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Principle of high accuracy for the nonlinear theory of the acceleration of electrons in a vacuum by lasers at relativistic intensities

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Abstract

Acceleration of electrons by lasers in a vacuum was considered impossible based on the fact that plane-wave and phase symmetric wave packets cannot transfer energy to electrons apart from Thomson or Compton scattering or the Kapitza–Dirac effect. The nonlinear nature of the electrodynamic forces of the fields to the electrons, expressed as nonlinear forces including ponderomotion or the Lorentz force, permits an energy transfer if the conditions of plane waves in favor of the beams and/or the phase symmetry are broken. The resulting electron acceleration by lasers in a vacuum is now well understood as "free wave acceleration", as "ponderomotive scattering", as "violent acceleration", or as "vacuum beat wave acceleration". The basic understanding of these phenomena relates to an *accuracy principle of nonlinearity* for explaining numerous discrepancies on the way to the mentioned achievement of "vacuum laser acceleration", which goes beyond the well-known experience of necessary accuracy in both modeling and experimental work experiences among theorists and experimentalists in the field of nonlinearity. From mathematically designed beam conditions, an absolute maximum of electron energy per laser interaction has been established. It is shown here how numerical results strongly (both essentially and gradually) depend on the accuracy of the used laser fields for which examples are presented and finally tested by the criterion of the absolute maximum.

1. INTRODUCTION

Considerable attention is given to the aim of how to accelerate electrons by lasers—preferably in vacuum—to very high energies. The very high electric fields **E** in a laser beam are by orders of magnitudes higher than in the classical particle accelerators, for example, the presently reached (Perry & Mourou, 1994; Cowan *et al.*, 1999) petawatt laser pulses are focused in vacuum to intensities $I = 10^{20}$ W/cm^2 corresponding to an amplitude of the high frequency electric field of 2.7×10^{11} V/cm, about one hundred times higher than the electric field in a H atom at the Bohr radius. One of the first conferences (Channel, 1982) to use these fields for particle acceleration by lasers well appreciated (Sessler, 1982) that ions can be accelerated to 0.5 GeV energy immediately (Begay *et al.*, 1983; Haseroth & Hora, 1996) by irradiation of solid targets where dielectric plasma effects cause a shrinking of the laser beam by relativistic self-focusing (Hora, 1975, 1991; Esarey *et al.*, 1997) to diameters of about half the wavelength (Häuser *et al.*, 1992), as measured (Basov *et al.*, 1987) and understood from wave optics (Castillo *et al.*, 1984) including a soliton mechanism (Häuser *et al.*, 1992*a*).

For the acceleration of electrons it was underlined (Sessler, 1982) that the (transversal) **E**-field of the laser light goes into the wrong direction. Though we are focusing here only on the laser acceleration of electrons in a vacuum, we should mention marginally that there was an early and extensive discussion on how the addition of plasma-effects with their longitudinal electric fields (in the interesting direction) can be used. This includes plasma wave effects as the beat-wave (Tajima, 1985), wake-field (Katsouleas *et al.*, 1989), or the laser-driven large amplitude longitudinal pseudowave (Eliezer *et al.*, 1995) acceleration. These mechanisms, without deciding which of them, later produced small numbers

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of accelerated electrons in the range of a few MeV (Kitagawa *et al.*, 1992).

The mechanism of laser acceleration of electrons in a vacuum was excluded in the earlier discussions by the well-known result that a plane wave-packet of electromagnetic radiation with symmetric phase properties can never transfer energy to a free electron if either Thomson or Compton scattering, or the Kapitza–Dirac effect is ignored. This result (Sessler, 1982, 1988) is well known in literature as an exact solution of the Maxwellian equations since the fifties, as reproduced later (Scheid & Hora, 1989). This fact, from an exact solution of the equation of motion of electrons in a wave-packet of infinitely spread plane-waves is known as the Lawson–Woodward theorem. Since any electromagnetic field can be produced by linear superposition of plane-waves, any electron acceleration in vacuum was excluded (Sessler, 1982).

Discussions with Lawson (1989) were about the trapping of electrons in a vacuum within the intensity minima of standing wave or interference fields (Hora, 1988*b*), and how the electrons are accelerated by moving the intensity minima, whether the electrons are really moved and do not slip through the intensity maxima. It was then shown by an extensive numerical work (Cicchitelli *et al.*, 1990) with convergences only after up to the thirteenth iteration, and realizing that the motion was only of a third order effect, that the electrons trapped in the intensity minima are not statically pushed to the minima as Weibel (1957) suggested, but that there is a dynamic bouncing of the electron motion between equivalent field potentials. It was convincingly shown that the motion of the interference field carries on the electrons and results in an acceleration in agreement with the trivial calculation.

With respect to the argument of Sessler (1982) in the sense also of the Lawson–Woodward theorem there was the non-trivial question that if a phase symmetric plane-wave packet cannot accelerate electrons, and since any electromagnetic field can be produced by linear superposition of infinite plane waves, that there can never be an acceleration of electrons by laser fields in a vacuum. This argument overlooked the fact that the superposition was linear while the electromagnetic forces to the electrons are basically nonlinear (Hora, 1969, 1985) as clarified (Hora 1996, 2000) by distinguishing between the Lorentz forces from ponderomotive processes and how this is generalized in the nonlinear force (Hora, 1969, 1985, 1991, 1996, 2000). The complexity of the physics in connection with the classical ponderomotive force and the nonlinear force has been elaborated (Hora, 1996, 1999), and apart from clarifying numerous points, several open questions were underlined especially with a very short time interaction (Hora *et al.*, 1996).

One key question for the laser acceleration of electrons in a vacuum was the breaking the symmetry of "plane-waves" and/or "phase-symmetry in a wave-packet" such that laser acceleration of electrons in a vacuum does happen, contrary to the before mentioned arguments. An active modulation of the phase by electro-optical crystals for the superposition of two laser wave-fields was elaborated (Hora, 1988*a*), see Figure 1, as a first step, especially highlighted (Evans, 1988) as further evaluated by Du and Xu (1999).

