Robotica (2003) volume 21, pp. 581–587. © 2003 Cambridge University Press
DOI: 10.1017/S0263574702004903 Printed in the United Kingdom

Synthesis of a complete sagittal gait cycle for a five-link biped robot

Xiuping Mu and Qiong Wu

Department of Mechanical and Industrial Engineering, University of Manitoba, Winnipeg, Manitoba (Canada) R3T 5V6
e-mail: cwu@cc.umanitoba.ca

(Received in Final Form: October 30, 2002)

SUMMARY
This paper presents a method for synthesising the joint profiles for a planar five-link biped walking on flat ground. Both single support and double support phases are considered. The joint profiles have been determined based on constraint equations cast in terms of step length, step period, maximum step height and so on. A special constraint equation is developed to eliminate the destabilising effect of the impact (heel strike) occurring in the system. Other advantages of our joint profiles include system stability during the double support phase and repeatability of gait. The method of formulating compatible trajectories of the hip and swing limb is employed. We demonstrate the advantages of this method over the one of direct formulation of the joint profiles in that it not only significantly simplifies the problem by de-coupling the biped into three subsystems (a trunk and two lower limbs), but also allows the incorporation of certain constraints without drastically increasing the complexity of the constraint equations. The effectiveness of the proposed method is demonstrated using computer simulations. We believe that this research can provide a valuable tool for generating motion patterns of bipedal gait.

KEYWORDS: Biped robot; Gait cycle; Single and double support phases; Joint profile generation; Hip trajectory; Swing limb trajectory; Stability and repeatability of walking.

1. INTRODUCTION
Bipedal robots are a class of walking robots that imitate human locomotion. They have greater mobility than conventional movable robots, especially when climbing stairs, stepping over obstacles or walking on uneven surfaces. The design of joint profiles is a crucial step in the development of bipedal robots; however, there is a lack of systematic methods for synthesizing the joint profiles and most of the previous work has been based on trial and error.[1–3] Vukobratovic *et al.*[4] and Zerrugh[5] have studied locomotion by using human walking data to prescribe the motion of the lower limbs. Two immediate problems arise when using human walking data directly. Firstly, a complex dynamic model is required. Secondly, the designer has no freedom to synthesize the joint profiles based on tangible gait characteristics such as walking speed, step length and step elevation. None of the aforementioned approaches give a well-defined method for the design of joint profiles. Hurmuzlu[6] developed a parametric formulation that ties together the constraint functions and joint profiles. The constraint functions are cast in terms of coherent physical characteristics of gait; these functions have been used to generate the joint profiles of a 5-link biped during a single support phase. In light of prevailing locomotion literature, Hurmuzlu's method[6] fills the gap in the design of walking machines regarding the specification of constraint functions. However, to have continuous and repeatable gait, the postures at the beginning and end of each step have to be identical. This requires the selection of specific initial conditions, constraint functions and their associated gait parameters. This selection can be extremely challenging, if not impossible. This issue has not been addressed in Hurmuzlu's work.[6] Ma and Wu[7] developed a general necessary and sufficient condition for repeatable gait where the highest order of the differential equations among the constraint equations is one. This condition provides a guideline for selecting constraint functions and their associated gait parameters in the context of producing repeatable gait based on Hurmuzlu's method.[6] However, finding repeatable gait when the constraint equations involve higher order differential equations still remained unsolved. The restriction of creating repeatable gait and the lack of rules for selecting proper initial angles, constraint functions and their gait parameters have severely limited the applications of Hurmuzlu's method.[6]

The above problem of selecting proper initial conditions to generate repeatable gait can be remedied by using numerical methods by approximating the joint angles by Fourier series expansion,[8] time polynomial functions[9,10] or periodic spline interpolation.[11,12] One advantage of this technique is that extra constraints, such as repeatability of gait, can be easily included by adding the coefficients to the polynomials. Disadvantages include the facts that the computing load is high for large bipedal systems and the selection of the polynomials may impose undesirable features to the joint profiles.

Chow and Jacobson[13] studied optimal biped locomotion and first drew attention to the hip motion and suggested that the hip trajectory be synthesized prior to joint profiles. The advantage of this method is the de-coupling of the biped into three sub-systems – a torso and two lower limbs. Although this method has been used to synthesize walking patterns for bipeds,[12] it has not attracted much attention.

Since a biped robot tips over easily, it is important to consider stability during the design of joint profiles. Methods have been proposed[12,14,15] for synthesizing walking patterns based on the concept of the zero moment point (ZMP).[4] The ZMP is defined as the point on the ground about which the sum of the moments of all the active forces equals zero. If the ZMP is within the convex hull of all contact points between the feet and the ground, the bipedal robot can walk. In previous work,[14,15] the ZMP trajectory was first designed and joint profiles were then derived. The advantage of this technique is that the stability can be guaranteed. However, not all desired ZMP can be achieved due to limited hip motion and, even if it can be achieved, large hip acceleration often results from the desired ZMP, which makes the control task difficult. Another problem is that the ZMP method cannot be used for the single support phase unless the supporting foot is modeled as a separate link.

