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NOTES

A NOTE ON SKILL-STRUCTURE SHOCKS, THE SHARE OF THE HIGH-TECH SECTOR, AND ECONOMIC GROWTH DYNAMICS

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By means of an endogenous growth model of directed technical change with vertical and horizontal R&D, we study a transitional-dynamics mechanism that is consistent with the changes in the shares of the high- versus the low-tech sectors found in recent European data. Under the hypothesis of a positive shock in the proportion of high-skilled labor, the technological-knowledge bias channel leads to unbalanced sectoral growth with a noticeable shift of resources across sectors. A calibration exercise suggests that the model is able to account for up to from 50 to about 100 percent of the increase in the share of the high-tech sector observed in the data from 1995 to 2007. However, the model predicts that the dynamics of the share of the high-tech sector has no significant impact on the dynamics of the economic growth rate.

Keywords: Industry Dynamics; High Tech; Low Tech; Directed Technical Change; Economic Growth

1. INTRODUCTION

Over more than a decade, European politicians have emphasized the need to increase the share of the high-tech sector as part of the European growth strategy [see, e.g., Johansson et al. (2007) and European Commission (2010) on the "Lisbon Strategy 2000–2010" and "Europe 2020 Strategy")]. But although two complementary measures of industry structure are of interest to assess the relative

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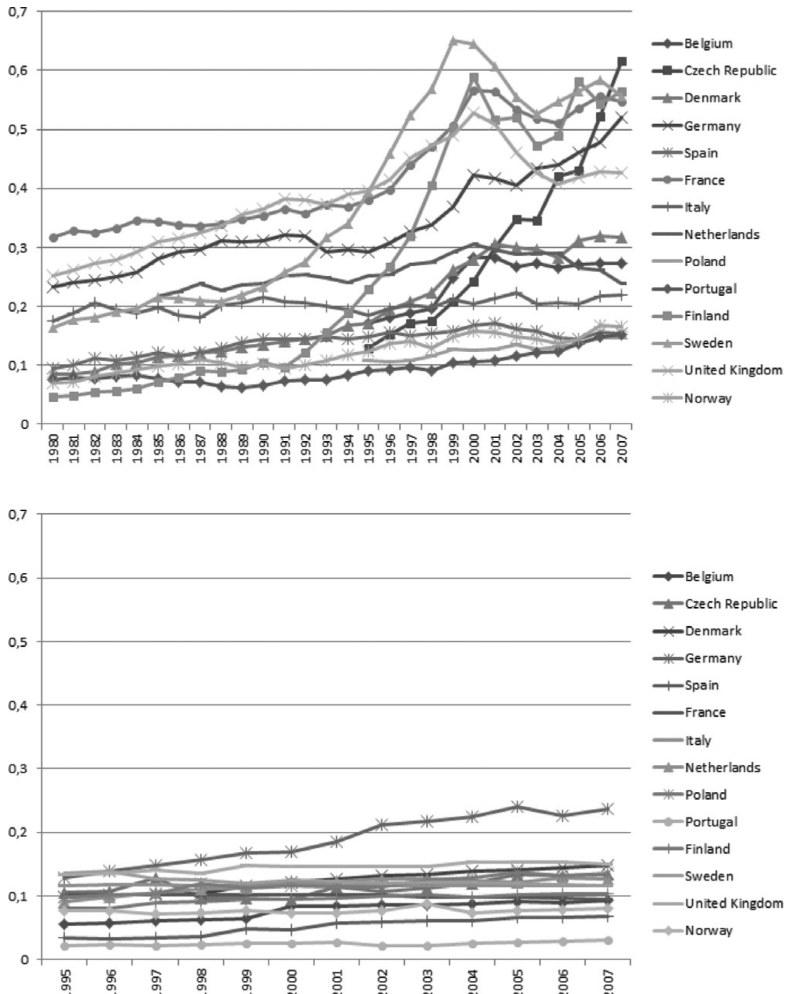

FIGURE 1. The share of the high-tech sectors through time: relative production (upper panel) and the relative number of firms (lower panel) according to the high-tech low-tech OECD classification in 14 European countries. Source: Eurostat on-line database on Science, Technology and Innovation—table "Economic statistics on high-tech industries and knowledge-intensive services at the national level," available at http://epp.eurostat.ec.europa.eu.

performance of the high-tech sector—the share of the high-tech sector with respect to production and the share with respect to the number of firms—casual empiricism has mainly focused on the latter and highlighted its slow growth. Notably, available data show that the performance of the production share has been clearly better. Figure 1 depicts the time-series data for *relative* production (production in the high-versus the low-tech manufacturing sector) over the period 1980–2007 and the *relative* number of firms (the number of firms in the high- versus the low-tech sector) over the period 1995–2007 for 14 European countries.[1] Considering the longest period with available data for both variables (1995–2007), we find that the average annual growth rate was positive for both relative production and the relative number of firms, but that the former exceeded the latter by 0.52 percentage points/year (1.22 percent/year versus 0.70 percent/year, computed as cross-country weighted averages).

What are the factors underlying the dynamics of the share of the high-tech sector in Europe in recent decades? And what can be expected with respect to the impact of that dynamics on economic growth? In our paper, we address these questions from the perspective of transitional dynamics within an extended theoretical model of endogenous growth. We conjecture that the disparity between the dynamics of production and of the number of firms is due to the asymmetric role played by the extensive and the intensive margins of industrial growth, where the former pertains to the creation of new products/firms and the latter to the increase of product quality of existing products and, thereby, of production per firm. Therefore, although we share the general view that industrial growth proceeds both along an intensive and an extensive margin in the long run, we expect a rich interaction between the two margins for shorter time horizons, namely in response to structural shocks. Having in mind (i) the observed specificity of the high- and low-tech sectors regarding the proportion of high-skilled labor[2] and (ii) the swift change in the skill structure measured by the proportion of high-skilled labor found in the data between the 1980s and the 1990s across a number of developed countries [see, e.g., Barro and Lee (2010)],[3] we emphasize in particular the hypothesis of a shock in the form of an increase in the relative supply of skills (i.e., the ratio of high- to low-skilled workers). This shock is transmitted through a mechanism of directed technical change and has an asymmetric impact on the intensive and the extensive margin, both within and across the high- and the low-tech sectors. As explained further in the following, the different natures of the intensive and the extensive margin should play a central role here.

To uncover the analytical mechanism through which the empirical evidence can be accommodated, we develop a general equilibrium growth model that incorporates endogenous directed technical change with vertical R&D (increase of product quality) and horizontal R&D (creation of new products/firms). Following Gil et al. (2013), we consider an R&D specification that allows us to endogenize the rates of intensive and extensive growth, and thereby production and the number of firms.

We analyze the transitional dynamics by considering the effects of a unanticipated one-off shock in the relative supply of skills. The theoretical results are consistent with the time-series data depicted by Figure 1. That is, there is an increase in the share of the high-tech sectors in terms both of production and of the number of firms, paralleled by an increase in production per firm relatively to the low-tech sectors. The former result stems from the positive response of the two measures of industry structure to the shock through the technological-knowledge bias channel (a larger market, measured by high-skilled labor, expands profits and thus the incentives to allocate resources to both types of R&D in the high-tech sectors), whereas the latter is explained by the higher complexity and congestion costs impinging on horizontal R&D (reflecting its physical nature vis-á-vis the immateriality of vertical R&D), which slow down and dampen the response of the number of firms relative to that of production. According to a calibration exercise, the model is able to account for up to from 50 to roughly 100 percent of the increase in the share of the high-tech sectors observed in the European data from 1995 to 2007.

On the other hand, we show that when the initial shock to the relative supply of skills is isolated as the driver of change in the industry structure, the model predicts that the economic growth rate will experience, at best, a mild level effect. Indeed, as a significant shift of economic activity from low- to high-tech sectors occurs over transition, the aggregate growth rate remains approximately unchanged.

The remainder of the paper has the following structure. In Section 2, we present the model. In Section 3, we detail the comparative dynamics results and carry out a calibration exercise. Section 4 concludes.

