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THE ZERO LOWER BOUND AND CRUDE OIL AND FINANCIAL MARKETS SPILLOVERS

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We investigate mean and volatility spillovers between the crude oil market and the debt, stock, and foreign exchange markets. In doing so, we estimate a four-variable VARMA–GARCH model with a BEKK representation and also examine the possible effects of monetary policy at the zero lower bound by including a dummy variable in both the conditional mean and variance equations. We find that the crude oil market and the financial markets are tightly interconnected and that monetary policy at the zero lower bound has strengthened their linkages.

Keywords: Mean and Volatility Spillovers, Structural Breaks, VARMA–GARCH BEKK Model

1. INTRODUCTION

In this paper, we take a financial market perspective and examine the relationship between the price of oil and three financial variables—the interest rate, stock price, and exchange rate—building on a growing literature that analyzes the link between the oil market and markets for financial assets. We are motivated by the fact that over the past decade there has been a closer link between oil prices and asset prices, raising the question of whether oil has itself become a financial asset, with its price reacting to and influencing other assets in financial markets.

The link between oil prices and stock prices has already been analyzed in the literature—see, for example, Jones and Kaul (1996), Park and Ratti (2008), and Sadorsky (1999, 2001, 2012). More recently, Kilian and Park (2009) estimate the global crude oil market model of Kilian (2009), augmented with a stock market variable, and report that the response of stock prices to oil price shocks depends on the nature of the oil price shocks. They treat the price of crude oil as endogenous and model changes in the real price of crude oil as arising from three different sources: shocks to the global supply of crude oil, shocks to the global demand for all industrial commodities (including crude oil) that are driven by the global business cycle, and oil-market specific demand shocks (also referred to as

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precautionary demand shocks). They show that demand and supply shocks driving the global crude oil market jointly account for 22% of the long-run variation in U.S. real stock returns.

There is also a growing literature analyzing the relationship between oil prices and exchange rates. See, for example, Amano and van Norden (1998), Sadorsky (2000), Chen and Chen (2007), and Chen et al. (2010), with the latter showing that the floating exchange rates of small commodity exporters, including Australia, Canada, New Zealand, South Africa, and Chile, with respect to the U.S. dollar in some cases have surprisingly robust power in forecasing the global prices of their commodity exports. In contrast, Alquist et al. (2013), find that the trade-weighted nominal U.S. exchange rate lacks predictive power with respect to the price of oil.

Finally, there is a large literature that investigates whether the economic effects of oil price changes also depend on how monetary policy responds. See, for example, Herrera and Pesavento (2009) and Kilian and Lewis (2011). In this regard, in the past, when oil prices rose prior to recessions so did interest rates, and as has been argued by Bernanke et al. (1997) it was the increase in the interest rate that led to the downturn. However, this view has been challenged by Hamilton and Herrera (2004), who argue that contractionary monetary policy plays only a secondary role in generating the contractions in real output and that it is the increase in the oil price that directly leads to contractions. Moreover, recent work by Kilian and Vigfusson (2011) shows that the original estimates presented in Bernanke et al. (1997) in support of a feedback from monetary policy are inconsistent and were constructed in a way that exaggerates the effects of oil price shocks. Moreover, the link to interest rates need not rely only on monetary policy, as Barsky and Kilian (2002) argue that interest rates affect storage behavior in oil markets, and Bodenstein et al. (2012), in the context of a global dynamic stochastic general equilibrium model with endogenous oil markets, show that U.S. monetary policy responses depend on the source of the observed oil price fluctuations.

In this paper, we use a multivariate volatility model to investigate mean and volatility spillovers between the crude oil market and the three financial markets—the debt, stock, and foreign exchange markets. Multivariate volatility models, first proposed by Bollerslev et al. (1988), are becoming standard in economics and finance, because they allow for rich dynamics in the variance-covariance structure of time series, making it possible to model spillovers in both the values and the conditional variances of the series under study. They can be used to investigate a large number of issues in economics and finance. For example, as Bauwens et al. (2006, p. 79) put it, "is the volatility of a market leading the volatility of other markets? Is the volatility of an asset transmitted to another asset directly (through its conditional variance) or indirectly (through its conditional covariances)? Does a shock on a market increase the volatility on another market, and by how much? Is the impact the same for negative and positive shocks of the same amplitude?"