From prevailing locomotion literature on joint profile generation for bipedal walking, it has been noted that several important issues need to be investigated further. Firstly, most studies have focused on motion generation during a single support phase, while the double support phase has received less attention. The double support phase plays an important role in keeping a biped stable walking at a wide range of speeds, and thus cannot be neglected. Secondly, impact, occurring at the transition between the single support and double support phases, has a significant effect on the stability of the bipedal system. Thus, it is important to synthesize gait patterns that minimize the effects of the impact on the joint profiles. Other important issues, which need to be considered in gait design, include system stability and repeatability of gait.

The objective of this paper is to propose a method for synthesizing joint profiles. The important issues, which have not been previously investigated properly, will be considered. We will revisit the method of designing compatible hip and swing limb trajectories, which has the advantage of simplifying the problem by de-coupling the biped into three subsystems. The joint profiles for a full gait cycle including both single support and double support phases will be designed. A special constraint is imposed on the hip and swing limb motion such that smooth joint profiles will be generated at all times, including the transition between the single and double support phases to eliminate the destabilizing effects caused by the impact. The constraint functions and gait parameters are to be chosen to generate repeatable gait and, furthermore, certain gait parameters will be selected such that the largest stability region during the double support phase will be obtained. The paper is organized as follows. In Section 2, we formulate the constraints for the hip and swing limb trajectories for both the single support and double support phases, and solve these trajectories using time polynomials. The explicit relationship among the joint angles and the hip and swing tip trajectories is also given in this section. Simulation results are presented in Section 3 to demonstrate the proposed method. Conclusions and further extensions are given in Section 4.

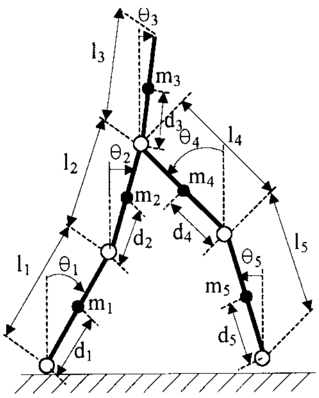


Fig. 1. Five-link biped model.

2. DESIGN OF JOINT PROFILES

2.1. *Walking cycle*

The bipedal system studied in this work has five rigid links connected by pin joints as shown in Figure 1. One link represents the upper body and two links are used for each lower limb, representing a thigh and shank, where l_i $(i=1,2,\ldots,5)$ are the link lengths that $l_4=l_2$ and $l_5=l_i$, and $\theta_i(t)$ $(i=1,2,\ldots,5)$ is the absolute angle between the ith link and the vertical plane. The biped moves steadily in the sagittal plane; that is, the joint profile is continuous and repeatable. There is an actuator located at each joint. Although the feet are not included in the model, we assume that a torque can be applied at each ankle.

A complete step cycle can be divided into a single support phase and a double support phase. The single support phase is characterized by one limb (the swing limb) moving in the forward direction while another limb (the stance limb) is pivoted on the ground. This phase begins with the swing limb tip leaving the ground and terminates with the swing limb touching the ground. Its time period is denoted as T_S. In the double support phase, both lower limbs are in contact with the ground while the body moves forward slightly. The time period of this phase is denoted as T_D. In the following step, the roles of the swing and stance limbs are exchanged.

It has been noted that the joint profiles can be determined if compatible trajectories for the hip and the tip of the swing limb can be prescribed. The compatible hip and swing limb trajectories should satisfy the condition that the distance between the hip and each tip of the lower limb is less than the length of the whole limb and greater than the difference between the lengths of the shank and thigh at any time. This condition guarantees the existence of the joint profiles during the whole walking period. If we further assume that both knees can only bend in one direction, the joint profile will be uniquely determined by the trajectories of the hip and swing limb.

From the viewpoint of natural human walking, it is desirable that the torso is kept directly upward. Giving the trajectory $\theta_3(t)=0$ in both single and double support phases, our main task here is to synthesize the joint profiles of the lower limbs. To satisfy bipedal walking under various ground conditions, it is appropriate to design the trajectory for the tip of the swing limb first, followed by the compatible hip trajectory. Note that the solution is valid

https://doi.org/10.1017/S0263574702004903 Published online by Cambridge University Press

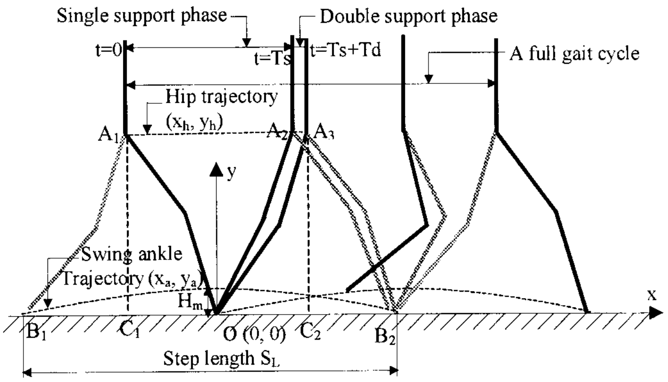


Fig. 2. Full gait cycle of a five-link biped walking in the sagittal plane.

when the no-slip condition at the contact point is satisfied, i.e. $\left|\frac{F_y}{F_x}\right| < \mu$, where, μ is the coefficient of friction between the lower limb tip and the ground.