2. THE MODEL

The model used herein is based on Acemoglu and Zilibotti (2001) and on Gil et al. (2013).[4]

2.1. Production and Price Decisions

The final good is produced by a continuum of firms, indexed by $n \in [0, 1]$, to which two substitute technologies are available: the "Low" (respectively "High") technology uses a combination of low-skilled labor, L (high-skilled labor, H) and a continuum of labor-specific intermediate goods indexed by $\omega_L \in [0, N_L]$ ($\omega_H \in [0, N_H]$).[5] Aggregate output at time t is defined as $Y_{tot}(t) = \int_0^1 P(n,t)Y(n,t)dn$. The production technology for each final-good firm n is

$$Y(n,t) = A\left\{\int_0^{N_L(t)} \left[\lambda^{j_L(\omega_L,t)} \cdot X_L(n,\omega_L,t)\right]^{1-\alpha} d\omega_L\right\}[(1-n)\cdot l \cdot L(n)]^{\alpha}$$
$$+ A\left\{\int_0^{N_H(t)} \left[\lambda^{j_H(\omega_H,t)} \cdot X_H(n,\omega_H,t)\right]^{1-\alpha} d\omega_H\right\}[n \cdot h \cdot H(n)]^{\alpha}, \quad (1)$$

where $0 < \alpha < 1$, $h > l \geq 1$, $\lambda > 1$, and $A > 0$. $L(n)$ and $H(n)$ are the labor inputs used by n, and $\lambda^{j_m(\omega_m,t)} \cdot X_m(n,\omega_m,t)$ is the input of the m-specific intermediate good ω_m measured in efficiency units at time t.[6] Final producers take the price of their final good, $P(n,t)$, wages, $W_m(t)$, and input prices $p_m(\omega_m,t)$ as given. From the usual profit maximization conditions, we determine the demand of intermediate good ω_m by firm n. There is an endogenous threshold $\bar{n}$, which follows

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from equilibrium in the inputs markets, $\bar{n}(t) = (1+\{\mathcal{H}/\mathcal{L}\cdot[Q_{\mathrm{H}}(t)/Q_{\mathrm{L}}(t)]\}^{1/2})^{-1}$, where $\mathcal{L} \equiv lL$, $\mathcal{H} \equiv hH$, and $L = \int_0^{\bar{n}} L(n)dn$, $H = \int_{\bar{n}}^1 H(n)dn$. The aggregate quality index

$$Q_m(t) = \int_0^{N_m(t)} q_m(\omega_m, t)d\omega,\ \ q_m(\omega_m, t) \equiv \lambda^{j_m(\omega_m,t)\left(\frac{1-\alpha}{\alpha}\right)},\ \ m \in \{L, H\} \quad \mathbf{(2)}$$

measures technological knowledge in each m-technology sector. Thus, $Q \equiv Q_{\mathrm{H}}/Q_{\mathrm{L}}$ measures the technological-knowledge bias. $\bar{n}(t)$ can be related to the ratio of the price indices of final goods, $P_{\mathrm{L}}(t)$ and $P_{\mathrm{H}}(t)$, produced with L- and H-technologies, $P_{\mathrm{L}}(t) = P(n,t)\cdot(1-n)^{\alpha} = \exp(-\alpha)\cdot\bar{n}(t)^{-\alpha}$ and $P_{\mathrm{H}}(t) = P(n,t)\cdot n^{\alpha} = \exp(-\alpha)\cdot[1-\bar{n}(t)]^{-\alpha}$.

The intermediate-good sector is characterized by monopolistic competition. The firm in industry $\omega_m \in [0, N_m(t)]$ fixes the price $p_m(\omega_m, t)$ but faces an isoelastic demand curve, $X_{\mathrm{L}}(\omega_{\mathrm{L}}, t) = \int_0^{\bar{n}(t)} X_{\mathrm{L}}(n, \omega_{\mathrm{L}}, t)dn$ or $X_{\mathrm{H}}(\omega_{\mathrm{H}}, t) = \int_{\bar{n}(t)}^1 X_{\mathrm{H}}(n, \omega_{\mathrm{H}}, t)dn$. Profit in ω_m is $\pi_m(\omega_m, t) = [p_m(\omega_m, t) - 1]\cdot X_m(\omega_m, t)$, and the profit-maximizing price is a constant markup over marginal cost, $p_m(\omega_m, t) \equiv p = 1/(1-\alpha) > 1,\ m \in \{L, H\}$. From the markup, we find the optimal intermediate-good production, $X_m(\omega_m)$, and thus the optimal profit accrued by the monopolist in ω_m, $\pi_m(\omega_m, t) = \pi_{0m}\cdot P_m(t)^{1/\alpha}\cdot q_m(\omega_m, t)$, $m \in \{L, H\}$, where $\pi_{0L} \equiv \mathcal{L}A^{1/\alpha}\alpha(1-\alpha)^{2/\alpha}/(1-\alpha)$, and $\pi_{0H} \equiv \mathcal{H}A^{1/\alpha}\alpha(1-\alpha)^{2/\alpha}/(1-\alpha)$ are positive constants.

Total intermediate-good optimal production, $X_{\mathrm{tot}}(t) \equiv X_{\mathrm{L}}(t) + X_{\mathrm{H}}(t) \equiv \int_0^{N_{\mathrm{L}}(t)} X_{\mathrm{L}}(\omega_{\mathrm{L}})d\omega_{\mathrm{L}} + \int_0^{N_{\mathrm{H}}(t)} X_{\mathrm{H}}(\omega_{\mathrm{H}})d\omega_{\mathrm{H}}$, and total final-good optimal production, $Y_{\mathrm{tot}}(t) \equiv Y_{\mathrm{L}}(t) + Y_{\mathrm{H}}(t) \equiv \int_0^{\bar{n}(t)} P(n,t)Y(n,t)dn + \int_{\bar{n}(t)}^1 P(n,t)Y(n,t)dn$, are respectively $X_{\mathrm{tot}}(t) = \chi_X\Gamma(t)$ and $Y_{\mathrm{tot}}(t) = \chi_Y\Gamma(t)$, where $\chi_X \equiv A^{1/\alpha}(1-\alpha)^{2/\alpha}$, $\chi_Y \equiv A^{1/\alpha}(1-\alpha)^{2(1-\alpha)/\alpha}$, and $\Gamma(t) \equiv P_{\mathrm{L}}(t)^{1/\alpha}\cdot\mathcal{L}\cdot Q_{\mathrm{L}}(t) + P_{\mathrm{H}}(t)^{1/\alpha}\cdot\mathcal{H}\cdot Q_{\mathrm{H}}(t)$.

Finally, by considering the condition that the real wage, W_m, must equal the marginal productivity of labor in equilibrium in the m-technology sector $m \in \{L, H\}$, we get the skill premium, $W(t) \equiv W_{\mathrm{H}}(t)/W_{\mathrm{L}}(t) = (h/l)(\mathcal{H}/\mathcal{L})^{-1/2}(Q(t))^{1/2}$.

2.2. R&D

Potential entrants devote the final good to either horizontal or vertical R&D, directed to either the high- or the low-skilled labor-specific technology. Each new design (a new variety or higher-quality good) is granted a patent, and successful R&D leads to the set-up of a new firm. There is perfect competition among entrants and free entry.

Vertical R&D. A successful innovation will instantaneously increase the quality index in ω_m from $q_m(\omega_m, t) = q_m(j_m)$ to $q_m(j_m + 1) = \lambda^{(1-\alpha)/\alpha}q_m(j_m)$. At equilibrium, each new successful innovator substitutes the incumbent monopolist.

Let $I_m^i(j_m) > 0$ denote the Poisson arrival rate of vertical innovations by potential entrant i in industry ω_m when the highest quality is j_m. The rate $I_m^i(j_m)$ is independently distributed across firms, across industries, and over time and depends on the flow of resources $R_{vm}^i(j_m)$ committed by i at time t. Aggregating across i in ω_m, we get $R_{vm}(j_m) = \sum_i R_{vm}^i(j_m)$ and $I_m(j_m) = \sum_i I_m^i(j_m)$, and thus

$$I_{\mathrm{L}}(j_{\mathrm{L}}) = R_{vL}(j_{\mathrm{L}}) / \Phi_{\mathrm{L}}(j_{\mathrm{L}}) \text{ and } I_{\mathrm{H}}(j_{\mathrm{H}}) = R_{vH}(j_{\mathrm{H}}) / \Phi_{\mathrm{H}}(j_{\mathrm{H}}), \tag{3}$$

where $\Phi_{\mathrm{L}}(j_{\mathrm{L}}) = \zeta \cdot q_{\mathrm{L}}(j_{\mathrm{L}}+1)$, $\Phi_{\mathrm{H}}(j_{\mathrm{H}}) = \zeta \cdot q_{\mathrm{H}}(j_{\mathrm{H}}+1)$, and $\zeta > 0$. An R&D complexity effect is considered in (3) [e.g., Barro and Sala-i-Martin (2004, ch. 7)]: the higher the level of quality, q_m, the costlier it is to introduce a further jump in quality.

Under free entry, we get the no-arbitrage conditions

$$r(t) + I_{\mathrm{L}}(t) = \frac{\pi_0 \cdot \mathcal{L} \cdot P_{\mathrm{L}}(t)^{\frac{1}{\alpha}}}{\zeta}, \; r(t) + I_{\mathrm{H}}(t) = \frac{\pi_0 \cdot \mathcal{H} \cdot P_{\mathrm{H}}(t)^{\frac{1}{\alpha}}}{\zeta}, \tag{4}$$

where r is the real interest rate, $\pi_0 \equiv \pi_{0L}/\mathcal{L} = \pi_{0H}/\mathcal{H}$, and $I_m(\omega_m, t) = I_m(t)$.

