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Effect of self-focused cosh Gaussian laser beam on the excitation of electron plasma wave and particle acceleration

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Abstract

This work presents an investigation of the self-focusing of a high-power laser beam having cosh Gaussian intensity profile in a collissionless plasma under weak relativistic-ponderomotove (RP) and only relativistic regimes and its effect on the excitation of electron plasma wave (EPW), and particle acceleration process. Nonlinear differential equations have been set up for the beam width and intensity of cosh Gaussian laser beam (CGLB) and EPW using the Wentzel-Kramers-Brillouin and paraxial-ray approximations as well as fluid equations. The numerical results are presented for different values of decentered parameter '*b*' and intensity parameter '*a*' of CGLB. Strong self-focusing is observed in RP regime as compared with only relativistic nonlinearity. Numerical analysis shows that these parameters play crucial role on the self-focusing of the CGLB and the excitation of EPW. It is also found that the intensity/amplitude of EPW increases with *b* and *a*. Further, nonlinear coupling between the CGLB and EPW leads to the acceleration of electrons. The intensity of EPW and energy gain by electrons is significantly affected by including the ponderomotive nonlinearity. The energy of the accelerated electrons is increased by increasing the value of '*b*'. The results are presented for typical laser and plasma parameters.

Keywords: Cosh Gaussian laser beam; Electron plasma wave; Laser–plasma interaction; Particle acceleration; Self-focusing

1. INTRODUCTION

With the availability of ultra-intense short pulse lasers (femtosecond pulse duration with powers in multi–TW-PW), it is possible to investigate the nonlinear interaction of highly intense laser beams with plasmas in relativistic regime where quiver velocity of electrons is equal to the velocity of light. The interaction of intense short laser pulse with plasma is a topic of current theoretical and experimental research interest due to its importance in potential applications such as particle acceleration (Macchi *et al*., 2013; Yang *et al*., 2014) fast ignition in inertial confinement fusion (Tabak *et al.,* 1994; Hora *et al*., 2008), and new radiation sources (Zhang & Yu, 2011; Li *et al*., 2012). Among these applications, charged particle acceleration for producing ultra-high energy electrons in plasma is one of the most attractive topics in the relativistic laser–plasma interaction research. In this process, self-focusing of intense laser beam and the generation of large amplitude relativistic electron plasma wave (EPW) are important in laser produced plasma for ultrahigh gradient particle acceleration. For these applications, it is highly desirable that laser beam propagates longer distance (over several Rayleigh lengths) in the plasmas without loss of the energy. Therefore, self-focusing of ultra-intense short laser beam in plasma and its propagation up to as much possible Rayleigh length is a very important research issue (Ren *et al*., 2001; Kant *et al*., 2011; 2012; Zhu *et al*., 2012; Patil *et al.,* 2013; Nanda & Kant, 2014a, b).

In laser–plasma interaction, self-focusing plays an important role in the guidance of laser beams in plasmas and arises when the refractive index of the plasma being an increasing function of the intensity (Sodha *et al.*, 1974; Boyd *et al.*, 2008). The propagation of high-intensity light pulses in plasmas encounters various nonlinear phenomena such as filamentation, stimulated Raman scattering, stimulated Brillouin scattering, harmonic generation, etc. (Kruer, 1988; Sprangle & Esarey, 1991; Rawat *et al*., 2014), which may destroy the desired coherence and are detrimental to the success of above applications. Apart of this, various low- and high-frequency waves such as electron plasma wave, ion acoustic wave, upper and lower hybrid wave etc., are generated in plasma, which can be used for particle acceleration and plasma heating (Chen, 1984). All of these processes are

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strongly affected by self-focusing of laser beam. The result of self-focusing of an intense laser beam in plasma is the further enhancement of laser intensity over large distances in comparison with the Rayleigh length. Self-focusing of intense laser beam in plasma is mainly due to two mechanisms that is, relativistic and relativistic-ponderomotive (RP), which depends on the time scale of the laser pulse (Brandi *et al.*, 1993a, b). The relativistic mechanism affects the dielectric constant via the increase of electron mass from the relativistic quiver velocity of electrons due to the electric field of the intense laser beam and accelerate the electrons, while in RP mechanism electrons are expelled from the high-intensity region by the ponderomotive force and the nonlinearity in dielectric constant of the plasma comes from electron mass variation due to the laser intensities and due to changes in electron density on account of the ponderomotive force. Therefore, in comparison with only relativistic nonlinearity the dynamics of the propagation of laser beams in plasmas is expected to be drastically affected due to cumulative effects of RP nonlinearity. It is also important to mention that relativistic effect and ponderomotive nonlinearity contribute to focusing on a femtosecond time scale at very high intensity.