2.2. *Trajectories of the swing limb*

The trajectory of the tip of the swing limb during the single support phase is a significant factor in bipedal walking. In this section, we develop constrained equations that can be used for solving the swing limb trajectory. This trajectory is denoted by the vector X_a: $(x_a(t), y_a(t))$, where $x_a(t)$ and $y_a(t)$ are the coordinates of the swing limb tip position with the origin of the coordinate system located at the tip of the supporting limb (see Figure 2). We use a third order polynomial and a fifth order polynomial functions for x_a and y_a, respectively. They are shown below:

$$Xa: \begin{cases} x_a(t)=a_0+a_1t+a_2t^2+a_3t^3 \\ y_a(t)=b_0+b_1t+b_2t^2+b_3t^3+b_4t^4+b_5t^5 \end{cases} \quad 0 \le t \le T_S \quad (1)$$

We next develop constraint equations that can be used for solving the coefficients, a_i and b_j ($i=0, \ldots, 3$ and $j=0, \ldots, 5$). We cast the gait patterns in terms of four basic quantities: step length, S_L, step period for the single support phase, T_S, maximum clearance of the swing limb, H_m, and its location S_m. Other constraints used for designing the swing limb motion are repeatable gait and the minimization of the effect of impact. The constraint relations are described as follows:

(1) ***Geometrical constraints:*** The swing limb has to be lifted off the ground at the beginning of the step cycle and has to be landed back at the end of it. We will enforce this condition by the following equations:

$$y_a(0)=0 \quad (2)$$

$$y_a(T_S)=0 \quad (3)$$

(2) ***Maximum clearance of the swing limb:*** During the swing phase, the tip of the swing limb has to stay clear off the ground to avoid accidental contact. In some previous work[6,16] the parabolic relation between $x_a(t)$ and $y_a(t)$ has been assumed. Although this relation is the simplest form that allows prescription of the desired step length and the tip maximum clearance independently, it is unlikely to satisfy the requirement of repeatable gait once the double support phase is considered. In this work, we synthesize the swing limb trajectory by setting the following relations:

$$x_a(T_m)=S_m \quad (4)$$

$$y_a(T_m)=H_m \quad (5)$$

$$\dot{y}_a(T_m)=0 \quad (6)$$

where H_m is the maximum clearance of the swing limb, S_m is the x-coordinate of the swing limb tip corresponding to the maximum clearance, and T_m is the time instant when the tip of the swing limb reaches to the maximum clearance. Note that T_m is not prescribed.

(3) ***Repeatability of the gait:*** The requirement for repeatable gait demands that the initial posture and velocities be identical to those at the end of the step. Based on the necessary and sufficient condition for repeatable gait developed in our previous work[7] we have ΔA_1B_1O to be identical to ΔA_3OB_2, which guarantees that $B_1O=OB_2=S_L/2$ (see Figure 2). Furthermore, since during the double support phase both tips remain stationary, the initial velocities in both horizontal and vertical direction must be zero. Subsequently, the following relations must hold:

$$x_a(0)=-\frac{S_L}{2} \quad (7)$$

$$x_a(T_s)=\frac{S_L}{2} \quad (8)$$

$$\dot{x}_a(0)=0 \quad (9)$$

$$\dot{y}_a(0)=0 \quad (10)$$

(4) ***Minimization of the effect of impact:*** During locomotion, when the swing limb contacts the ground (heel strike), impact occurs, which causes sudden changes in the joint angular velocities. The effect of the impact on the joint angular velocities is shown by equation A3 in the Appendix for the case where the impact is perfectly plastic (i.e. the tip of the swing limb has zero velocity after impact). Thus, by keeping the velocity of the swing limb tip zero before the impact, the sudden jump in the joint angular velocities can be eliminated. The above conditions lead to:

$$\dot{x}_a(T_S)=0 \quad (11)$$

$$\dot{y}_a(T_S)=0 \quad (12)$$

Equations (2) through (12) can be used for solving ten polynomial coefficients a_i, b_j ($i=0, \ldots, 3$ and $j=0, \ldots, 5$) and T_m. The trajectory of the swing limb with the above coeff cients satisfies the requirements of repeatable gait with no destabilizing effect from the impact and with prescribed gait parameters.

