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LIQUIDITY, PRICES, SEIGNIORAGE, AND THE TRANSITION FROM BARTER TO FIAT MONEY

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The government-led transition from barter to fiat money and its possible failure are analyzed in the Kiyotaki-Wright model when prices are determined endogenously by the strategic bargaining process. The transition is shown to be more inflationary as the government becomes less patient and less credible. The possibility of breakdown in fiat money due to uncertainty in the size of government and its patience implies longer transition path and higher expected inflation. An application to the transition from local currencies to currency integration shows that local governments are tempted to issue more currency to extract seigniorage from foreign as well as home agents. As long as the degree of economic integration is sufficiently large, an increasing frequency of trading opportunities implies lower price levels and higher welfare relative to the local currency regime. It is when the two countries are fully integrated that the world economy with the unified currency achieves the highest welfare.

Keywords: Seigniorage, Prices, Transition to Fiat Money, Currency Integration

1. INTRODUCTION

In the early history of fiat money, the transition from barter to fiat money depended crucially on the movement of prices or purchasing powers of money. Barter may have reappeared to some extent when the fiat money regime breaks down into a gigantic inflation, with prices doubling every day. In this case money is such a poor store of value that even the very short period of time that must elapse between receiving and spending money is too long to hold it, so that some transactions are best conducted by barter.[1] In ancient China, for instance, the repetition over several hundred years of failures in a transition to fiat money was due to the fact that the people lost confidence in paper money with the continuous rise in prices following the overissues of money to meet financial deficits [Yang (1952)].

The goal of this paper is to examine the importance of prices in understanding the transition from barter to fiat money and its possible breakdown. Recently, Ritter (1995) used the search-theoretic monetary model of Kiyotaki and Wright (1993) to provide an equilibrium account of the transition from barter to fiat money

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regimes. The explanation relies on the intervention of a self-interested government that must be able to promise credibly to limit the issue of money. To achieve credibility, the government must offset the benefits of seigniorage by internalizing some of the aggregate externalities generated by the issue of fiat money. The government's patience and the extent of its involvement or size in the economy are key determinants of whether the transition can be accomplished.

However, following the assumption by Kiyotaki and Wright (1993) that both commodities and money are indivisible, relative prices are constrained to be either 0 or 1. To make a complete explanation of the transition from barter to fiat money regimes, it is necessary to model money as a nontrivial unit of account by relaxing the indivisibility assumption. Trejos and Wright (1995) and Shi (1995) relax the assumption of indivisible commodities, while maintaining the indivisibility of money.[2] Following Nash (1950) and Rubinstein (1982), they explicitly incorporate a bilateral bargaining process into the model to determine the price level endogenously, which leads to additional insights concerning both the existence of stationary monetary equilibrium and the welfare of agents in the equilibrium. In particular, they show that the welfare-maximizing value of money stock balances the trade-off between providing liquidity and raising the price level. However, neither Trejos and Wright (1995) nor Shi (1995) relates the existence of stationary monetary equilibrium to the feasible transition path from barter to fiat money regime.

This paper reconsiders the transition from barter to fiat money when prices are determined endogenously by the strategic bargaining process. The basic structure follows that of Ritter (1995), including subgame perfection as the equilibrium concept, except that goods are now divisible and there is a disutility cost of production as in Trejos and Wright (1995). However, unlike Trejos and Wright (1995), who examine the dynamics under the assumption of myopic bargaining, the transition dynamics of prices are characterized by forward-looking bargaining solutions during the transition which involves a finite length of time periods.

First, it is optimal for government to issue money as soon as possible during the transition, and hence Ritter's (1995) "pedal-to-metal" money growth still holds in a framework in which prices are determined by bilateral bargaining. The government chooses money supply along the transition to maximize its welfare, which depends on purchasing powers of money or prices. An agent holding production opportunities (i.e., seller) trades her output for money that can only be spent in the future, and hence the quantity of output that a seller is willing to produce in any given period depends on its value in the following period. Backward induction, starting from the last period of the transition for a given value of holding money in the steady state, implies that a forward-looking bargaining solution of price in a transition period is determined independently of the government's current money supply decision. Therefore, for a given price, the impatient government maximizes seigniorage by issuing money as rapidly as possible.

Second, the equilibrium money stock decreases with the size of the government and its patience. This is a generalization of Ritter (1995) in the sense that the equilibrium money stock is now determined by considering its effect on the purchasing powers of money or the prices, in addition to providing liquidity and seigniorage benefits. For instance, a larger government coalition internalizes at a greater extent the aggregate externality of issuing money, and hence enjoys less seigniorage. In equilibrium, this should be matched by a decrease in the liquidity provision net of the price effect, which implies a decrease in the money stock.

Further, the equilibrium level of money stock is lower than the one in the fixed-price case, which maximizes frequency of trade (or liquidity effect) and seigniorage. The search friction as well as the future discount implies an inefficiently low quantity of output in exchange for a unit of money. Moreover, the purchasing power of money increases when liquidity is relatively scarce, but declines eventually as the economy's money stock increases further. Considering that the government would want to push the equilibrium quantity of output toward the efficient level, the "inflation effect" of the money supply on the already inefficiently low quantity of output implies that the equilibrium money stock should be lower than in the fixed-price regime.

This also imposes further restrictions, relative to the fixed-price case, on the feasible parameter values for the government's patience and its size, which determine the transition from barter to fiat money. For a given money stock, the inefficiently low quantity of output implies a lower utility in the steady state than in the fixed-price model. Hence, for a given welfare level in the barter equilibrium, the government would have a greater incentive to return to barter than in the fixed-price regime. To sustain monetary exchange, therefore, the government is required to be more patient and more credible. Further, the government with higher patience and credibility follows a less inflationary transition path.

To examine the causes of breakdowns or disruptions in fiat money as observed in the history of fiat money, uncertainty is introduced in the size of government and in its patience: The first captures "political uncertainty" in terms of the random extent to which government takes into account negative aggregate externality in its currency issues, while the second captures its random expenditure needs. Now, there exists a positive probability with which a sufficiently large (negative) shock to the size of government or its patience will cause breakdown in fiat money. The uncertainty in the transition to fiat money implies a longer transition path, accompanied by a higher expected inflation along the transition. This also implies that, to sustain the monetary equilibrium, the government is required to be more involved or credible and more patient than without the possible breakdown in fiat money.

The equilibrium account of the transition from barter to fiat money is also applied to the currency integration in the world economy where there are initially two regions or countries isolated from each other with their own local currencies. As the two countries start integrating with the unified currency, each government's incentive to extract seigniorage from foreigners as well as home agents yields a larger equilibrium money stock relative to the one under complete isolation. Further, with a small degree of economic integration, higher money stocks also

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imply higher price level or lower purchasing power of money than under the local currency regime.

When the lack of economic integration is severe enough, the welfare of a representative agent under the unified currency is lower than that under the local currency regime. Despite the higher money stocks in the unified currency regime, the increasing frequency of trading opportunity in the more integrated world implies lower price level and higher welfare relative to the isolated world with local currencies. It is when the two countries are fully integrated that the world economy with the unified currency achieves the highest welfare. As long as the world economy is not completely integrated, an inefficiency arises in the sense that the lack of economic integration imposes additional frictions on monetary exchange with the unified currency.

The paper is organized as follows. Section 2 reviews a barter equilibrium and a stationary monetary equilibrium in the standard search model of money with divisible commodities. Section 3 characterizes a monetary equilibrium that takes into account the transition from barter to fiat money, including the transition dynamics of prices. An application to the transition from local currencies to currency integration is discussed in Section 4, followed by some concluding remarks in Section 5.

2. THE MODEL AND STATIONARY EQUILIBRIUM

The model economy is a version of the Kiyotaki-Wright (1993) model with divisible commodities. Time is discrete and the horizon is infinite. There is a continuum of infinitely lived agents whose population is normalized to one. There are two types of objects: commodities and money. In the initial period, a fraction M of agents are endowed (at random) with one unit of money each. Money cannot be consumed or privately produced, and it is storable but indivisible. Commodities are divisible and produced by individual agents. Assume that agents cannot hold money and production opportunities at the same time.[3] This is equivalent to assuming unit-inventory technology by which agents can only carry one object—either money or commodities—at a time. This implies a simple stationary distribution of money holdings: In any given period, a fraction M of population holds exactly one unit of money each, and the remaining $1 - M$ holds production opportunities. Agents holding money are referred to as money traders or *buyers*, whereas those with production opportunities are referred to as commodity traders or *sellers*. Agents can produce as soon as they become sellers.

There is a continuum of commodities, and each agent is capable of producing only one type of commodity. Each agent desires to consume a proportion of commodities produced $x \in (0, 1]$, and consumption of one's own output does not yield any utility. Assume that x is the same for all agents. Then, in any match between the two agents (e.g., agent 1 and agent 2), independence and symmetry ensure that (i) the probability that agent 1 will produce what agent 2 wants to consume is x, and (ii) the probability that agent 1 desires what agent 2 produces and agent 2

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desires what agent 1 produces (i.e., double coincidence of wants) is x^2. Assume that individual agents' trading histories are private information. This, together with the other assumptions made here, prevents the "double-coincidence problem" from being overcome with credit arrangement. An agent receives utility $u(Q)$ from consuming Q units of her consumption goods, while its production incurs (disutility) cost of $c(Q)$. Assume that $u(0) = c(0) = 0$, $u'(0) > c'(0) = 0$, $u'(Q) > 0$, $c'(Q) > 0$, $u''(Q) \leq 0$, and $c''(Q) > 0$.

