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Inelastic Compton scattering of photons by bound atomic electrons in weakly coupled plasmas

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Abstract. Plasma screening effects on inelastic Compton scattering of photons by bound atomic electrons of hydrogenic target ions in weakly coupled plasmas are investigated. The particle interaction potential in weakly coupled plasmas is obtained using the Debye–Hückel model. The screened wave functions and energy eigenvalues for the ground and excited states of the target ion are obtained using the Ritz variational method. The expression for the lowest-order transition matrix element is obtained from a two-photon process associated with terms quadratic in the vector potential **A**. The inelastic Compton scattering cross-section from the 1s ground state to the 2p excited state is obtained as a function of the incident photon energy, including plasma screening effects. It is found that plasma screening effects significantly reduce the inelastic Compton scattering cross-section.

1. Introduction

Compton scattering (Heitler 1954; Bethe and Salpeter 1957; Gould 1965, 1972, 1979, 1984; Osborn 1988; Longair 1992; McGuire 1997) of photons by free or bound electrons can serve as a probe of the atomic environment, since spectra of radiation emitted by or scattered from astrophysical and laboratory plasmas provide information about these plasmas. The scattering of photons on bound atomic electrons (Osborn 1988; McGuire 1997) has been a subject of special interest in many areas of physics, such as atomic and plasma physics, because it is quite sensitive to the details of atomic structure and the correlation effects between atomic electrons. When the energy of the incident photon is greater than the excitation threshold of the target atom, the final state of the target atom can be different from the initial state, and this process is called inelastic Compton scattering (sometimes Raman scattering). The inelastic Compton scattering of photons by bound electrons in atoms in dense plasmas may be different from that by free atoms owing to plasma screening effects. In dense laboratory and astrophysical plasmas, the range of the Debye length Λ is $\Lambda \geqslant 10a_Z$ (Jung 1993), where a_Z ($\equiv a_0/Z = \hbar^2/Zme^2$) is the Bohr radius of a hydrogenic ion with nuclear charge Z and m is the electron rest mass, since the electron densities n_e and temperatures T_e are known to be around 10^{20}–10^{23} cm^{-3} and 10^7–10^8 K respectively. These plasmas can be classified as weakly coupled plasmas, since the ratio of the average Coulomb energy to the random kinetic energy is much less than unity. In weakly coupled plasmas, the Debye–Hückel model (Jung 1995a; Jung and Cho 1995; Jung and Yoon 1996) has been found to be quite reliable for describing particle interactions. Radiative decay from the 2p state to the ground 1s state in plasmas is known to be a useful tool in the study of plasma parameters (Jung 1995b; Hong and Jung 1996). As a result of absorption due to the inelastic Compton effect in dense plasmas, line absorption occurs mainly in the transmission windows below the absorption edges (Salzmann 1998), where the line wings are the most important absorption mechanisms. This property may substantially change the opacities of plasmas. Therefore, in this paper, we consider plasma screening effects on inelastic Compton scattering on the $1s \to 2p$ transition in a hydrogenic target ion in order to investigate plasma screening effects on the lowest optically allowed excitation process. Relativistic effects for bound-state wave functions are only of relative order $(Z\alpha)^2$, and we shall ignore them since we restrict ourselves to hydrogenic wave functions with $Z\alpha \ll 1$, where α $(= e^2/\hbar c \approx \frac{1}{137})$ is the fine-structure constant. The screened bound wave functions and corresponding energy eigenvalues of a hydrogenic target ion with nuclear charge Z are obtained by the perturbation and Ritz variational (Jung 1993) methods. The expression for the transition-matrix element near resonance is obtained by the basic lowest-order two-photon perturbation Hamiltonian.

In Sec. 2, we discuss the transition matrix element and the differential cross-section for the inelastic Compton scattering process. In Sec. 3, we obtain the screened atomic wave functions and energy eigenvalues using the Ritz variational method. In Sec. 4, we derive the inelastic Compton scattering cross-section for $1s \to 2p_0$ $(m = 0)$ excitation, and investigate plasma screening effects on the total inelastic scattering cross-section. The results show that plasma screening effects on inelastic Compton scattering in dense plasmas substantially reduce the scattering cross-section. The results are summarized in Sec. 5.