The next step was to study plane-waves with a completely asymmetric phase by using "rectified" laser fields where only half waves were used while the following half waves were eliminated as in ac-rectifiers (Scheid & Hora, 1989). The advantage was that exact solutions for the single electron motion of the electrons using the relativistic Lorentz force were derived. These results were used to calculate the *maximum energy* an electron achieves during such half-wave interaction in order to find the laser pulse parameters for TeV electron generation, for example, by injection of electrons into boxlike laser beam profiles (Häuser *et al.*, 1992*b*). When using Gaussian-like beams, an enormous acceleration was seen if the beam width had a minimum value and a single wavelength pulse was running over the initially resting electron. The final electron energy was very close to the before mentioned maximum energy (Häuser *et al.*, 1994*a*). It was essential that the earlier discovered Maxwellian exact laser fields (Hora, 1981; Cicchitelli *et al.*, 1990), including the necessary longitudinal component, had to be used.

The longitudinal components for a single beam acceleration was decreasing the electron energy (Häuser *et al.*, 1994*a*) against the initial expectation and contrary to the scheme of a two-beam crossing acceleration scheme (Caspers & Jensen, 1991; Scully, 1990; Takeuchi & Sugihara, 1998). All these results were considered with hesitation in view of the linear superposition argument of Sessler (1988) or the alternatively formulated Lawson–Woodward theorem. A breakthrough of confidence to the laser acceleration of electrons in a vacuum appeared in the work of Woodward *et al.* (1996) where essentially the same results, and about the same gained maximum electron energies (modified since the longitudinal laser field was not included), were derived independently and in a different way than before (Häuser *et al.*, 1994*a*).

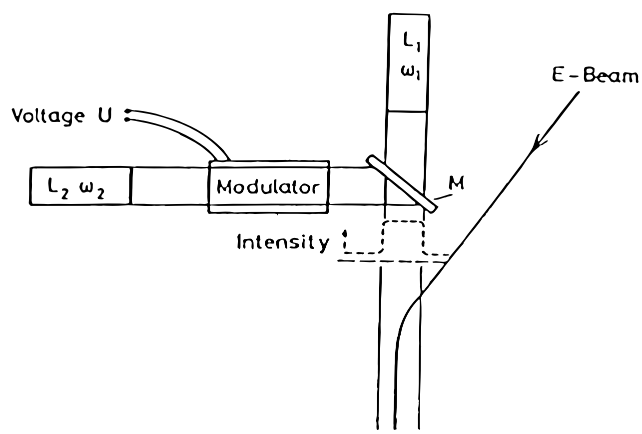

Fig. 1. Superposition of laser beams L_1 and L_2 with frequencies ω_1 and ω_2 by a mirror M and using an active phase modulation by applying a voltage U to an electro-optical modulator, causing a controlled motion of the minima of the interference field into which electrons from a beam are injected for acceleration by the acceleration of the intensity minima (Hora, 1988*a*).

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The final persuasion for the vacuum acceleration of electrons by lasers in a vacuum was given by the experiment at Limeil–Valenton (Lefebvre *et al.*, 1998; Malka *et al.*, 1997) where the gain of MeV energy by electrons interacting with lasers in a vacuum was measured. Very extensive computations followed (Hartemann *et al.*, 1998; Wang *et al.*, 1998) such that there is no more doubt that the mechanism does work. This acceleration was described (Sprangle *et al.*, 1996) as "vacuum-laser acceleration" or as "violent acceleration" (Wang *et al.*, 1998), in the scheme of "free-wave acceleration" (Woodward *et al.*, 1996), as "ponderomotive scattering" (Hartemann *et al.*, 1998), or as the "vacuum beat-wave acceleration" (Sprangle *et al.*, 1996).

It is curious why the experimental results of the electron acceleration by a laser in a vacuum (Malka *et al.*, 1997; Lefebvre, 1998) were not accepted (McDonald, 1998) as facts. The difficulties in understanding this acceleration were known from the beginning (Hora, 1988*a*; Evans, 1988) of these considerations, while the fully clear theory had been published before (Häuser *et al.*, 1994*a*) or some other convincing theoretical facts had been given, for example, by Mora *et al.* (1998). The earlier assumed Compton scattering (McDonald, 1989) was found identical to Lorentz acceleration (Ho *et al*., 2000). Also, the earlier work (Häuser *et al.*, 1992*b*; Scheid & Hora, 1989) using half-wave interaction are now supported (Rau *et al.*, 1997) in view of the experimentally verified, ingenious realization of rectified laser beams (Bonvalet *et al.*, 1995) of subcycle laser pulses (Raman *et al.*, 1996).

While the basic mechanism of the laser acceleration of electrons in a vacuum is now settled (Sprangle *et al.*, 1996; Hartemann *et al.*, 1998; Wang *et al.*, 1998) since the first breakthrough in 1988 (Hora, 1988*a*; Evans, 1988) there is an interesting problem to be considered when comparisons between experiments, theory, and computations are to be done now in the details for the next measurements. It has been experienced before (Hora, 1981; Cicchitelli *et al.*, 1990) that a wide range of discrepancies can appear if nonlinear processes are to be analyzed theoretically.

Inaccuracy problems are well known in physics, especially when treating nonlinear problems, as remarks by Feynman show (Hora, 1996, 2000). We have to point out here that these inaccuracies are not gradually only by adding or by neglecting higher order terms or by differences of the results by few percentages. We have to realize that a basically new phenomenon has appeared in nonlinear physics where the addition or subtraction of very tiny quantities can change the results from completely right into wrong, or effects from "yes" into "no" contrary to observations. This will be more detailed in the following section. This paper then describes a similar example in the theory of nonlinear physics with the case of the relativistic electron acceleration by lasers in a vacuum (Hoelss, 1998, Hoelss *et al.*, 1998, 1999) as will be reported here in more detail.

This experience should teach how cautiously one has to proceed with the theory of the nonlinear processes. On the other hand, it opens a door of systematically discovering basically new effects of nonlinearity as a very new dimension of exploration in physics, in contrast to the view of the saturation of physics knowledge articulated by Stephen Hawkings and Carl Friedrich von Weizsäcker, as shown in section 6.3 of Hora (2000). This, again, is the reason that the next section first discusses the "principles of nonlinearity" before the mentioned results of our studies on the accuracy of computations for the laser acceleration of electrons in a vacuum will be presented in the following sections.

