1}}_{H(\bar{d})}} .$$

Whereas the left-hand side decreases with  $A$ , the right-hand side increases with  $\bar{d}$  if  $\alpha(1 + \delta) > 1$ . Thus, this condition is a sufficient assumption to get  $\partial\bar{d}/\partial A < 0$  and, thus,  $\partial s/\partial A > 0$ . ■