An experimental and theoretical study of self-focusing of intense laser beam in plasma and its effect on the generation of EPW and particle acceleration process under different time scale of laser pulse has been investigated by many authors in the past decades (Antonsen & Mora, 1992; Borisov *et al.*, 1992; Borghesi *et al.*, 1997; Chessa *et al.*, 1998; Malka *et al.*, 2000; Kumar *et al.*, 2006; Gupta & Suk, 2007; Priyanka *et al.*, 2013). It has been observed in many experiments that relativistic self-focusing is an efficient way to guide a laser pulse over distances much longer than the Rayleigh length (Borisov *et al.*, 1992; Borghesi *et al.*, 1997; Malka *et al.*, 2000). Pioneering works on the generation of a high accelerating field and energetic electron beams have been reported in the literature (Tajima & Dawson, 1979; Joshi *et al.*, 1984; Gorbunov & Kirsanov, 1987; Kitagawa *et al.*, 1992; Modena *et al.*, 1995; Wang *et al.*, 2000; Malka *et al.*, 2002). In order to generate a large amplitude plasma wave and a high-accelerating gradient, the laser pulses must be focused to a small spot size. Tajima and Dawson (1979) reported a novel concept of particle acceleration known as laser plasma accelerators, which utilizes plasma waves excited by intense laser–beam interactions with plasmas for particle acceleration. Joshi *et al.* (1984) experimentally observed the electron acceleration from the breaking of relativistic plasma waves in which the relativistic plasma wave was excited by an intense, short duration laser pulse. Gupta and Suk (2007) highlighted the importance of relativistic mass effect and magnetic field on self-focusing during laser electron acceleration in magnetized plasma. Kumar *et al.* (2006) have reported the effect of a relativistically intense Gaussian laser pulse on the propagation of EPW and particle acceleration under relativistic and RP regimes. Priyanka *et al.* (2013) have recently studied the effect of self-focusing of rippled Gaussian laser beam on the generation of EPW and particle acceleration process under RP regime.

Much of the earlier work on self-focusing and associated phenomena in laser–plasma interaction have been carried out for Gaussian irradiance distribution of the laser beams (Akhmanov *et al.*, 1968; Esarey *et al.*, 1997; Sharma *et al.*, 2004; Kumar *et al.*, 2006), however, various spatial profile of laser beam viz. super Gaussian, hollow Gaussian, Bessel, triangular profiles etc., behave differently in plasmas and has been used to study the laser–plasma interaction. Apart of these, cosh Gaussian intensity profile of laser beam (decentered Gaussian beam) that can be focus earlier than Gaussian beam, has been a subject of considerable interest due to its high utility in the field of nonlinear interactions (Patil *et al.*, 2009, 2010; Chen *et al.*, 2011), complex optical systems (Tovar & Casperson, 1998; Lu *et al.*, 1999), and turbulent atmosphere (Ji *et al.*, 2006; Chu *et al.*, 2007). The main feature of considered cosh Gaussian laser beam (CGLB) is having more power than that of Gaussian laser beams and high intensity near the axis of propagation. It generates flat top beam profiles (Konar *et al.*, 2007), which are useful in many applications where same intensity of laser beams for long time is required. Only a few investigations have been reported on the self-focusing of cosh Gaussian beams. Gill *et al.* (2011) investigated the self-focusing and self-phase modulation of CGLB with relativistic and ponderomotive nonlinearities through a variational approach and found that a large value of absorption coefficient weakens the self-focusing effect in the absence of decentered parameter. Patil and Takale (2013) reported the effect of weakly RP force on self-focusing of a CGLB propagating in plasma using the Wentzel-Kramers-Brillouin (WKB) and paraxial approximations through a parabolic equation approach. They observed strong self-focusing for higher decentered parameters in the weakly RP case as compared with the relativistic only case. The study of self-focusing of CGLBs in collisionless magnetoplasma under plasma density ramp in applied magnetic field has been recently reported by Nanda and Kant (2014a, b). In this study, plasma density ramp and decentered parameter play a vital role to enhance the self-focusing effect. Kant and Wani (2015) have also recently studied the self-focusing of CGLB in plasma by taking into account the effect of plasma density ramp and linear absorption through parabolic equation approach under paraxial approximation.

The effect of self-focused CGLB on the excitation of EPW and particle acceleration under weakly RP regime has not been studied so far. In the present work, self-focusing of CGLB in plasma and its effect on the generation of EPW and particle acceleration have been studied in the presence of weakly RP nonlinearity and the results are compared with taking only relativistic nonlinearity. The organization of this paper is as follows: In Section 2, we study the self-focusing of the CGLB in collsionless plasma. We set up the equations for beam width parameter for cosh-Gaussian beam profile propagating in the plasmas and self-trapped mode in the presence of RP nonlinearity by applying WKB and paraxial approximations (Akhmanov *et al*., 1968; Sodha *et al*., 1974), and solve them numerically by applying initial conditions. Section 3 presents the analytical model for the generation of EPW and particle acceleration process. Section 4 deals with the numerical results and discussions of the study. The main conclusions are summarized in Section 5.

2. PROPAGATION OF CGLBS IN PLASMA

Let us consider the propagation of a linearly polarized CGLB of frequency ω_0 along the z-direction through collisionless plasma. The field distribution of the beam is given by (Casperson *et al*., 1997; Lu *et al*., 1999; Lu & Luo, 2000)

$$E(r,z) = \frac{E_0}{2f}\exp\left(\frac{b^2}{4}\right) \times \left[\exp\left\{-\left(\frac{r}{r_0 f}+\frac{b}{2}\right)^2\right\} + \exp\left\{-\left(\frac{r}{r_0 f}-\frac{b}{2}\right)^2\right\}\right] \quad (1)$$

where r is the radial coordinate of the cylindrical coordinate system, r_0 is the initial beam width, b is the decentered parameter of the beam, f is the dimensionless beam-width parameter of the laser beam in plasma, which measures axial intensity and width of the beam and E_0 is the amplitude of the electric field at the central position of $r = z = 0$.

The propagation of the CGLB in a collisionless plasma is governed by the wave equation

$$\nabla^2 E - \nabla(\nabla\cdot E) + \frac{\omega_0^2}{c^2}\varepsilon E = 0 \quad (2)$$

where ε is the effective dielectric function of the plasma and c is the speed of light in free space.