If we equate the effective rates of return for the two R&D sectors by assuming (4), another no-arbitrage condition obtains:

$$I_{\mathrm{H}}(t) - I_{\mathrm{L}}(t) = \frac{\pi_0}{\zeta}\left[\mathcal{H} \cdot P_{\mathrm{H}}(t)^{\frac{1}{\alpha}} - \mathcal{L} \cdot P_{\mathrm{L}}(t)^{\frac{1}{\alpha}}\right]. \tag{5}$$

Solving (3) for $R_{vm}(\omega_m, t) = R_{vm}(j_m)$ and aggregating across industries ω_m, we determine total vertical R&D, $R_{vm}(t) = \zeta \cdot \lambda^{(1-\alpha)/\alpha} \cdot I_m(t) \cdot Q_m(t), \; m \in \{L, H\}$.

Horizontal R&D. A successful innovation will instantaneously increase the number of varieties. Let $\dot{N}_m^e(t)$ denote the contribution to the instantaneous flow of new m-specific intermediate goods by potential entrant e, $\eta_m(t)$ the R&D cost in units of the final good, and $R_{hm}^e(t)$ the flow of resources devoted to horizontal R&D by e at time t. Aggregating across e, we get $R_{hm}(t) = \sum_e R_{hm}^e(t)$ and $\dot{N}_m(t) = \sum_e \dot{N}_m^e(t)$, and thus

$$\dot{N}_m(t)/N_m(t) = R_{hm}(t)/\eta_m(t), \; m \in \{L, H\}. \tag{6}$$

We assume that $\eta_m(t)$ is increasing in both the number of existing varieties, $N_m(t)$, and the number of new entrants, $\dot{N}_m(t)$, $\eta_m(t) = \phi \cdot N_m(t)^{1+\sigma} \cdot \dot{N}_m(t)^{\gamma}$, where $\sigma > 0$, $\gamma > 0$ and $\phi > 0$. An R&D complexity effect is considered in (6) [e.g., Barro and Sala-i-Martin (2004, Ch. 6)]: the larger the number of existing varieties, N_m, the costlier it is to introduce new varieties. It is noteworthy that the elasticity regulating the horizontal-R&D complexity costs is larger than the one in the vertical-R&D case (i.e., $1+\sigma > 1$), in line with what should be expected bearing in mind the distinct nature of the two types of R&D (physical versus immaterial). The dependence of η on $\dot{N}$ (a congestion effect) implies that new varieties are brought to the market gradually, instead of through a lumpy adjustment [e.g., Geroski (1995)].

Every horizontal innovation results in a new intermediate good whose quality level is drawn randomly from the distribution of existing varieties. Thus, the expected quality level of the horizontal innovator is $\bar{q}_m(t) = \int_0^{N_m(t)} q_m(\omega_m, t) d\omega_m / N_m(t) = Q_m(t)/N_m(t)$. Under free entry, we get the no-arbitrage condition

$$r(t) + I_m(t) = \frac{\bar{\pi}_m(t)}{\eta_m(t)/N_m(t)}, \ m \in \{L, H\}, \tag{7}$$

where $\bar{\pi}_m = \pi_{0m} P_m^{1/\alpha} \bar{q}_m$.

Intra-sector no-arbitrage conditions. No arbitrage in the capital market requires that the two types of investment—vertical and horizontal R&D—yield equal rates of return. Thus, by equating the effective rate of return $r + I_m$ for both types of R&D, from (4) and (7), we get the intra-sector no-arbitrage conditions

$$\bar{q}_m(t) = \frac{Q_m(t)}{N_m(t)} = \frac{\eta_m(t)}{\zeta \cdot N_m(t)}, \ m \in \{L, H\}. \tag{8}$$

Equation (8) can be equivalently recast as

$$\dot{N}_m(t) = x_m(Q_m(t), N_m(t)) \cdot N_m(t), \ m \in \{L, H\}, \tag{9}$$

where

$$x_m(Q_m, N_m) \equiv \left(\frac{\zeta}{\phi}\right)^{\frac{1}{\gamma}} \cdot Q_m^{\frac{1}{\gamma}} \cdot N_m^{-\frac{\sigma+\gamma+1}{\gamma}}. \tag{10}$$

On the other hand, time-differentiating (2) and using (9) yields

$$\dot{Q}_m(t) = (\Xi \cdot I_m(t) + x_m(Q_m(t), N_m(t))) \cdot Q_m(t), \ m \in \{L, H\}, \tag{11}$$

where $\Xi \equiv \lambda^{(1-\alpha)/\alpha} - 1$ denotes the quality shift. The vertical innovation rate is endogenous and will be determined as an economywide function later.

2.3. Households

The economy is populated by a fixed number of infinitely lived households that consume and collect income from investments in financial assets and from labor. Households inelastically supply low-skilled, L, or high-skilled labor, H. Consumers choose the path of final-good aggregate consumption $\{C(t), t \geq 0\}$ to maximize a standard discounted lifetime utility function, subject to the usual flow budget constraint and transversality condition. The Euler equation for consumption is standard:

$$\dot{C}(t) = \frac{1}{\theta} \cdot (r(t) - \rho) \cdot C(t), \tag{12}$$

where $\rho > 0$ is the subjective discount rate and $\theta > 0$ is the inverse of the intertemporal elasticity of substitution.

2.4. Macroeconomic Aggregation and Equilibrium Innovation Rates

By aggregating the households' flow budget constraint and using (8), we get the aggregate flow budget constraint $Y_{\mathrm{tot}}(t) = C(t) + X_{\mathrm{tot}}(t) + R_h(t) + R_v(t)$, where $R_h = \sum_{m=\mathrm{L,H}} R_{hm}$ and $R_v = \sum_{m=\mathrm{L,H}} R_{vm}$. Substituting the expressions for the aggregate outputs and total R&D expenditures, and using (8) and (9), we get the endogenous vertical-innovation rate at equilibrium in the L-technology sector,

$$\begin{aligned} &I_{\mathrm{L}}(Q_{\mathrm{L}}, Q_{\mathrm{H}}, N_{\mathrm{L}}, N_{\mathrm{H}}, C, I_{\mathrm{H}}) \\ &\quad = \frac{1}{\zeta \cdot \lambda^{\frac{1-\alpha}{\alpha}}} \left[\chi \cdot \left([P_{\mathrm{H}}(Q_{\mathrm{H}}, Q_{\mathrm{L}})]^{\frac{1}{\alpha}} \cdot \mathcal{H} \cdot \frac{Q_{\mathrm{H}}}{Q_{\mathrm{L}}} + [P_{\mathrm{L}}(Q_{\mathrm{H}}, Q_{\mathrm{L}})]^{\frac{1}{\alpha}} \cdot \mathcal{L} \right) - \frac{C}{Q_{\mathrm{L}}} \right] \\ &\quad - \frac{Q_{\mathrm{H}}}{Q_{\mathrm{L}}} \cdot I_{\mathrm{H}} - \frac{1}{\lambda^{\frac{1-\alpha}{\alpha}}} \cdot \left[\frac{Q_{\mathrm{H}}}{Q_{\mathrm{L}}} \cdot x_{\mathrm{H}}(Q_{\mathrm{H}}, N_{\mathrm{H}}) + x_{\mathrm{L}}(Q_{\mathrm{L}}, N_{\mathrm{L}}) \right], \end{aligned} \tag{13}$$

where $\chi \equiv \chi_Y - \chi_X = A^{1/\alpha}(1-\alpha)^{2/\alpha}[(1-\alpha)^{-2} - 1] > 0$. If we further use (5) to eliminate I_{H} from (13), we get $I_{\mathrm{L}} \equiv I_{\mathrm{L}}(Q_{\mathrm{L}}, Q_{\mathrm{H}}, N_{\mathrm{L}}, N_{\mathrm{H}}, C)$. From (4), and under the condition $I_{\mathrm{L}} > 0$, we get the rate of return to capital as $r(Q_{\mathrm{L}}, Q_{\mathrm{H}}, N_{\mathrm{L}}, N_{\mathrm{H}}, C) = r_{0m} - I_m(Q_{\mathrm{L}}, Q_{\mathrm{H}}, N_{\mathrm{L}}, N_{\mathrm{H}}, C)$, where $r_{0L} \equiv \pi_0 \mathcal{L} P_{\mathrm{L}}^{1/\alpha}/\zeta$ and $r_{0H} \equiv \pi_0 \mathcal{H} P_{\mathrm{H}}^{1/\alpha}/\zeta$.