Our methodological approach represents an original contributions to the literature; to the best of our knowledge, no study has considered the modeling of the oil price, interest rate, stock price, and exchange rate in a systems context, with a focus on both first (mean) and second (volatility) moment linkages. We also contribute to the literature by examining the effects of unconventional monetary policy in the aftermath of the global financial crisis. In this regard, in the aftermath of the global financial crisis, conventional monetary policy has been ineffective, because the policy rate has reached the zero lower bound and cannot be driven below zero. The Federal Reserve and many other central banks around the world have resorted to unconventional monetary policy (quantitative easing, long-term bond purchases, and managing expectations) in order to lower long-term interest rates and stimulate their economies. We examine the possible effects of monetary policy at the zero lower bound by including a dummy variable in both the conditional mean and variance equations of our four-variable VARMA–GARCH model with a BEKK representation to allow for possible parameter shifts and capture the effects of monetary policy when the policy rate hits the zero lower bound.

The paper is organized as follows. Section 2 discusses the data and investigates their time series properties. Sections 3 and 4 describe the empirical method and present the results. The final section concludes the paper.

2. DATA AND BASIC FACTS

We use monthly data for the United States over the period from January 1979 to February 2015. For the oil price series (o_t), we use the composite refiners' acquisition cost of crude oil (RAC), as compiled by the U.S. Department of Energy. This price index is a weighted average of domestic and imported crude oil costs, including transportation and other fees paid by refiners, and therefore measures the price of crude oil as an input to production. For the interest rate series (i_t), we use the 3-month Treasury bill rate, obtained from the Federal Reserve Economic Database (FRED) maintained by the Federal Reserve Bank of St. Louis. The S&P 500 index (s_t) is obtained from Yahoo Finance and the series for the nominal effective exchange rate (e_t) from the IMF International Financial Statistics.

Table 1 presents summary statistics for the log levels of the series ($\ln o_t$, $\ln i_t$, $\ln s_t$, and $\ln e_t$) as well as for the first differences of the logs ($\Delta \ln o_t$, $\Delta \ln i_t$, $\Delta \ln s_t$, and $\Delta \ln e_t$). In general, the p-values for skewness and kurtosis point to significant deviations from symmetry and normality with both the logged series and the first logged differenced series. In fact, the Jarque and Bera (1980) test statistic, distributed as a $\chi^2(2)$ under the null hypothesis of normality, rejects the null hypothesis.

One interesting feature of the data is the contemporaneous correlation between the different series. These correlations are reported in Table 2 for the log levels (in panel A) and the first differences of the log levels (in panel B). To determine whether these correlations are statistically significant, we follow Pindyck and Rotemberg (1990) and perform a likelihood ratio test of the hypothesis that the correlation matrix is equal to the identity matrix. The test statistic is

$$-2\ln\left(|R|^{N/2}\right),$$

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TABLE 1. Summary statistics

Series	Mean	Variance	*p*-values Skewness	Kurtosis	Normality
		A. Log levels			
Oil price	3.418	0.429	0.000	0.001	0.000
T-bill rate	0.826	3.164	0.000	0.000	0.000
S&P 500 index	6.330	0.802	0.000	0.000	0.000
Exchange rate	4.427	0.127	0.000	0.069	0.000
		B. First differences of log levels			
Oil price	0.003	0.005	0.000	0.000	0.000
T-bill rate	−0.014	0.046	0.000	0.000	0.000
S&P 500 index	0.007	0.002	0.000	0.000	0.000
Exchange rate	0.003	0.0002	0.132	0.003	0.004

Note: Sample period, monthly observations, 1979:1–2015:2.

TABLE 2. Contemporaneous correlations

	A. Log levels				B. First differences of log levels			
	Oil price	T-bill rate	S&P 500	Exchange rate	Oil price	T-bill rate	S&P 500	Exchange rate
Oil price	1	0.292	0.985	0.982	1	0.165	−0.008	−0.213
T-bill rate	0.292	1	0.337	0.388	0.165	1	0.039	0.035
S&P 500	0.985	0.337	1	0.997	−0.008	0.039	1	−0.126
Exchange rate	0.982	0.388	0.997	1	−0.213	0.035	−0.126	1
	$\chi^2(6) \to +\infty$				$\chi^2(6) = 43.009$			

Note: Sample period, monthly data, 1979:1–2015:2.

where $|R|$ is the determinant of the correlation matrix and N is the number of observations. This test statistic is distributed as χ^2 with $0.5q(q-1)$ degrees of freedom, where q is the number of series. The test statistic is very large with a p-value of 0.000 for the logged values and 43.009 with a p-value of 0.000 for the first differences of the logs. Clearly, the hypothesis that these data series are uncorrelated is rejected. The correlation patterns documented in Table 2 manifest in the graphical representation of the logged levels of the series in Figure 1; the shaded area represents the period over which the zero lower bound constraint on the policy rate is binding.