The effective dielectric constant of the plasma at frequency ω_0 is given by

$$\varepsilon = \varepsilon_0 + \varphi(E.E^*) \quad (3)$$

where ε_0 and ϕ represent the linear and nonlinear parts of dielectric constant respectively. The liner part of dielectric constant of the plasma can be expressed as

$$\varepsilon_0 = 1 - \frac{\omega_{p0}^2}{\omega_0^2} \quad (4)$$

where ω_{p0} is the plasma frequency given by $\omega_{p0} = 4\pi n_0 e^2/m_0$, with e and m_0 (in the absence of external field) are the electronic charge and rest mass respectively, n_0 is the equilibrium electron density in the absence of laser beam. In the presence of an intense laser field, the RP force on the electrons modifies the electron density. The modified electron density profile of plasma due to RP force can be written as (Brandi *et al.*, 1993a, b; Kumar *et al*., 2006)

$$F_P = -m_0 c^2 \nabla(\gamma_0 - 1) \quad (5)$$

where γ_0 is called the relativistic factor defined as follows

$$\gamma_0 = \left[1 + \frac{e^2}{c^2 m_0^2 \omega_0^2} E.E^*\right]^{\frac{1}{2}}$$

The modified electron density (n_T) due to RP force is given by (Brandi *et al*., 1993a; Kumar *et al*., 2006)

$$n_T = n_0 + n_2 = n_0 + \frac{c^2 n_0}{\omega_{p0}^2}\left(\nabla^2\gamma_0 - \frac{(\nabla\gamma_0)^2}{\gamma_0}\right) \quad (6)$$

where n_2 is the nonlinear variation of the density together with the relativistic correction that is, the second order correction in the electron density equation and

$$\frac{n_T}{n_0} = 1 + \frac{c^2 a}{\omega_{p0}^2 4f^2}\left[\frac{1}{\gamma_0}AC + \frac{1}{\gamma_0}r^2B^2 - \frac{2a}{4f^2\gamma_0^3}A^2r^2B^2\right] \quad (7)$$

where

$$A = \exp\left[-\left(\frac{r^2}{f^2r_0^2}+\frac{br}{fr_0}\right)\right] + \exp\left[-\left(\frac{r^2}{f^2r_0^2}-\frac{br}{fr_0}\right)\right]$$

$$B = \exp\left[-\left(\frac{r^2}{f^2r_0^2}+\frac{br}{fr_0}\right)\right]\left(-\frac{2}{f^2r_0^2}-\frac{b}{frr_0}\right) + \exp\left[-\left(\frac{r^2}{f^2r_0^2}-\frac{br}{fr_0}\right)\right]\left(-\frac{2}{f^2r_0^2}+\frac{b}{frr_0}\right)$$

$$C = \exp\left[-\left(\frac{r^2}{f^2r_0^2}+\frac{br}{fr_0}\right)\right] \times\left(-\frac{4}{f^2r_0^2}-\frac{b}{frr_0}+\frac{4r^2}{f^4r_0^4}+\frac{b^2}{f^2r_0^2}+\frac{4br}{f^3r_0^3}\right) + \exp\left[-\left(\frac{r^2}{f^2r_0^2}-\frac{br}{fr_0}\right)\right] \times\left(-\frac{4}{f^2r_0^2}+\frac{b}{frr_0}+\frac{4r^2}{f^4r_0^4}+\frac{b^2}{f^2r_0^2}-\frac{4br}{f^3r_0^3}\right)$$

The nonlinear dielectric constant of the plasma (in the presence of RP nonlinearity) is given by

$$\varphi(E.E^*) = \frac{\omega_{p0}^2}{\omega_0^2}\left(1 - \frac{n_T}{n_0\gamma}\right) \quad (8)$$

Expanding the dielectric constant around $r = 0$ in Eq. (3) by Taylor expansion, one can write

$$\varepsilon = \varepsilon_f + \gamma_1 r^2$$

where,

$$\varepsilon_f = \varepsilon_0 + \Omega^2\left[1 - \frac{1}{\gamma_0} - \frac{c^2 a}{\omega_0^2 f^4 r_0^2}(b^2-4)\right] \quad (9)$$

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$$\gamma_1 = -\frac{\omega_{p0}^2}{\omega_0^2}\left[\frac{a}{\gamma_0^3 f^4 r_0^2} + \frac{c^2 a}{\omega_{p0}^2 f^2}\left(\frac{(16-4b^2)}{\gamma_0^2 f^4 r_0^4} + \frac{a(2b^2-16)}{\gamma_0^4 r_0^4 f^6}\right)\right] \quad (10)$$

Here, $\Omega = \omega_{p0}/\omega_0$, and $a = \alpha E_0^2$ is the intensity parameter.

In order to solve Eq. (2), the second term [i. e. $\nabla(\nabla . E)$] on left hand side can be neglected provided that $(c^2/\omega_0^2)|(1/\varepsilon)\nabla^2 \text{In}\varepsilon| \ll 1$ (Sodha *et al.*, 1974). Thus

$$\nabla^2 E + \frac{\omega_0^2}{c^2}\varepsilon E = 0 \quad (11)$$

Using (WKB) approximation and following Akhmanov *et al.* (1968) and Sodha *et al.* (1974), the solution of Eq. (11) can be written as

$$E = A(r, z)\exp(-ik_0 z) \quad (12)$$

where $A(r, z)$ is the complex amplitude of the electric field E, $k_0 = (\omega_0/c)(\varepsilon_0)^{1/2}$ is the wave number. Substituting the value of E into Eq. (11), we obtain the following equation

$$-2ik_0\frac{\partial A}{\partial z} + \left(\frac{\partial^2}{\partial r^2} + \frac{\partial^2}{\partial z^2} + \frac{1}{r}\frac{\partial}{\partial r}\right)A + \frac{\omega_0^2}{c^2}\varphi(E.E^*)A = 0 \quad (13)$$