https://doi.org/10.1017/S0263574702004903 Published online by Cambridge University Press

2.3. Trajectories of the hip

Hip motion has a signifcant effect on the stability of the bipedal system.[13] Here, the trajectory of the hip is designed for single support and double support phases separately. The hip position is denoted as X_{hS}: $(x_{hS}(t), y_{hS}(t))$ for the single support phase and X_{hD}: $(x_{hD}(t), y_{hD}(t))$ for the double support phase. Third order polynomial functions are used for x_{hS} and X_{hD}. Together with the general function of vertical hip motion, they are shown below:

$$\begin{array}{ll} x_{hS}(t) = c_0 + c_1 t + c_2 t^2 + c_3 t^3 & 0 \le t \le T_S \\ x_{hD} \;\; = d_0 + d_1 t + d_2 t^2 + d_3 t^3 & 0 \le t \le T_D \\ y_{hS}(t) = y_h(t) & 0 \le t \le T_S \\ y_{hD}(t) = y_h(t) & 0 \le t \le T_D \end{array} \tag{13}$$

We next develop constraint equations, which include the following additional quantities: positions of the hip at the beginning of the single and double support phases, S_{S0} and S_{D0}, step period for the double support phase, T_D, and the height of the hip, H_h. Aside from the constraints of repeatable gait and minimization of the effect of impact, the stability of bipedal walking during the double support phase is also considered. The constraint relations are described as follows:

(1) ***Vertical hip motion:*** One desirable feature of biped gait is to keep the vertical motion of the gravity center to a minimum, which requires minimal vertical motion of the hip. For the sake of simplicity, we assume Y_{hS} and Y_{hD} to be constant at any time during the whole gait cycle, i.e.

$$y_{hS}(t) = H_h \tag{14}$$

$$y_{hD}(t) = H_h \tag{15}$$

(2) ***Repeatability of the gait:*** Based on the necessary and sufficient condition for repeatable gait developed in our previous work,[7] ΔA_1B_1O must be identical to ΔA_3OB_2, which guarantees that $OC_1 = B_2C_2 = S_{S0}$ (See Figure 2). Furthermore, the hip velocities at the beginning and the end of the step must be the same. Thus, the following relations must hold:

$$x_{hS}(0) = -S_{S0} \tag{16}$$

$$x_{hD}(T_D) = \frac{1}{2} S_L - S_{S0} \tag{17}$$

$$\dot{x}_{hS}(0) = V_{h1} \tag{18}$$

$$\dot{x}_{hS}(T_D) = V_{h1} \tag{19}$$

where V_{h1} is the hip velocity at the beginning of each step, which will be determined later.

(3) ***Continuity of the gait:*** The horizontal displacements of the hip during the single and double support phases must be continuous, which leads to:

$$x_{hS}(T_S) = S_{D0} \tag{20}$$

$$x_{hD}(0) = S_{D0} \tag{21}$$

(4) ***Minimization of the effect of impact:*** As discussed before, it is desirable to design the hip and swing limb trajectory such that the effect of impact can be removed. To satisfy this requirement, we set the hip velocities at the end of the single support phase and the beginning of the double support phase to be identical. Thus, we have:

$$\dot{x}_{hS}(T_S) = V_{h2} \tag{22}$$

$$\dot{x}_{hD}(0) = V_{h2} \tag{23}$$

V_{h2} will be determined later.

(5) ***Stability of the gait during the double support phase:*** The horizontal velocity of the hip is the main factor that affects the stability of a bipedal robot walking in a sagittal plane.[12,13] Previous research focused on deriving the hip trajectories to achieve a desired ZMP. The disadvantages are that not all desired ZMP trajectories can be attained and the hip acceleration may be very large. In this work, we propose the following procedure to determine optimal V_{h1} and V_{h2}:

(i) generate a series of smooth x_{hS} and x_{hD} by selecting various V_{h1} and V_{h2} and
(ii) determine the optimal V_{h1} and V_{h2} (x_{hS} and x_{hD}) with the largest stability margin.

The stability margin is defined as the minimum distance between the ZMP and the boundary of the stable region, which is the line between the tips of the supporting limbs. Equations (16) through (23) and the stability constraints can be used to solve the coefficients, c_i and d_i ($i = 0, \ldots, 3$), and the two velocities V_{h1} and V_{h2}.

2.4. Biped joint profile

Using the hip and swing leg tip trajectories design, the joint angle profile can be expressed as follows:

$$\begin{cases} \theta_1(t) = arcsin\left(\dfrac{a_1C_1 + B_1\sqrt{A_1^2 + B_1^2 - C_1^2}}{A_1^2 + B_1^2}\right) \\ \theta_2(t) = \theta_1(t) + arcsin\left(\dfrac{A_1 cos(\theta_1(t)) - B_1 sin(\theta_1(t))}{l_2}\right) \\ \theta_3(t) = 0 \\ \theta_4(t) = arcsin\left(\dfrac{A_4C_4 + B_4\sqrt{A_4^2 + B_4^2 - C_4^2}}{A_4^2 + B_4^2}\right) \\ \theta_5(t) = \theta_4(t) + arcsin\left(\dfrac{A_4 cos(\theta_4(t)) - B_4 sin(\theta_4(t))}{l_5}\right) \end{cases} \tag{24}$$

Table I. Parameters of the bipedal robot.