2.5. The Balanced-Growth Path

The equilibrium paths can be obtained from the dynamical system (9), (11), and (12), given $Q_m(0)$ and $N_m(0)$, and the transversality condition. As the functions in the dynamical system are homogeneous, a balanced growth path (BGP) exists only if (i) the asymptotic growth rates of consumption and of the quality indices are constant and equal to the economic growth rate, $g_C = g_{Q_{\mathrm{L}}} = g_{Q_{\mathrm{H}}} = g$; (ii) the asymptotic growth rates of the number of varieties are constant and equal, $g_{N_{\mathrm{L}}} = g_{N_{\mathrm{H}}}$; (iii) the vertical-innovation rates and the final-good price indices are asymptotically trendless, $g_{I_{\mathrm{L}}} = g_{I_{\mathrm{H}}} = g_{P_{\mathrm{L}}} = g_{P_{\mathrm{H}}} = 0$; and (iv) the asymptotic growth rates of the quality indices and the number of varieties are monotonically related, $g_{Q_{\mathrm{L}}}/g_{N_{\mathrm{L}}} = g_{Q_{\mathrm{H}}}/g_{N_{\mathrm{H}}} = (\sigma + \gamma + 1)$, $g_{N_m} \neq 0$, $m \in \{\mathrm{L}, \mathrm{H}\}$. Observe, from (9), that $x_m = g_{N_m}$ is always positive if $N_m > 0$.

By considering the growth rate of the number of varieties, x_m, as defined by (10), the consumption rate, $z_{\mathrm{L}} \equiv C/Q_{\mathrm{L}}$, and the technological-knowledge bias, $Q \equiv Q_{\mathrm{H}}/Q_{\mathrm{L}}$, we get an equivalent dynamical system in detrended variables,

$$\dot{x}_{\mathrm{L}} = \left[\frac{\Xi}{\gamma} \cdot I_{\mathrm{L}} - \left(\frac{\sigma + \gamma}{\gamma} \right) \cdot x_{\mathrm{L}} \right] \cdot x_{\mathrm{L}}, \tag{14}$$

$$\dot{z}_{\mathrm{L}} = \left[\frac{1}{\theta} \cdot (r_{0L} - \rho) - \left(\frac{1}{\theta} + \Xi \right) \cdot I_{\mathrm{L}} - x_{\mathrm{L}} \right] \cdot z_{\mathrm{L}}, \tag{15}$$

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$$\dot{x}_{\mathrm{H}} = \left[\frac{\Xi}{\gamma} \cdot I_{\mathrm{L}} - \left(\frac{\sigma + \gamma}{\gamma} \right) \cdot x_{\mathrm{H}} + \frac{\Xi}{\gamma} \cdot \frac{\pi_0}{\zeta} \cdot \left(\mathcal{H} \cdot P_{\mathrm{H}}^{\frac{1}{\alpha}} - \mathcal{L} \cdot P_{\mathrm{L}}^{\frac{1}{\alpha}} \right) \right] \cdot x_{\mathrm{H}}, \quad \mathbf{(16)}$$

$$\dot{Q} = \left[\Xi \cdot \frac{\pi_0}{\zeta} \cdot \left(\mathcal{H} \cdot P_{\mathrm{H}}^{\frac{1}{\alpha}} - \mathcal{L} \cdot P_{\mathrm{L}}^{\frac{1}{\alpha}} \right) + x_{\mathrm{H}} - x_{\mathrm{L}} \right] \cdot Q, \quad \mathbf{(17)}$$

where $I_{\mathrm{L}} \equiv I_{\mathrm{L}}(Q, x_{\mathrm{L}}, x_{\mathrm{H}}, z_{\mathrm{L}}) = I_{\mathrm{L}}(Q_{\mathrm{L}}, Q_{\mathrm{H}}, N_{\mathrm{L}}, N_{\mathrm{H}}, C)$, $I_{\mathrm{H}} \equiv I_{\mathrm{H}}(Q, x_{\mathrm{L}}, x_{\mathrm{H}}, z_{\mathrm{L}}) = I_{\mathrm{H}}(Q_{\mathrm{L}}, Q_{\mathrm{H}}, N_{\mathrm{L}}, N_{\mathrm{H}}, C)$, $P_{\mathrm{L}} \equiv P_{\mathrm{L}}(Q) = P_{\mathrm{L}}(Q_{\mathrm{L}}, Q_{\mathrm{H}})$, and $P_{\mathrm{H}} \equiv P_{\mathrm{H}}(Q) = P_{\mathrm{L}}(Q_{\mathrm{L}}, Q_{\mathrm{H}})$. These equations, together with the transversality condition and the initial conditions $x_{\mathrm{L}}(0)$, $x_{\mathrm{H}}(0)$, and $Q(0)$, describe the transitional dynamics and the BGP, by jointly determining $x_{\mathrm{L}}(t)$, $z_{\mathrm{L}}(t)$, $x_{\mathrm{H}}(t)$, and $Q(t)$. Thus, we can determine the level variables $N_m(t)$, $C(t)$, and $Q_{\mathrm{L}}(t)$ (respectively $Q_{\mathrm{H}}(t)$) for a given $Q_{\mathrm{H}}(t)$ ($Q_{\mathrm{L}}(t)$).

PROPOSITION 1. *Let* $\tilde{r}_{0m} - \rho > 0$, *and* $0 < \frac{\Xi}{\theta}(\tilde{r}_{0m} - \rho)/[\Xi(\sigma + \gamma + 1) + (\sigma + \gamma)/\theta] < \overline{x} \equiv \chi \cdot \tilde{\Gamma}/(\tilde{Q}_{\mathrm{L}}\mathcal{Z})$, $m \in \{\mathrm{L}, \mathrm{H}\}$. *The interior steady state,* $(\tilde{x}_{\mathrm{L}}, \tilde{z}_{\mathrm{L}}, \tilde{x}_{\mathrm{H}}, \tilde{Q})$, *exists and is unique, with:*[7]

$$\tilde{Q} = \tfrac{\mathcal{H}}{\mathcal{L}}, \quad \tilde{x}_{\mathrm{L}} = \tilde{x}_{\mathrm{H}} = \frac{\frac{\Xi}{\theta}(\tilde{r}_{0m} - \rho)}{\Xi\,(\sigma + \gamma + 1) + \frac{1}{\theta}\,(\sigma + \gamma)},$$
$$\tilde{z}_{\mathrm{L}} = \chi \left(\tilde{P}_{\mathrm{H}}^{\frac{1}{\alpha}} \mathcal{H} \tilde{Q} + \tilde{P}_{\mathrm{L}}^{\frac{1}{\alpha}} \mathcal{L} \right) - \left(\zeta \lambda^{\frac{1-\alpha}{\alpha}} \tilde{I}_{\mathrm{L}} + \zeta \tilde{x}_{\mathrm{L}} \right) \left(\tilde{Q} + 1 \right),$$

where

$$\tilde{I}_{\mathrm{L}} = \tilde{I}_{\mathrm{H}} = \left(\tfrac{\sigma+\gamma}{\Xi}\right) \tilde{x}_{\mathrm{L}} = \left(\tfrac{\sigma+\gamma}{\Xi}\right) \tilde{x}_{\mathrm{H}};\ \tilde{r}_{0L} \equiv \pi_0 \mathcal{L} \tilde{P}_{\mathrm{L}}^{\frac{1}{\alpha}}/\zeta = \tilde{r}_{0H} \equiv \pi_0 \mathcal{H} \tilde{P}_{\mathrm{H}}^{\frac{1}{\alpha}}/\zeta;$$
$$\tilde{P}_{\mathrm{L}} = \exp(-\alpha) \cdot (1 + \mathcal{H}/\mathcal{L})^{\alpha};\ \tilde{P}_{\mathrm{H}} = \exp(-\alpha) \cdot \left[1 + (\mathcal{H}/\mathcal{L})^{-1}\right]^{\alpha};$$
$$\tilde{\Gamma} \equiv \tilde{P}_{\mathrm{L}}^{\frac{1}{\alpha}} \cdot \mathcal{L} \cdot \tilde{Q}_{\mathrm{L}} + \tilde{P}_{\mathrm{H}}^{\frac{1}{\alpha}} \cdot \mathcal{H} \cdot \tilde{Q}_{\mathrm{H}};\ \text{and}\ \mathcal{Z} \equiv \zeta\,[(\sigma + \gamma)/\Xi + \sigma + \gamma + 1]\,(\tilde{Q} + 1).$$

The preceding equations represent a steady-state equilibrium with balanced growth in the usual sense, such that the endogenous growth rates are positive, $\tilde{g}_{N_{\mathrm{L}}} = \tilde{g}_{N_{\mathrm{H}}} = \tilde{x}_m > 0$ and $\tilde{g}_{Q_{\mathrm{L}}} = \tilde{g}_{Q_{\mathrm{H}}} = \tilde{g} = (\sigma + \gamma + 1)\tilde{x}_m > 0$. Thus, our model predicts, under a sufficiently productive technology, a BGP with constant positive growth rates, where $g > g_{N_m}$ due to the growth of intermediate-good quality, $\Xi \cdot I_m$ (see (11)).