Further, the variation of A can be expressed as

$$A = A_0(r, z)exp(-ik_0 S_0) \quad (14)$$

where A_0 and S_0 are the real function of r and z (S_0 being the eikonal of the beam). Substituting the expression for A into Eq. (13) and separating real and imaginary parts, the following set of equations is obtained

$$2\frac{\partial S_0}{\partial z} + \left(\frac{\partial S_0}{\partial z}\right)^2 = \frac{1}{k_0^2 A_0}\left(\frac{\partial^2}{\partial r^2} + \frac{\partial^2}{\partial z^2} + \frac{1}{r}\frac{\partial}{\partial r}\right)A_0 + \frac{1}{\varepsilon_0}\varphi(E.E^*) \quad (15)$$

$$\frac{\partial A_0^2}{\partial z} + \frac{\partial A_0^2}{\partial r}\frac{\partial S_0}{\partial r} + A_0^2\left(\frac{1}{r}\frac{\partial S_0}{\partial r} + \frac{\partial^2 S_0}{\partial r^2}\right) = 0 \quad (16)$$

The solution of the above coupled equations can be written as (Akhmanov *et al.*, 1968; Sodha *et al.*, 1974)

$$S_0 = \varphi_0(z) + \frac{r^2}{2}\beta(z) \quad (17)$$

where

$$\beta(z) = \frac{\omega_0}{c}\frac{\varepsilon_{f0}^{1/2}}{f}\frac{df}{dz}, \quad \text{and } \varphi_0(z) \text{ is the axial phase.}$$

and the intensity of the beam as

$$A_0^2 = \frac{E_0^2}{4f^2}\exp\left(\frac{b^2}{2}\right)\left[\exp\left\{-\left(\frac{r}{r_0 f} + \frac{b}{2}\right)^2\right\} + \exp\left\{-\left(\frac{r}{r_0 f} - \frac{b}{2}\right)^2\right\}\right]^2 \quad (18)$$

Using Eq. (18) in Eq. (15), and equating the coefficients of r^2 on both sides of the resulting equation, we obtain the equation governing the beam width parameter f as

$$\frac{d^2 f}{d\xi^2} = \left(\frac{12 - 12b^2 - b^4}{3f^3}\right) - \left[\left(\frac{\omega_{p0}^2 r_0^2}{c^2}\right)\frac{a}{\gamma_0^3 f^3} + \frac{a}{f}\left(\frac{(16-4b^2)}{\gamma_0^2 f^4} + \frac{a(2b^2-16)}{\gamma_0^4 f^6}\right)\right] \quad (19)$$

where ($\xi = z/k_0 r_0^2$) is the dimensionless distance of propagation. Equation (12) describes the beam width of CGLB with the distance of propagation in collisionless plasma, when both relativistic and ponderomotive nonlinearities are operative.

2.1. Self-Trapped Mode

The initial conditions for the propagation of CGLB in the uniform waveguide/self-trapped mode are: $d^2f/d\xi^2 = 0$ with $df/d\xi = 0$, $\xi = 0$ and $f = 1$. This condition is known as critical condition under which CGLB propagates in plasma without convergence or divergence. We obtain a relation between the dimensionless initial beam width parameter ($\rho_0 = r_0\omega_{p0}/c$) and critical values of power of the beam ($a = \alpha E_{00}^2$) in the presence of RP nonlinearity by substituting $d^2f/d\xi^2 = 0$ in Eq. (19). The general expression for determination of critical threshold for various values of b can be obtained by using the critical condition. Eq. (19) is simplified for self-trapped mode and is given as follows:

$$\rho_0^2 = \left(\frac{12 - 12b^2 - b^4}{3a}\right)\gamma_0^3 - \left[(16 - 4b^2)\gamma_0 + \frac{a(2b^2-16)}{\gamma_0}\right] \quad (20)$$

The critical condition reflects that the CGLB undergoes self-focusing when the condition $d^2f/dz^2 < 0$ is satisfied, whereas for $d^2f/dz^2 > 0$, the CGLB either oscillatory or steady state self-focusing.

3. GENERATION OF EPW AND PARTICLE ACCELERATION

Nonlinear interaction of the EPW with the self-focused CGLB leads to their excitation. It is clear from the earlier analysis that the CGLB can be self-focused in plasma if the initial laser power is larger than the critical power and the change in background density due to ponderomotive force and the relativistic effects. In this process, the laser beam intensity is very intense and the plasma density is also changed due to the ponderomotive force. Thus, the amplitude of EPW, which depends on the background electron density is modified. The magnitude of EPW can be obtained by using the equation of continuity, the equation of motion and Poisson's

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equation. Following the standard techniques, one obtains the general equation governing the electron density variation in the EPW (neglecting the contribution of the ions) as (Purohit *et al.*, 2012):

$$\frac{\partial^2 N_e}{\partial t^2} + 2\Gamma_e \frac{\partial N_e}{\partial t} - \upsilon_{th}^2 \nabla^2 N_e + \frac{\omega_{p0}^2}{\gamma}\frac{n_T}{n_0} N_e = 0 \quad (21)$$

where Γ_e is the Landau damping coefficient for EPW (Krall & Trivelpiece, 1973), υ_{th} is the electron thermal velocity. Using the WKB and paraxial approximations (Akhmanov *et al.*, 1968; Sodha *et al.*, 1974), the solution of Eq. (21) can be expressed as

$$N_e = N_{e0}(r, z)\exp[i(\omega t - kz)] \quad (22)$$

where N_{e0} is the slowly varying real function of r and z, ω and k are the frequency and the propagation vector of the EPW related by the following dispersion relation

$$\omega^2 = \omega_{p0}^2 + k^2 \upsilon_{th}^2 \quad (23)$$

Substituting Eqs. (22) and (23) into Eq. (21) one gets

$$-N_{e0}\omega^2 + 2\Gamma_e N_{e0} i\omega - \upsilon_{th}^2\left(\frac{\partial^2 N_{e0}}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r}\right) \times N_{e0} + 2ik\upsilon_{th}^2 \frac{\partial N_{e0}}{\partial z} + k^2\upsilon_{th}^2 N_{e0} + \frac{n_T}{n_0}\frac{\omega_{p0}^2}{\gamma} N_{e0} = 0 \quad (24)$$