In the beginning of each period, agents meet pairwise and at random. When an appropriate buyer and seller meet they bargain over q, the quantity of output to be exchanged for a unit of money. The unit-inventory restriction implies that barter is only possible between sellers. One of the agents in pairwise meeting is chosen at random to propose a value of q, to which the other can respond by either accepting or rejecting the offer. If the offer is rejected, they wait for another round when someone is again chosen at random to make an offer, and so the process continues. It is assumed that agents never meet other potential trading partners during the interval between a rejection and the next offer. When they come to an agreement, they produce the agreed-upon quantity of commodities and trade for consumption.[4]

As is standard in the strategic bargaining model of Rubinstein (1982), in equilibrium no agent chooses to terminate the bargaining process, all offers are made so that they are accepted, and all negotiations are completed in the first round at a quantity that depends on whether the seller or the buyer makes a proposal. However, as the time between rounds in the bargaining process goes to zero, the quantity approaches the Nash (1950) solution, which does not depend on who makes an offer first. This paper will use the generalized Nash bargaining solution to determine price in each period.

2.1. Barter Equilibrium

Let V_B denote the expected utility or value function for an agent in a barter equilibrium and q^B denote quantity of barter exchange. Then V_B is determined by

$$V_B = x^2 \left[u(q^B) - c(q^B) + \beta V_B\right] + (1 - x^2)\beta V_B, \qquad \textbf{(1)}$$

where $\beta \in (0, 1)$ is the discount factor. The quantity of exchange in a barter equilibrium, q^B, is determined by the Nash bargaining solution:

$$q^B = \arg\max[u(q^B) - c(q^B)]^2 \quad \text{subject to} \quad u(q^B) - c(q^B) \geq \beta V_B, \qquad \textbf{(2)}$$

which implies $u'(q^B) = c'(q^B)$. With $x^2[u(q^B) - c(q^B)] > 0$ (from the assumptions on u and c) so that barter is preferred to autarky (i.e., no trade) in every period, a *barter equilibrium* consists of (V_B, q^B) such that: (i) q^B solves the bargaining problem (2); and (ii) V_B satisfies (1).

2.2. Stationary Monetary Equilibrium

The expected utilities or value functions for commodity and money traders, denoted respectively as V_G and V_M, are

$$V_G = (1-M)x^2[u(q^B) - c(q^B)] + Mx[-c(q) + \beta(V_M - V_G)] + \beta V_G, \quad \textbf{(3)}$$

$$V_M = (1-M)x[u(q) + \beta(V_G - V_M)] + \beta V_M, \quad \textbf{(4)}$$

where q denotes the quantity of output in exchange for a unit of money, or the purchasing power of money. The value function of holding production opportunities in equation (3) consists of the expected payoffs from (i) barter exchange with another commodity trader; and (ii) monetary exchange with a money trader. In equation (4), the expected utility of holding money is equal to the probability of meeting a commodity trader times the expected gain from such a meeting. For given q and q^B, it can be shown that $-c(q) + \beta V_M \geq \beta V_G$, which means that commodity traders voluntarily accept money, if and only if

$$x \leq \frac{(1-M)u(q) - (z-M)c(q)}{(1-M)[u(q^B) - c(q^B)]}, \quad \textbf{(5)}$$

where $z \equiv [1-\beta(1-x)]/\beta x > 1$. Notice that, for a given $M \in (0,1)$, $x > 0$ requires β to be sufficiently large for money to be accepted by commodity traders.

Given V_G and V_M, the purchasing power of money, q, is determined by the following solution of Nash bargaining between a buyer and a seller:

$$q = \arg\max[u(q) + \beta V_G][-c(q) + \beta V_M] \quad \textbf{(6)}$$

subject to the individual rationality or participation constraints

$$u(q) + \beta V_G \geq \beta V_M, \quad \textbf{(7)}$$

$$-c(q) + \beta V_M \geq \beta V_G. \quad \textbf{(8)}$$

A *stationary monetary equilibrium* consists of (q, V_G, V_M) such that (i) q solves the bargaining problem (6) subject to the constraints (7) and (8), taking V_G and V_M as given; and (ii) V_G and V_M satisfy (3) and (4), taking q as given. Trejos and Wright (1995, Proposition 3) show that two types of stationary monetary equilibria, constrained and unconstrained, coexist if $\beta > \tilde{\beta} \equiv \tilde{\beta}(M, x)$. An *unconstrained* stationary equilibrium consists of q, which solves the unconstrained Nash problem (6). In a *constrained* stationary equilibrium, sellers are indifferent between accepting or rejecting money; that is, the constraint (8) holds with an equality.

This paper focuses on the unconstrained equilibria. Some key properties of the unconstrained stationary equilibrium are reviewed as benchmarks that will be compared later with those of a monetary equilibrium incorporating the transition from barter to fiat money.[5]

First, for any $\beta \in (0,1)$, $q < q^B$ and $q \to q^B$ as $\beta \to 1$. That is, q is inefficiently low because commodity traders produce q for money that can only be spent in the

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future and, since they discount future consumption (i.e., $\beta < 1$), they are willing to provide less than if they could use money for immediate consumption. This implies that $q \to q^B$ as $\beta \to 1$.[6] Further, as $\beta \to 1$, $z \to 1$ in (5) and hence $-c(q) + \beta V_M \geq \beta V_G$ for any $x \in (0, 1]$.

Let the steady-state welfare be defined as $W = MV_M + (1 - M)V_G$, which is the long-run value function for the average agent, not conditional on the current status. Differentiating the welfare in the unconstrained equilibrium with respect to M yields

$$(1-2M)[u(q)-c(q)] + M(1-M)[u'(q)-c'(q)]\frac{\partial q}{\partial M} - 2(1-M)xK = 0, \quad \textbf{(9)}$$

where $K \equiv u(q^B) - c(q^B)$ and $\partial q/\partial M$ captures the effect of money stocks on the price level. An increase in M will increase the number of meetings between buyers and sellers—a "liquidity effect"—which can increase the amount of production and exchange. When M is relatively small, this is more likely to dominate the tendency for increases in M to reduce the value of money. However, as M increases, the price level eventually begins to rise. According to (9), the welfare-maximizing value of M balances the trade-off between providing liquidity and raising the price level in the unconstrained equilibrium.

3. TRANSITION FROM BARTER TO FIAT MONEY

Now, the "endogenous price" setup of Trejos and Wright (1995) is merged into the "transition-from-barter-to-money" framework of Ritter (1995), seeking for new insights into understanding characteristics underlying the transition from barter to fiat money regimes or possible breakdown in fiat money.

As shown by Ritter (1995), in a trading environment with search frictions due to the lack of double coincidence in a pairwise meeting, the transition from barter equilibrium to a stationary monetary equilibrium cannot be achieved by the actions of rational individuals issuing money. An agent holding production opportunities who meets someone with whom barter is impossible, but who can produce her consumption good, will always have an incentive to issue money. However, individual agents do not consider the aggregate externality they create by issuing money. This kind of coordination failure causes the issue of "too much" money and hence the collapse of any potential monetary equilibrium. Thus, valued money requires some form of institution that can limit the issue of money.

Assume that a single coalition of agents, called "the government," exists, consisting of a fraction $\alpha > 0$ of the population, where the constant α is assumed to be exogenously given. The government is given legal monopolies in the issuance of fiat money. These agents differ from the rest of the economy in that they can issue money identified with the government and the government can limit the issue of money by its members.[7]

Let M_t denote the money supply in period t and if one unit of M_t exchanges for q_t units of consumption, the implied price level at t is $p_t = 1/q_t$. Let $V_G(M_t)$

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and $V_M(M_t)$ denote, respectively, the value functions for a commodity trader and a money trader in period t. Let $m_t = (1 - M_t)x(1 - x)$ denote the probability that an agent holding production opportunities (i.e., a seller) will wish to issue money. Then, the money stock will evolve according to

$$M_{t+1} = M_t + \mu_t \alpha m_t (1 - M_t), \tag{10}$$

where $\mu_t \in [0, 1]$ is the fraction of government agents issuing money at t.

PROPOSITION 1. *Let T denote the last period of the transition from barter to fiat money. Then, $\mu_t = 1$ if $t < T$, $\mu_t \in [0, 1]$ if $t = T$, and $\mu_t = 0$ if $t > T$.*

Proof. Suppose the optimal monetary policy follows a transition path of finite length to a steady state (this conjecture is verified later). Let M denote the money supply target in the steady state. Then, the quantity of exchange at T is determined by the Nash bargaining solution given by

$$q_T = \arg\max[u(q_T) + \beta V_G(M)][-c(q_T) + \beta V_M(M)]$$

subject to $u(q_T) + \beta V_G(M) \geq \beta V_M(M)$ and $-c(q_T) + \beta V_M(M) \geq \beta V_G(M)$, where $V_G(M)$ and $V_M(M)$ are the steady-state value functions given by (3) and (4), respectively. Further, from (10),

$$M = M_T + \mu_T \alpha m_T (1 - M_T). \tag{10a}$$

Notice that, for the given steady-state money supply M, q_T depends on the future expected utilities of holding money and production opportunities in the steady state. Hence, q_T is determined independently of the government's money supply decision, μ_T, in (10a).

Now, the welfare of government members at T consists of the expected utilities of government members holding money and production opportunities:

$$\begin{aligned} W_T &= M_T\{(1 - M_T)x[u(q_T) + \beta V_G(M)] + [1 - (1 - M_T)x]\beta V_M(M)\} \\ &\quad + (1 - M_T)\{\mu_T m_T u(q_T) + s_T[-c(q_T) + \beta V_M(M)] \\ &\quad + (1 - s_T)\beta V_G(M)\} + b_T \\ &= M_T(1 - M_T)x[u(q_T) - c(q_T)] + \frac{1}{\alpha}(M - M_T)[u(q_T) - \alpha c(q_T)] \\ &\quad + \beta V_G(M) - M\beta[V_G(M) - V_M(M)] + b_T, \end{aligned}$$

where $s_T = M_T x + \alpha \mu_T m_T$ and $b_T = (1 - M_T)^2 x^2 [u(q_T^B) - c(q_T^B)]$ in the first equality, which, together with (10a), implies the second equality.