We also conduct a battery of unit root and stationary tests in panel A of Table 3 in the natural log of each of the series. In particular, we use the Augmented Dickey–Fuller (ADF) test [see Dickey and Fuller (1981)] and the Dickey–Fuller GLS test [see Elliot et al., (1996)], assuming both a constant and trend, to determine

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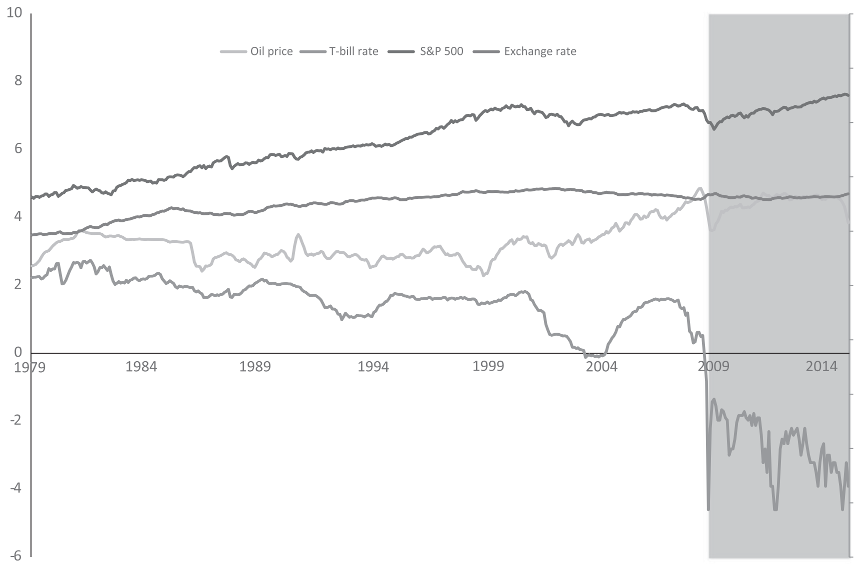

FIGURE 1. Logs of oil price, T-bill rate, S&P 500 index, and exchange rate.

TABLE 3. Unit root and stationary tests

Series	Test: ADF	DF-GLS	KPSS	Decision
	A. Logged levels			
Oil price	−2.472	−2.238	1.685	$I(1)$
T-bill rate	−1.533	−1.479	1.183	$I(1)$
S&P 500 index	−1.698	−1.482	1.551	$I(1)$
Exchange rate	−1.932	−0.510	1.897	$I(1)$
	B. Logged first differences			
Oil price	−11.718	−9.277	0.065	$I(0)$
T-bill rate	−17.046	−9.509	0.036	$I(0)$
S&P 500 index	−19.673	−5.552	0.058	$I(0)$
Exchange rate	−14.052	−7.963	0.087	$I(0)$

Notes: Sample period, monthly observations, 1979:1–2015:2. The 1% and 5% critical values are −4.023 and −3.441 for the ADF test, −3.529 and −2.988 for the DF-GLS test, and 0.216 and 0.146 for the KPSS test, respectively.

whether the series have a unit root. The optimal lag length is taken to be the order selected by the Bayesian information criterion (BIC), after we assume a maximum lag length of 4 for each series. Moreover, given that unit root tests have low power against trend stationary alternatives, we also use the KPSS test [see Kwiatkowski

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et al. (1992)] to test the null hypothesis of stationarity around a trend. As shown in panel A of Table 3, the null hypothesis of a unit root is rejected at conventional significance levels by both the ADF and DF-GLS test statistics. Moreover, the null hypothesis of trend stationarity can be rejected at conventional significance levels by the $\hat{\eta}_\tau$ KPSS test. We thus conclude that each of the series is nonstationary, or integrated of order one, $I(1)$. In panel B of Table 3, we repeat the unit root and stationarity tests using the first differences of the logarithms of the series. The null hypotheses of the ADF and DF-GLS tests are in general rejected and the null hypothesis of the KPSS test cannot be rejected, suggesting that the first differences of the logarithms of the series are stationary, or integrated of order zero, $I(0)$.