Link	Mass (kg)	Moment of inertia (kgm^2)	Length (m)	Location of centre of mass to lower joint (m)
Torso (3)	14.79	3.30×10^{-2}	0.486	0.282
Thigh (2, 4)	5.28	3.30×10^{-2}	0.302	0.236
Leg (1, 5)	2.23	3.30×10^{-2}	0.332	0.189

where for single support phase, we have:

$$A_1 + x_{hS}(t),\ B_1 = y_{hS}(t),\ c_1 = \frac{A_1^2 + B_1^2 + l_1^2 - l_2^2}{2l_1}$$

$$A_4 = x_{hS}(t) - x_{aS}(t),\ B_4 = y_{hS}(t) - y_{aS}(t),\ C_4 = \frac{A_4^2 + B_4^2 + l_4^2 - l_5^2}{2l_4}$$

For double support phase, we have:

$$A_1 = x_{hD}(t) \qquad B_1 = y_{hD}(t) \quad C_1 = \frac{A_1^2 + B_1^2 + l_1^2 - l_2^2}{2l_1}$$

$$A_4 = \frac{1}{2}S_L - x_{hD}(t) \quad B_4 = y_{hD}(t) \quad C_4 = \frac{A_4^2 + B_4^2 + l_4^2 - l_5^2}{2l_4}$$

3. SIMULATIONS

In this section, a joint profile for a five-link biped walking on flat ground with both single and double support phases are determined using the method discussed in Section 2. The values of the parameters m_i, I_i, l_i, and d_i, of the five-link biped robot are listed in Table I and the walking speed is chosen 1.06 m/s with step length $S_L = 0.72\ m$, $T_S = 0.6s$, $T_D = 0.1s$, $H_m = 0.05\ m$ and $S_m = 0\ m$.

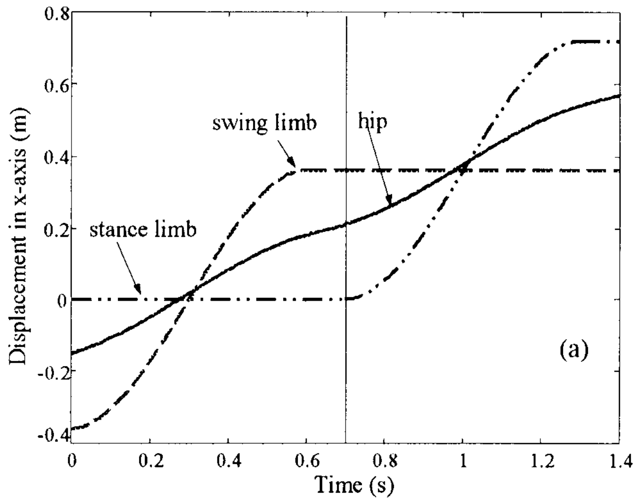
Figure 3a shows the horizontal displacements of the hip and the tips of both the swing and stance limbs versus time. Figure 3b shows the trajectories of the tip of the swing limb. All the trajectories are smooth; i.e., all the velocities are continuous. Figure 4 shows the motion of the lower limb joints for two steps. Figure 4a shows the profiles of the joint angles and Figure 4b shows the angular velocities during the single and double support phases. It can be seen that the both joint angles and their angular velocities are repeatable, the velocities are continuous at the instant of impact, and the sudden changes in the angular velocities, often occurring in other work, have been removed.

Figure 5 shows the horizontal displacements of the gravity centre of the biped, hip and ZMP during the double support phase. The grey area represents the stability region with the top and bottom lines representing the locations of the two feet. It can be seen that the gravity centre, the hip, and especially the ZMP remain approximately at the centre of the stability region, which ensures the largest stability margin and greatest stability.

Figure 6 shows a stick diagram of the five-link bipedal model walking on flat ground. From this diagram, one can observe the overall motion of the biped during both the single and double support phases. The solid lines represent the single support phase while the dashed lines represent the double support phase. The stance limb propels the upper body forward while the upper body is maintained in the upright position. The posture of the biped at the end of each step is close to that at the beginning of each step, which indicates that the repeatability condition is satisfied. Thus, the joint profiles designed based on the method proposed in this paper are acceptable.