2. Differential cross-section

Let $|i\rangle$ be the ground state, with energy eigenvalue E_i, of an atom whose centre of mass is assumed to be fixed at the origin, and let $|f\rangle$ be the first excited state, with energy eigenvalue E_f. For inelastic Compton scattering, the initial and final states of the total atom and radiation system are given respectively by $|\phi_i\rangle = |i; \mathbf{k}_i \hat{\boldsymbol{\epsilon}}_i\rangle$ and $|\phi_f\rangle = |f; \mathbf{k}_f \hat{\boldsymbol{\epsilon}}_f\rangle$, where $\mathbf{k}_i$ and $\mathbf{k}_f$ are the photon wave vectors and $\hat{\boldsymbol{\epsilon}}_i$ and $\hat{\boldsymbol{\epsilon}}_f$ are the photon polarization unit vectors. From the Fermi Golden Rule in the Coloumb gauge (McGuire 1997), the differential scattering cross-section for inelastic Compton scattering is found to be

$$d\sigma_{fi} = \frac{2\pi}{\hbar c} |M_{fi}|^2 \, \delta(E_f - E_i) \frac{d^3 \mathbf{k}_f}{(2\pi)^3}, \tag{1}$$

where M_{fi} is the transition-matrix element

$$M_{fi} = \langle f; \mathbf{k}_f \hat{\boldsymbol{\epsilon}}_f | H_I | i; \mathbf{k}_i \hat{\boldsymbol{\epsilon}}_i \rangle. \tag{2}$$

Here H_I $(= e^2 A^2/mc^2)$ is the lowest-order two photon perturbation Hamiltonian (Cohen-Tannoudji et al. 1989), where $\mathbf{A}$ is the vector potential of the field. For weak fields, such a two-photon process is known to be associated with terms quadratic in $\mathbf{A}$. For resonant Raman or resonant Compton scattering at photon energies close to the resonance energy, the $(\mathbf{p}\cdot\mathbf{A})^2$ term is dominant, where $\mathbf{p}$ $(=(\hbar/i)\boldsymbol{\nabla})$ is the momentum operator. However, at high photon energies, the $(\mathbf{p}\cdot\mathbf{A})^2$ term can be almost neglected, and the leading A^2 term dominates Compton scattering (McGuire 1997).

After some algebra, the differential Compton scattering cross-section for inelastic scattering from an arbitrary initial state $|i\rangle$ to an arbitrary final state $|f\rangle$ is then given by

$$\frac{d\sigma_{fi}}{d\Omega}=\frac{\omega_f}{\omega_i}r_0^2\,|\,(\hat{\boldsymbol{\epsilon}}_i\cdot\hat{\boldsymbol{\epsilon}}_f)\,\langle f|\,e^{i(\mathbf{k}_i-\mathbf{k}_f)\cdot\mathbf{r}}\,|\,i\rangle|^2, \tag{3}$$

where $d\Omega$ is the solid angle about the direction $\mathbf{k}_f$, r_0 $(=e^2/mc^2)$ is the classical electron radius, and ω_i and ω_f are the initial and final photon energies respectively. The expression (3) is valid for arbitrary initial and final states, including transitions of multiple electrons. In the following section, we shall discuss plasma screening effects on the atomic wave functions. However, we consider inelastic Compton scattering only on the $1\mathrm{s}\rightarrow 2\mathrm{p}_0(m=0)$ transition in a hydrogenic ion in order to investigate plasma screening effects on the lowest optically allowed excitation process. The inelastic Compton scattering cross-section is smaller than the Thomson scattering cross-section σ_T $(=\frac{8}{3}\pi r_0^2)$ because of the cancellation effect in the matrix element in (3). The cancellation effect including the plasma screening effect can be used in the study of plasma parameters.