3. THE ECONOMETRIC MODEL

Normally, the presence of unit roots would suggest logarithmic first differences as the correct data representation in our model. However, we find evidence of cointegration among the four series, using Johansen's (1988) maximum likelihood method. Vector error correction (VEC) models are often used with cointegrated series, because they allow for an explicit analysis of cointegrating relations. However, a vector autoregression (VAR) in levels is sufficient if the cointegrating relations are not the focus of study, as in our case. In fact, VAR and VEC are equivalent, as demonstrated by Lütkepohl (2004).

One issue in applied VAR analysis is that finite-lag order VAR models are usually fitted to data generated by VAR processes of possibly infinite-lag order. As noted by Inoue and Kilian (2002, p. 322), "the existence of finite-lag order VAR models is highly implausible in practice and often inconsistent with the assumptions of the macroeconomic model underlying the empirical analysis" — see also Braun and Mittnik (1993) and Lütkepohl (2005). However, infinite-lag order VAR models often can be represented by an invertible finite-order vector autoregressive moving-average (VARMA) structure, although such a structure could be very difficult to estimate using numerical methods.

In this paper, we use a VARMA(1,1) formulation in log-levels (multiplied by 100) for the mean equation, thus capturing features of the data generating process in a more parsimonious way, without adding a large number of parameters (or lagged variables). We also use the Baba, Engle, Kraft, and Kroner (BEKK) specification for the variance equation. In particular, our empirical model is a VARMA(1,1)–BEKK(1,1,1) model with break in coefficients since December 2008 when the zero lower is reached:

$$\boldsymbol{z}_t = \Phi + (\Gamma + \tilde{\Gamma} \times D)\boldsymbol{z}_{t-1} + (\Psi + \tilde{\Psi} \times D)\boldsymbol{\epsilon}_{t-1} + \boldsymbol{\epsilon}_t, \qquad (1)$$

where

$$\boldsymbol{\epsilon}_t | \Omega_{t-1} \sim t_v(\mathbf{0}, \boldsymbol{H}_t); \quad \boldsymbol{H}_t = \begin{bmatrix} h_{oo,t} & h_{oi,t} & h_{os,t} & h_{oe,t} \\ h_{io,t} & h_{ii,t} & h_{is,t} & h_{ie,t} \\ h_{so,t} & h_{si,t} & h_{ss,t} & h_{se,t} \\ h_{eo,t} & h_{ei,t} & h_{es,t} & h_{ee,t} \end{bmatrix},$$

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and

$$z_t = \begin{bmatrix} \ln o_t \\ \ln i_t \\ \ln s_t \\ \ln e_t \end{bmatrix}; \quad \epsilon_t = \begin{bmatrix} \epsilon_{o,t} \\ \epsilon_{i,t} \\ \epsilon_{s,t} \\ \epsilon_{e,t} \end{bmatrix}; \quad \Gamma = \begin{bmatrix} \gamma_{11} & \gamma_{12} & \gamma_{13} & \gamma_{14} \\ \gamma_{21} & \gamma_{22} & \gamma_{23} & \gamma_{24} \\ \gamma_{31} & \gamma_{32} & \gamma_{33} & \gamma_{34} \\ \gamma_{41} & \gamma_{42} & \gamma_{43} & \gamma_{44} \end{bmatrix};$$

$$\Psi = \begin{bmatrix} \psi_{11} & \psi_{12} & \psi_{13} & \psi_{14} \\ \psi_{21} & \psi_{22} & \psi_{23} & \psi_{24} \\ \psi_{31} & \psi_{32} & \psi_{33} & \psi_{34} \\ \psi_{41} & \psi_{42} & \psi_{43} & \psi_{44} \end{bmatrix};$$