The level variables are $\tilde{C} = \tilde{z}\tilde{Q}_{\mathrm{L}}$, $\tilde{N}_{\mathrm{L}} = (\zeta/\phi)^{1/(\sigma+\gamma+1)}(\tilde{x}_{\mathrm{L}})^{-\gamma/(\sigma+\gamma+1)}(\tilde{Q}_{\mathrm{L}})^{1/(\sigma+\gamma+1)}$, $\tilde{N}_{\mathrm{H}} = (\zeta/\phi)^{1/(\sigma+\gamma+1)}(\tilde{x}_{\mathrm{H}})^{-\gamma/(\sigma+\gamma+1)}[(\mathcal{H}/\mathcal{L})\tilde{Q}_{\mathrm{L}}]^{1/(\sigma+\gamma+1)}$, where $\tilde{Q}_{\mathrm{L}}$ is undetermined. From the expressions for X_{L} and X_{H} (see Section 2.1) and for N_{L} and N_{H} (preceding), we derive the steady-state expressions for relative production and the relative number of firms (i.e., the H- vis-á-vis the L-technology sector), $\tilde{X} \equiv (X_{\mathrm{H}}\tilde{/}X_{\mathrm{L}}) = \mathcal{H}/\mathcal{L}$, $\tilde{N} \equiv (N_{\mathrm{H}}\tilde{/}N_{\mathrm{L}}) = (\mathcal{H}/\mathcal{L})^{1/(\sigma+\gamma+1)}$. Finally, we get the steady-state skill premium $\tilde{W} = h/l$.

To characterize the interior steady state $(\tilde{x}_{\mathrm{L}}, \tilde{z}_{\mathrm{L}}, \tilde{x}_{\mathrm{H}}, \tilde{Q})$ in terms of local stability, we linearize the dynamical system (14)–(17) in a neighbourhood of $(\tilde{x}_{\mathrm{L}}, \tilde{z}_{\mathrm{L}}, \tilde{x}_{\mathrm{H}}, \tilde{Q})$

and obtain the following fourth-order system:

$$\begin{pmatrix} \dot{x}_{\mathrm{L}} \\ \dot{z}_{\mathrm{L}} \\ \dot{x}_{\mathrm{H}} \\ \dot{Q} \end{pmatrix} = \begin{pmatrix} a_{11}\tilde{x}_{\mathrm{L}} & a_{12}\tilde{x}_{\mathrm{L}} & a_{13}\tilde{x}_{\mathrm{L}} & \frac{\Xi}{\gamma}\left(\frac{\partial \tilde{I}_{\mathrm{L}}}{\partial Q}\right)\tilde{x}_{\mathrm{L}} \\ a_{21}\tilde{z}_{\mathrm{L}} & a_{22}\tilde{z}_{\mathrm{L}} & a_{23}\tilde{z}_{\mathrm{L}} & a_{24}\tilde{z}_{\mathrm{L}} \\ a_{31}\tilde{x}_{\mathrm{H}} & a_{32}\tilde{x}_{\mathrm{H}} & a_{33}\tilde{x}_{\mathrm{H}} & a_{34}\tilde{x}_{\mathrm{H}} \\ -\tilde{Q} & 0 & \tilde{Q} & -\Xi\mathcal{S}_1\tilde{Q} \end{pmatrix} \begin{pmatrix} x_{\mathrm{L}} - \tilde{x}_{\mathrm{L}} \\ z_{\mathrm{L}} - \tilde{z}_{\mathrm{L}} \\ x_{\mathrm{H}} - \tilde{x}_{\mathrm{H}} \\ Q - \tilde{Q} \end{pmatrix}. \qquad \mathbf{(18)}$$

The Jacobian matrix $J(\tilde{x}_{\mathrm{L}}, \tilde{x}_{\mathrm{H}}, \tilde{z}_{\mathrm{L}}, \tilde{Q})$, in (18), is evaluated at the steady state, where we define $a_{11} \equiv -\frac{1}{\gamma}(\frac{\Xi}{\Xi+1})\mathcal{S}_0 - \frac{\sigma+\gamma}{\gamma}$; $a_{12} \equiv -\frac{1}{\zeta}\frac{1}{\gamma}(\frac{\Xi}{\Xi+1})\mathcal{S}_0$; $a_{13} \equiv -\frac{1}{\gamma}(\frac{\Xi}{\Xi+1})\mathcal{S}_0\frac{\mathcal{H}}{\mathcal{L}}$; $a_{21} \equiv (\frac{1}{\theta} + \Xi)\frac{1}{\Xi+1}\mathcal{S}_0 - 1$; $a_{22} \equiv (\frac{1}{\theta} + \Xi)\frac{1}{\Xi+1}\frac{1}{\zeta}\mathcal{S}_0$; $a_{23} \equiv (\frac{1}{\theta} + \Xi)\frac{1}{\Xi+1}\frac{\mathcal{H}}{\mathcal{L}}\mathcal{S}_0$; $a_{24} \equiv \frac{\pi_0}{\theta\zeta}\frac{1}{2e}\mathcal{L} - (\frac{1}{\theta} + \Xi)(\frac{\partial \tilde{I}_{\mathrm{L}}}{\partial Q})$; $a_{31} \equiv a_{11} + \frac{\sigma+\gamma}{\gamma}$; $a_{32} = a_{12}$; $a_{33} \equiv a_{13} - \frac{\sigma+\gamma}{\gamma}$; $a_{34} \equiv a_{14} - \frac{\Xi}{\gamma}\mathcal{S}_1$, with $\mathcal{S}_0 \equiv 1/(1 + \frac{\mathcal{H}}{\mathcal{L}})$; $\mathcal{S}_1 \equiv \frac{\pi_0}{\zeta}\frac{1}{2e}\mathcal{L}\frac{1}{\tilde{Q}\mathcal{S}_0}$; $(\frac{\partial \tilde{I}_{\mathrm{L}}}{\partial Q}) = [\mathcal{S}_1\tilde{Q} - (\frac{1}{\Xi+1} + \frac{\sigma+\gamma}{\Xi})\tilde{x}_{\mathrm{H}}]\mathcal{S}_0 + \frac{1}{\zeta}\frac{1}{e}\mathcal{H}\frac{1}{\Xi+1}\chi\frac{1}{\tilde{Q}}$.

Because there are three predetermined variables, x_{L}, x_{H}, and Q, and one jump variable, z_{L}, saddlepath stability in the neighborhood of the interior equilibrium $(\tilde{x}_{\mathrm{L}}, \tilde{x}_{\mathrm{H}}, \tilde{z}_{\mathrm{L}}, \tilde{Q})$ requires that $J(\tilde{x}_{\mathrm{L}}, \tilde{x}_{\mathrm{H}}, \tilde{z}_{\mathrm{L}}, \tilde{Q})$ have three eigenvalues with a negative real part and one with a positive real part, hence implying that $\det(J(\tilde{x}_{\mathrm{L}}, \tilde{x}_{\mathrm{H}}, \tilde{z}_{\mathrm{L}}, \tilde{Q})) < 0$. However, as the latter condition is compatible with both one and three eigenvalues with negative real part, further conditions must be satisfied so that saddlepath stability applies. These conditions are particularly hard, if even possible, to check analytically, considering that $J(\tilde{x}_{\mathrm{L}}, \tilde{x}_{\mathrm{H}}, \tilde{z}_{\mathrm{L}}, \tilde{Q})$ is a 4×4 matrix with just one zero element [see, e.g., Eicher and Turnovsky (2001)].[8] In this context, we perform a numerical exercise to check the existence of three eigenvalues with negative real part and one with a positive real part for reasonable values of the parameters, and conclude that[9]

Remark 1. The interior steady state is locally saddlepath stable for the typical baseline parameter values, but also over a wide range of parameter sets.

Finally, it is noteworthy that, because the dimension of the stable manifold is larger than unity (it is three-dimensional), there are multiple independent sources of stability in the dynamical system, which interact among themselves. Thus, non-monotonic trajectories can emerge in the predetermined variables along transition even in the case of a linearized dynamical system.