3. Screened atomic states

When a hydrogenic ion with nuclear charge Z is placed in a dense weakly coupled plasma, the radial Schrödinger equation (Jung 1993) with Debye–Hückel interaction potential takes the form

$$-\frac{\hbar^2}{2m}\left[\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dR_{nl}}{dr}\right)-\frac{l(l+1)}{r^2}R_{nl}\right]-\frac{Ze^2}{r}e^{-r/\Lambda}R_{nl}=E_{nl}R_{nl}, \tag{4}$$

where R_{nl} and E_{nl} are respectively the screened radial wave function and energy eigenvalue of the nlth shell electron. In order to obtain simple approximate analytical solutions for (4), we choose the normalized trial 1s ground-state and 2p excited-state wave functions:

$$R_{1\mathrm{s}}(r)=2\beta_{1\mathrm{s}}^{-3/2}\,e^{-r/\beta_{1\mathrm{s}}}, \tag{5}$$

$$R_{2\mathrm{p}}(r)=\frac{1}{2\sqrt{6}}\beta_{2\mathrm{p}}^{-5/2}\,r\,e^{-r/2\beta_{2\mathrm{p}}}, \tag{6}$$

where the variational parameters $\beta_{1\mathrm{s}}$ and $\beta_{2\mathrm{p}}$ are the effective 1s and 2p Bohr radii and $\beta_{1\mathrm{s}}$, $\beta_{2\mathrm{p}}\rightarrow a_Z$ for $\Lambda\rightarrow\infty$, i.e. a pure Coulomb potential Ze^2/r. The energy expectation value for the ground state is obtained from (4) and (5):

$$\langle E_{1\mathrm{s}}(\beta_{1\mathrm{s}})\rangle=\frac{\hbar^2}{2m\beta_{1\mathrm{s}}^2}-\frac{Ze^2}{\beta_{1\mathrm{s}}(1+\beta_{1\mathrm{s}}/2\Lambda)^2}. \tag{7}$$

The solution for the parameter $\beta_{1\mathrm{s}}$ can be determined by minimization of (7), i.e. $\partial\langle E_{1\mathrm{s}}(\beta_{1\mathrm{s}})\rangle/\partial\beta_{1\mathrm{s}}=0$. Using the perturbation method for weak-screening cases, i.e. $\beta_{1\mathrm{s}}<\Lambda$, the approximate analytical solution for $\beta_{1\mathrm{s}}$ is given by

$$\beta_{1\mathrm{s}}\approx a_Z/\eta_{1\mathrm{s}}, \tag{8}$$

where the parameter η_{1s} $(\equiv 1-\frac{3}{4}(a_Z/\Lambda)^2+(a_Z/\Lambda)^3)$ represents the plasma-screening effect on the ground state. Equation (8) represents the plasma screening effect on the first Bohr radius ($n=1$). This second-order screening correction to a_Z is known to be quite reliable for our domain of interest of the Debye length, $\Lambda \geqslant 10a_Z$. Then the energy eigenvalue of the 1s ground state is obtained from (7) and (8):

$$E_{1s} = -Z^2 Ry\ (1-\delta_{1s}), \tag{9}$$

where δ_{1s} $(\approx 2a_Z/\Lambda-\frac{3}{2}(a_Z/\Lambda)^2+(a_Z/\Lambda)^3)$ represents the plasma-screening correction to the ground-state energy and Ry $(= me^4/2\hbar^2 \approx 13.6\,\mathrm{eV})$ is the Rydberg constant.

The energy expectation value for the 2p excited state is also given by (4) and (6):

$$\langle E_{2p}(\beta_{2p})\rangle = \frac{\hbar^2}{8m\beta_{2p}^2}-\frac{Ze^2}{4\beta_{2p}(1+\beta_{2p}/\Lambda)^4}. \tag{10}$$

The parameter β_{2p} can also be determined by minimization of (10), i.e. $\partial\langle E_{2p}(\beta_{2p})\rangle/\partial\beta_{2p}=0$. For weak-screening cases ($\beta_{2p}<\Lambda$), the approximate analytical solution for β_{2p} is found to be

$$\beta_{2p} \approx a_Z/\eta_{2p}, \tag{11}$$

where the parameter η_{2p} $(\equiv 1-10(a_Z/\Lambda)^2+40(a_Z/\Lambda)^3)$ represents the plasma screening effects on the 2p state. Then, after some straightforward manipulations, the energy eigenvalue for the 2p excited state becomes

$$E_{2p} = -Z^2 Ry\,(1-\delta_{2p}), \tag{12}$$

where δ_{2p} $(\approx 8(a_Z/\Lambda)-20(a_Z/\Lambda)^2+40(a_Z/\Lambda)^3)$ is the plasma screening effect on the 2p excited-state energy. In the following section, we shall discuss the plasma screening effects on the $1s\to 2p_0$ inelastic Compton scattering cross-section in weakly coupled plasmas.