$$\tilde{\Gamma} = \begin{bmatrix} \tilde{\gamma}_{11} & \tilde{\gamma}_{12} & \tilde{\gamma}_{13} & \tilde{\gamma}_{14} \\ \tilde{\gamma}_{21} & \tilde{\gamma}_{22} & \tilde{\gamma}_{23} & \tilde{\gamma}_{24} \\ \tilde{\gamma}_{31} & \tilde{\gamma}_{32} & \tilde{\gamma}_{33} & \tilde{\gamma}_{34} \\ \tilde{\gamma}_{41} & \tilde{\gamma}_{42} & \tilde{\gamma}_{43} & \tilde{\gamma}_{44} \end{bmatrix}; \quad \tilde{\Psi} = \begin{bmatrix} \tilde{\psi}_{11} & \tilde{\psi}_{12} & \tilde{\psi}_{13} & \tilde{\psi}_{14} \\ \tilde{\psi}_{21} & \tilde{\psi}_{22} & \tilde{\psi}_{23} & \tilde{\psi}_{24} \\ \tilde{\psi}_{31} & \tilde{\psi}_{32} & \tilde{\psi}_{33} & \tilde{\psi}_{34} \\ \tilde{\psi}_{41} & \tilde{\psi}_{42} & \tilde{\psi}_{43} & \tilde{\psi}_{44} \end{bmatrix},$$

with Ω_{t-1} being the information set available in period $t-1$, υ a parameter which characterizes the shape of the Student's t distribution, and D a dummy variable taking the value of zero before December 2008 and the value of 1 since December 2008 when the zero lower bound constraint on the policy rate is binding. That is,

$$D = \begin{cases} 0 & \text{before December 2008,} \\ 1 & \text{since December 2008.} \end{cases}$$

The BEKK(1,1,1) specification for the variance equation is a multivariate extension of the GARCH(1,1) process, and is as follows:

$$\begin{aligned} \boldsymbol{H}_t &= \boldsymbol{C}'\boldsymbol{C} + (\boldsymbol{B} + \widetilde{\boldsymbol{B}} \times D)' \boldsymbol{H}_{t-1} (\boldsymbol{B} + \widetilde{\boldsymbol{B}} \times D) \\ &\quad + \left(\boldsymbol{A} + \widetilde{\boldsymbol{A}} \times D\right)' \epsilon_{t-1}\epsilon'_{t-1} \left(\boldsymbol{A} + \widetilde{\boldsymbol{A}} \times D\right), \end{aligned} \quad (2)$$

$$\boldsymbol{A} = \begin{bmatrix} \alpha_{11} & \alpha_{12} & \alpha_{13} & \alpha_{14} \\ \alpha_{21} & \alpha_{22} & \alpha_{23} & \alpha_{24} \\ \alpha_{31} & \alpha_{32} & \alpha_{33} & \alpha_{34} \\ \alpha_{41} & \alpha_{42} & \alpha_{43} & \alpha_{44} \end{bmatrix}; \quad \widetilde{\boldsymbol{A}} = \begin{bmatrix} \tilde{\alpha}_{11} & \tilde{\alpha}_{12} & \tilde{\alpha}_{13} & \tilde{\alpha}_{14} \\ \tilde{\alpha}_{21} & \tilde{\alpha}_{22} & \tilde{\alpha}_{23} & \tilde{\alpha}_{24} \\ \tilde{\alpha}_{31} & \tilde{\alpha}_{32} & \tilde{\alpha}_{33} & \tilde{\alpha}_{34} \\ \tilde{\alpha}_{41} & \tilde{\alpha}_{42} & \tilde{\alpha}_{43} & \tilde{\alpha}_{44} \end{bmatrix};$$

$$\boldsymbol{B} = \begin{bmatrix} \beta_{11} & \beta_{12} & \beta_{13} & \beta_{14} \\ \beta_{21} & \beta_{22} & \beta_{23} & \beta_{24} \\ \beta_{31} & \beta_{32} & \beta_{33} & \beta_{34} \\ \beta_{41} & \beta_{42} & \beta_{43} & \beta_{44} \end{bmatrix}; \quad \widetilde{\boldsymbol{B}} = \begin{bmatrix} \tilde{\beta}_{11} & \tilde{\beta}_{12} & \tilde{\beta}_{13} & \tilde{\beta}_{14} \\ \tilde{\beta}_{21} & \tilde{\beta}_{22} & \tilde{\beta}_{23} & \tilde{\beta}_{24} \\ \tilde{\beta}_{31} & \tilde{\beta}_{32} & \tilde{\beta}_{33} & \tilde{\beta}_{34} \\ \tilde{\beta}_{41} & \tilde{\beta}_{42} & \tilde{\beta}_{43} & \tilde{\beta}_{44} \end{bmatrix},$$