Notice that (10a) implies $M_T = f(\mu_T)$, where $f'(\mu_T) < 0$ for a sufficiently small $x \in (0, 1)$.[8] Now, for given q_T, differentiating W_T with respect to μ_T yields

$$f'(\mu_T)(1-2M_T)x[u(q_T)-c(q_T)] - \frac{f'(\mu_T)}{\alpha}[u(q_T)-\alpha c(q_T)]$$
$$-2f'(\mu_T)(1-M_T)x^2\left[u\left(q_T^B\right)-c\left(q_T^B\right)\right] > 0,$$

as long as $x \in (0, 1)$ is sufficiently small. Hence, W_T increases with μ_T. Further, by backward induction, it can be shown that W_t increases with μ_t for $t \leq T$. Therefore, $\mu_t = 1$ for $t < T$ and $\mu_T = (M - M_T)/\alpha m_T(1 - M_T) \in [0, 1]$ for given M. Finally, for the given money supply target $M \in (0, 1)$, this implies a transition path of finite length to the monetary steady state. □

According to (10), where $\mu_t = 1$ for $t \leq T$, along the transition path, individual members of the government coalition are permitted to issue money at will (except the last period of the transition when only a fraction of the members are randomly chosen to issue money so as to reach the money supply target).

Notice that a unit of money that a commodity trader receives in exchange of output can only be spent in the future, and hence the quantity of output (or purchasing power of money) in any given period depends on the value of holding money in the following period. Since the currency seigniorage in a given transition period depends on the evolution of money stock according to (10), the government's optimal path of money supply via the choice of μ_t is to consider its possible effects on the purchasing powers of money, q_t, along the transition path. However, backward induction (starting from the last period T of the transition) implies that, for a given value of holding money in a monetary steady state, q_t is determined independently of μ_t during the transition. For given q_t, the impatient government will then maximize seigniorage by issuing money as soon as possible ($\mu_t = 1$ for $t \leq T$), and hence the optimal monetary policy follows a transition path of finite periods to the steady state with monetary exchange.[9]

3.1. Monetary Equilibrium

Since the transition from barter to fiat money requires the study of the economy outside steady states, the definition of a monetary equilibrium allows for the explicit treatment of subgames that start in a given period. A *monetary equilibrium* at t is a sequence of $\{M_\tau, q_\tau, V_G(M_\tau), V_M(M_\tau)\}_{\tau=t}^{\infty}$ such that, given the actions and expectations of all other agents, each agent who trades accepts money in exchange for commodities in $\tau \geq t$. In particular, without precommitment to a monetary policy by the government, the method developed by Ritter (1995) is followed to determine the optimal issuance of money, which is supported by the *subgame perfect monetary equilibrium* under proper conditions.

Let M denote the money supply target in the sense that M is the government's best option among the optimal path of money supply. Since precommitment is not possible, however, it is necessary to determine whether M is a Nash equilibrium of the subgame that begins in period $T+1$, where T is the last period of the transition to fiat money. Notice that the government has another choice: Once M is reached at T, the economy can be driven back to barter by issuing more money at $T+1$.

Without precommitment, this option must not be preferred by the government if money is to have value. That is, the following condition is required for M to be a monetary equilibrium:

$$V_B \leq W \equiv (1-M)V_G(M) + MV_M(M), \tag{11}$$

where V_B is the expected utility in the barter equilibrium as given by equation (1), and W is the subgame utility of the government if it does not issue more money. Moreover, from the government's point of view, no incentive to issue an additional amount of money at $T+1$ implies the following necessary condition for optimality of M:

$$\frac{\partial W_T(M_T, M_{T+1}, M_{T+1}, \ldots)}{\partial M_{T+1}} = 0,$$

where W_T is the welfare of the government from the standpoint of period T and the derivative is evaluated at $M_T = M_{T+1} = M$. Given that W_T involves one period of transition followed by a steady state, the above differentiation is based on the following[10]:

$$\begin{aligned}
&W_T(M_T, M_{T+1}, M_{T+1}, \ldots)\\
&\quad = M_T W_T^M(M_T, M_{T+1}, M_{T+1}, \ldots) + (1-M_T)W_T^G(M_T, M_{T+1}, M_{T+1}, \ldots)\\
&\quad = M_T\{(1-M_T)x[u(q_T) + \beta V_G(M_{T+1})] + [1-(1-M_T)x]\beta V_M(M_{T+1})\}\\
&\qquad + (1-M_T)\{\mu_T m_T u(q_T) + s_T[-c(q_T) + \beta V_M(M_{T+1})]\\
&\qquad + (1-s_T)\beta V_G(M_{T+1})\} + b_T\\
&\quad = M_T(1-M_T)x[u(q_T) - c(q_T)] + \frac{1}{\alpha}(M_{T+1} - M_T)[u(q_T) - \alpha c(q_T)]\\
&\qquad + \beta V_G(M_{T+1}) - M_{T+1}\beta[V_G(M_{T+1}) - V_M(M_{T+1})] + b_T,
\end{aligned} \tag{12}$$

where W_T^M and W_T^G are the expected utilities of government members holding money and production opportunities, respectively, and $s_T = M_T x + \alpha\mu_T m_T = M_T x + (M_{T+1} - M_T)/(1-M_T)$. The above expression for W_T implies the following necessary condition for optimality:

$$\begin{aligned}
&\frac{1}{\alpha}[u(q) - \alpha c(q)] + \frac{\beta x}{1-\beta}\bigg\{(1-2M)[u(q) - c(q)]\\
&\quad + M(1-M)[u'(q) - c'(q)]\frac{\partial q}{\partial M} - 2(1-M)x[u(q^B) - c(q^B)]\bigg\} = 0,
\end{aligned}$$

which can be rewritten as follows:

$$\begin{aligned}
&\frac{\beta x}{1-\beta}\left[\gamma x - (1-2M)(1-\gamma x) - M(1-M)\left(\frac{u'-c'}{u-c}\right)\frac{\partial q}{\partial M}\right]\\
&\quad = \frac{1}{\alpha}\left[1 + \left(\frac{1-\alpha}{\alpha}\right)\frac{c}{u-c}\right],
\end{aligned} \tag{13}$$

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where $\gamma = [u(q^B) - c(q^B)]/[u(q) - c(q)] > 1$ and the argument q is dropped for simplicity from u', c', u, and c. Now, the government's money supply target, M, is the solution to the above equation.

A *subgame perfect monetary equilibrium* at t consists of a sequence of $\{M_\tau, q_\tau, V_G(M_\tau), V_M(M_\tau)\}_{\tau=t}^{\infty}$ such that

(i) $\{M_\tau\}_{\tau=t}^{\infty}$ follows (10) with $\mu_\tau = 1$ for $\tau < T$, $\mu_T = (M - M_T)/\alpha m_T(1 - M_T)$, where M is determined by (13), and $M_{\tau+1} = M$ for $\tau \geq T$;

(ii) $\{q_\tau\}_{\tau=t}^{\infty}$ solves the following sequence of Nash bargaining problems at $\tau \geq t$:

$$q_\tau = \arg\max[u(q_\tau) + \beta V_G(M_{\tau+1})][-c(q_\tau) + \beta V_M(M_{\tau+1})] \tag{14}$$

subject to $u(q_\tau) + \beta V_G(M_{\tau+1}) \geq \beta V_M(M_{\tau+1})$ and $-c(q_\tau) + \beta V_M(M_{\tau+1}) \geq \beta V_G(M_{\tau+1})$, taking $\{V_G(M_{\tau+1}), V_M(M_{\tau+1})\}_{\tau=t}^{\infty}$ and $\{M_\tau\}_{\tau=t}^{\infty}$ as given;

(iii) $\{V_G(M_\tau), V_M(M_\tau)\}_{\tau=t}^{\infty}$ satisfy

$$\begin{aligned} V_G(M_\tau) &= (1 - M_\tau)x^2\left[u\left(q_\tau^B\right) - c\left(q_\tau^B\right)\right] + M_\tau x\{-c(q_\tau) \\ &\quad + \beta[V_M(M_{\tau+1}) - V_G(M_{\tau+1})]\} + \beta V_G(M_{\tau+1}), \end{aligned} \tag{15}$$

$$\begin{aligned} V_M(M_\tau) &= (1 - M_\tau)x\{u(q_\tau) + \beta[V_G(M_{\tau+1}) - V_M(M_{\tau+1})]\} \\ &\quad + \beta V_M(M_{\tau+1}), \end{aligned} \tag{16}$$

taking $\{q_\tau\}_{\tau=t}^{\infty}$ and $\{M_\tau\}_{\tau=t}^{\infty}$ as given; and

(iv) the option of returning to barter exchange is not preferred, as summarized by (11).

I first characterize the steady state consistent with the monetary equilibrium as defined above. Equation (13) implies that the steady-state money stock, M, is determined by balancing the liquidity provision net of the price effect of the money stock (left side of the equation) with its effect on the seigniorage benefits (right side of the equation). For a relatively large money stock, the price effect of an increase in the money stock is to dominate the liquidity effect. This drives down the equilibrium money stock to the level where the liquidity effect net of the price effect is matched by the seigniorage effect.