3. INDUSTRY AND AGGREGATE DYNAMICS

3.1. Comparative Dynamics

This section focuses on the change of the industry structure over time and on its relationship with the dynamics of the aggregate variables. We start by considering that the economy is in the (pre-shock) steady state; then we posit an unanticipated one-off shock in the proportion of high-skilled labor that shifts the steady state.[10] The dynamical system (14)–(17) describes the transitional dynamics after the

TABLE 1. Post-shock and pre-shock steady state values, corresponding to, respectively, $H/L = 0.19$ and $H/L = 0.10$

H/L	ζ	$\tilde{z}$	$\tilde{Q}$	$\tilde{x}_m$	$\tilde{g}$	$\tilde{I}_m$	$\tilde{r}$
0.19	0.42	0.1204	0.2600	0.0073	0.0248	0.0208	0.0572
0.10	0.42	0.0976	0.1300	0.0063	0.0214	0.0179	0.0521

shock, toward the new (post-shock) steady state of $(x_{\mathrm{L}}, x_{\mathrm{H}}, Q, z_{\mathrm{L}})$. This allows us to assess the industry dynamics, measured by the time path of the relative number of firms, $N(t) = [x_{\mathrm{H}}(t)/x_{\mathrm{L}}(t)]^{-\gamma/(\sigma+\gamma+1)} Q(t)^{1/(\sigma+\gamma+1)}$, relative production, $X(t) = (\mathcal{H}/\mathcal{L})^{1/2} Q(t)^{1/2}$, the sectoral growth rates, $g_{Q_{\mathrm{H}}}(t) = I_{\mathrm{L}}(t) \cdot \Xi + x_{\mathrm{L}}(t)$, $g_{Q_{\mathrm{L}}}(t) = I_{\mathrm{H}}(t) \cdot \Xi + x_{\mathrm{H}}(t)$, and the skill premium, $W(t) = (h/l)(\mathcal{H}/\mathcal{L})^{-1/2} Q(t)^{1/2}$. At the aggregate level, the dynamics is given by the time path of the economic growth rate, $g(t) = \{\mathcal{L}^{1/2} \cdot g_{Q_{\mathrm{L}}}(t) + [Q(t) \cdot \mathcal{H}]^{1/2} \cdot g_{Q_{\mathrm{H}}}(t)\} / \{\mathcal{L}^{1/2} + [Q(t) \cdot \mathcal{H}]^{1/2}\}$, and the real interest rate, $r(t) = \pi_0 \cdot \mathcal{L} \cdot P_{\mathrm{L}}(t)^{1/\alpha}/\zeta - I_{\mathrm{L}}(t) = \pi_0 \cdot \mathcal{H} \cdot P_{\mathrm{H}}(t)^{1/\alpha}/\zeta - I_{\mathrm{H}}(t)$.

Relative to the baseline case, a rise in H/L occurs by assuming a jump in high-skilled labor, H, from 0.1 to 0.19, whereas the low-skilled labor, L, is normalized to unity.[11] This then implies that the initial and the new steady state/BGP are characterized by, respectively, $H/L = 0.1$ and $H/L = 0.19$. These correspond to the average value of the proportion of the high skilled (measured by the ratio of college graduates to non-college graduates) in the 14 countries presented in Figure 1, as found in Barro and Lee (2010)'s data set for 1980 and 1995, respectively.[12]

As for the structural parameters, we let $\rho = 0.02$; $\theta = 1.5$; $A = 1$; $\phi = 1$; $\alpha = 0.6$; $\lambda = 2.5$; $\sigma = 1.2$; $\gamma = 1.2$; $l = 1.0$; $h = 1.3$.[13] Given that, along the BGP, we have $g_{Q_m} - g_{N_m} = (\sigma + \gamma) g_{N_m}$, we let $\sigma + \gamma = 2.4$ to match the ratio between the growth rate of the average firm size and the growth rate of the number of firms found in cross-sectional data for European countries in the period 1995–2007, whereas the values for l and h are in line with Afonso and Thompson (2011), also drawn from European data. Because it has no impact on the growth rates, ϕ was normalized to unity, whereas the values for θ, ρ, λ, and α were set in line with the standard literature [see, e.g., Barro and Sala-i-Martin (2004)]. The values of the remaining parameters, A and ζ, were chosen to calibrate the after-shock BGP economic growth rate, g, around 2.5 percent/year (see Table 1), matching the average of the per capita GDP growth rate across the 14 European countries over the period 1995–2007.

In what follows, we are interested in analyzing both the long-run effects (shift in the steady-state/BGP values) and their decomposition into short-run and transitional-dynamics effects of a unanticipated one-off increase in the amount of high-skilled labor, H, with low-skilled labor, L, remaining constant through time, as referred to previously (see Figure 2).

Industry dynamics: short-run effect. The increase in H generates an increase in resources available for R&D. However, the allocation of resources is unbalanced

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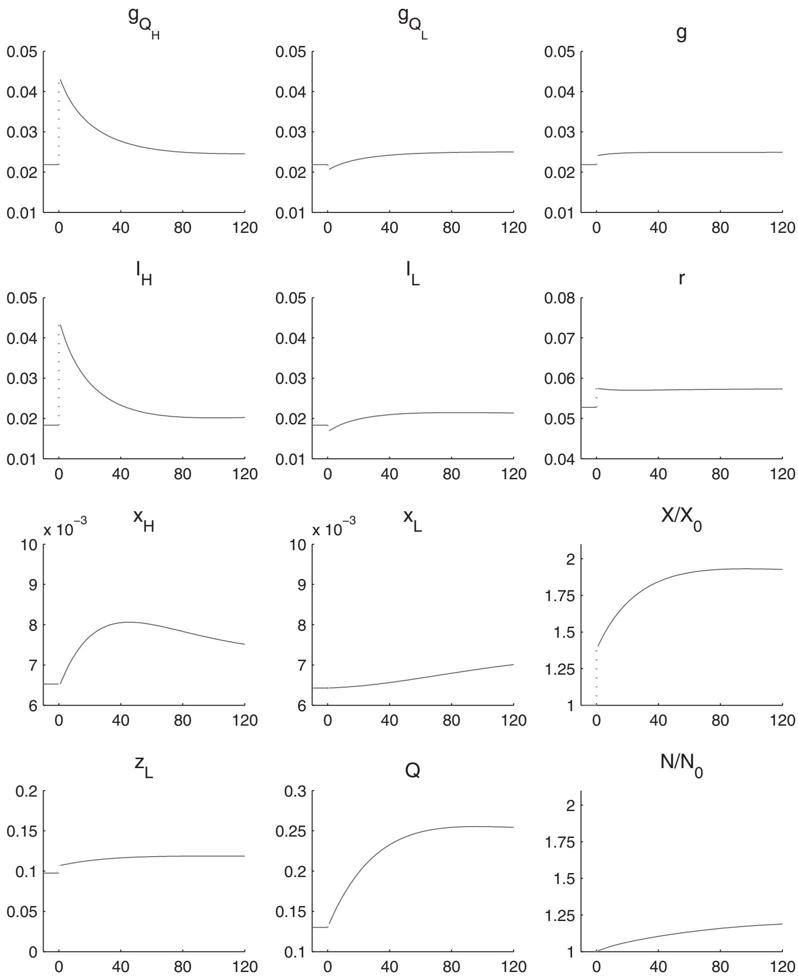

FIGURE 2. Short-run effects and transitional dynamics of the aggregate and industry-level variables as H increases from 0.1 to 0.19. Values for X and N are adjusted by their pre-shock initial values, $X(0) = X_0$ and $N(0) = N_0$, to facilitate comparison between the time variation of those two variables.

between sectors. The direct positive impact on the profitability of the production of intermediate goods in the H-technology sector more than compensates for the decrease in the price index, P_{H}, due to the fall in the marginal productivity of labor in that sector; thus, an increase in the vertical-innovation rate I_{H} occurs. The diversion of resources from the L-technology to the H-technology sector induces a fall in I_{L}, although only slightly because of the countervailing effect of the upward jump in the price index, P_{L}. As a result, the sectoral growth rate in the

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H-technology sector, g_{Q_H}, jumps upward, while the growth rate in the L-technology sector, g_{Q_L}, experiences a small shift downward.[14]

Industry dynamics: transitional-dynamics and long-run effects. After the initial jump, g_{Q_H} takes a downward path, while g_{Q_L} follows an upward path; the former reflects the behavior of the *intensive* margin, I_H, which more than compensates for the extensive margin, x_H; in contrast, the increase in g_{Q_L} reflects the behavior of both I_L and x_L. After the initial level effect, we have $I_\mathrm{H} > I_\mathrm{L}$, which implies an increase in the technological-knowledge bias, Q; then P_H decreases and P_L increases toward the new steady state and, thus, responding to this feedback effect, I_H decreases and I_L increases. In turn, x_H and x_L rise because of the increase in both Q_H and Q_L (given $I_\mathrm{H} > 0$ and $I_\mathrm{L} > 0$), reflecting the complementarity between the horizontal-entry rate and the technological-knowledge stock (see (9)); however, the fact that $I_\mathrm{H} > I_\mathrm{L}$ means that the costs pertaining to horizontal entry are only slightly compensated for in the L-technology sector at the beginning of the transition path (see (14) and (16)), whereas the opposite occurs in the other sector, explaining the different shapes of the time paths of x_H and x_L. Because $x_\mathrm{H} > x_\mathrm{L}$ throughout transition, the relative number of firms, N, rises but at a decreasing rate, reflecting the congestion effects in horizontal R&D. In turn, the increase in relative production, X, commanded by Q, is faster because of the absence of congestion effects in vertical R&D. In the long run, both the relative number of firms and relative production increase relative to the pre-shock steady state.