Further, we express N_{e0} as

$$N_{e0} = N_{e00}[-ikS(r, z)] \quad (25)$$

where N_{e00} is the real function of its argument and S is the eikonal of the EPW. Substituting N_{e0} into Eq. (24) and separating real and imaginary parts of the resulting equation, we obtain

$$2\frac{\partial S}{\partial z} + \left(\frac{\partial S}{\partial r}\right)^2 = \frac{1}{k^2 N_{e00}}\left(\frac{\partial^2 N_{e00}}{\partial r^2} + \frac{1}{r}\frac{\partial N_{e00}}{\partial r}\right) + \frac{\omega_{p0}^2}{k^2\upsilon_{th}^2}\left(1 - \frac{1}{\gamma}\frac{n_T}{n_0}\right) \quad (26)$$

$$\frac{\partial N_{e00}^2}{\partial z} + \frac{\partial S}{\partial r}\frac{\partial N_{e00}^2}{\partial r} + N_{e00}^2\left(\frac{\partial^2 S}{\partial r^2} + \frac{1}{r}\frac{\partial S}{\partial r}\right) + \frac{2\Gamma_e \omega N_{e00}^2}{k\upsilon_{th}^2} = 0 \quad (27)$$

To solve the coupled Eqs. (26) and (27), we assume the initial radial variation of the density perturbation to be

$$N_{e00}^2\big|_{z=0} = \left(\frac{N_{10}}{2}\right)^2 \exp\left(\frac{b^2}{2}\right) \times \left[\exp\left\{-\left(\frac{r}{a_0} + \frac{b}{2}\right)^2\right\} + \exp\left\{-\left(\frac{r}{a_0} - \frac{b}{2}\right)^2\right\}\right]^2 \quad (28)$$

where a_0 is the initial beam width of the plasma wave and N_{10} is the initial density associated with the EPW at $r = 0$. The solution of Eqs. (26) and (27) can be written as (Akhmanov *et al.*, 1968)

$$S = \frac{r^2}{2}\beta(z) + \varphi(z)$$

$$N_{e00} = \frac{N_{10}}{2f_e}\exp\left(\frac{b^2}{4}\right) \times \left[\exp\left\{-\left(\frac{r}{r_0 f_e} + \frac{b}{2}\right)^2\right\} + \exp\left\{-\left(\frac{r}{r_0 f_e} - \frac{b}{2}\right)^2\right\}\right]\exp(-k_i z) \quad (29)$$

where $k_i = \Gamma_e \omega / k\upsilon_{th}^2$ and f_e is the dimensionless beam width parameter for EPW. Using Eq. (29) in Eq. (26) and equating the coefficients of r^2 on both sides by using the normalized distance ($\xi = z/kr_0^2$), we obtain:

$$\frac{d^2 f_e}{d\xi^2} = \left(\frac{12 - 12b^2 - b^4}{3a_0^4 f_e^3}\right) r_0^4 - f_e\left(\frac{c^2}{\upsilon_{th}^2}\right) \times \left[\frac{\omega_{p0}^2 r_0^2 a}{c^2\gamma_0^3 f^3} + \frac{a(16 - 4b^2)}{\gamma_0^2 f^6} + \frac{a^2(2b^2 - 16)}{\gamma_0^4 f^8}\right] \quad (30)$$

Equations (29) and (30) respectively give the amplitude of the density perturbation at finite z (intensity of EPW) and the coupling of the EPW with the CGLB when both relativistic and ponderomotive nonlinearities are operative.

The excited EPW by an intense CGLB transfer its energy to electrons and accelerates them [last term on the left hand side of Eq. (21)]. The energy gain (per unit rest mass energy of electron) by the electron is given by

$$\gamma = \left(1 - \frac{\upsilon^2}{c^2}\right)^{-1/2} \quad (31)$$

Differentiating and putting $d(m\upsilon)/dt = ikm_0 c^2 \varphi_1$, we get

$$\frac{d\gamma}{dt} = -ik\upsilon.\varphi_1 \quad (32)$$

where ϕ_1 $(=e\phi/m_0 c^2)$ is the dimensionless electrostatic potential of EPW. Using the Posisson equation, the expression for ϕ_1 is given by

$$\varphi_1 = \frac{i\omega_{p0}^2}{c^2 k^2 f_e}\exp(-k_i z)\sin(kz)$$

The first order differential Eq. (32) has been solved numerically, where we have used f_e from Eq. (30).