2. THE ACCURACY PRINCIPLE OF NONLINEARITY

In order to understand the needs for a *complete* representation of the structure of the laser field for the interaction and acceleration of the free electron, we first refer to the problems of accuracy in the treatment of nonlinear physics as they appeared when the theory was given for the experiment of Boreham (Boreham & Hora 1979) [and extended by Meyerhofer *et al.* (1996), Meyerhofer (1997), and Salamin & Faisal (1998)] where the energy of electrons emitted sidewise from a laser focus in tenuous He gas was measured.

The Nd glass laser beam was focused to an intensity near 10^{16} W/cm^2 and the emitted electrons had a maximum energy of about 1 keV, exactly what was expected from the nonlinear force acceleration (Hora, 1969, 1985, 1991, 1996) given by the then applicable ponderomotive potential (Hora, 2000). The fact that this is no potential, in general, has been explained (Hora 1969, 1996, 2000) and treated (Hora 1996). The electron emission perpendicular to the laser beam had to be independent from the polarization of the laser as measured. A little forward direction was concluded (Hora *et al.*, 1983) and experimentally confirmed later with higher laser intensities by Meyerhofer *et al.* (1996) and Meyerhofer (1997).

The problem appeared when the electron emission was to be described by a single particle motion instead of using the ponderomotive potential. The eightlike motion showed a drift in the direction of the **E**-field at linear polarization of the laser beam arriving at the same translative energy as from the ponderomotive potential and as measured. Using the transverse magnetic field **H** and the transverse field for **E** for the motion in the **H**-direction, the electrons did not receive any gain of translative motion contrary to the ponderomotive force description, and contrary to the experiments. What was wrong? Contrary to the plane waves with purely transversal **E**- and **H**-fields, these transversal fields in a beam with radial decay are not the complete Maxwellian exact solutions. Using the paraxial approximation, Carter (1972) and later Lax *et al.* (1978) had discovered that electromagnetic beams in a vacuum do have longitudinal components contrary to the usual knowledge in electromagnetic and optic theory. In the case of the Boreham experiment, however, not an approximation but only the Maxwellian exact solution had to be applied. For a linear radial decay of the transversal **E**-field

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of a beam, the longitudinal components could be calculated exactly without approximation by elementary functions (Hora, 1981). This was first presented in 1979 (Castillo *et al.*, 1981; Schwarz, 1981).

The result was very surprising: the addition of *the very tiny longitudinal component* to the calculation of the electron motion *changed the result from not any net acceleration into the large acceleration* with the same translative energy as from the ponderomotive potential.

A more sophisticated evaluation of the polarization independence of the Boreham & Hora (1979) experiment with Gaussian laser beams was possible only if the Maxwellian exact laser field, including the longitudinal components and composed by the linear superposition of plane waves was used, while the transversal laser components alone resulted in a strong polarization dependence (Cicchitelli *et al.*, 1990).

This was the first experience that nonlinear physics needs the most accurate ingredients of linear physics if correct predictions are to be reached. Usually it is assumed when treating a nonlinear problem that one can simply drop higher order terms, etcetera, and that some inaccuracy of a few percent have to be taken into account. If one knows the experimental result to be interpreted, this method of approximations may well be possible just to reproduce the observation. But in the case of prediction for experiments from nonlinear theory, the situation is different: one has to work with the highest possible accuracy. It is then possible, however, to predict phenomena in physics about which nobody could have dreamed! This is the reason why the expectation of a saturation of physics according to the very refined philosophy presented by Stephen Hawking or Carl Friedrich von Weizsäcker (see Hora, 2000), is not true and physics is really at a basically new beginning of dawn, thanks to the just demonstrated accuracy principle of nonlinearity.

This is a consequence in the numerical work, and the difficulties experienced in differences and discrepancies observed when working on laser acceleration of electrons by lasers. It has been shown that the Maxwellian exact laser field derived with the methods of Cicchitelli *et al.* (1990) can differ in a nearly invisible way from an approximate field description. But the violent acceleration of electrons in both cases can be exceedingly different (Hoelss, 1998; Hoelss *et al.*, 1998, 1999) and will be demonstrated in the following sections. This is a warning for all computations in this field and indicates that extensive investigations will be needed in the future for solving the laser-electron interaction problem in a vacuum generally. To the history of the longitudinal laser field component, Carter (1972) used the paraxial approximation similar to Lax *et al.* (1975) or with an easier derivation of their results by Davis (1979). Quesnel and Mora (1998) underlined how these approximations were improved to higher orders (Agrawal & Patanyak, 1979), up to the fifth order corrections (Barton & Alexander, 1989). The fact, however, that the fully exact solutions are necessary, can be seen in a transparent way (Hora, 1981) using elementary functions only (however for an exotic triangular laser beam) to establish the experimentally proven polarization independence of the Boreham (1979) experiment. When using a Gaussian beam (Cicchitelli *et al.*, 1990), iterations up to the 14th order were necessary to reach the necessary convergence, resulting then in the earlier known polarization independence. A consequence of the longitudinal field is discussed with respect to the Evans–Vigier field and the rest mass of the photon (Novak, 1983; Argyris *et al.*, 1998).

This situation makes it understandable why the research in this field can be confusing and needs a new kind of a categorical imperative of research in physical sciences: work even more accurate in nonlinear physics than in linear physics (contrary to some earlier assumptions about nonlinear research). This will cause a much more complicated and difficult situation for the research (especially on the computational side). It is a special challenge to work into these new kinds of nonlinear predictions, from which unimaginable new scientific and innovative discoveries can be consequently and systematically concluded. This was explained as one part of the new physics together with Haken's synergetic and the deterministic chaos theory after this century's achievements, the quantum mechanics and the relativity seem to be nearly completed (Hora, 1998). "Nearly" refers here to some caution as expressed by Dirac's (1978) first Australasian lectures with respect to expected novelties in quantum theory and relativity.