Aggregate dynamics. The economic growth rate, g, and the real interest rate, r, experience only a very slight increase along the transition path; thus, the long-run effect of an increase in H results almost entirely from the short-run response to the exogenous shock. The stability of the aggregate variables over transition reflects the opposing movements of the sectoral growth rates, g_{Q_H} and g_{Q_L}, in the case of g,[15] and the parallel movements of the vertical innovation rate, I_m, and the price index, P_m, within each m-technology sector, in the case of r.

3.2. Discussion and Calibration

As the economy evolves toward the new steady state, there is a shift of economic activity between sectors. For the baseline values of the parameters, relative production, X, and the relative number of firms, N, increase a total of, respectively, 13.0 and 12.4 percentage points over the long run (i.e., variation between the initial and the final steady states), whereas the economic growth rate, g, and the real interest rate, r, increase a total of, respectively, 0.34 and 0.51 percentage points. Thus, the aggregate variables remain roughly unchanged, which implies, in particular, that the share of the high-tech sector has roughly a null correlation with the economic growth rate over the adjustment.

Also, the speed of adjustment to a positive shock in the relative supply of skills may be quite different across variables (see Figure 3). The speeds of convergence

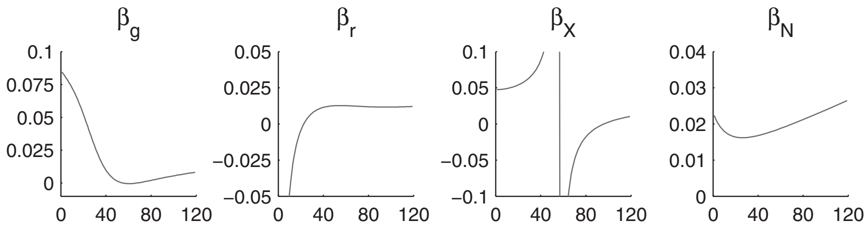

FIGURE 3. Time profile of the speed of convergence for the variables of interest (aggregate and industry levels), measured as $\beta_y(t) = -\dot{y}(t)/(y(t) - \tilde{y})$ [see Eicher and Turnovsky (2001)], where $\tilde{y}$ is the BGP value of a given variable y and $y \in \{g, r, X, N\}$.

are also time-varying for each variable. Overall, our model implies a speed of convergence toward the new steady state that is faster at the sectoral than at the aggregate level. In the first 50 years, the variable with the greatest speed is relative production, and from then on it is the relative number of firms. This result is mainly explained by the asymmetric impact of the complexity and congestion costs on vertical and horizontal R&D, which then implies different speeds of convergence of the two industry-structure variables.

Moreover, Figure 2 suggests that our theoretical results are qualitatively consistent with the empirical facts of industry dynamics. According to the time series data for the 14 European countries in Figure 1, both measures of industry structure are growing over time, but with relative production outpacing the relative number of firms. Our results are also characterized by a rising technological-knowledge bias and thus an increasing skill premium over transition, a prediction that also seems to be corroborated by the available data for the same set of European countries.[16]

But we also wish to assess the quantitative relevance of our transitional-dynamics mechanism in addressing the observed distinct performance of relative production and the relative number of firms. Thus, we extend our calibration exercise as follows. We consider the shock on H/L as a change from 1980 to 1995, as in Section 3.1, with a one-year lag for the impact on the industry structure. However, given the uncertainty regarding the timing and intensity of the shock, we also include a scenario with a 5-year lag for the impact on the industry structure and, thus, in which the shock on H/L is measured considering the time span from 1980 to 1990 [in this case, we let H/L increase from 0.1 to 0.15, according to the data on the 14 countries in Barro and Lee (2010)].

This exercise is run by setting ρ, θ, α, λ, ζ, and $\sigma + \gamma$ to their baseline values. However, the parameter that regulates the horizontal R&D congestion cost, γ, is crucial for determining the speed of convergence of N and X and hence their growth rates per period over transition. Given the lack of empirical guidance regarding this parameter, we consider, as a sensitivity analysis, different values for γ (and thus for σ) in the interval (0; 2.4) (recall that $\sigma + \gamma = 2.4$ in the baseline scenario). Figure 4 shows a monotonic relationship between γ and the predicted values for the transitional growth rates of N and X. As expected, lower values of γ

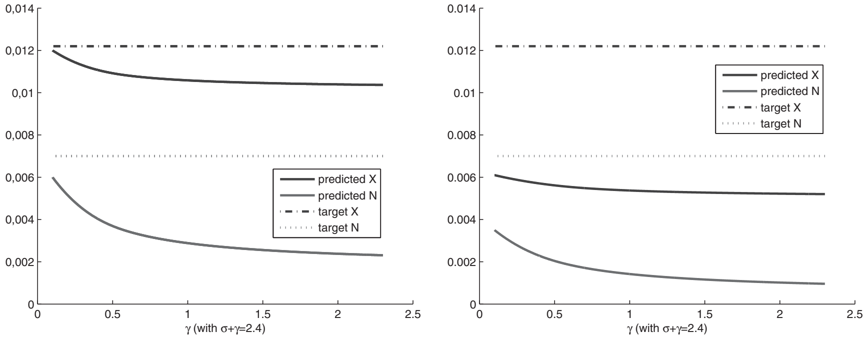

FIGURE 4. Calibration exercise for the annual growth rates of relative production and the relative number of firms. "Target X" and "Target N" are the observed values for the annual growth rates over the period 1995–2007, computed as the log differences of the weighed averages of each variable across 14 European countries (see country data and source description in Figure 1). "Predicted X" and "Predicted N" are the predicted values for the transitional growth rates. In the left (right) panel, they were obtained by considering the dynamic equations for $N(t)$ and $X(t)$, from $t = 1$ to $t = 13$ ($t = 5$ to $t = 17$), where $t = 0$ corresponds to the year 1994 (1990), after an initial shock that increased H/L from 0.1 to 0.19 (from 0.1 to 0.15). Parameter values are $\rho = 0.02$, $\theta = 1.5$, $\alpha = 0.6$, $\lambda = 2.5$, $\zeta = 0.42$, and $\sigma + \gamma = 2.4$, as in Figure 2, whereas γ takes values in the interval [0.1; 2.3].

yield higher growth rates of both N and X, but with a stronger effect on the former as γ approaches the lower boundary, because shifts in that parameter impact the horizontal entry cost directly and the vertical entry cost only indirectly.

Table 2 summarizes the results of the calibration exercise by considering the baseline and the upper and lower values for γ. The results show that, under the hypothesis of an initial increase in relative supply of skills from 0.1 to 0.19 and a 1-year lag impact (from 0.1 to 0.15 and 5-year lag impact), the model accounts for, respectively, 84 to 99% and 33 to 90% (41 to 50% and 13 to 50%) of the average annual growth rate of relative production and of the relative number of firms observed in the data. As a robustness check, we look into the ability of the model to replicate the dynamic behavior of the skill premium and find that the model accounts for from 93 to over 100% (47 to 56%) of its annual growth rate observed in the data.

4. CONCLUDING REMARKS

This paper shows that, under the hypothesis of a positive shock in the proportion of high-skilled labor, the technological-knowledge bias channel leads to unbalanced sectoral growth, whereas the aggregate variables are roughly unchanged. The theoretical results are qualitatively and quantitatively consistent with the increase in the share of the high-tech sectors found in time-series data, computed as a weighed average across 14 European countries. However, importantly, the model

TABLE 2. Calibration exercise for the annual growth rates of relative production, the relative number of firms, and the skill premium

	Relative production (annual growth rate, %)	Relative number of firms (annual growth rate,%)	Skill premium (annual growth rate,%)
	H/L increases from 0.1 to 0.19 (1-year lag)		
$\gamma = 2.3$	1.03	0.23	0.87
$\gamma = 1.2$(baseline)	1.18	0.34	0.99
$\gamma = 0.1$	1.21	0.63	1.06
	H/L increases from 0.1 to 0.15 (5-year lag)		
$\gamma = 2.3$	0.50	0.09	0.44
$\gamma = 1.2$(baseline)	0.56	0.14	0.49
$\gamma = 0.1$	0.61	0.35	0.53
Observed	1.22	0.70	0.94

Notes: Observed and predicted values for the growth rates of the former two are computed as described in Figure 4. The observed values for the growth rates of the skill premium cover the period 2002–2006 (see data description in fn. 16), whereas the predicted values are based on the dynamic equation for $W(t)$. In the upper (lower) panel, the values were obtained by considering the time path from $t = 8$ to $t = 12$ ($t = 12$ to $t = 16$), where $t = 0$ corresponds to the year of 1994 (1990). Parameter values are the same as in Figure 4.

predicts that the dynamics of the share of the high-tech sector has no significant impact on the economic growth rate. Therefore, inasmuch as the change in industry structure is mainly driven by a shift in the proportion of high-skilled workers, our results suggest that raising the share of the high-tech sector may be largely ineffective in stimulating economic growth.