4. NUMERICAL RESULTS AND DISCUSSION

The present study is mainly divided into two phases: (1) Propagation of CGLB in collisionless plasma in the presence

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of RP nonlinearity (RPNL)/only relativistic nonlinearity (RNL), and (2) the effect of the self-focused CGLB on the propagation of EPW and particle acceleration process. Strong self-focusing of CGLB in plasma has been observed when ponderomotive nonlinearity is included in comparison with only relativistic nonlinearity, and Gaussian profile of laser beam, which significantly affected the generation of EPW and particle acceleration. The following set of laser and plasma parameters have been used for carrying out the numerical solution of the problem:

$\omega_0 = 1.778 \times 10^{15}$ rad/s, $r_0 = 20$ µm, $\omega_{p0} = 0.05\omega_0$, $a_0 = 10$ µm, $v_{th} = 0.1c$, $b = 0$, 0.5, 0.9 and 1.5, and $a = 0.1$, 0.5 and 1.

Equation (19) describes the variation in beam width parameter with normalized distance of propagation ξ $(=z/kr_0^2)$ and it is solved by the following boundary conditions: $f_0|z = 0 = 1$ and $df_0/dz = 0$. The right hand side of Eq. (19) contains two terms, where the first one is the diffractive term responsible for the divergence of the laser beam, and the second is a nonlinear term arising due to RP nonlinearity. The self-focusing/de-focusing of the CGLB is determined by the relative magnitude of nonlinear and diffraction terms. The nonlinear term depends on the intensity parameter (a), decentered parameter (b) and plasma density. It is interesting to note that on setting $b = 0$ in Eq. (19), self-focusing of Gaussian laser beam is observed. Furthermore, if $(n_T/n_0 = 1)$ is inserting in Eq. (8), one can get the equation for relativistic self-focusing of CGLB in plasma. In order to understand the dynamics of the propagation of CGLB in plasma, a comparison with only relativistic regime and Gaussian beam is essentially important. Figure 1 depicts the variation of the beam width parameter (f) of CGLB with the dimensionless distance of propagation (ξ) with the RP nonlinearity and the only relativistic nonlinearity of self-focusing for the constant value of $a = b = 0.5$. It is obvious that self-focusing is strong and occurs earlier when both relativistic and ponderomotive nonlinearities are taken into account in comparison with taking only relativistic nonlinearity. Figure 2a, 2b respectively illustrate the variation of the beam width parameter (f) of CGLB with the dimensionless distance of propagation (ξ) for different values of b and a, when both relativistic and ponderomotive nonlinearities are operative. It is clear from Figure 2a that the rate of self-focusing increases when we increase the value of decentered parameter 'b', while for $b = 1.5$; the beam width parameter (f) decreases steeply and thus supports the results of Gill *et al.* (2011). However, when $b = 0$, strong oscillatory self-focusing effect is observed at certain higher value of normalized distance. The rate of self-focusing is faster for CGLB with RP nonlinearity because the power and intensity of CGLB is relatively higher than Gaussian laser beam. Moreover, it is also observed from Figure 2b that with increase in the intensity of laser beam, there is an increase in self-focusing. This is because, as we increase the intensity of the laser beam, nonlinear refractive term in Eq. (19) dominate the diffractive term and hence there is an increase in focusing of the beam at higher intensity. The tendency of CGLB that is, converge earlier than Gaussian beam, leads us to choose the profile of CGLB to various applications.

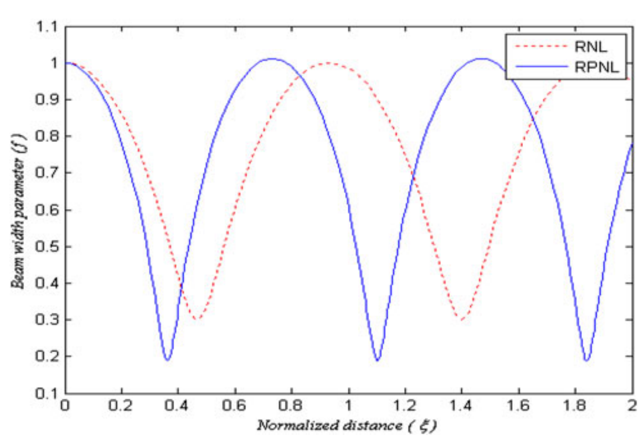

Fig. 1. Variation of the beamwidth parameter (f) of cosh Gaussian laser beam with normalized propagation distance (ξ) for $a = b = 0.5$.

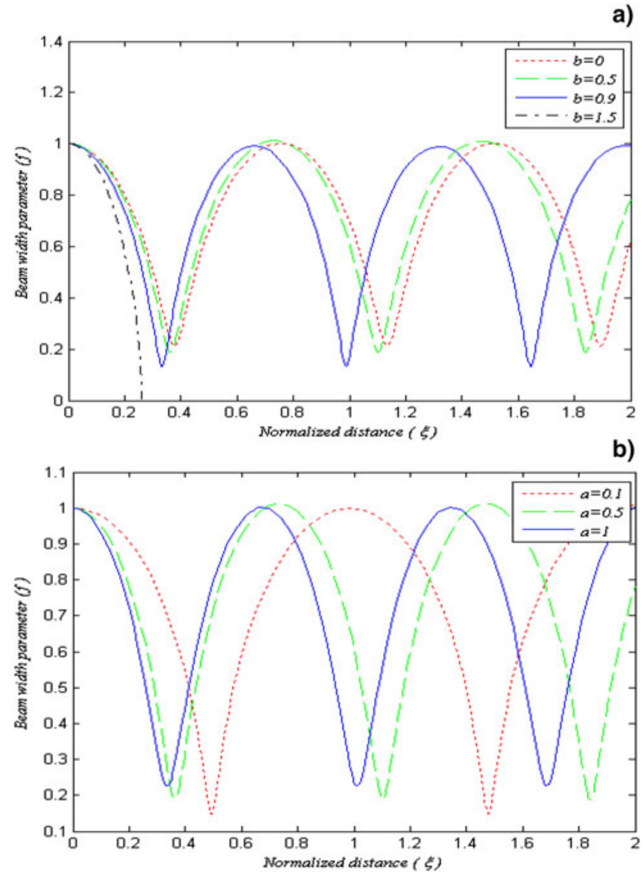

Fig. 2. (a)Variation of beam width parameter (f) with normalized propagation distance (ξ) for cosh-Gaussian (different values of decentered parameter b) and Gaussian laser beam ($b = 0$) with $a = 0.5$, when both relativistic and ponderomotive nonlinearities are taken into account. (b) Variation of beam width parameter (f) with normalized propagation distance (ξ) for different values of intensity parameter (a) with constant value of decentered parameter (b), when relativistic-ponderomotive nonlinearity is operative.