4. Inelastic Compton cross-section

In inelastic Compton scattering from the 1s ground state to the 2p excited state, the final photon frequency ω_f is given by

$$\omega_f = \omega_i-\frac{E_{2p}-E_{1s}}{\hbar} = \omega_i-\frac{3}{4\hbar}Z^2 Ry\,(1-\Delta_{2p,\,1s}), \tag{13}$$

where $\Delta_{2p,\,1s}$ $(\approx \frac{14}{3}a_\Lambda^2-12a_\Lambda^3)$ is the plasma screening effect on the excitation threshold energy and a_Λ $(\equiv a_Z/\Lambda)$ is the scaled reciprocal Debye length. Since the energy correction $\Delta_{2p,\,1s}$ is positive and less than unity for $\Lambda \geqslant 10a_Z$, the final photon frequency is increased only to the plasma screening effect for a given incident photon energy.

After some algebra, the matrix element for $1s\to 2p_0$ ($m=0$) excitation is given by

$$\langle 2p_0|\,e^{i(\mathbf{k}_i-\mathbf{k}_f)\cdot\mathbf{r}}\,|1s\rangle = i4\sqrt{2}\,\frac{\eta_{1s}^{3/2}\,\eta_{2p}^{5/2}}{a_Z^4}\,\frac{(\xi/a_Z)\,|\mathbf{k}_i-\mathbf{k}_f|}{[(\xi/a_Z)^2+|\mathbf{k}_i-\mathbf{k}_f|^2\,]^3}, \tag{14}$$

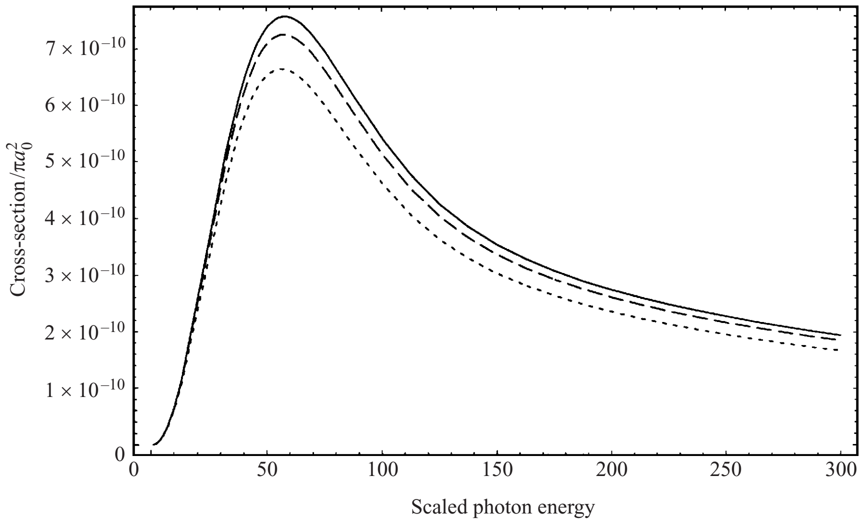

Figure 1. The $1\text{s} \to 2\text{p}_0$ inelastic Compton scattering cross-section σ_C, in units of πa_0^2, for $Z = 2$ as a function of the scaled photon energy ε_i ($\equiv \hbar\omega_i / Z^2 Ry$). The solid line represents the cross-section for $a_\Lambda = 0$, i.e. neglecting plasma screening effects. The dashed line represents the cross-section for $a_\Lambda = 0.05$. The dotted line represents cross-section for $a_\Lambda = 0.1$.