The philosophical aspect clarifies why there was such a confusion about the violent acceleration of electrons by lasers in a vacuum between the highest level experts as Lawson (1989) and Sessler (1982, 1988), the CERN team (Caspers & Jensen, 1991), Scully (1990) and why it took some time from the first steps (Hora, 1988*a*, 1988*b*) to reach the convincing basis of this acceleration (Hartemann *et al.*, 1998; Sprangle *et al.*, 1996; Wang *et al.*, 1998) finally funded experimentally (Malka *et al.*, 1997) as a basis for the energetic beginning of extensive research in the field. It has been explained before that Einstein's (1917) discovery of the laser principle in 1916 was exactly the same kind of prediction of a completely unexpected and unimaginable phenomenon for which indeed Einstein's exceptional ingenuity was necessary.

There are other examples of how the ignorance of the mentioned new nonlinear principle resulted in fundamental errors (Hora, 1991, 2000). When the problem of generating inexhaustive, low cost, safe and clean nuclear fusion energy was proposed around 1951, the use of beam fusion was considered by prominent physicists like E.O. Lawrence, Sir Mark Oliphant, Salisbury and others. But this scheme was knocked down by Spitzer (Hora, 1987, 1991, 1992, 1996, 2000) by the argument of the much smaller cross sections of nuclear fusion reactions compared with electron collisions causing preferential heating and a little fusion reactions therefore preventing an exothermic reaction. This argument of Spitzer was correct in mathematics, physics, and logic. Nevertheless it was completely wrong! The reason is that Spitzer was using linear physics and not nonlinear physics. The argument of beam fusion is correct with nonlinear physics (Hora, 1987) what became clear many years after 1951 when the laser had been realized.

3. A SCALE FOR MAXIMUM ELECTRON ENERGY

We first summarize some earlier results from exact calculations about the maximum energy an electron can obtain in a laser field. The calculation (Scheid & Hora, 1989) for a fully rectified laser field of infinite plane waves shows exact solutions for the motion of the electron inside the first half wavelength. The electron is moved first to the side, that is, perpendicular to the direction of the wave along the **E**-field (linearly polarized in the x direction) and then in a bent motion in forward propagation direction (z-direction) due to the Lorentz force by the **H**-field (in the y-direction). At very high laser intensities, the relativistic motion of the electron is driven nearly completely into the direction of the laser radiation.

The exact analytical solutions (Scheid & Hora, 1989) for this half wave acceleration of the initially resting electron result in a relativistic γ-value or a translative energy E for Nd glass lasers of 1053 nm wave length in

$$\gamma = 1 + 1.62087 \times 10^{-18}\, I \quad (I \text{ in W/cm}^2) \tag{1}$$

$$E = 8.283 \times 10^{-13}\, I \text{ eV}. \tag{2}$$

During this acceleration the electron performs a sideways motion in the direction of the electric field (for $\gamma \gg 1$)

$$x = 4.74 \times 10^{-14}\, I^{1/2} \text{ cm} \tag{3}$$

and a motion along the direction of the propagation of the laser field

$$z = 3.20 \times 10^{-23}\, I \text{ cm} \tag{4}$$

Taking a boxlike cross section of the plane wave with a square side of x, we express the γ-value and the translative energy E of the electron in terms of the laser power $P = Ix^2$:

$$\gamma = 1 + [P/(8.551 \times 10^8)]^{1/2} \quad (P \text{ in W}) \tag{5}$$

$$E = [P/(3.275 \times 10^{-3})]^{1/2} \text{ eV}. \tag{6}$$

These results implied the important condition that the cross section of interaction is exactly determined by the value of the sideways motion x. Equations (5) and (6) are independent of the wavelength.

These values are the highest possible energies E that an electron can achieve in the laser field by an exact plane wave and half wavelength interaction. Figure 2 shows the result where the intensity I is independent of the wavelength. The forward motion of the electron in the z-direction Eq. (4) is given by X and the sideways motion along the electric field

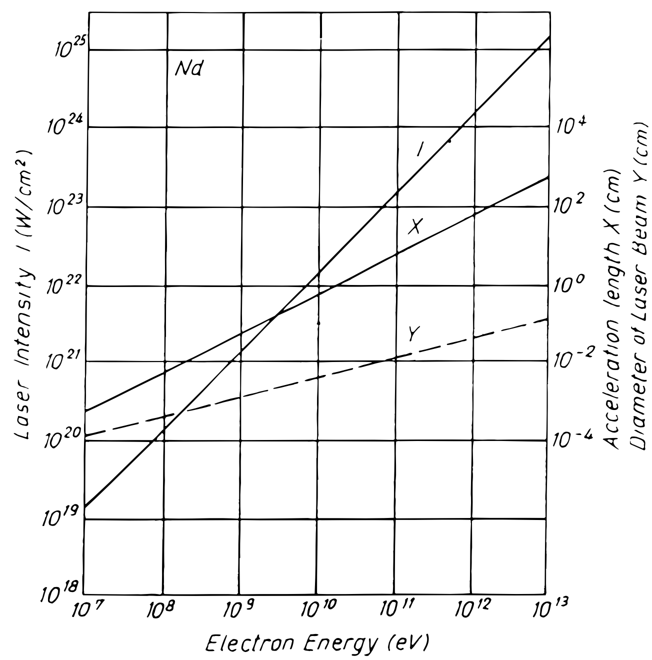

Fig. 2. Maximum electron energies gained by electrons by a sideways injection in a nearly boxlike laser pulse of given intensity I with the necessary minimum width Y (corresponding to x in the preceding formulas) resulting in an acceleration length X (corresponding to z) before the electron is ejected from the beam after a half-wave length interaction (Häuser *et al.*, 1992*a*).

[x-direction, Eq. (3)] by Y for Nd glass lasers. One example is the question of the values for reaching TeV electron energy without speculating how to rectify the laser wave and how to produce a half-wave pulse. The result is that one needs a laser intensity of $I = 1.21 \times 10^{24}$ W/cm^2; the sideways extension is $x = 0.521$ mm. The length of interaction, that is, the path along the electron is carried by this pancake-like laser pulse (see Fig. 1 in Hora, 2000) at nearly the speed of light and receiving its energy by shifting from the front edge to the end edge of the laser half-wave, is $z = 38.6$ cm. Due to the sideways motion, the laser power $P = Ix^2$ is then 3.275×10^{21} W = 3275 Exawatts. Such an accelerator with a length of only 39 cm instead of the dozens of kilometers of classical linacs indeed needs enormous laser capacities which finally may result in lower costs than the costs of a linac. For the TeV electrons it was estimated that the energy loss by bremsstrahlung is sufficiently low and that a luminosity of up to 10^{33} cm^{-2}s^{-1} may be achieved (Häuser *et al.*, 1992*a*).