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In order to study the propagation of CGLB in a plasma, we numerically analyze the dependence of dimensionless initial beam width parameter (ρ_0) as a function of critical values of power of the beam ($a = \alpha E_0^2$) for various values of b when both relativistic and ponderomotive nonlinearities are taken into account. We have solved Eq. (20) numerically and the results are presented in the form of Figure 3, which shows the variation of ρ_0 with a for different values of b. It is observed that for uniform waveguide propagation of CGLB, ρ_0 exponentially decreases with a. It is also observed from the figure that with increase in the value of decentered parameter the rate of initial decreases in ρ_0 with a is faster. It shows that highest laser power is required for the Gaussian laser beam ($b = 0$) to propagate in uniform wave guide mode. Thus, CGLBs are more suitable for propagation in uniform waveguide mode at lowest laser power.

The EPW excited due to nonlinear coupling between the CGLB and plasma with RP nonlinearity. This coupling arises on account of the change in the background density that is, the motion of the charge carrier will be modified due to the RP force in the plasma. Thus, the amplitude of EPW, which depends upon the background electron density, gets strongly coupled to the laser beam. Eq. (29) describes the modified density profile of plasma, when the paraxial approximation is taken into consideration. The intensity profile of the EPW depends on the beam widths (f_e and f) of EPW and the CGLB. We have solved Eq. (29) with the help of Eq. (30) numerically to obtain the amplitude of the density perturbation at finite z for typical laser and plasma parameters. The results are displayed in Figure 4a–4c, which show that the EPW gets excited due to nonlinear coupling with high power laser beam in the presence of RP and only relativistic nonlinearities. In Figure 4a, we have comparatively studied the variation in the intensity of EPW with normalized distance, when RP/only relativistic nonlinearities are operative for $a = b = 0.5$. It is obvious that the intensity of EPW get enhanced when both nonlinearities are present. It is also clear from Figure 4b, 4c, the intensity of EPW increases with increases the valves of decentered parameter (b) and

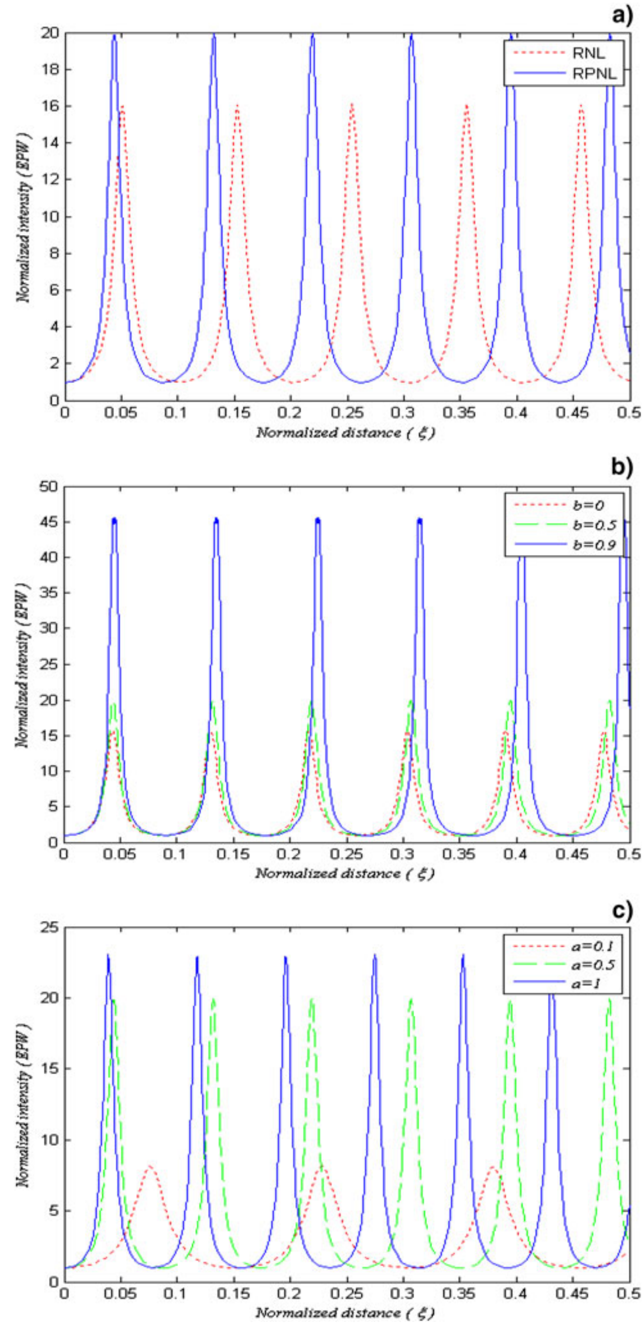

Fig. 4. (a) Variation of electron plasma wave intensity with normalized distance of propagation (ξ) for $a = b = 0.5$. (b) Variation of electron plasma wave intensity with normalized distance of propagation (ξ) for cosh-Gaussian (different values of decentered parameter b) and Gaussian laser beam ($b = 0$) with $a = 0.5$, when both relativistic and ponderomotive nonlinearities are taken into account. (c) Variation of electron plasma wave intensity with normalized distance of propagation (ξ) for different values of intensity parameter (a) with constant value of decentered parameter (b), when relativistic-ponderomotive nonlinearity is operative.