where $\boldsymbol{C}$, $\boldsymbol{B}$, $\widetilde{\boldsymbol{B}}$, $\boldsymbol{A}$, and $\widetilde{\boldsymbol{A}}$ are 4×4 matrices with $\boldsymbol{C}$ being a triangular matrix to ensure positive definiteness of $\boldsymbol{H}$. This specification allows past volatilities, $\boldsymbol{H}_{t-1}$, as well as lagged values of $\epsilon_{t-1}\epsilon'_{t-1}$ to show up in estimating the current volatility of the T-bill rate, the stock price, the oil price and the exchange rate. Since the $\boldsymbol{H}$ matrix is symmetric, equation (2) produces 10 unique equations modeling the

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dynamic variances of the oil price, the T-bill rate, the stock price, and the exchange rate, as well as the covariances between them.

We refrain from adding additional explanatory variables, since our model already contains 68 mean equation parameters, 74 variance equation parameters, and the distribution shape parameter, υ, for a total of 143 free parameters. It is to be noted that we do not impose any restriction on υ, but we estimate it together with the other parameters, thus capturing the fat tails in the distribution of the errors (as $\upsilon \to \infty$, the distribution becomes the normal distribution and the fat tails disappear). However, we impose the restriction $\tilde{\gamma}_{11} = \tilde{\psi}_{11} = \tilde{\alpha}_{11} = \tilde{\beta}_{11} = 0$, thus not allowing the crude oil data generating process to be affected by the zero lower bound constraint.

To address our research question, we capture the dynamics of the system by the Γ, Ψ, $\boldsymbol{A}$, and $\boldsymbol{B}$ coefficient matrices before the zero lower bound was reached, and by $\Gamma + \tilde{\Gamma}$, $\Psi + \tilde{\Psi}$, $\boldsymbol{A} + \widetilde{\boldsymbol{A}}$, and $\boldsymbol{B} + \widetilde{\boldsymbol{B}}$ during the zero lower bound period.

4. EMPIRICAL EVIDENCE

The four-variable VARMA(1,1)–BEKK(1,1,1) specification with a structural break in December 2008, consisting of equations (1) and (2), is estimated in Estima RATS 9.0 using the Maximum Likelihood method. We use the BFGS (Broyden, Fletcher, Goldfarb, and Shanno) estimation algorithm, which is recommended for GARCH models, combined with the derivative-free Simplex preestimation method. Table 4 reports the coefficients estimates obtained (with p-values in parentheses), the estimate of the Student's t distribution shape parameter, υ, and key diagnostics for the standardized residuals

$$z_{jt} = \frac{\epsilon_{jt}}{\sqrt{h_{jt}}},$$

for $j = \ln o, \ln i, \ln s$ and $\ln e$.

We discuss the estimation results of the mean equation first and only focus on those coefficients which are statistically significant at the 95% level. It is to be noted that our discussion is only in terms of predictability, and not about causal relationships, since our model is nonstructural. Moreover, we are not addressing the identification of shocks, given the scope of our research questions. The autoregressive coefficients in the Γ matrix are all significant and close to one along the main diagonal, suggesting that for each of the four markets, today's performance is a useful predictor of tomorrow's performance. In addition, we find significant spillover effects to the crude oil, bond, and stock markets, but there is no evidence of spillovers from the crude oil, bond, and stock markets to the foreign exchange market. In particular, the current price of crude oil is affected by last period's stock price and exchange rate; a higher S&P 500 index leads to an increase in the price of oil ($\gamma_{13} = 0.025$ with a p-value of 0.002) whereas a stronger U.S. dollar leads to a decline in the price of oil ($\gamma_{14} = -0.052$ with a p-value of 0.000).