The just mentioned values are the absolute maximum energies one can reach with the mentioned intensities and powers for a half-wave acceleration. In practical cases one cannot have the rectified pulses and not the boxlike cross sections of plane waves. Going to a cylindrical laser beam with the radius x, the maximum γ-value or the energy E of the electron after a half-wave is (Häuser *et al.*, 1994*a*,*b*) (independent of wavelength)

$$\gamma = 1 + [P/(2.69 \times 10^9)]^{1/2} \; (P \text{ in W}) \tag{6a}$$

$$E = [P/(1.03 \times 10^{-2})]^{1/2} \text{ eV}. \tag{7}$$

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But even this rectified wave field is not easily possible. Using a Gaussian beam of one full wavelength with symmetric phase, nevertheless the sideways motion of the electron into areas of lower intensity does result in an energy loss of a few or several percent below the mentioned maximum values. If the calculation includes the longitudinal components of the laser field, again a reduction by about 40% of the electron energy occurs (Häuser *et al.*, 1992*a*, 1994*a*).

The result is that acceleration of the electrons in the laser field up to the range of the maximum energies can be expected. For the 2-petawatt laser at Livermore (Perry & Mourou, 1994; Cowan *et al.*, 1999) the maximum electron energy to be gained per interaction is given by

$$E_{\max} = 441 \text{ MeV (for 2-petawatt laser independent of wave length)}, \tag{8}$$

which again may be reduced to 60% or a similar value when the exact laser field with the longitudinal components will be applied. The minimum radius of the beam for this interaction is $x = 0.011$ mm for a wavelength of 1053 nm. The use of this minimum radius, given by Eq. (3), is essential for the design of experiments. If the radius is too small, the maximum electron energy, Eqs. (6) or (7) or a little below these values can never be reached. Larger focus values may provide an electron motion within several wavelengths of the laser beam and may result even in higher values than the given here single half-wave exact maximum values.

There is a simple understanding for the acceleration of the electrons expressed by Mori (1999). Since the magnetic field cannot transfer energy to the electron but can only bend in a sideways motion due to the electric field, it is interesting that all the before mentioned results can be explained in an energy gain of the electron by integrating **E** along the sideways motion. The result is exactly the energy which the electron receives by the complicated final motion into the forward direction caused by the Lorentz-force mechanism. The only question is how far does the integration go in the *x*-direction. The answer is indeed the very complicated motion described (Hora, 1988*a*; Häuser *et al.*, 1994*a*; Sprangle *et al.*, 1996; Hartemann *et al.*, 1998; Wang *et al.*, 1998), resulting in the energies in the range of of Eqs. (2), (6) or (7). Therefore the energy input by the electrical laser field indeed goes in the wrong direction but thanks to the Lorentz force the electron trajectory is bent in the direction of the laser beam.

As an example for the agreement of the calculated values of Figure 2 with measurements, we mention the following detailed analysis of the Umstadter (1996; Umstadter *et al.*, 1996) experiment. The experiment was producing more than 10^8 30 MeV electrons when a 30 TW laser pulse hits an atmospheric pressure gas puff. This was the right order of the number of electrons expected from the electron acceleration by lasers in a vacuum discussed here, contrary to the much smaller numbers of MeV electrons measured before (Kitagawa *et al.*, 1992). The measurement of Umstadter *et al.* (Umstadter, 1966; Umstadter *et al.*, 1996) could immediately be explained (Hora *et al.*, 1997) by the acceleration of the electrons in the "vacuum", including relativistic self focusing (Hora, 1975, 1991; Hora *et al.*, 1978; Jones *et al.*, 1982; Castillo *et al.*, 1984; Basov *et al.*, 1987; Lezius *et al.*, 1998; Häuser *et al.*, 1992*a*; Esarey *et al.*, 1997; Hain *et al.*, 1997; Kumar *et al.*, 1998). The number of all electrons in the volume of the focus with a radius fulfilling the optimum condition of Eq. (3) is 10^9 Hora *et al.* (1997) which is close to the measured number of Umstadter *et al.* (Umstadter, 1996; Umstadter *et al.*, 1996). The energy of the electrons following Eq. (7) is near 50 MeV which has to be reduced by 40% due to the longitudinal laser field, arriving just at the measured 30 MeV. Exactly the same mechanism is the basis of the explanation of similar experiments using self focusing by a numerical phase-cell description (Kalashnikov *et al.*, 1994; Meyer-ter-Vehn *et al.*, 1999).

4. PARAXIAL APPROXIMATION AND EXACT PRESENTATION OF THE LASER FIELD

Taking the complexity of the accuracy principle of nonlinearity into account, it is no surprise that some attempts for the numerical computation of the energy gained by an electron in a laser beam may be much lower than the experimental value, though the initial conditions of position and energy of the electron before the interaction and the phase of the interaction are further parameters to be taken into account. The energy gained by an initially resting electron, when put into a Nd glass laser beam of 1.2×10^{24} W/cm^2 and 0.168 mm half-width radius with lateral coordinates *x* (along the electrical field) and *y* (along the magnetic field) is shown in Figure 3 (Hoelss, 1998). The parameters were chosen so that a maximum energy towards TeV would be reached. The longitudinal laser fields were all Maxwellian exact. It should be mentioned that these results are similar to that calculated before for the corresponding conditions of a carbon dioxide laser pulse (Häuser *et al.*, 1994a, see Fig. 3), though these calculations had only an approximation for the longitudinal laser field. We note here that the little deviations of the longitudinal field for the carbon dioxide case, from the exact case, do not affect the result remarkably. We also underline that the maximum energy of the electrons shown in Figure 3 reach the order of magnitude of the absolute maximum value of Figure 2 since the energies reached for the case of the realistic acceleration is about half of the absolute maximum.