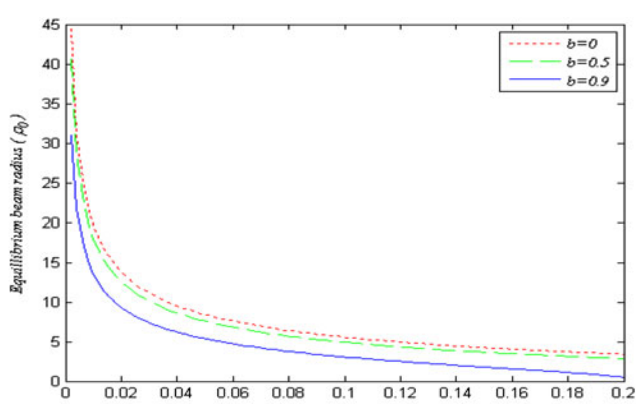

Fig. 3. Variation of the dimensionless initial beam width (ρ_0) as a function of intensity parameter (a) for different values of decentered parameter (b).

the intensity of CGLB (a). This is because the intensity of EPW depends directly on the magnitude of the self-focusing of main CGLB and EPW.

To study the effect of self-focused CGLB and the intensity of EPW on the energy gain by electrons at finite z, we have

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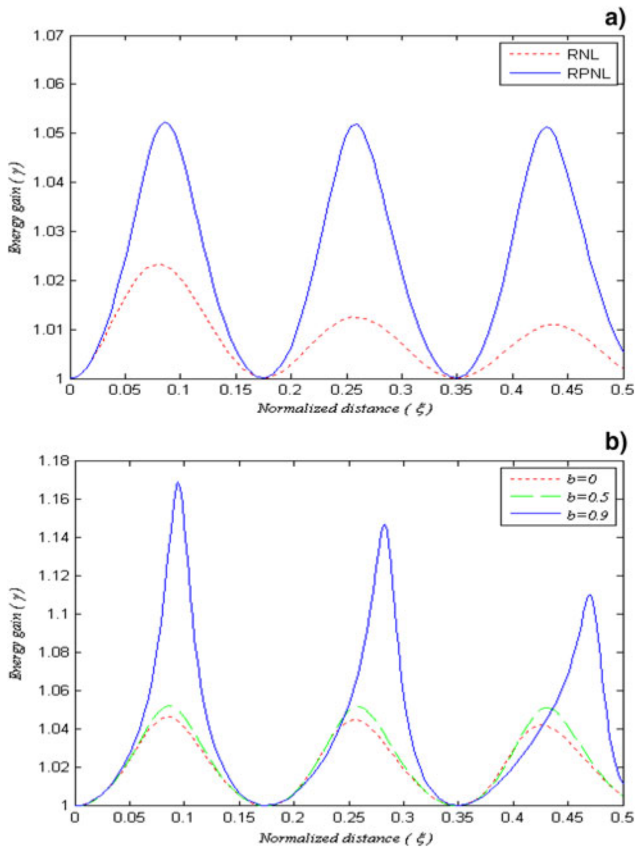

Fig. 5. (a) Variation of energy gain (γ) by the electron with normalized distance of propagation (ξ) for $a = b = 0.5$. (b) Variation of energy gain (γ) by the electron with normalized distance of propagation (ξ) for different values of decentered parameter (b) with constant value of intensity parameter (a), when relativistic-ponderomotive nonlinearity is operative.

solved Eq. (32) numerically with the help of Eq. (30). The energy gain by electrons depends on the self-focusing of CGLB in plasma as well as the amplitude of EPW. Figure 5a represents the variation of energy gain with the normalized distance of propagation (ξ), when both RP and only relativistic nonlinearities are operatives. It is evident from Figure 5a that the maximum energy gain by electrons, gets enhanced when both nonlinearities are present. This is due to strong self-focusing of CGLB and the enhancement of EPW intensity in comparison with only relativistic nonlinearity. Figure 5b reflects the effect of decentered parameter (b) on the energy gain with the normalized distance of propagation (ξ). It is observed that maximum energy gain by electrons is significantly increased by increasing the value of b. This is due to the fact that, with an increase in the value of b, self-focusing of CGLB in plasma and the amplitude of EPW enhanced significantly.

5. CONCLUSION

In summary, we have investigated the propagation of CGLB in plasma under weak RP and only relativistic regimes using the WKB and paraxial approximations. The effect of self-focused CGLB on the excitation of EPW and particle acceleration has been studied. It is found that the self-focusing of the CGLB is very strong and occurs earlier than Gaussian laser beam. Strong and early self-focusing ability of cosh-Gaussian beam depends on the decentered parameter (b) and the intensity of laser beam. The intensity of EPW increases with the increase in value of decentered parameter (b) and intensity of laser beam. It is also observed that the electron gain by electrons depends on the magnitude of self-focusing of CGLB and EPW intensity. This study reflects the usefulness of the propagation of CGLB in plasma with RP nonlinearity over only relativistic nonlinearity and Gaussian profile of the laser beam. These results are relevant in laser–plasma interaction experiments, where an intense laser beam can be used for plasma heating and particle acceleration by beat wave process.

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