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Some results for relativistic space-times with spherical symmetry

Malcolm William Cook

This thesis has two parts; each dealing with a separate aspect of the one problem; namely a certain class of spherically symmetric solutions to the Einstein gravitational field equations.

The first consists of one chapter and deals with gravitational aberrations in stellar images, by studying some aspects of light propagation in a space-time described by a Robertson-Walker metric with radial perturbations. This is published in detail elsewhere [*2*].

The main results of this work, divided into three chapters, comprise Part II.

In Chapter II we find that in some approximate spherically symmetric solutions (related to those of Chapter I) to the field equations, the sign of the "spatial curvature" (usually regarded as fixed in uniform model theory) may depend on "time". Approximate solutions describing a distribution of matter with uniform density but with non-uniform pressure also exhibit this feature, and in Chapter III, we develop the corresponding exact solutions, namely,

$$ds^2 = g_{00}dt^2 - c^{-2}g_{11}\left[dr^2+r^2\left(d\theta^2+\sin^2\theta d\phi^2\right)\right] ,$$

where

$$g_{00} = \left[1+\tfrac{1}{4}\alpha(t)\gamma(t)r^2\right]^2\left[1+\tfrac{1}{4}\alpha(t)r^2\right]^{-2}$$

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and

$$g_{11} = S^2(t)\left[1+\tfrac{1}{4}\alpha(t)r^2\right]^{-2} ,$$

and where

$$\gamma = 1 - (S d\alpha/dt)(\alpha dS/dt)^{-1} .$$

[The arbitrary functions α and S are to be determined by a two-point specification of equations connecting the physical state variables.]

We proceed to investigate some properties of these solutions; and find that closed global distributions develop regions of negative pressure in conjunction with a change of sign of the "spatial curvature". We conclude this chapter with a preliminary investigation of isolated distributions; and in particular those with vanishing boundary pressure.

In Chapter IV is an application of the work in the preceding chapter to distributions whose "equation of state" varies with the radial coordinate (in particular, distributions with a radiation core and an incoherent particle matter boundary). This class of models may be specified by the relations:

$$np(0, t) = c^2\rho(t) ;$$

$$p(r^*, t) = mp(0, t) ;$$

where p, ρ, and r^* are respectively the pressure, density, and boundary coordinate of the distribution, and m and n are numbers (although in general they may be functions of time) satisfying:

$$0 \le m < 1 , \text{ and } n \ge 3 .$$

A further investigation of properties of these distributions is carried out with emphasis on their geometry; asymptotic behaviour with time; approach to singular states; and behaviour of their boundaries with respect to an observer in the external field.

A recently propounded view [1] of quasi-stellar objects leads us to regard these as possibly having some features in common with distributions of the above type; and we investigate whether some of their properties, notably their characteristically high redshifts, might be explainable relativistically. A discussion of some of the obvious limitations of the models used, which are seen as being in a sense minimial cases for any actual distributions, is then given.

References

[1] Halton Arp, "Observational paradoxes in extragalactic astronomy", *Science (USA)* **174** (1971), 1189-1200.

[2] M.W. Cook, "On gravitational aberrations in stellar images", *Austral. J. Phys.* **25** (1972), 749-758.

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