# A TORSION-INDUCED COUPLING BETWEEN ELECTRIC AND MAGNETIC FIELDS IN THE SOLAR CONVECTION ZONE

METOD SANIGA

*Astronomical Institute, Slovak Academy of Sciences, Tatranská Lomnica, Czechoslovakia*

**Abstract.** A striking formal analogy between sunspots and the type II superconductor magnetic vortex has been pointed out by Saniga (1990). On the basis of this similarity, it was shown that, by postulating the existence of a complex-valued scalar field on the Sun (a "Higgs-like field"), many features of sunspots can be reproduced. Here this formal analogy is exploited further to show that the *Abelian-Higgs sunspot* acquires a non-zero electric charge in the background of a curved space-time endowed with a constant, completely antisymmetric torsion tensor. The space-time torsion is also shown to relate the spot's electric and magnetic fileds.

**Key words:** Sunspot – Differential geometry – Quantum field theory

In two recent papers (Saniga and Klačka, 1992a, b; Saniga, 1992c) an *Abelian Higgs model* of a sunspot in the background of a curved space-time has been investigated. It has been shown, in particular, that the amplitude of the *Higgs-like field* might induce a non-metricity of the Sun's space-time manifold in the regions occupied by sunspots; one can arrive at this conclusion by requiring that the postulated Abelian Higgs field be globally conformally symmetric (Saniga, 1992c).

In the above mentioned papers it has been assumed that the space-time is endowed with a symmetric affine connection. In the present contribution I shall go further and investigate what happens if the latter assumption is relaxed; i.e, if a space-time with a non-zero torsion is considered. In order to keep things simple, I shall assume that the manifold is metrically flat, the only source of the curvature being then its torsion.

Let us start from an Abelian Higgs configuration of fields in a flat Minkowski space-time, which is represented by the action $S^{\odot} = \int \mathcal{L}_f^{\odot} d^4x$, with the Lagrangian density $\mathcal{L}_f^{\odot}$ of the form (Saniga, 1990, 1992a,b) [1]

$$\mathcal{L}_f^{\odot} = -\frac{1}{4} F_{\rho\tau}F^{\rho\tau} + \frac{1}{2}\left(\frac{\partial\Phi}{\partial x^{\iota}} + igA_{\iota}\Phi\right) \times \left(\frac{\partial\Phi^*}{\partial x^{\rho}} - igA_{\rho}\Phi^*\right)\eta^{\iota\rho} - \frac{\lambda}{4}\left(\Phi\Phi^* - \frac{m^2}{\lambda}\right)^2 . \tag{1}$$

Assume a metrically flat space-time endowed with a completely antisymmetric torsion tensor $Q^{\sigma}_{.\mu\nu}$,

$$Q^{\sigma}_{.\mu\nu} = \frac{1}{4}\eta^{\sigma\kappa}\varepsilon_{\kappa\mu\nu\rho}\breve{Q}^{\rho} = \frac{1}{4}\Theta\eta^{\sigma\kappa}\varepsilon_{\kappa\mu\nu\rho}q^{\rho}, \tag{2}$$

where $\varepsilon_{\kappa\mu\nu\rho}$ is a unit, completely antisymmetric tensor of the fourth-rank ($\varepsilon_{1234} = +1$), $\Theta$ is a constant, and $q^{\rho}$ is a constant fourth-vector. Provided that $\Theta$ is a small quantity, using the principle of minimal coupling of physical fields to the curvature of space-time (the so called "strong principle of equivalence"; e.g., Misner *et al.*,

[1] Explanation of the symbols and the notation used throughout this paper can be found in Saniga, 1992a, and Saniga and Klačka, 1992a.



*F. Krause et al. (eds.), The Cosmic Dynamo,* 269–270.

1973, or Weinberg, 1972), the above Lagrangian density can be easily generalized to the form

$$\mathcal{L}_c^{\odot} = \mathcal{L}_f^{\odot} - \frac{\Theta}{4}\varepsilon^{\kappa\mu\nu} A_\kappa F_{\mu\nu}, \tag{3}$$

where $\varepsilon^{\kappa\mu\nu} \equiv \varepsilon^{\kappa\mu\nu\rho} q_\rho$, and $\varepsilon^{\kappa\mu\nu\rho} \equiv \eta^{\kappa\alpha}\eta^{\mu\beta}\eta^{\nu\gamma}\eta^{\rho\delta}\varepsilon_{\alpha\beta\gamma\delta}$.

The Lagrangian density (3), which contains a Chern-Simons-like term (see, e.g., Jackiw *et al.*, 1990), is very interesting from the physical point of view, for it implies a non-zero electric charge for the "generalized Abelian Higgs sunspot". In order to see that, let us use the Maxwell-like set of equations resulting from Eq. (3),

$$\frac{\partial F^{\rho\kappa}}{\partial x^\kappa} = +i\frac{\mathrm{g}}{2}\left(\Phi\frac{\partial \Phi^*}{\partial x^\kappa} - \Phi^*\frac{\partial \Phi}{\partial x^\kappa}\right)\eta^{\rho\kappa} + \mathrm{g}^2 A^\rho \Phi\Phi^* - \frac{1}{2}\Theta\varepsilon^{\rho\mu\nu}F_{\mu\nu} =$$

$$\equiv j^\rho - \frac{1}{2}\Theta\varepsilon^{\rho\mu\nu}F_{\mu\nu}, \tag{4}$$

where $j^\rho$ represents the electric current density of the "ordinary Abelian Higgs spot". It can be readily seen that $j^\rho$ is a conserved quantity ($\partial j^\rho/\partial x^\rho = 0$) since $2^{-1}\Theta\varepsilon^{\rho\mu\nu}\partial F_{\mu\nu}/\partial x^\rho \equiv 0$. Restricting ourselves to the case of a cylindrically symmetric spot, and taking the unit vector $q^\rho$ to be space-like and pointing along the spot's symmetry axis (the $z$-axis), then the time component of Eqs. (4) reads

$$\frac{\partial F^{0i}}{\partial x^i} = j^0 - \frac{1}{2}\Theta\varepsilon^{0ij}F_{ij} = j^0 - \Theta F_{xy}. \tag{5}$$

Now, the electric field strength $F^{0i}$ of a sunspot is, like the magnetic field strength $F_{ij}$, of finite range (Saniga and Klačka, 1992c), and so the left-hand side of the Eq. (5) yields zero upon a space integration of the latter. Thus, one finally obtains $Q^{\mathrm{el}} = \Theta\Psi$, where $Q^{\mathrm{el}} \equiv \int j^0 dxdy$ is the total electric charge (per unit length) of the spot, and $\Psi \equiv \int F_{xy}dxdy$ is its total magnetic flux. Hence, it follows that a non-zero electric charge appears in the "generalized Abelian Higgs spot model" due to a non-zero torsion of the Sun's space-time. Furthermore, the result $Q^{\mathrm{el}} = \Theta\Psi$ tells us that the torsion relates the spot's electric and magnetic fields. This turns out to be a possible connection between the electromagnetic properties of the sunspot and the geometry of underlying space-time manifold.

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