Notice that the price-effect term associated with $(\partial q/\partial M)$ leads to the departure of M from the fixed-price model of Ritter (1995), while the seigniorage $[u(q) - \alpha c(q)]/\alpha$ is the difference between (13) and the steady-state-only counterpart (9) in Trejos and Wright (1995). For example, in the fixed-price model where $u(q) = u(q^B) = u$, $c(q) = c(q^B) = 0$, and $(\partial q/\partial M) = 0$, the necessary condition (13) is equivalent to the one of Ritter (1995) where the equilibrium money stock, denoted as $\bar{M}$, is

$$\bar{M} = \frac{1}{2}\left(\frac{1 - 2x}{1 - x}\right) + \frac{1}{2\alpha}\frac{1}{x(1 - x)}\left(\frac{1 - \beta}{\beta}\right), \tag{17}$$

which consists of the liquidity effect and the seigniorage benefits of the equilibrium money stock.

With regard to the properties of the equilibrium money stock, equation (13) implies the following relationship between M and (α, β).

PROPOSITION 2. *As long as $u'(q)/c'(q)$ is sufficiently close to 1 for a given q, M decreases with α and β.*

Proof. In equation (13), an increase in α causes a decrease in the seigniorage benefits. In equilibrium, this should be matched by a decrease in the money stock, which lowers the frequency of trade net of the price effect, as long as $u'(q) - c'(q)$ is kept small for a given q. Similarly, it can be shown that M satisfying (13) decreases with β. ■

This is a generalization of Ritter (1995) in the sense that the inverse relationship between (α, β) and M is extended in the presence of the price effect, in addition to the liquidity and seigniorage benefits. A larger or more patient government considers at a greater extent the aggregate externality of issuing money, and hence must offset the seigniorage benefits. In equilibrium, this should be balanced by a decrease in the liquidity provision net of the inflation effect, which implies a decrease in the money stock. However, if the marginal cost of production is relatively small for a given quantity of output so that the inflation effect "dominates" the liquidity effect, equation (13) implies that a decrease in the seigniorage benefits following an increase in α can be matched by an increase in M.

PROPOSITION 3. *For a small x, $M \leq \bar{M}$.*

Proof. In the fixed-priced model, the equilibrium condition (11) is reduced to

$$x \leq (1 - \bar{M})(1 - x), \tag{18}$$

whereas, in the present model with flexible price, it becomes

$$x \leq (1 - M)\left(\frac{1}{\gamma} - x\right). \tag{19}$$

Since $\gamma > 1$, the comparison of (19) with (18) implies $M \leq \bar{M}$ for a given x. ■

In the fixed-price model where trade involves one-for-one swap of indivisible commodities or money, an agent gets a constant level of utility from consuming one unit of her consumption good. Once the assumption of indivisible commodities is relaxed, agents can bargain over the quantity of output in exchange for a unit of money. However, the search friction along with the discounting of future consumption yields an inefficiently low quantity of output in exchange for a unit of money in the sense that $u'(q) > c'(q)$, implying $q < q^B$ where q^B is determined by $u'(q^B) = c'(q^B)$. Hence, the government would want q to increase toward q^B. In doing that, the government must take into account the inflation effect of the money supply on the already inefficiently low q, in addition to the frequency of trade and seigniorage that are maximized at $\overline{M}$ in the fixed-price case. Therefore, the negative effect of the money stock on q implies that the equilibrium money stock, M, is lower than the one with fixed price.

In terms of satisfying the equilibrium condition (11), an inefficiently low quantity of output relative to barter exchange implies a lower value of W for a given money stock than in the fixed-price model. Hence, once M is reached for a given x, the government would have a greater incentive to go back to barter exchange by printing more money. Therefore, to have monetary exchange preferred by the government, the government should be more patient and more credible than in the fixed-price model economy. This is also consistent with the inverse relationship between (α, β) and the equilibrium money stock (Proposition 2). Let $\overline{\alpha}$ and $\overline{\beta}$ denote, respectively, the size of government and its patience in the fixed-price equilibrium. Then, this is summarized by the following proposition.

PROPOSITION 4. *As long as $u'(q)/c'(q)$ is sufficiently close to 1 for a given q,*

$$(\alpha, \beta) \geq (\overline{\alpha}, \overline{\beta}).$$

Unlike in the fixed-price model, the analytic solution for M such as (17) is not readily available. Numerical solutions are computed assuming the following functional forms for utility and variable cost: $u(q) = q$ and $c(q) = \kappa q^2$, where $\kappa > 0$. Combining (11) and (13), panels (A) and (B) of Figure 1 depict numerically the region of the parameters α and β in which the unconstrained monetary equilibrium exists for $x = 0.2$. The dashed line indicates a boundary for the existence of the fixed-price monetary equilibrium.

First, as claimed in Proposition 4, when the marginal cost of production is relatively high for a given quantity of output, the consideration of the inflation effect shrinks the set of feasible values (α, β) for the monetary equilibrium relative to the fixed-price case of Ritter (1995) (Panel A, where $\kappa = 1.0$), whereas the opposite is the case when the marginal cost of production is relatively small (Panel B, where $\kappa = 0.7$).[11] Moreover, the higher values of (α, β) in Panel A are consistent with the lower-equilibrium money stocks relative to the fixed-price model, as illustrated in Panel C.

3.2. Equilibrium Transition Path

Noting that the transition involves finite periods T, the forward-looking solution of equilibrium transition path can be computed by backward induction, starting from the last period of the transition, T. For a given x, a pair of (α, β) is selected, under which the monetary equilibrium exists. For the equilibrium money stock, M, the bargaining solution in the last period T of the transition is given by

$$q_T = \arg\max[u(q_T) + \beta V_G(M)][-c(q_T) + \beta V_M(M)]$$

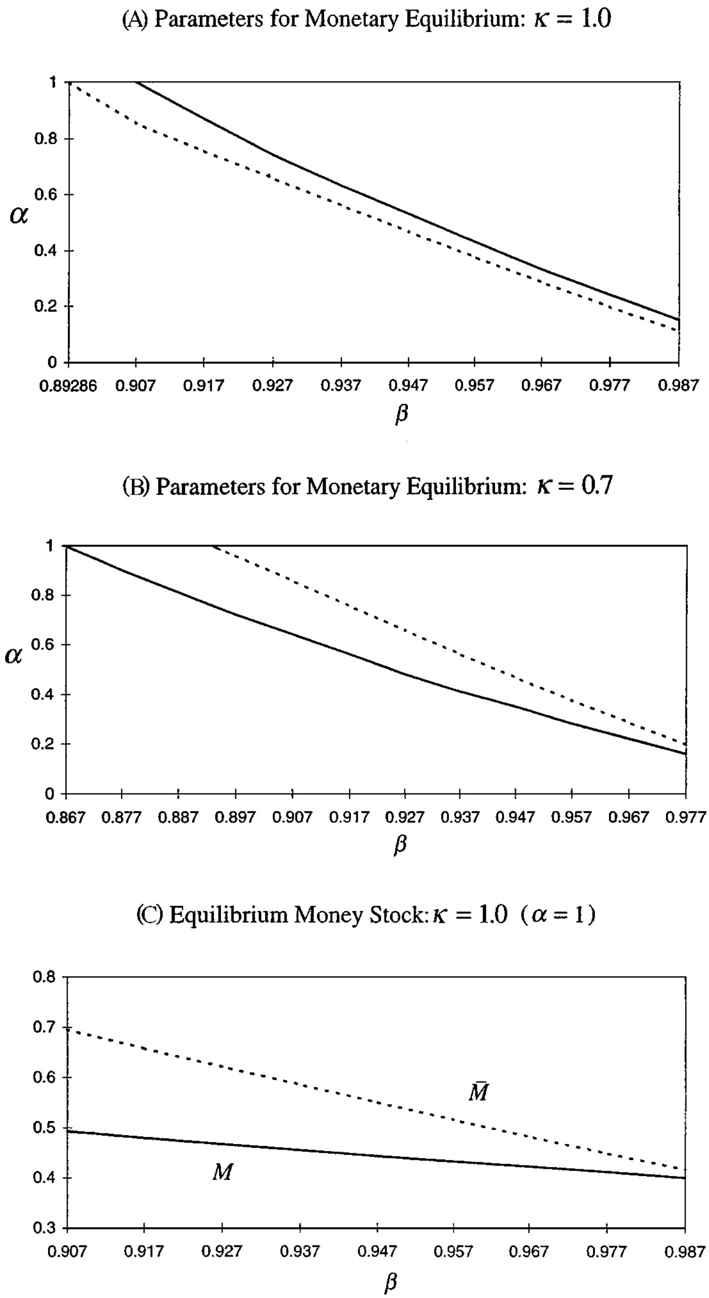

Figure 1. Unconstrained monetary equilibrium.

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subject to $u(q_T) + \beta V_G(M) \geq \beta V_M(M)$ and $-c(q_T) + \beta V_M(M) \geq \beta V_G(M)$, where $V_G(M)$ and $V_M(M)$ are given by the respective value functions in the monetary steady state. The value functions at T, $V_G(M_T)$ and $V_M(M_T)$, now solve (15) and (16) with the money stock M_T obtained from solving $M_T + \alpha x(1-x)(1-M_T)^2 - M = 0$ in (10) where μ_T is set equal to 1.[12] As for $T-1$, the bargaining solution q_{T-1} can be obtained as analogous to q_T, with the value functions $V_G(M_T)$ and $V_M(M_T)$ determined as above. The value functions, $V_G(M_{T-1})$ and $V_M(M_{T-1})$, now solve (15) and (16) with the money stock M_{T-1} solving $M_{T-1} + \alpha x(1-x)(1-M_{T-1})^2 - M_T = 0$ from (10) where $\mu_{T-1} = 1$ and M_T is determined as above. The same procedures can be applied for $t = T-2, T-3, \ldots$.