TABLE 4. The four variables VARMA–BEKK model with oil price, T-bill rate, S&P 500 index, and exchange rate

A. Conditional mean equation

$$\Gamma = \begin{bmatrix} 0.961\ (0.000) & 0.005\ (0.605) & 0.025\ (0.002) & -0.052\ (0.000) \\ -0.006\ (0.397) & 0.986\ (0.000) & 0.045\ (0.000) & -0.134\ (0.000) \\ 0.001\ (0.839) & 0.003\ (0.512) & 0.990\ (0.000) & 0.023\ (0.016) \\ -0.001\ (0.559) & 0.001\ (0.680) & 0.003\ (0.284) & 0.987\ (0.000) \end{bmatrix}; \tilde{\Gamma} = \begin{bmatrix} 0.000 & 0.050 (0.001) & -0.061\ (0.060) & 0.136\ (0.005) \\ 0.151\ (0.409) & -0.439\ (0.000) & -1.291\ (0.000) & 1.607\ (0.000) \\ 0.001\ (0.961) & -0.016\ (0.003) & -0.041\ (0.009) & 0.058\ (0.003) \\ -0.012\ (0.002) & -0.007\ (0.004) & 0.012\ (0.041) & -0.012\ (0.136) \end{bmatrix};$$

$$\Psi = \begin{bmatrix} 0.565\ (0.000) & -0.025\ (0.186) & -0.030\ (0.365) & -0.033\ (0.642) \\ 0.019\ (0.452) & 0.445\ (0.000) & -0.084\ (0.042) & -0.180\ (0.116) \\ -0.122\ (0.000) & -0.016\ (0.459) & -0.072\ (0.166) & -0.387\ (0.006) \\ -0.010\ (0.285) & 0.003\ (0.448) & -0.022\ (0.194) & 0.361\ (0.000) \end{bmatrix}; \tilde{\Psi} = \begin{bmatrix} 0.000 & -0.007 (0.750) & 0.557\ (0.000) & 0.094\ (0.813) \\ 0.564\ (0.094) & -0.003\ (0.975) & 1.080\ (0.188) & 8.852\ (0.000) \\ 0.169\ (0.000) & 0.024\ (0.293) & -0.048\ (0.587) & 0.388\ (0.205) \\ -0.024\ (0.037) & 0.005\ (0.347) & -0.063\ (0.028) & 0.119\ (0.103) \end{bmatrix}.$$

B. Residual diagnostics

	Mean	Variance	$Q(4)$	$Q^2(4)$
z_{o_t}	−0.054	1.160	(0.003)	(0.863)
z_{i_t}	−0.128	1.003	(0.002)	(0.360)
z_{s_t}	−0.047	1.015	(0.430)	(0.160)
z_{e_t}	−0.037	0.919	(0.819)	(0.420)

C. Student's t distribution shape

v = 6.168 (0.000)

D. Conditional variance-covariance structure

$$\boldsymbol{A} = \begin{bmatrix} 0.398\ (0.000) & 0.063\ (0.081) & 0.009\ (0.685) & -0.022\ (0.002) \\ 0.017\ (0.490) & 0.700\ (0.000) & 0.003\ (0.883) & 0.021\ (0.001) \\ 0.022\ (0.622) & -0.133\ (0.007) & 0.079\ (0.011) & -0.002\ (0.881) \\ -0.093\ (0.221) & 0.302\ (0.107) & 0.022\ (0.845) & 0.064\ (0.046) \end{bmatrix}; \tilde{\boldsymbol{A}} = \begin{bmatrix} 0.000 & -1.373\ (0.000) & 0.126\ (0.005) & -0.025\ (0.043) \\ -0.046\ (0.115) & -0.413\ (0.000) & 0.028\ (0.157) & -0.021\ (0.003) \\ 0.730\ (0.000) & -0.380\ (0.659) & 0.353\ (0.001) & 0.078\ (0.008) \\ 1.636\ (0.008) & -3.244\ (0.384) & 3.264\ (0.000) & 0.389\ (0.002) \end{bmatrix};$$

$$\boldsymbol{B} = \begin{bmatrix} 0.934\ (0.000) & -0.026\ (0.069) & 0.074\ (0.000) & -0.012\ (0.078) \\ 0.001\ (0.892) & 0.780\ (0.000) & -0.022\ (0.153) & 0.009\ (0.060) \\ 0.013\ (0.475) & -0.058\ (0.064) & 0.914\ (0.000) & -0.167\ (0.000) \\ -0.085\ (0.285) & -0.160\ (0.100) & 1.588\ (0.000) & 0.793\ (0.000) \end{bmatrix}; \tilde{\boldsymbol{B}} = \begin{bmatrix} 0.000 & -1.358\ (0.000) & -0.082\ (0.047) & -0.047\ (0.000) \\ 0.073\ (0.000) & -0.118\ (0.080) & 0.048\ (0.006) & -0.027\ (0.000) \\ -0.341\ (0.001) & 0.799\ (0.226) & -0.262\ (0.000) & 0.158\ (0.000) \\ 2.151\ (0.000) & 13.291\ (0.000) & -1.663\ (0.000) & -0.377\ (0.000) \end{bmatrix}.$$

Note: Sample period, monthly data: 1979:01-2015:02. Numbers in parentheses are tail areas of tests.