We demonstrate now examples of the computations with different representations of the laser field. Figure 4 shows (Hoelss, 1998) the result for a laser field from the exact calculation using the superposition of plane waves, according to Cicchitelli *et al.* (1990). The electric field component E_x depending on the radius ρ of the beam is shown in Figure 4a with its maximum value at the beam propagation length $z = 0$ of the focal center, and how this changed following the propagation of the beam along higher values of *z*. The Gaussian decay of E_x at $z = 0$ along the coordinates *x* and *y* is shown in Figure 4b. The longitudinal component of the

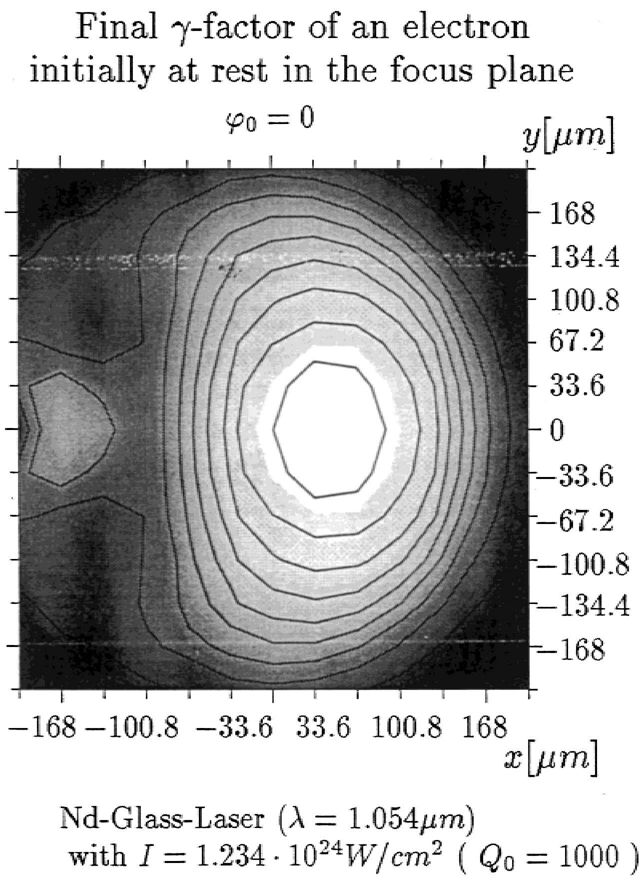

Fig. 3. A Gaussian Nd glass laser pulse of intensity $I = 1{,}234 \times 10^{24}$ W/cm^2 and 0.168 mm half-width radius hits an electron at a phase $\phi_o = 0$ initially located in a position given by the coordinates x and x with respect to the beam cross section gaining an energy expressed by γ of 450,000 at the innermost closed curve, the next of 400,000, etcetera ($\gamma = 1.96 \times 110^6$ corresponds to an electron energy of 1 TeV). All field components are Maxwellian exact.

electric field, E_z at $z = 0$ depending on x and y is shown in Figure 4c and the then necessary longitudinal component H_z of the magnetic field in Figure 4d.

When calculating the same field from the paraxial approximation based on an angular spectrum method, the fields appear visibly at the very same diagrams as shown in Figure 4. However, when looking to the values of div**E** and rot**H**, see Figure 5, the paraxial approximation results in values different from zero, while the Maxwellian exact values are necessarily zero.

It is then no surprise, that the trajectories of the electron motion are very different between the calculation with the exact field, and that using the approximation. Figure 6 shows the trajectories for the motion of electrons in a stationary (time independent) Nd glass laser beam (1053 nm wave length) of intensity $I = 1.23 \times 10^{22}$ W/cm^2 and a beam width of 0.016 mm. The trajectory for the exact calculation differs very strongly from the angular spectrum paraxial (Osman *et al.*, 1999) approximation (Hoelss *et al.*, 1999).

5. PHASE DEPENDENCE OF THE RELATIVISTIC ACCELERATION OF ELECTRONS IN THE LASER FIELDS IN A VACUUM

Apart from the use of an exact or approximating description of the laser field for the electron acceleration in a vacuum, there is a strong dependence on the phase of the laser field when injecting the electron in a stationary beam. This can be seen from the case described in the following (Wang *et al.*, 1998). Figure 7 describes the geometry of the injection of an electron in an extra-axial position into the laser beam. There can be an inelastic interaction as shown by the dotted trajectories and an elastic interaction shown by the fully drawn line, where a deviation of the propagation of the electron by an angle *f* occurs. The time dependence of the position x of the moving electron and of its γ-value is shown in Figure 8. The injected electron has an initial energy of 25 MeV which is nearly unchanged (apart from a very little increase) after the interaction in the case of the elastic interaction while the inelastic interaction results in an electron energy of 1.5 GeV.

The criterion as a necessary condition for the correct result can be seen by comparison with the boxlike calculation including a reduction by the longitudinal field of a beam instead of a box. The highest possible energy is then 1.86 GeV with a value not too much higher than the achieved value of 1.5 GeV, which resulted from a series of computations until the optimized conditions for the selection of the parameters of phase, direction, and energy for the injection of the electron into the stationary laser beams were found.

As an example of how the resulting electron energies depend on the phase of the laser field, the calculation of the final electron energies (expressed by *g*-values) at the same conditions as Figure 3, but with a phase of 0.51 radian instead of zero, are shown in Figure 9.

The experience was that cases had been found where whatever initial conditions for the electrons were chosen, there was nearly no acceleration in the laser field. The essential result of these treatments is that as long as the mentioned maximum electron energies are not nearly achieved, either the used approximation of the laser field is not sufficiently accurate, or the initial conditions for the computations are not optimized, or both insufficiencies are determining the results in discrepancy to the experiments.