For $\beta = 0.99$ and $x = 0.1$, Panel A of Figure 2 illustrates the transition paths of inflation, q_t/q_{t+1} for $t \leq T$, when $\alpha = 0.7$ and 0.9. The functional forms for utility and variable costs are the same as in Figure 1 with $\kappa = 1$: $u(q) = q$ and $c(q) = q^2$. Along the transition path, the larger, and hence more credible, government yields lower inflation. Further, as illustrated in panel B, where $\alpha = 0.9$ and $x = 0.1$, the economy with more patient agents (including the government members) follows a less inflationary transition path in a monetary equilibrium. For a given money stock, as individual agents discount their future consumption more heavily (lower β), money becomes a relatively poor store of value, and hence the period of time that must elapse between receiving and spending money becomes essentially longer. This implies lower purchasing power of money since sellers are willing to produce less quantity of output in exchange for money than if they are more patient.

3.3. Breakdown of Fiat Money

So far I have investigated the properties of monetary equilibrium which takes an explicit account of the transition from barter to fiat money. However, the history of fiat money had seen breakdowns or interruptions in the fiat money regime accompanied by considerable inflations following urgent expenditure needs of government or its lack of recognition that the control of its ability to create money was a public concern.[13] To examine the causes of breakdowns or disruptions in fiat money, I introduce uncertainty in the size of government and in its patience: The first captures a "political uncertainty" in terms of the random extent to which government represents the whole population by taking into account negative externality in its currency issues, whereas the second captures its random expenditure needs.

Let the size of government at t be $\alpha_t = \alpha - \epsilon_t$, where ϵ_t is an identically and independently distributed (i.i.d.) random variable over time with two possible realizations, $\epsilon_t \in \{0, \epsilon\}$, and $\text{Prob}[\epsilon_t = \epsilon] = \pi \in (0, 1)$. Similarly, let the stochastic level of patience at t be $\beta_t = \beta - \eta_t$, where η_t is an i.i.d. random variable over time with $\eta_t \in \{0, \eta\}$ and $\text{Prob}[\eta_t = \eta] = \rho \in (0, 1)$. For a given realization of α_t during a transition path, Proposition 1 still holds so that the transition to fiat money involves a finite length of time period. Now, in the last period T of the transition to fiat money,

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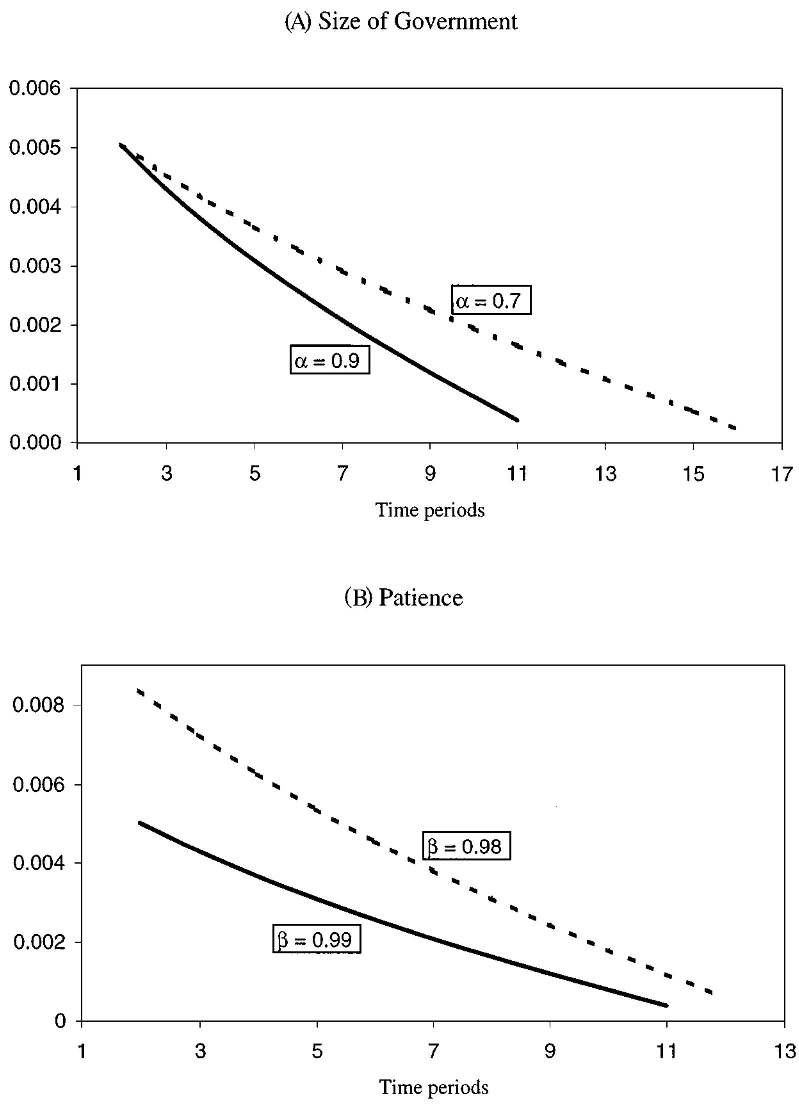

Figure 2. Transition paths of inflation.

the model economy is subject to (i) a possible reduction in the size of government, $\alpha_T = \alpha - \epsilon$; or (ii) a positive expenditure shock, $\beta_T = \beta - \eta$. In either of the two or both situations, for sufficiently large ϵ or η, the government will prefer the option of returning to barter exchange by issuing more money. This implies that, for given $\epsilon > 0$ and $\eta > 0$ with the associated probabilities (i.e., π and ρ), the government is required to be more representative of the whole population (i.e., more credible) and more patient than without the possible breakdown in fiat money. For example,

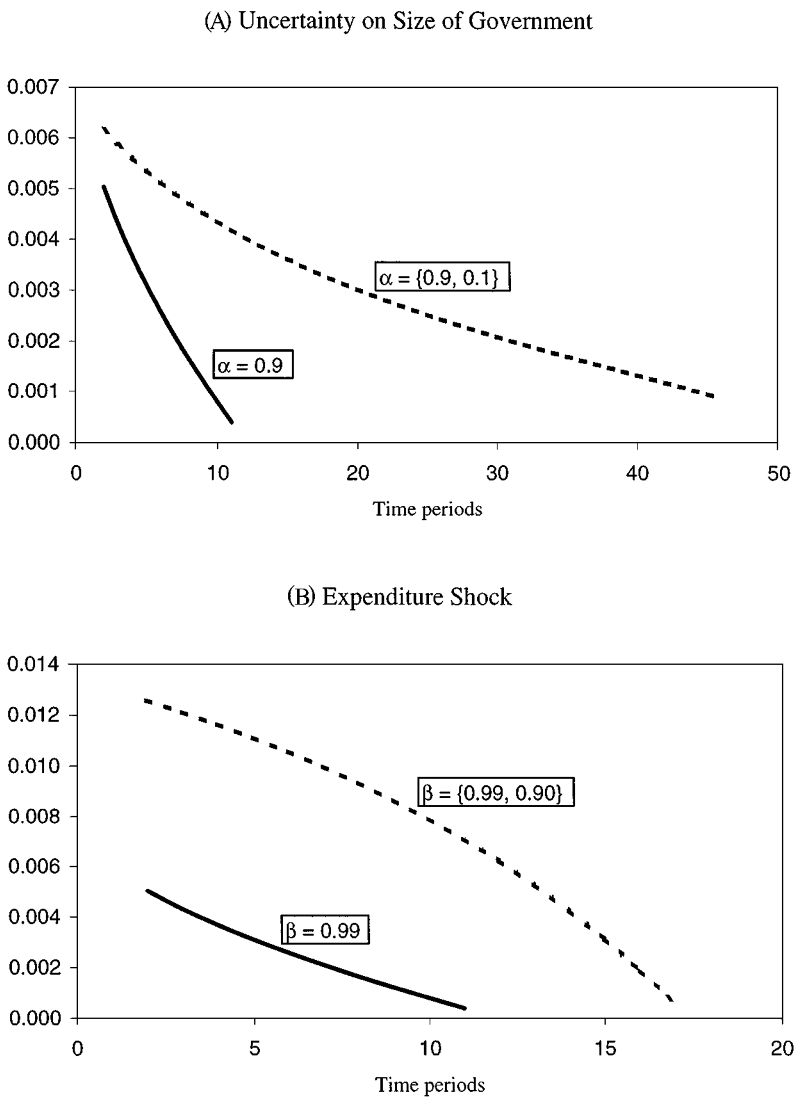

FIGURE 3. Uncertainty and transition paths of expected inflation.

a government that experiences a rise in impatience due to urgent expenditure needs should be larger in order to maintain the monetary equilibrium.

Figure 3 illustrates transition paths of *expected* inflation under uncertainty in (α, β), along with the ones without uncertainty, where $x = 0.1$, $\alpha = 0.9$, $\epsilon = 0.8$, $\beta = 0.99$, $\eta = 0.09$, and $\pi = \rho = 0.5$. The functional forms for utility and variable cost are $u(q) = q$ and $c(q) = q^2$. Notice that the possibility of breakdown in fiat money following the shock to the size of government (panel A) or its impatience (panel B) implies a longer transition path and higher expected inflation along the transition.[14]

4. CURRENCY INTEGRATION

The equilibrium account of the transition from barter to fiat money is now applied to the currency integration in the world economy. I extend the basic structure of Matsuyama et al. (1993) and Trejos and Wright (1998) to allow for the transition from isolation with local currencies to the unified currency. The world economy is populated by a continuum of infinitely lived agents with unit mass. The agents are now divided in two regions, Home and Foreign. Let $n \in (0, 1)$ be the size of the Home population, and hence $(1-n) \in (0, 1)$ will be the size of the Foreign population. Let $\theta = (1-n)/n$ denote the relative size of the Foreign country.