However, the spillover effects change after the policy rate hits the zero lower bound in December 2008, as is indicated by the $\tilde{\Gamma}$ matrix. Regarding the oil price equation, we find that a higher interest rate could lead to an increase in the price of oil, since $\tilde{\gamma}_{12} = 0.05$ with a p value of 0.001. Also, the intertemporal correlation between the oil price and the exchange rate changes when the zero lower bound constraint on the policy rate is binding, since in that case an appreciation of the U.S. dollar leads to a higher oil price ($\gamma_{14} + \tilde{\gamma}_{14} = -0.052 + 0.136 = 0.084$). We also find a new spillover effect from the crude oil market to the foreign exchange market, as $\tilde{\gamma}_{41} = -0.012$ with a p value of 0.002, implying that a higher oil price will lead to a depreciation of the U.S. dollar. Overall, we find that spillovers running from the financial variables to the price of oil are bigger when the zero lower bound constraint is binding.

The moving average coefficients along the diagonal of the Ψ matrix are moderate and significant, except that for the stock price, suggesting that each of the crude oil price, interest rate, and exchange rate series is consistent with a typical ARMA process. The off-diagonal elements of the Ψ matrix indicate the spillover effects across the four markets. There is no evidence of shock spillovers from each of the financial markets to the crude oil market, except in the case of the stock market and when the zero lower bound occurs (as $\tilde{\psi}_{13} = 0.557$ with a p-value of 0.000). However, oil price shocks affect the stock market negatively at normal times ($\psi_{31} = -0.122$ with a p-value of 0.000) and ambiguously when the zero lower bound is reached (as $\tilde{\psi}_{31} = 0.169$ with a p-value of 0.000).

Regarding volatility linkages, all the "own-market" coefficients in the $\boldsymbol{A}$, $\widetilde{\boldsymbol{A}}$, $\boldsymbol{B}$, and $\widetilde{\boldsymbol{B}}$ matrices are statistically significant and the estimates suggest a high degree of persistence. There are no spillover ARCH effects from the oil market to the bond and stock markets before December 2008, but we find statistically significant spillover ARCH effects when the zero lower bound occurs. In particular, an unexpected shock in the crude oil market increases the volatility of the bond and stock markets after the zero lower bound is reached, since $\tilde{\alpha}_{12} = -1.373$ (with a p-value of 0.000) and $\tilde{\alpha}_{13} = 0.126$ (with a p-value of 0.005). Moreover, the spillover ARCH effect from the oil market on the foreign exchange market increases when the zero lower bound is reached, since $\tilde{\alpha}_{14} = -0.025$ (with a p-value of 0.043), implying an ARCH effect of $(0.022 + 0.025)^2$.

We also find statistically significant spillover GARCH effects when the zero lower bound occurs. There is evidence for volatility spillovers from the crude oil market to the bond and foreign exchange markets, with $\tilde{\beta}_{12} = -1.358$ (with a p-value of 0.000) and $\tilde{\beta}_{14} = -0.047$ (with a p-value of 0.005), respectively. We also find volatility spillovers running from each of the financial markets to the crude oil market. Overall, we find that the volatility spillovers across markets are enhanced when the zero lower bound occurs than when it is not, suggesting that unconventional monetary policy at the zero lower bound has strengthened the linkages between the crude oil and financial markets.

5. CONCLUSION

We have investigated mean and volatility spillovers between the crude oil market and three financial markets—the debt, stock, and foreign exchange markets—by estimating a VARMA model with a BEKK representation. We have also examined the possible effects of monetary policy at the zero lower bound by including a dummy variable in both the conditional mean and variance equations. The four-variable VARMA–BEKK model analyzed, our focus on both first- and second-moment linkages, and the incorporation of a structural break to capture the effects of monetary policy when the policy rate hits the zero lower bound all represent original contributions to the literature.

We find that the crude oil market and the financial markets are tightly interconnected and that monetary policy at the zero lower bound has strengthened their linkages.

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