4. CONCLUSIONS

In this paper, a systematic approach has been presented for synthesizing joint profiles for a five-link biped walking in the sagittal plane. Unlike most of the previous work



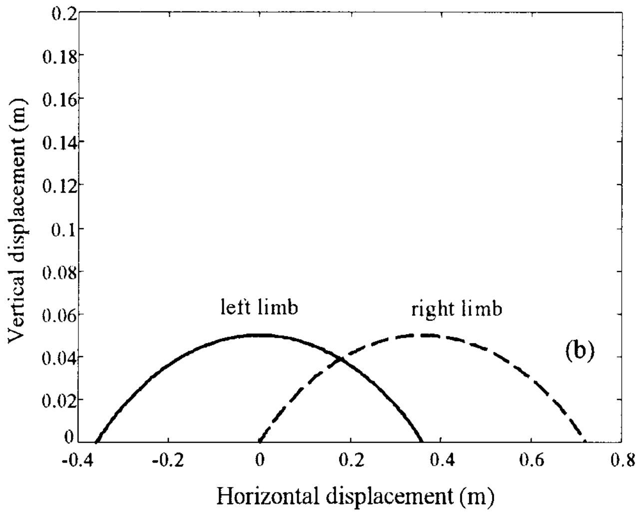


Fig. 3. Designed movements of the hip and lower limb (a) *x-t* and (b) *y-x*.

https://doi.org/10.1017/S0263574702004903 Published online by Cambridge University Press

focusing on the single support phase, our model includes both single and double support phases, which gives the biped a wider range of walking speeds and more stable locomotion. The joint profiles have been determined based on constraint equations cast in terms of step length, step period, maximum step height, and so on. Due to the impact (heel strike) occurring in the system, the angular velocities are discontinuous in most of the previous work. We developed a special constraint to eliminate the destabilising effect of the impact on bipedal motion. Thus, our joint profiles have continuous angular velocities throughout the whole gait cycles. Other advantages of our designed joint profiles included system stability during the double support phase and repeatability of gait. Computer simulations were carried out to demonstrate the effectiveness of the proposed method.

The method of formulating the compatible trajectories for the hip and the swing limb was revisited. This method has the advantage of de-coupling the biped into three sub-systems, namely a trunk and two lower limbs. As compared with directly synthesizing the joint angles, this method not only significantly simplified the problem, but also allowed us to incorporate certain constraints directly to the hip and swing limb trajectories without much increase in the complexity of the constraint equations. For example, we demonstrated that it is straight-forward to impose constraints on the hip and swing limb trajectories to eliminate the destabilizing effect from the impact occurring at the heel strike. We also showed the effectiveness of designing the hip motion directly to enhance system stability, which is one of the primary concerns in the development of bipedal robots.

We believe that this research can provide a valuable tool for generating motion patterns of bipedal gait, and it is a stepping stone for the use of more complicated constraint functions. One limitation of the presented work is that a rigorous stability analysis of the bipedal system during the single support phase cannot be carried out. This is because the ZMP method is not applicable to the models where both feet are simplified as points. Furthermore, even for a bipedal system where the feet are modelled as separate links,

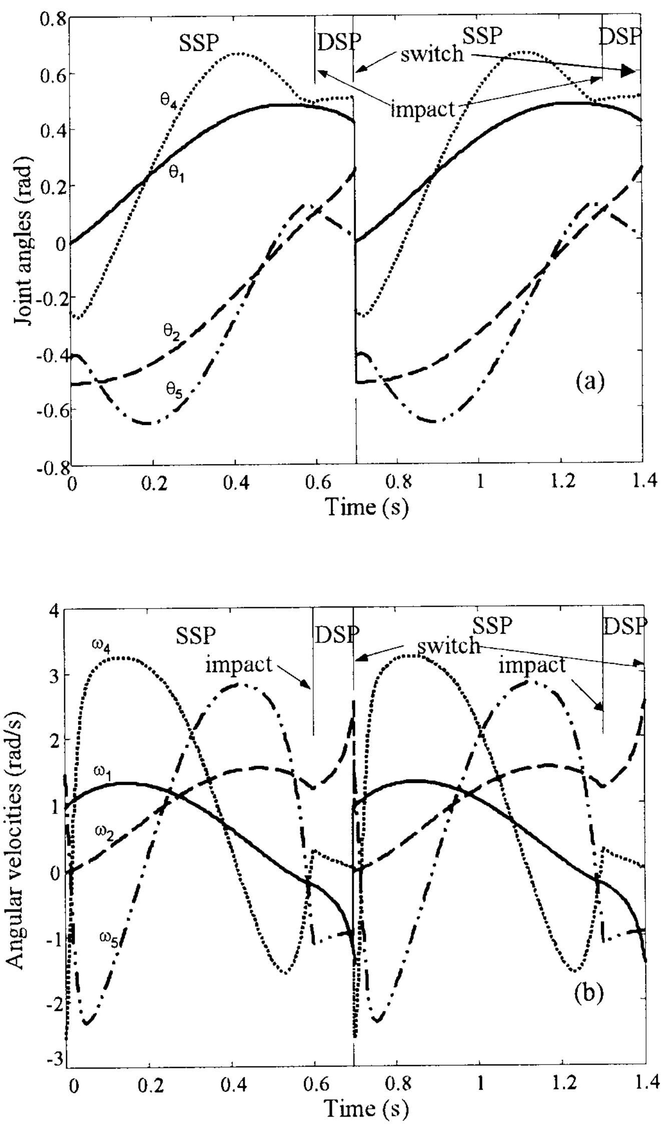


Fig. 4. Designed joint angles and angular velocities, (a) joint angles, (b) angular velocities.