For inventory and money holdings, let M_h and M_f denote, respectively, the fraction of Home agents holding the Home and Foreign currency. The fraction of Home agents without currency (and hence holding production opportunities) is then $1 - M_h - M_f$, so that the inventory distribution among Home agents can be summarized by $X = (M_h, M_f, 1 - M_h - M_f)$. Similarly, the inventory distribution among Foreign agents can be summarized by $X^* = (M_h^*, M_f^*, 1 - M_h^* - M_f^*)$, where M_h^* and M_f^* denote, respectively, the fraction of Foreign agents holding Home and Foreign currency. Let M and $M^* \in (0, 1)$ denote, respectively, the supply of Home currency per Home agent and the supply of Foreign currency per Foreign agent. Then, $nM = nM_h + (1-n)M_h^*$ and $(1-n)M^* = nM_f + (1-n)M_f^*$.

Each period, a Home agent meets another Home agent with probability n, whereas she meets a Foreign agent with probability $\delta(1-n)$, where $\delta \in [0, 1]$. The parameter $\delta \in [0, 1]$ represents the relative frequency of meeting a foreigner (i.e., an agent of a different region), such that a pair of agents who live in different regions or countries meet less frequently than a pair of agents who live in the same country.

Further, the relative probabilities of meeting foreigners versus locals are equal to $\delta\theta$ for a Home agent and δ/θ for a Foreign agent, so that they depend on both the relative frequency of meeting a foreign agent and the relative country size. Notice that an increase in δ does not reduce the chance of meeting one's fellow citizen and instead results in a higher frequency of trading opportunity. This is the sense in which δ captures the *degree of economic integration*. Finally, for simplicity, both Home and Foreign agents face the same $x \in (0, 1)$, which represents the extent of double coincidence in the exchange of commodities.

4.1. Stationary Equilibrium with Unified Currency

I first consider a stationary monetary (Nash) equilibrium in which the two currencies are unified and become perfect substitutes. That is, (i) a Home agent trades her production good for both the Home and Foreign currencies, and trades both currencies for her consumption good; and (ii) a Foreign agent trades her production

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good for both the Home and Foreign currencies, and trades both currencies for her consumption good.

In the steady state, a complete mixing of inventories is achieved, $X = X^*$, and hence $M_h = M_h^* = nM$ and $M_f = M_f^* = (1-n)M^*$. Let V_g, V_h, and V_f be the value functions of a Home agent who holds production opportunities, the Home currency, and the Foreign currency, respectively. Let q_j^i denote the quantity of output produced by a "seller" from country j in exchange for the unified currency held by a "buyer" from country i, where i, $j =$ Home (h), Foreign (f). Further, let q^B denote the quantity of exchange under barter. Then, given q_j^i and q^B, the value functions are

$$\begin{aligned} V_g = {} & x^2[n + \delta(1-n)][1 - nM - (1-n)M^*][u(q^B) - c(q^B)] \\ & + (xn)nM\left[-c\left(q_h^h\right) + \beta(V_h - V_g)\right] + x\delta(1-n)nM \\ & \times \left[-c\left(q_h^f\right) + \beta(V_h - V_g)\right] + (xn)(1-n)M^*\left[-c\left(q_h^h\right) + \beta(V_f - V_g)\right] \\ & + x\delta(1-n)(1-n)M^*\left[-c\left(q_h^f\right) + \beta(V_f - V_g)\right] + \beta V_g, \end{aligned} \tag{20}$$

$$\begin{aligned} V_h = {} & (xn)n(1-M)\left[u\left(q_h^h\right) + \beta(V_g - V_h)\right] \\ & + x\delta(1-n)n(1-M)\left[u\left(q_f^h\right) + \beta(V_g - V_h)\right] \\ & + (xn)(1-n)(1-M^*)\left[u\left(q_h^h\right) + \beta(V_g - V_h)\right] \\ & + x\delta(1-n)(1-n)(1-M^*)\left[u\left(q_f^h\right) + \beta(V_g - V_h)\right] + \beta V_h, \end{aligned} \tag{21}$$

$$\begin{aligned} V_f = {} & (xn)n(1-M)\left[u\left(q_h^h\right) + \beta(V_g - V_f)\right] \\ & + x\delta(1-n)n(1-M)\left[u\left(q_f^h\right) + \beta(V_g - V_f)\right] \\ & + (xn)(1-n)(1-M^*)\left[u\left(q_h^h\right) + \beta(V_g - V_f)\right] \\ & + x\delta(1-n)(1-n)(1-M^*)\left[u\left(q_f^h\right) + \beta(V_g - V_f)\right] + \beta V_f, \end{aligned} \tag{22}$$

where $\beta \in (0, 1)$ is the common discount factor for both Home and Foreign agents. The expected utility of a Home producer in equation (20) is equal to the expected payoffs from (i) meeting another Home or Foreign producer; (ii) meeting a Home or Foreign agent holding Home currency; and (iii) meeting a Home or Foreign agent holding Foreign currency. The value functions of holding the Home and the Foreign currency, in equations (21) and (22), respectively, can be explained similarly. Notice that equations (21) and (22) imply that $V_h = V_f$. One can similarly define the value functions of a Foreign agent, (V_g^*, V_h^*, V_f^*), which satisfy the relations analogous to equations (20) through (22) where $V_h^* = V_f^*$. Let $V_h = V_f = V_m$ and $V_h^* = V_f^* = V_m^*$.

Now, for given (V_g, V_m) and (V_g^*, V_m^*), the purchasing power of the unified currency is determined by

$$q_j^i = \arg\max_q \left[u(q) + \beta V_g^i\right]\left[-c(q) + \beta V_m^j\right] \tag{23}$$

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subject to the participation constraints

$$u(q) + \beta V_g^i \geq \beta V_m^i, \tag{24}$$

$$-c(q) + \beta V_m^j \geq \beta V_g^j. \tag{25}$$

A *stationary monetary equilibrium with the unified currency* consists of q_j^i, (V_g, V_m), and (V_g^*, V_m^*) such that (i) q_j^i solves the bargaining problem (23) subject to the constraints (24) and (25), taking (V_g, V_m) and (V_g^*, V_m^*) as given; and (ii) (V_g, V_m) and (V_g^*, V_m^*) satisfy, respectively, (20) and either (21) or (22), and their counterparts in the Foreign country, taking q_j^i as given.

4.2. Transition from Local Currencies to Unified Currency

Let $\alpha_h \in (0, 1)$ and $\alpha_f \in (0, 1)$ denote, respectively, the size of government in the Home and Foreign countries. Each government has the legal monopoly in the production of local currency. Initially, the two countries are completely isolated from each other, so that the world economy looks like two of the economy discussed in Section 3.

Let $m_t = (1 - M_{ht} - M_{ft})x(1 - x) = [n(1 - M_t) + (1 - n)(1 - M_t^*)]x(1 - x)$ denote the probability that a Home government agent holding production opportunities will wish to issue Home currency. Then, as analogous to (10), the Home government will find it optimal to supply its currency according to

$$M_{t+1} = M_t + \mu_t \alpha_h m_t \left[n(1 - M_{ht} - M_{ft}) + \delta(1 - n)\left(1 - M_{ht}^* - M_{ft}^*\right)\right], \tag{26}$$

with μ_t as specified in Proposition 1, where T is the last period of the transition to the unified currency. Similarly, the Foreign government follows the money supply path given by

$$M_{t+1}^* = M_t^* + \mu_t^* \alpha_f m_t^* \left[\delta n(1 - M_{ht} - M_{ft}) + (1 - n)\left(1 - M_{ht}^* - M_{ft}^*\right)\right], \tag{27}$$

where μ_t^* is specified similarly to μ_t with the last period of the transition T^* and $m_t^* = (1 - M_{ht}^* - M_{ft}^*)x(1 - x)$.

Notice that, initially, under complete isolation, $M_{ht} = M_t$, $M_{ft} = M_{ht}^* = 0$, $M_{ft}^* = M_t^*$, and $\delta = 0$, in which case, both money supply paths, (26) and (27), are reduced to their counterparts in the isolated world with local currencies. As the two countries start integrating with the unified currency, the temptation for each government to issue more currency increases because the local government can extract seigniorage from both Home and Foreign agents. Hence, an equilibrium account of the transition from isolation to the unified currency will imply a larger equilibrium money stock than under the local currency regime.

Analogously to the preceding section, the optimal money supply targets at the Home and Foreign countries, denoted M and M^*, respectively, are determined by

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the following necessary conditions, respectively:

$$\frac{\partial W_T(M_T, M_{T+1}, M_{T+1}, \ldots)}{\partial M_{T+1}} = 0, \quad \frac{\partial W_T^*\left(M_T^*, M_{T+1}^*, M_{T+1}^*, \ldots\right)}{\partial M_{T+1}^*} = 0, \quad \textbf{(28)}$$

where W_T and W_T^* are, respectively, the welfare of the Home and Foreign governments from the standpoint of the last transition period T, and the derivatives are evaluated respectively at $M_T = M_{T+1} = M$ and $M_T^* = M_{T+1}^* = M^*$.

