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On the electrical conductivity of semiclassical two-component plasmas

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Abstract. Starting from a memory-function formalism coupled with the Green–Kubo formula and an approximate expression for the generalized Coulomb logarithm, the electric conductivity of a dense high-temperature hydrogen plasma is studied. A pseudopotential model, taking account of short-range quantum effects and long-range screening-field effects, is employed to include quantum mechanical and polarization effects. An analytical formula for the Coulomb logarithm is proposed when the thermal de Broglie wavelengths are rather smaller than the Debye radius. A minimum in the curve of electrical conductivity is found and some physical evidence for its appearance is produced.

1. Introduction

Theoretical studies of the thermodynamic and transport properties of dense high-temperature plasmas, encountered in inertial-confinement fusion experiments and in the interiors of main-sequence stars, are of great importance. Knowledge of these properties proves to be necessary for the description of various processes. For the time being, to obtain this knowledge, both computer simulation methods, i.e. Monte Carlo experiments (see e.g. [1]) and molecular dynamics (see e.g. [2]), and standard theoretical approaches such as the Ornstein–Zernike relation (see e.g. [3]), Green functions (see e.g. [4]), and the density-response formalism (see e.g. [5–7]), are widely used.

In this paper, we study the transport properties of semiclassical two-component plasmas that are typical of the solar interior and inertial confinement fusion. Such plasmas generate pressures of approximately 10^5 Mbar and temperatures of about 10^7 K. As a result, quantum mechanical and polarization effects play significant roles in determining the characteristics of the plasma medium. It should also be stressed here that under these conditions, a plasma is found to be in a weakly coupled state and nonideality effects are irrelevant to its consideration. For strongly coupled plasmas, the local field correction must be taken into account (see e.g. [8,9]).

2. Dimensionless plasma parameters.

In this paper, a two-component plasma consisting of ions (with electric charge Ze, mass m_i, and number density n_i) and electrons (with electric charge $-e$, mass m_e, and number density $n_e = Zn_i$) is considered. The ionic subsystem of the plasma may be characterized by the average interparticle spacing

$$a = \left(\frac{3}{4\pi n_i}\right)^{1/3} \tag{2.1}$$

and the dimensionless Coulomb coupling parameter

$$\Gamma = \frac{(Ze)^2}{ak_BT}, \tag{2.2}$$

where T is the plasma temperature and k_B is Boltzmann's constant.

The electron subsystem may also be described by two dimensionless parameters: the density parameter

$$r_s = \left(\frac{3}{4\pi n_e}\right)^{1/3} \frac{m_e e^2}{\hbar^2} \tag{2.3}$$

and the degeneracy parameter

$$\theta = \frac{k_BT}{E_F} = 2\left(\frac{4}{9\pi}\right)^{2/3} Z^{5/3}\frac{r_s}{\Gamma}, \tag{2.4}$$

where E_F is the Fermi energy of electrons and $\hbar$ is Planck's constant.

In this paper, a hydrogen plasma ($Z = 1$) with $r_s \sim 1$ and $\Gamma \sim 1$ is studied. From (2.4), it follows that $\theta \sim 1$, and consequently degeneracy effects should be taken into account.

3. Pseudopotential model

In the theory of a semiclassical fully ionized plasma, effective potentials are used to investigate the thermodynamic and transport properties. These pseudopotentials conventionally mimic quantum effects of diffraction and symmetry at short distances [10, 11]. In particular, Deutsch and co-workers proposed the following form of the effective potential of plasma particle interaction:

$$\varphi_{ab}(r) = \frac{e_a e_b}{r}\left[1 - \exp\left(-\frac{r}{\lambda_{ab}}\right)\right] + \delta_{ae}\delta_{be}k_BT\ln 2\exp\left(-\frac{r^2}{\lambda_{ee}^2\pi\ln 2}\right), \tag{3.1}$$

where e_a and e_b are the electric charges of the interacting pair, $\lambda_{ab}=\hbar/(2\pi\mu_{ab}k_BT)^{1/2}$ is the thermal de Broglie wavelength, $\mu_{ab} = m_a m_b/(m_a + m_b)$ is the reduced mass of the interacting pair, and δ_{ab} is the Kronecker delta.

There also exist another type of pseudopotentials (such as the Debye–Hückel potential and that proposed by Baimbetov et al. [12, 13]), which simulate long-range polarization effects at large distances.