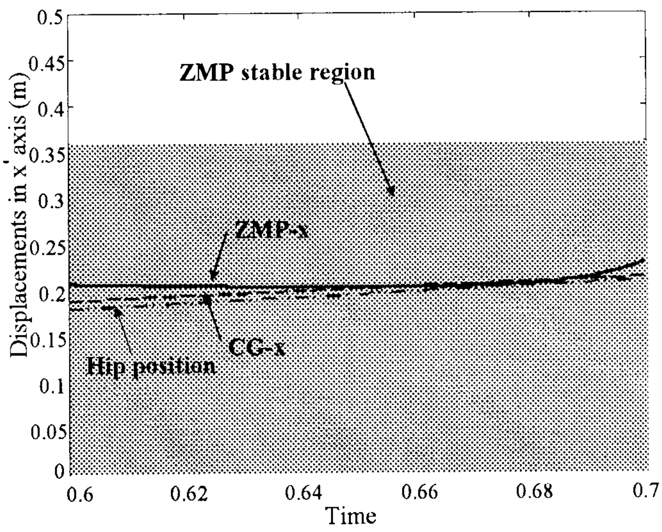


Fig. 5. CG, hip and ZMP trajectories during the double support phase.

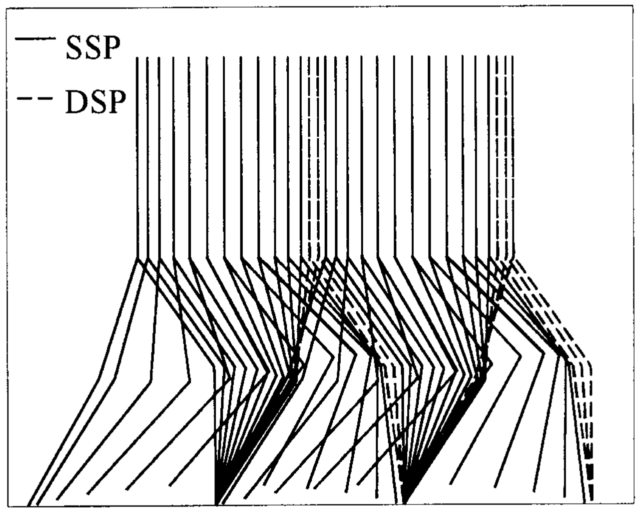


Fig. 6. Stick diagram of bipedal walking.

walking patterns generated based on ZMP trajectories are often not natural and not desirable due to high accelerations ofthe hip, which makes the control task difficult.[12] Thus, an immediate extension of the presented work is the development of a proper stability criterion for bipeds regardless of whether the feet are modeled as separate links or points, and for generating more natural gait patterns. Research on this subject is in progress.

