Construzione degli assintoti della conica inviluppata dalle rette AA′, BB′, ... congiungenti i punti corrispondenti di due punteggiate projettive $r \equiv AB......,r' \equiv A'B'......$

Le coppie di tangenti parallele determinano sopra una tangente fissa r una involuzione di punti AA_1, BB_1,... il cui punto centrale R è il punto in cui r tocca la conica. Percio, se, in r, si prende $\overline{RP}^2 = \overline{RQ}^2 = RA.RA_1$, saranno P,Q i punti d'intersezione di r cogli assintoti.

La conica sia adunque data mediante due rette punteggiate projettive, sia S il punto ad esse commune, R il punto di contatto della prima, e T il punto della stessa prima punteggiata che corrisponde all' infinito della seconda. Allora prendendo nella prima retta $\overline{RP}^2 = \overline{RQ}^2 = RS.RT$, i punti P,Q appartengono agli assintoti.

On a Problem in Partition of Numbers.

By Professor CHRYSTAL.

At a recent meeting of the Royal Society of Edinburgh, Professor Tait proposed and solved the following problem :—

To calculate the number of Partitions of any number that can be made by taking any number from 2 up to another given number.

Let us denote by ${}_nP_r$ the number of partitions of r obtained by taking any of the numbers 2, 3, 4,......$(n-1)$, n. In the particular case $n=7$, $r=10$, the actual partitions are $3+7$, $4+6$, $5+5$; $2+2+6$, $2+3+5$, $2+4+4$, $3+3+4$; $2+2+2+4$, $2+2+3+3$; $2+2+2+2+2$; ten in all. Hence ${}_7P_{10}=10$.

The object proposed here is not to find an analytical expression for ${}_nP_r$, but to give a process for quickly calculating a table of double entry for it. The following has some advantages over the method given by Professor Tait although the result is in reality much the same.

$$\text{Since } \frac{1}{(1-x^2)(1-x^3)......(1-x^n)} = (1+x^2+x^{2.2}+x^{3.2}+......)$$
$$\times(1+x^3+x^{2.3}+x^{3.3}+......)$$
$$\times\cdots\cdots\cdots\cdots\cdots\cdots$$
$$\times(1+x^n+x^{2n}+x^{3n}+......);$$

we have obviously

$$\frac{1}{(1-x^2)(1-x^3)\ldots\ldots(1-x^n)} = 1 + {}_nP_1x + {}_nP_2x^2 + \ldots\ldots + {}_nP_rx^r + \ldots\ldots;$$

$$\frac{1}{(1-x^2)(1-x^3)\ldots\ldots(1-x^n)(1-x^{n+1})} = 1 + {}_{n+1}P_1x + {}_{n+1}P_2x^2 + \ldots\ldots + {}_{n+1}P_rx^r + \ldots\ldots;$$

whence

$$(1-x^{n+1})(1 + {}_{n+1}P_1x + {}_{n+1}P_2x^2 + \ldots\ldots) = 1 + {}_nP_1x + {}_nP_2x^2 + \ldots\ldots.$$

Equating Coefficients we have

$$\begin{array}{ll} {}_{n+1}P_1 = {}_nP_1 & {}_{n+1}P_{n+1} = {}_nP_{n+1} + 1 \\ {}_{n+1}P_2 = {}_nP_2 & {}_{n+1}P_{n+2} = {}_nP_{n+2} + {}_nP_1 \\ \cdots\cdots = \cdots\cdots & \cdots\cdots\cdots\cdots = \cdots\cdots\cdots\cdots \\ {}_{n+1}P_n = {}_nP_n & {}_{n+1}P_{n+r} = {}_nP_{n+r} + {}_nP_{n-r+1}. \end{array}$$

Remembering that

$${}_2P_0 = 1, \quad {}_2P_1 = 0, \quad {}_2P_2 = 1, \quad {}_2P_3 = 0, \ \&c.,$$

we can, therefore, tabulate (see fig. 32) the values of ${}_nP_r$ on a piece of paper ruled into squares, as follows:—First, write in the upper line 1,0,1,0,1,0, &c. Through the second 1 draw the diagonal EF, then the numbers in the part of any column under this diagonal are simply the numbers *on the diagonal* repeated over and over again. These need not be written down. The lines to the right of the column are filled in thus—place a piece of paper cut in the form ABDC, with AB on the line GK, AC along a perpendicular to GK, and the blank in the line to be filled next to the last step of BD on that line. Then the blank is filled by adding the number above it to the number lowest in position on the immediate left of AC, whether that number lie in the first row 1,0,1,0,1, &c, or on the diagonal EF, or in the part of the line we are dealing with which has been already filled in. As ABDC is placed in the figure, the 25th square of the 20th line has just been filled in by adding 376 to 2.

La Tour d' Hanoï.

By R. E. Allardice, M.A., and A. Y. Fraser, M.A.

§ 1.—The following account of this problem is taken from the *Journal des Débats* for December 27th, 1883.

La poste nous a apporté ces jours-ci une petite boîte en carton peint, sur laquelle on lit: *la Tour d' Hanoï*, véritable casse-tête

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