6. CONCLUDING REMARKS

In view of the rather fundamental aspects about the *accuracy principle of nonlinearity* expressed in this paper, it seems to be indicated to underline the following. There will be no change in the settled parts of physics. Newton's mechanics for calculating the motion of planets including Einstein's modifications will always be correct, or the fact that the time dilatation in a spacecraft flying nearly with the speed of light

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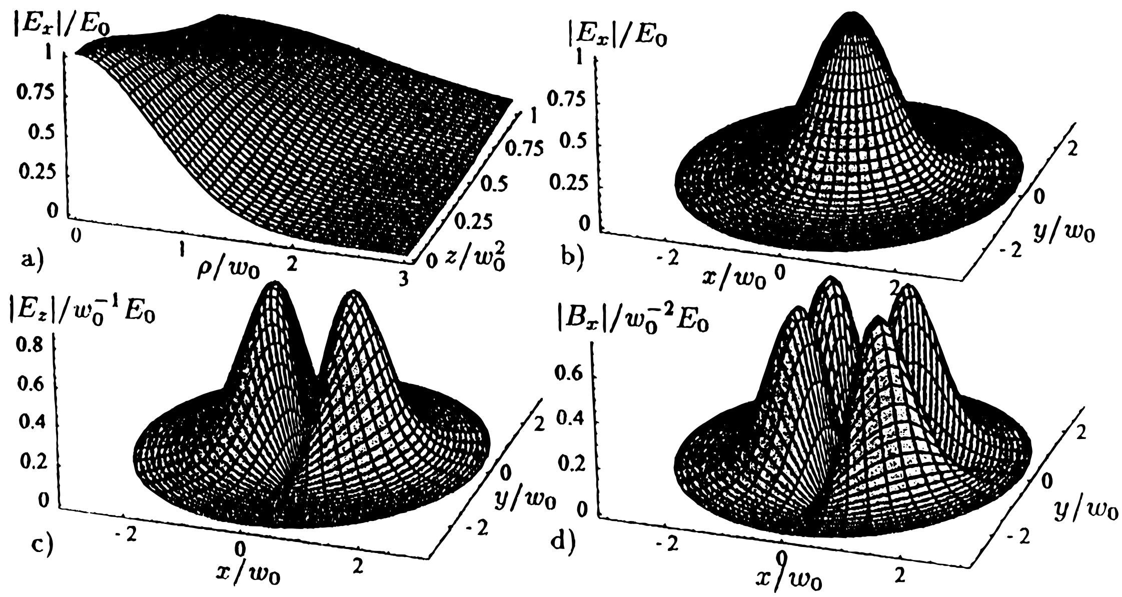

Fig. 4. Beam profiles (Hoelss *et al.*, 1999) for the exact calculation of a laser field with a Gaussian transversal component E_x depending on the radius ρ developing along the propagation direction z (a) having a dependence on the coordinates x and y of the cross section in the focal areas (b). The Maxwellian exact following longitudinal component of the electric field, E_z has a cross section shown in (c) and the then necessary magnetic field needs a component B_x additional to the Gaussian transversal component B_y (d).

will result in much less aging of the astronaut than for his twin on the earth, or the quantum mechanical calculation of spectral lines. The problems came when Spitzer's logically, mathematically, and physically correct prediction in 1951 of the impossibility of beam fusion and the need of magnetically confined fusion energy production appeared to be nevertheless completely wrong because he was using linear physics and missed nonlinearity.

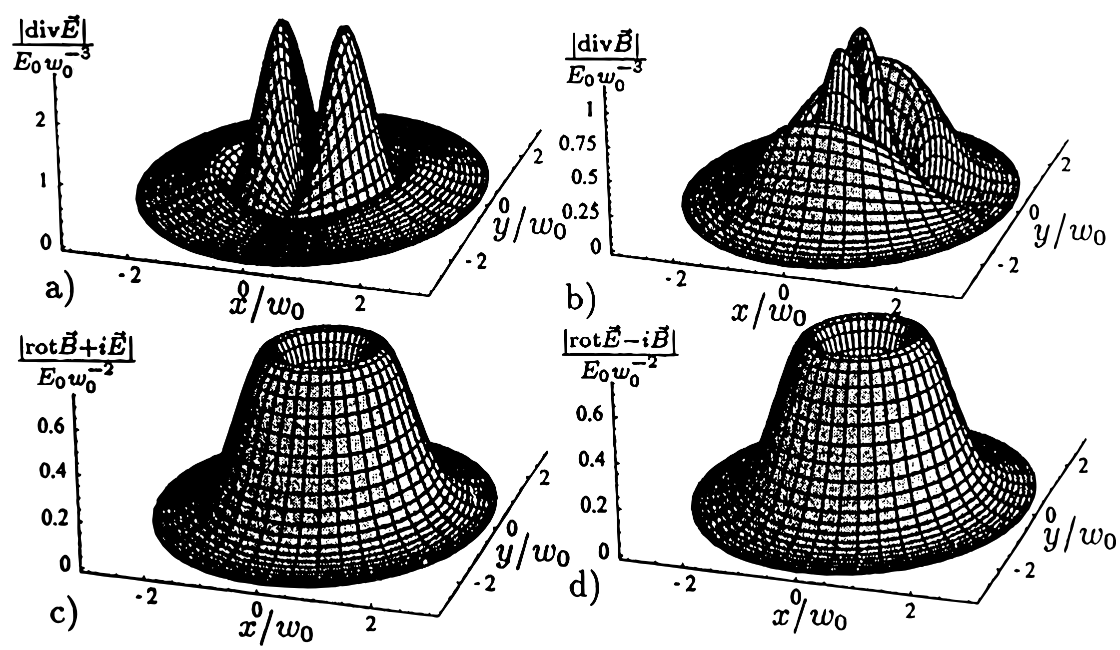

Fig. 5. The fields from the paraxial angular spectrum approximation (Hoelss *et al.*, 1999) look very similar to the exact solutions of Figure 4, but when evaluating div**E** and rot**H**, the values shown in the diagrams are different from zero while the exact solutions are zero.