It would be interesting to extend our model to a setting in which total factor productivity (TFP) growth rates were not homogeneous across the high- and low-tech sectors along the BGP, and analyze the implications of the cross-sector differences in TFP growth rates for the dynamics of sectoral input reallocation and of the economic growth rate. However, it should be noted that in this case, and in line with the recent literature on structural change [e.g., Ngai and Pissarides (2007)], the unbalanced sectoral growth may induce an ever-increasing (-decreasing) share of the sector with higher (lower) TFP growth. In contrast, in the model developed in this paper, when the BGP is (asymptotically) reached, balanced growth at both the aggregate and the sectoral level is established, and thus no sector ever vanishes, as seems to be the case empirically.

NOTES

1. The source is the Eurostat on-line database on Science, Technology and Innovation, available at http://epp.eurostat.ec.europa.eu, where the OECD classification of high- and low-tech sectors is used [see Hatzichronoglou (1997)]. By crossing the data on both variables—production and the number of firms—and considering a minimum time span of 12 years (which is the maximum time span available for the number of firms), we end up with a sample of 14 European countries.

2. Empirical evidence suggests that high-tech sectors are more intensive in high-skilled labor than low-tech sectors. For instance, according to the data for the average of the European Union (27 countries, 2007), 30.9% of employment in the high-tech manufacturing sectors is high-skilled ("college graduates"), against 12.1% of employment in the low-tech sectors (source: Eurostat on-line database on Science, Technology and Innovation, http://epp.eurostat.ec.europa.eu).

3. According to Barro and Lee (2010)'s data set, the proportion of the high-skilled (measured by the ratio of college to non-college graduates) in the 10 countries with available data for relative production depicted by Figure 1 accelerated from 4.11 to 5.76 percent in 1980–1995 and then slowed down from 3.35 to 0.51 percent in 1995–2007.

4. Sections 2.1 to 2.4 are an abridged version of the corresponding sections in Gil et al. (2015).

5. Because the data show that the high-tech sectors are more intensive in high-skilled labor than the low-tech sectors (see fn. 2), we take the high- and low-skilled labor-specific intermediate-good sectors in the model as the theoretical counterpart of the high- and low-tech sectors.

6. Henceforth, we will also refer to the "m-specific intermediate-good sector" as the "m-technology sector".

7. The proof is available in Gil et al. (2015).

8. Because the characteristic polynomial for the linearized system (18) is of the form $P_o(\beta) = \beta^4 + b_3\beta^3 + b_2\beta^2 + b_1\beta + b_0$, where β denote the characteristic roots of matrix $J(\tilde{x}_{\mathrm{L}}, \tilde{x}_{\mathrm{H}}, \tilde{z}_{\mathrm{L}}, \tilde{Q})$, and the coefficients b_{4-k}, $k = 1, .., 4$, equal the sum of the kth-order principal minors (in particular, $b_0 = \det(J)$ and $b_3 = -\mathrm{tr}(J)$), those conditions rely on the solution for a quartic equation (see, e.g., King, 2000; Brito, 2004). Considering partitions in the space of b_{4-k} for the number of pairs of complex eigenvalues, it can be shown that the necessary and sufficient conditions for the existence of three eigenvalues with negative real part and one with a positive real part are: (i) for zero complex eigenvalues, $b_0 < 0$ and $(b_1 < 0, b_2 < 0 \text{ or } b_2 > 0, b_3 > 0)$; (ii) for one pair of complex conjugate eigenvalues, $b_0 < 0$ and $(b_3 > 0, h_1 < 0 \text{ or } b_1 < 0, b_3 > 0, h_1 = 0 \text{ or } b_1 < 0, h_1 > 0)$, where $h_1 = b_0 b_3^2 + b_1^2 - b_1 b_2 b_3$.

9. To numerically verify the local saddlepath stability, we: (i) assume a sensible interval of variation for each parameter value and (ii) rerun the computation of the eigenvalues of matrix J, in (18), by letting a given parameter take the values in that interval, whereas the other parameters are set to their baseline values, presented in Section 3. This experimentation shows that there are always three eigenvalues with negative real part and one with a positive real part for the considered broad range of parameter values, thus satisfying the conditions for local saddlepath stability stated in fn. 8. The results are available in Gil et al. (2015).

10. We run a VAR model in order to test Granger causality between the proportion of high-skilled labor and the share of production of the high-tech sector, for the period 1980-1995. Our results (which are available in Gil et al., 2015) support causality running from the share of the high skilled to the share of production of the high-tech sector in European data.

11. Available data suggest that increases in H have clearly been larger than those in L over time. For instance, the annual average variation of college graduates (the usual proxy for high-skilled labor) and non-college graduates (the proxy for the low-skilled) was, respectively, 5.04 and 0.15%, computed as the average of the 14 European countries presented in Figure 1 for the period 1980–1995 period. The data are from Barro and Lee (2010)'s data set.

12. The first year (1980) is determined by data availability for production, whereas the final year (1995) was chosen by observing that by that time there was a significant acceleration of the share of the high-skilled and of the share of production of the high-tech sector (see fn. (3) and Figure 1).

13. The value of the discount rate, ρ, implies that each period in our model represents a year.

14. Notice that, because x_{L}, x_{H} and Q are pre-determined variables, they do not experience any short-term response to the exogenous shock.

15. In fact, because g is a weighed average of $g_{Q_{\mathrm{H}}}$ and $g_{Q_{\mathrm{L}}}$, with the weight being a function of Q, i.e., the share of the H-technology sector in terms of the technological-knowledge stock, Q also plays a direct role in the dynamics of g. More specifically, the effect of the relatively intense fall in $g_{Q_{\mathrm{H}}}$ is dampened by the increase in Q over transition.

16. In the 14 European countries considered in Figure 1, the skill premium – measured as the mean annual earnings of the college graduates employed in manufacturing vis-á-vis the mean annual earnings of the non-college graduates – increased from an average of 1.678, in 2002, to 1.742, in 2006, implying a growth rate of 0.94 percent/year (*Source*: Eurostat on-line database on Science, Technology and Innovation, http://epp.eurostat.ec.europa.eu).

REFERENCES

Acemoglu, D. and F. Zilibotti (2001) Productivity differences. *Quarterly Journal of Economics* 116(2), 563–606.

Afonso, O. and M. Thompson (2011) Costly investment, complementarities and the skill premium. *Economic Modelling* 28, 2254–2262.

Barro, R. and X. Sala-i-Martin (2004) *Economic Growth*, 2nd ed. Cambridge, MA: MIT Press.

Barro, R.J. and J.-W. Lee (2010) A New Data Set of Educational Attainment in the World, 1950–2010. NBER working paper 15902, pp. 1–47.

Brito, P. (2004) Local Dynamics for Four-Dimensional Models: A Complete Characterization. Mimeo, ISEG, Universidade de Lisboa.

Eicher, T. and S. Turnovsky (2001) Transitional dynamics in a two-sector non-scale growth model. *Journal of Economic Dynamics and Control* 25, 85–113.

European Commission (2010) Europe 2020—A Strategy for Smart, Sustainable and Inclusive Growth. Discussion paper, Communication from the Commission.

Geroski, P. (1995) What do we know about entry? *International Journal of Industrial Organization* 13, 421–440.

Gil, P.M., O. Afonso, and P. Vasconcelos (2015) Skill-structure shocks, the share of the high-tech sector and economic growth dynamics. *FEP Working Papers* 554, 1–47.

Gil, P.M., P. Brito, and O. Afonso (2013) Growth and firm dynamics with horizontal and vertical R&D. *Macroeconomic Dynamics* 17(7), 1438–1466.

Hatzichronoglou, T. (1997) Revision of the High-Technology Sector and Product Classification. OECD/STI working paper 2, pp. 1–26.

Johansson, B., C. Karlsson, M. Backman, and P. Juusola (2007) The Lisbon Agenda from 2000 to 2010. CESIS Electronic working paper, 106, pp. 1–46.

King, R.B. (2000) *Beyond the Quartic Equation*, Modern Birkhauser reprint ed. Boston: Birkhäuser.

Ngai, R. and C. Pissarides (2007) Structural change in a multi-sector model of growth. *American Economic Review* 97(1), 429–443.