where $\xi \equiv \eta_{1\text{s}} + \frac{1}{2}\eta_{2\text{p}}$. Then, after averaging the initial polarization, the total inelastic Compton scattering cross-section in units of πa_0^2 from the 1s state to the 2p_0 state is found to be

$$\frac{\sigma_C}{\pi a_0^2} = 2^5\, \eta_{1\text{s}}^3\, \eta_{2\text{p}}^5\, \xi^2\, \alpha^4 \left[1 - \frac{3(1 - \Delta_{2\text{p},\,1\text{s}})}{4\varepsilon_i}\right] \times \int_0^\pi d\theta\, \frac{\sin\theta\,(1 + \cos^2\theta)\,(\bar{k}_i^2 + \bar{k}_f^2 - 2\bar{k}_i\,\bar{k}_f \cos\theta)}{(\xi^2 + \bar{k}_i^2 + \bar{k}_f^2 - 2\bar{k}_i\,\bar{k}_f\cos\theta)^6}, \quad (15)$$

where ε_i ($\equiv \hbar\omega_i / Z^2 Ry$) is the scaled incident photon energy, θ is the angle between $\mathbf{k}_i$ and $\mathbf{k}_f$, $\bar{k}_i \equiv k_i a_Z$, and $\bar{k}_f \equiv k_f a_Z$. The $1 + \cos^2\theta$ term in (15) is known as the angular phase for Thomson scattering (Shu 1991).

In order to explicitly investigate the total plasma screening effects on total inelastic Compton scattering from the 1s state to the 2p_0 state, specifically, we consider three cases of the Debye lengths: $a_\Lambda = 0.1$, 0.05 and 0 (i.e. $\Lambda = 10a_Z$, $20a_Z$ and ∞), and we assume that $Z = 2$, since our non-relativistic result (15) is valid for $Z\alpha \ll 1$. In Fig. 1, we show the inelastic Compton scattering cross-section, in units of πa_0^2, from the 1s state to the 2p_0 state, including plasma screening effects, as a function of the scaled photon energy ε_i for $\hbar\omega_i > E_{2\text{p}} - E_{1\text{s}}$. The unscreened inelastic Compton scattering cross-section is also illustrated in Fig. 1. Numerical values of the inelastic Compton scattering cross-section in units of πa_0^2 are listed in Table 1. Plasma screening effects reduce the inelastic Compton scattering cross-section (e.g. $\equiv 14\%$ for $a_\Lambda = 0.1$, $\approx 5\%$ for $a_\Lambda = 0.05$). It is found that plasma screening effects are less than 14% for our domain of interest of the Debye length $\Lambda \geqslant 10a_Z$. A recent investigation (Jung 1998) shows that the plasma screening effect on the total photoionization cross-section is known to be less than 10% for $\Lambda \geqslant 10a_Z$. Hence the plasma screening effect on inelastic Compton scattering is found to be stronger than that on

Table 1. Numerical values of the $1s \to 2p_0$ inelastic Compton scattering cross-section in units of πa_0^2 for $Z = 2$ and ε_i ($\equiv \hbar\omega_i / Z^2 Ry$) = 100 and 200.

a_Λ	$\sigma_C(\varepsilon_i = 100)/\pi a_0^2$	$\sigma_C(\varepsilon_i = 200)/\pi a_0^2$
0.1	4.6302×10^{-10}	2.3605×10^{-10}
0.05	5.1493×10^{-10}	2.6158×10^{-10}
0	5.4221×10^{-10}	2.7500×10^{-10}

photoionization, since two screened bound states are involved in inelastic Compton scattering while only one bound state is involved in photoionization. The emissivity (Sarazin 1988) due to a line collisionally excited by the inelastic Compton process will be reduced, since the inelastic Compton scattering cross-section is reduced owing to plasma screening effects.

5. Conclusions

We have investigated plasma screening effects on inelastic Compton scattering of photons by bound atomic electrons from the 1s ground state to the 2p state of hydrogenic ions in weakly coupled plasmas. The particle interaction in weakly coupled plasmas was obtained using the Debye–Hückel interaction potential. The screened atomic wave functions and energy levels for the 1s ground and 2p excited states of the hydrogenic ion in weakly coupled plasmas were obtained using the Ritz variational and perturbation methods. The expression for the transition matrix element (M_{fi}) was obtained using the basic lowest-order two-photon perturbation Hamiltonian ($H_I \propto A^2$). Plasma screening effects significantly reduce the inelastic Compton scattering cross-section, and it was also found that the plasma screening effects are less than 14% for our domain of interest of the Debye length, $\Lambda \geqslant 10a_Z$. These results provide useful information on inelastic Compton scattering processes in dense plasmas.

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