Considering that W_T involves one period of transition followed by a steady state, the welfare of the Home government as of T can be written as

$$\begin{aligned}
&W_T(M_T, M_{T+1}, M_{T+1}, \cdots) \\
&\quad = M_{hT}W_{hT} + M_{fT}W_{fT} + (1 - M_{hT} - M_{fT})W_{gT} \\
&\quad = \left[nM_T + (1-n)M_T^*\right]xn\left[n(1-M_T) + (1-n)\left(1-M_T^*\right)\right] \\
&\qquad \times \left[u\left(q_{h,T}^h\right) - c\left(q_{h,T}^h\right)\right] + \left[nM_T + (1-n)M_T^*\right]x\delta(1-n) \\
&\qquad \times \left[n(1-M_T) + (1-n)\left(1-M_T^*\right)\right]\left[u\left(q_{f,T}^h\right) - c\left(q_{h,T}^f\right)\right] \\
&\qquad + \frac{M_{T+1} - M_T}{\alpha_h}\left[\frac{n}{n+\delta(1-n)}u\left(q_{h,T}^h\right) + \frac{\delta(1-n)}{n+\delta(1-n)}u\left(q_{f,T}^h\right)\right. \\
&\qquad \left. - \frac{n\alpha_h}{n+\delta(1-n)}c\left(q_{h,T}^h\right)\right] - \left(M_{T+1}^* - M_T^*\right)\frac{\delta(1-n)}{n+\delta(1-n)}c\left(q_{h,T}^f\right) \\
&\qquad + \left[(M_{T+1} - M_T)\frac{n}{n+\delta(1-n)} + \left(M_{T+1}^* - M_T^*\right)\frac{\delta(1-n)}{\delta n + (1-n)}\right] \\
&\qquad \times \beta(V_{m,T+1} - V_{g,T+1}) + \beta V_{g,T+1} + \left[nM_T + (1-n)M_T^*\right] \\
&\qquad \times \beta(V_{m,T+1} - V_{g,T+1}) + \{x[n + \delta(1-n)] \\
&\qquad \times [1 - nM - (1-n)M^*]\}^2[u(q^B) - c(q^B)],
\end{aligned}$$

where W_{hT}, W_{fT}, and W_{gT} are, respectively, the expected utilities of government members holding Home currency, Foreign currency, and production opportunities. One can similarly write the expected utility of the Foreign government, $W_T^*(M_T^*, M_{T+1}^*, M_{T+1}^*, \ldots)$. Notice that, with complete isolation of the two countries, W_T and W_T^* are simplified to (12) in Section 3.

Further, once the money supply targets (M and M^*) are reached, the local governments should not prefer the option of returning to isolation with local currencies:

$$W_A + W_A^* \leq W + W^*, \quad \textbf{(29)}$$

where W_A and W_A^* are, respectively, the expected utilities of the Home and Foreign agents under isolation with local currencies, whereas $W \equiv (1 - M_h - M_f)V_g + (M_h + M_f)V_m$ and $W^* \equiv (1 - M_h^* - M_f^*)V_g^* + (M_h^* + M_f^*)V_m^*$ are, respectively,

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the subgame expected utilities of the Home and Foreign agents, evaluated at the money supply targets, $M_h = M_h^* = nM$ and $M_f = M_f^* = (1-n)M^*$.

In the absence of precommitment on the part of the local governments, a *subgame perfect monetary equilibrium with the unified currency* at t consists of a sequence of $\{q_{j,\tau}^i\}_{\tau=t}^{\infty}$, $\{M_\tau, V_{g\tau}, V_{m\tau}\}_{\tau=t}^{\infty}$, and $\{M_\tau^*, V_{g\tau}^*, V_{m\tau}^*\}_{\tau=t}^{\infty}$ such that

(i) $\{M_\tau, M_\tau^*\}_{\tau=t}^{\infty}$ follow, respectively, (26) and (27), where the money supply targets at the Home and Foreign countries are determined by (28) with $M_{\tau+1} = M$ for $\tau \geq T$ and $M_{\tau+1}^* = M^*$ for $\tau \geq T^*$;

(ii) $\{q_{j,\tau}^i\}_{\tau=t}^{\infty}$ solves the following sequence of Nash bargaining problems at $\tau \geq t$:

$$q_{j,\tau}^i = \arg\max_{q_\tau}\left[u(q_\tau) + \beta V_{g,\tau+1}^i\right]\left[-c(q_\tau) + \beta V_{m,\tau+1}^j\right]$$

subject to $u(q_\tau) + \beta V_{g,\tau+1}^i \geq \beta V_{m,\tau+1}^i$ and $-c(q_\tau) + \beta V_{m,\tau+1}^j \geq \beta V_{g,\tau+1}^j$, taking $\{V_{g\tau}, V_{m\tau}\}_{\tau=t}^{\infty}$, $\{V_{g\tau}^*, V_{m\tau}^*\}_{\tau=t}^{\infty}$, and $\{M_\tau, M_\tau^*\}_{\tau=t}^{\infty}$ as given;

(iii) $\{V_{g\tau}, V_{m\tau}\}_{\tau=t}^{\infty}$ and $\{V_{g\tau}^*, V_{m\tau}^*\}_{\tau=t}^{\infty}$ satisfy

$$\begin{aligned}
V_{g,\tau} &= x^2[n+\delta(1-n)]\left[1 - nM_\tau - (1-n)M_\tau^*\right]\left[u\left(q_\tau^B\right) - c\left(q_\tau^B\right)\right] \\
&\quad + (xn)nM_\tau\left[-c\left(q_{h,\tau}^h\right) + \beta(V_{m,\tau+1} - V_{g,\tau+1})\right] \\
&\quad + x\delta(1-n)nM_\tau\left[-c\left(q_{h,\tau}^f\right) + \beta(V_{m,\tau+1} - V_{g,\tau+1})\right] \\
&\quad + (xn)(1-n)M_\tau^*\left[-c\left(q_{h,\tau}^h\right) + \beta(V_{m,\tau+1} - V_{g,\tau+1})\right] \\
&\quad + x\delta(1-n)(1-n)M_\tau^*\left[-c\left(q_{f,\tau}^h\right) + \beta(V_{m,\tau+1} - V_{g,\tau+1})\right] + \beta V_{g,\tau+1}, \\
V_{m,\tau} &= (xn)n(1-M_\tau)\left[u\left(q_{h,\tau}^h\right) + \beta(V_{g,\tau+1} - V_{m,\tau+1})\right] \\
&\quad + x\delta(1-n)n(1-M_\tau)\left[u\left(q_{f,\tau}^h\right) + \beta(V_{g,\tau+1} - V_{m,\tau+1})\right] \\
&\quad + (xn)(1-n)\left(1-M_\tau^*\right)\left[u\left(q_{h,\tau}^h\right) + \beta(V_{g,\tau+1} - V_{m,\tau+1})\right] \\
&\quad + x\delta(1-n)(1-n)\left(1-M_\tau^*\right)\left[u\left(q_{f,\tau}^h\right)\right. \\
&\quad \left. + \beta(V_{g,\tau+1} - V_{m,\tau+1})\right] + \beta V_{m,\tau+1},
\end{aligned}$$

and their counterparts in the Foreign country, taking $\{q_{j,\tau}^i\}_{\tau=t}^{\infty}$ and $\{M_\tau, M_\tau^*\}_{\tau=t}^{\infty}$ as given; and

(iv) the option of returning to local currencies is not preferred, as summarized by (29).

Figure 4 depicts the equilibrium solutions and the implied welfare in the steady state as a function of the degree of economic integration, $\delta \in [0, 1]$. The functional forms for utility and variable cost are $u(q) = q$ and $c(q) = q^2$. The parameters are set as $x = 0.2$, $\beta = 0.98$, $\alpha_h = \alpha_f = 0.7$, and $n = 0.5$. First, as illustrated in Panel A, each government's incentive to extract seigniorage from foreigners as well as home agents yields a larger equilibrium money stock (per agent in the world economy)

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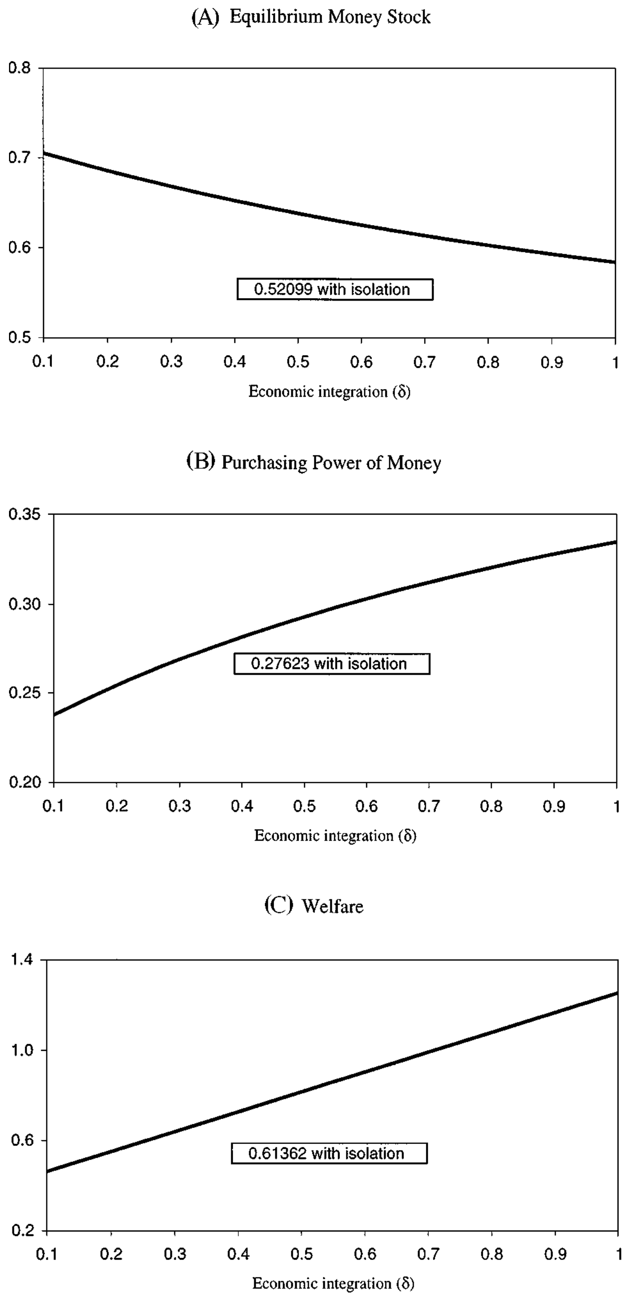

FIGURE 4. Currency integration.