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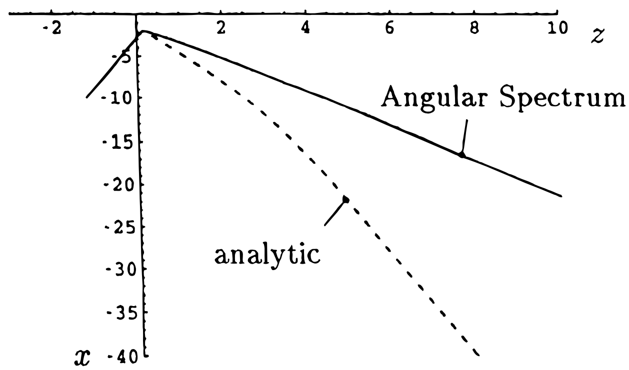

Fig. 6. Demonstration of the very different trajectories of electrons moving through an exact laser fields (analytic) and that of the angular spectrum paraxial approximation (Hoelss, 1998) for a Nd glass laser beam of $I = 1.23 \times 10^{22}$ W/cm^2 intensity and beam width of 0.016 mm.

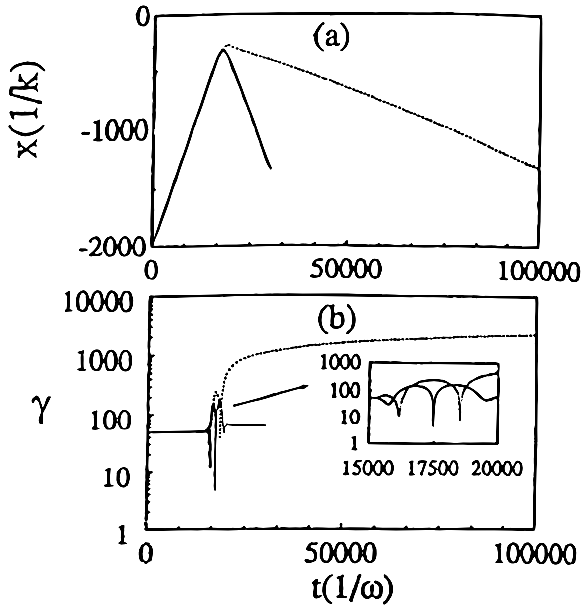

Fig. 8. Example (Wang *et al.*, 1998) of an inelastic interaction (dotted line) of an electron at an extra-axial injection into an electron beam showing the position x (measured in reciprocal wave numbers k) and the actual γ-value depending on the time t, compared with the elastic interaction (fully drawn line). The calculation is for a Nd glass laser beam of intensity $I = 1.23 \times 10^{22}$ W/cm^2 of 0.033 mm diameter. The electron with inelastic interaction gains an energy $E = 1.5$ GeV.

The same was with the prediction (Hora, 1988*a*) of the strong acceleration of electrons by laser fields in a vacuum. Its impossibility was claimed by highly renown and influential experts while its truth was shown experimentally (Malka *et al.*, 1997). The theoretical prediction, however, could be based on fully exact theory and computations only. This was experienced by evaluation of the polarization independence of the experiment of Boreham & Hora (1979) where the mathematical transparent calculations with an exotic (triangular) laser beam showed that the neglecting of a very tiny nonlinear component could change the whole result from completely wrong into correct (Hora, 1981).

It is then no surprise that relativistic computations of electron acceleration by lasers in a vacuum can result in totally different predictions if the mathematical ingredients are not fully exact as shown again in the examples of this paper. This result, on the other hand demonstrated that the new nonlinear physics—if done with sufficient accuracy—will permit predictions of phenomena of which nobody could have dreamed before.

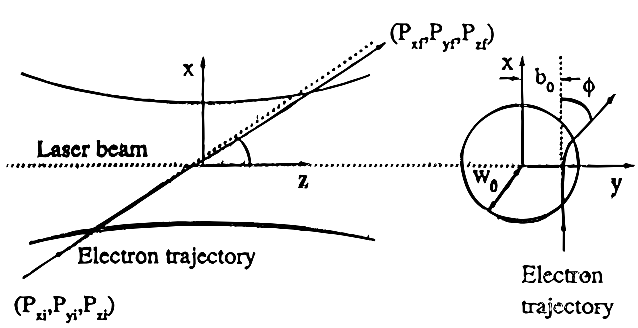

Fig. 7. Dependence of the electron motion at injection into a stationary laser beam at an extra-axial position. The dotted trajectory of the electron corresponds to an inelastic interaction and the fully drawn trajectory with a bending of the electron trajectory by an angle ϕ shows the case of an elastic interaction (Wang *et al.*, 1998).

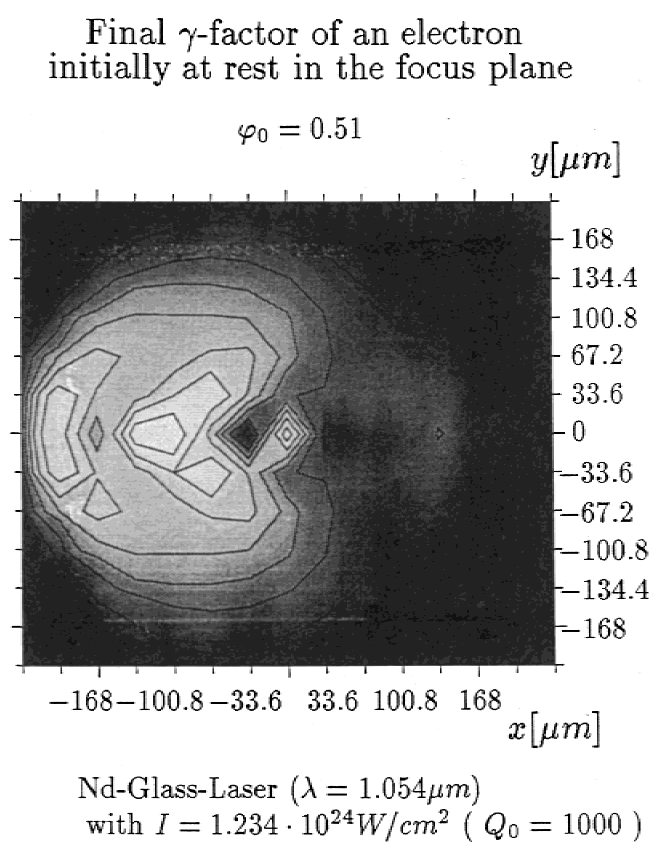

Fig. 9. Same conditions as in Figure 3 but with an initial phase of $\phi_0 = 0.52$ radian for the interaction of the laser beam with the electron.

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