References

1. S. Tzafestas, M. Raibert and C. Tzafestas, "Robust Sliding-mode Control Applied to a 5-Link Biped Robot", *Journal of Intelligent and Robotic Systems* **15**, 67–133 (1996).
2. J. Furusho and M. Masubuchi, "Control of a Dynamical Biped Locomotion System for Steady Walking", *Journal of Dynamic Systems, Measurement, and Control* **108**, 111–118 (1986).
3. J. Furusho and M. Masubuchi, "A Theoretically Motivated Reduced Order Model for the Control of Dynamic Biped Locomotion", *Journal of Dynamic Systems Measurement, and Control* **109**, 155–163 (1987).
4. M. Vukobratovic, B. Borovac, D. Surla and D. Stokic, *Scientific Fundamentals of Robotics 7. Biped Locomotion: Dynamics Stability, Control and Application* (Springer-Verlag, New York, 1990).
5. M.Y. Zarrugh and C.W. Radcliffe, "Computer Generation of Human Gait Kinematics", *Journal of Biomechanicas* **12**, 99–111 (1979).
6. Y. Hurmuzlu, "Dynamics of Bipedal Gait: Part I – Objective Functions and the Contact Event of a Planar Five-Link Biped", *Journal of Applied Mechanics* **60**, 331–336 (1993).
7. B. Ma and Q. Wu, "Parameter Study of Repeatable Gait for a Planar Five-Link Biped" **20**, Part 5, 493–498 (2002).
8. G. Cabodevila and G. Abba, "Quasi-Optimal Gait for A Biped Robot Using Genetic Algorithm", *IEEE International Conference on Systems, Man, and Cybernetics – Computational Cybernetics and Simulation* **4**, 3960–3965 (1997).
9. C. Chevallereau and Y. Aoustin, "Optimal Reference Trajectories for Walking And Running of a Biped Robot", *Robotica* **19**, Part 1, 557–569 (2001).
10. E. Red "A Dynamic Optimal Trajectory Generator for Cartesian Path Following", *Robotica* **18**, Part 1, 451–458 (2000).
11. C. Shih, "Gait Synthesis for a Biped Robot", *Robotica* **15**, 599–607 (1997).
12. Q. Huang, K. Yokoi, S. Kajita, K. Kaneko, H. Arai, N. Koyachi and K. Tanie, "Planning Walking Patterns for a Biped Robot", *IEEE Transactions on Rohotics and Automation* **17**, 280–289 (2001).
13. C.K. Chow and D.H. Jacobson, "Studies of Locomotion via Optimal Programming", *Mathematical Biosciences* **10**, 239–306 (1971).
14. C.L. Shih, S. Churng, T.T. Lee and W.A. Gruver, "Trajectory Synthesis and Physical Admissibility for a Biped Robot During the Single-Support Phase", *Proc. of the 1990 IEEE International Conference on Robotics and Automation* (l990) pp. 1646–1651.
15. K. Hirai, M. Hirose, Y. Haikawa and T. Takenaka "The Development of Honda Humanoid Robot", *Proc. of the 1998 IEEE International Conference on Robotics and Automation* (1998) pp. 1321–1326.
16. M.Y. Cheng and C.S. Lin, "Dynamic Biped Robot Locomotion on Less Structured Surfaces", *Robotica* **18**, Part 1, 163–170 (2000).
17. Y.F. Zheng and H. Hemami, "Impact Effect of Biped Contact with the Environment", *IEEE Transactions on Systems, Man and Cyhernetics* **3**, 437–443 (1984).

APPENDIX: DYNAMIC MODEL OF A PLANAR FIVE-LINK BIPED

We will show the dynamic equations for a planar five-link biped model during single support phase, double support phase and impact. By applying Lagrange's method, the equation of motion during the single support phase can be written in the following general form:

$$\underline{D}(\underline{\theta})\underline{\ddot{\theta}} + \underline{H}(\underline{\theta}, \underline{\dot{\theta}}) + \underline{G}(\underline{\theta}) = \underline{T}_{\theta} \qquad \text{(A1)}$$

where $\underline{D}(\underline{\theta})$ is the 5×5 positive definite and symmetric mass matrix, $\underline{H}(\underline{\theta}, \underline{\dot{\theta}})$ and $\underline{G}(\underline{\theta})$ are 5×1 matrices of centrifugal and Coriolis terms and gravity terms, and $\underline{\theta}$, $\underline{\dot{\theta}}$, $\underline{\ddot{\theta}}$, $\underline{T}_{\theta}$ are the 5×1 vectors of generalized coordinates, velocities, accelerations and actuator torques, respectively. The detailed terms can be found elsewhere[1] and are not repeated here.

The motion during the double support phase can be derived based on the Lagrange dynamic equations with constraint conditions in the following form:

$$\begin{aligned} \underline{D}(\underline{\theta})\underline{\ddot{\theta}} + \underline{H}(\underline{\theta}, \underline{\dot{\theta}}) + \underline{G}(\underline{\theta}) &= \underline{T}_{\theta} + \underline{J}^T \underline{\lambda} \\ \underline{\lambda} &= -(\underline{JD}^{-1}\underline{J}^T)^{-1}(\underline{JD}^{-1}(\underline{T}_{\theta} - \underline{h} - \underline{G}) + \underline{\dot{J}}\underline{\dot{\theta}}) \end{aligned} \qquad \text{(A2)}$$

where $\underline{\lambda}$ is the Lagrangian multiplier and $\underline{J}$ is the Jacobian matrix.

At the end of the single support phase, the swing limb contacts the ground surface. The generalized velocities will be subject to a sudden change resulting from the impact event. During the impact, there will be impulsive forces between the contact points of both lower limbs and the ground. The velocities of the two contact points immediately after impact are zero under the assumption of perfectly plastic impact. This event is defined as double impact. Using the time interval integration method,[17] the corresponding equations describing the new angular velocity after double impact can be written as:

$$\underline{\dot{\theta}}^{+}_{impact} = \underline{\dot{\theta}}^{-} + \underline{D}^{-1}\underline{J}^T[\underline{JD}^{-1}\underline{J}^T]^{-1}\{-\dot{x}_a^{-} - \dot{y}_a^{-}\}^T \qquad \text{(A3)}$$

where $\underline{\dot{\theta}}^{+}_{impact}$ and $\underline{\dot{\theta}}^{-}$ are the 5×1 vectors of velocity immediately after and before the impact, respectively.

https://doi.org/10.1017/S0263574702004903 Published online by Cambridge University Press