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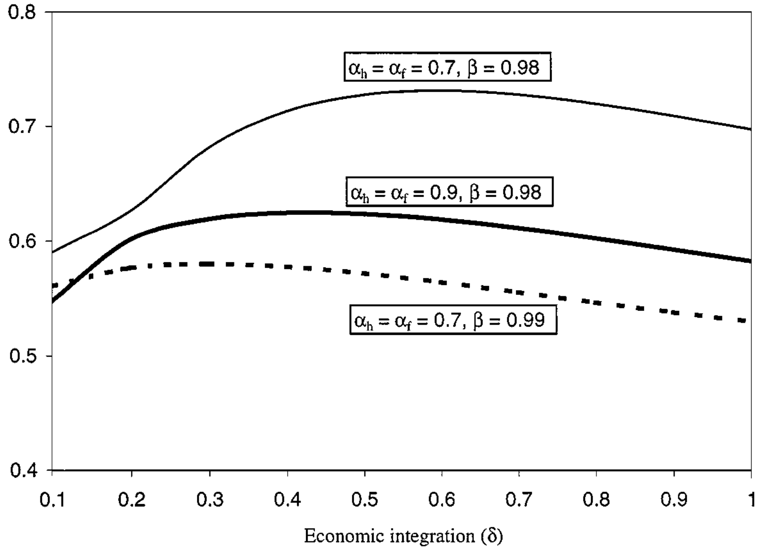

FIGURE 5. Equilibrium money stocks and (α, β): $n = 0.7$.

relative to the one under complete isolation with local currencies only. With a small degree of economic integration, this also implies higher price level or lower purchasing power of money than under the local currency regime (see Panel B).[15]

Second, the more integrated world economy (i.e., an increase in δ) implies (i) a higher frequency of trading opportunity for a given money stock; and (ii) the temptation for each country to issue more currency to exploit seigniorage from agents in both countries. For the two countries of equal size (i.e., $\theta = (1-n)/n = 1$ or $n = 0.5$), the first effect "dominates" the second effect for $\delta \in (0, 1]$, so that the equilibrium money stock in the world economy decreases with δ. For $\theta \neq 1$, the second effect outweighs the first for a small degree of economic integration, so that the equilibrium money stock increases for a small $\delta \in (0, 1]$; whereas the opposite is the case for a large δ (see Figure 5). As the difference in the country size increases, the dominance of the second effect becomes more prevalent, and hence the equilibrium money stock tends to increase for a wider range of δ. Further, as agents become more patient (i.e., a higher β) or the sizes of Home or Foreign governments increase (i.e., a larger α_h or α_f), an increase in δ tends to imply a fall in the equilibrium money stock for a given n because the higher frequency of trading opportunity for a given money stock becomes more important than the (immediate) seigniorage benefit from issuing more currency.

Finally, when the lack of economic integration is severe enough [i.e., a sufficiently small $\delta \in (0, 1)$] for given $(x, \beta, \alpha_h, \alpha_f, n)$, the lower value of currency (relative to isolation with local currencies) as well as the lower frequency of trading opportunity implies that the welfare of a representative agent with the unified currency is lower than that under local currencies [i.e., the inequality in (29) is reversed], so that the local governments would prefer the option of returning to local currencies. In Panel C of Figure 4, for example, the local currency regime would be preferred for $\delta \in (0, 0.2]$. Despite the higher money stock (relative to isolation) in the more integrated world economy, the increasing frequency of trading opportunity implies lower price or higher purchasing power of money and higher welfare relative to the local currency regime.

It is when the two countries are fully integrated ($\delta = 1$) that the world economy with the unified currency achieves the highest welfare. As long as the world economy is *not* completely integrated ($\delta < 1$), an inefficiency arises in the sense that, despite the unified currency, an incomplete economic integration reduces the frequency of trading opportunity. Essentially, the lack of economic integration imposes additional frictions on monetary exchange in the world economy.

5. CONCLUDING REMARKS

Using a random matching model of endogenous money such as those of Trejos and Wright (1995) and Ritter (1995), this paper has examined the importance of prices in the transition from barter to fiat money and its possible breakdown. The equilibrium money stock in the fixed-price model is shown to be overstated because it maximizes the frequency of trade and seigniorage, without considering the effect on price changes of the money supply. In the presence of search friction, the quantity of output in exchange for a unit of money is inefficiently low in the monetary equilibrium, and hence the price effect implies the lower equilibrium money stock so that it yields an equilibrium quantity of output closer to the efficient level.

This also imposes further restrictions on the feasible parameter values for the government's patience and its size in the model economy. For a given extent of the double-coincidence-of-wants problem, the inefficiently low quantity of output relative to barter exchange implies a lower expected utility than in the fixed-price model, and hence the government would have a greater incentive to go back to barter exchange by printing more money. To sustain a monetary equilibrium, therefore, the government is required to be more patient and more credible than in the model economy where price is fixed. Further, the less patient government extracts more resources via seigniorage, and therefore yields the more inflationary transition to fiat money regime. The possibility of breakdown in fiat money due to political uncertainty implies a longer transition path and a higher expected inflation during the transition.

Finally, in an application to the transition from local currencies to currency integration, local governments attempt to issue more currency for the purpose of extracting seigniorage from both home and foreign agents, which yield higher equilibrium money stocks than in an isolated world economy. For a small degree of economic integration, higher money stocks also imply higher price level. As long as the world economy is sufficiently integrated, a relatively higher frequency of trading opportunity implies lower price level and higher welfare than in the world with local currencies. It is with the complete economic integration that the world economy with the unified currency achieves highest welfare.

NOTES

1. Another common reason for barter is that prices are not allowed to adjust to equilibrium. Mendershausen (1949) documented the appearance of this type of barter during the Allied occupation of Germany after World War II.

2. There have been analyses of the model with divisible money and no unit-inventory constraint such as those by Green and Zhou (1998), Camera and Corbae (1996), and Shi (1997).

3. This assumption is to guarantee that money holders cannot trade with the other money traders, which will rule out the case in which agents would not want to inventory commodities in equilibrium.

4. I follow the timing convention of Ritter (1995) where agents' production and consumption occur in the same period. Most of the Kiyotaki-Wright models assume that agents decide on their inventory at the end of a period and carry it over to the following period for trade.

5. Although some different results arise in the existence of constrained and unconstrained equilibria when the transition is explicitly considered, most of the findings in the unconstrained equilibrium also hold qualitatively in the constrained equilibrium. The details are available upon request.

6. Trejos and Wright (1995) also show the same result with no direct barter in which the unique stationary monetary equilibrium is unconstrained.

7. Burnell and Kim (1997a) examine a search model in which agents differ in their discount of future consumption, and they characterize the size of government as an outcome of the majority-voting equilibrium policy. Burnell and Kim (1997b) study the endogenous formation of monetary coalitions and the equilibrium money stock by allowing for an entry of a new coalition to create its own currency.

8. Applying the implicit function theorem to (10a),

$$f'(\mu_T) = -\frac{\alpha x(1-x)(1-M_T)^2}{1-2\mu_T \alpha x(1-x)(1-M_T)} < 0,$$

as long as $x \in (0, 1)$ is sufficiently small.

9. The "pedal-to-metal" result ($\mu_t = 1$ if $t \leq T$) also holds in the fixed-price model of Ritter (1995).

10. Note that a government member would be indifferent between barter and money issue, but a trading partner would not be. Further, a government member would be indifferent between accepting and refusing to take money issued by the other members. It is assumed that, if barter is possible, it takes place, and a member of the government always accepts money issued by the other members.

11. In contrast to the findings of Trejos and Wright (1995), constrained equilibria do not necessarily coexist with unconstrained equilibria. In particular, for β close to 1, a constrained equilibrium does not exist, but an unconstrained equilibrium does. In the constrained equilibrium where commodity sellers get a zero surplus from trade [i.e., $-c(q) + \beta V_M = \beta V_G$], the equilibrium condition (11) is reduced to $0 < \beta x^2[u(q^B) - c(q^B)] \leq (1-\beta)c(q)$. Note that, as $\beta \to 1$, the government prefers to drive the economy back to barter exchange by issuing slightly more money. As sellers get more patient, their net surplus from trade,$-c(q) + \beta V_M - \beta V_G$, is more likely to be positive since V_M rises with β. Hence, as β approaches 1, it becomes more difficult to sustain a constrained monetary equilibrium where a seller's net surplus from trade is zero. Instead, the government would return to the barter, which guarantees strictly positive net surplus from the direct exchange of commodities.

12. In general, $\mu_T \in [0, 1]$, depending on the money supply target, $M \in (0, 1)$. The numerical example in Figure 2 is computed also with $\mu_t = 1$ for $t \leq T$.

13. For instance, in ancient China, failures in a transition to fiat money involved the continuous fall in purchasing power of money following the overissue of money to finance the government's expenditure needs [Yang (1952)]. In the United Kingdom, during the period of its first fiat money regime between 1797 and 1819 (which coincided with the French wars), the Bullion Committee found that the first major inflations were caused by excessive issues of bank notes by the Bank of England

to support the credit of the government. Newlyn and Bootle (1978) noted that, at this time, there was no clear recognition by the Directors of the Bank of England, acting simply as a commercial bank, that they had public responsibilities. A recommendation of the Bullion Committee was to end the fiat money regime and return to a fully automatic gold standard, which continued until 1931.

14. As in the preceding section, the forward-looking solutions of transition paths are computed backward, starting from the last period of the transition.

15. It can be shown that, for a given degree of economic integration, the money stock in a stationary monetary equilibrium with the unified currency (as characterized in Section 4.1) is lower than its counterpart in Panel A. This is because of the seigniorage possibility in the latter.

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