Starting from the pseudopotential (3.1), Arkhipov et al. [14,15] constructed a new pseudopotential model that incorporates both quantum and polarization effects. Specifically, the Fourier transform of this pseudopotential is written as

$$\tilde{\Phi}_{ee}(k) = \frac{4\pi e^2}{\Delta}\left\{\frac{1}{k^2(1+k^2\lambda_{ee}^2)} + \frac{1}{k^4 r_{Di}^2}\left[\frac{1}{(1+k^2\lambda_{ee}^2)(1+k^2\lambda_{ii}^2)} - \frac{1}{(1+k^2\lambda_{ei}^2)^2}\right]\right.$$
$$\left. + A\left(1 + \frac{1}{k^2 r_{Di}^2(1+k^2\lambda_{ii}^2)}\right)\exp\left(-\frac{k^2}{4b}\right)\right\}, \tag{3.2}$$

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$$\tilde{\Phi}_{ii}(k) = \frac{4\pi Z^2 e^2}{\Delta} \left\{ \frac{1}{k^2(1+k^2\lambda_{ii}^2)} + \frac{1}{k^4 r_{De}^2} \left[\frac{1}{(1+k^2\lambda_{ee}^2)(1+k^2\lambda_{ii}^2)} - \frac{1}{(1+k^2\lambda_{ei}^2)^2} \right] + \frac{A}{k^2 r_{Di}^2 (1+k^2\lambda_{ii}^2)} \exp\left(-\frac{k^2}{4b}\right) \right\}, \quad (3.3)$$

$$\tilde{\Phi}_{ei}(k) = -\frac{4\pi Z e^2}{\Delta} \frac{1}{k^2(1+k^2\lambda_{ei}^2)}, \quad (3.4)$$

where

$$A = \frac{k_B T \ln 2\sqrt{\pi}\, b^{-3/2}}{4e^2}, \quad r_{De} = \left(\frac{k_B T}{4\pi n_e e^2}\right)^{1/2} \quad \text{and} \quad r_{Di} = \left(\frac{k_B T}{4\pi n_i Z^2 e^2}\right)^{1/2}$$

are the Debye screening radii of electrons and ions respectively, and

$$\Delta = 1 + \frac{1}{k^2 r_{De}^2(1+k^2\lambda_{ee}^2)} + \frac{1}{k^2 r_{Di}^2(1+k^2\lambda_{ii}^2)} + \frac{1}{k^2 r_{De}^2 k^2 r_{Di}^2} \left[\frac{1}{(1+k^2\lambda_{ee}^2)(1+k^2\lambda_{ii}^2)} - \frac{1}{(1+k^2\lambda_{ei}^2)^2} \right] + \frac{A}{r_{De}^2} \left(1 + \frac{1}{k^2 r_{Di}^2 (1+k^2\lambda_{ii}^2)}\right) \exp\left(-\frac{k^2}{4b}\right). \quad (3.5)$$

Expressions for $\Phi_{ab}(r)$ in ordinary configuration space can be obtained from (3.2)–(3.5) with the aid of the Fourier transform:

$$\Phi_{ab}(r) = \frac{1}{(2\pi)^3} \int d\mathbf{k}\, \tilde{\Phi}_{ab}(k) \exp(i\,\mathbf{k}\cdot\mathbf{r}).$$

Subject to $\lambda_{ii}, \lambda_{ei}, \lambda_{ee} \ll r_{Di}, r_{De}$, the Fourier transformation can be performed analytically, and gives rise to the following simple form:

$$\Phi_{ab}(r) = \frac{e_a e_b}{r} \left[\exp\left(-\frac{r}{r_D}\right) - \exp\left(-\frac{r}{\lambda_{ab}}\right) \right] + \delta_{ae}\delta_{be} k_B T \ln 2 \exp\left(-\frac{r^2}{\lambda_{ee}^2 \pi \ln 2}\right), \quad (3.6)$$

This expression differs from (3.1) by the presence of the $\exp(-r/r_D)$ term in the brackets instead of 1.

4. Electrical conductivity

It is well known that the static structure factors are important characteristics of physical systems and may be used for calculation of both thermodynamic functions and transport properties. Arkhipov and Davletov [14] proposed the following analytical expression for $S_{ab}(k)$:

$$S_{ab}(k) = \delta_{ab} - \frac{\sqrt{n_a n_b}}{k_B T} \tilde{\Phi}_{ab}(k), \quad (4.1)$$

where $\tilde{\Phi}_{ab}(k)$ is expressed through (3.2)–(3.5).

After neglecting terms quadratic is λ_{ab}/r_D in the case of $\lambda_{ii}, \lambda_{ei}, \lambda_{ee} \ll r_{Di}, r_{De}$,

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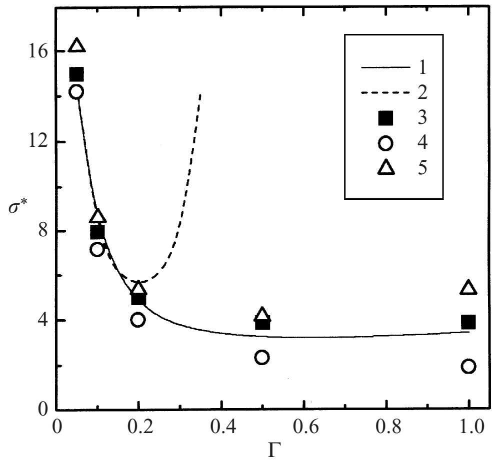

Figure 1. Normalized electrical conductivity σ^* of a hydrogen plasma at $r_s = 0.4$: (1) (4.5) with (4.1); (2) (4.5) with (4.7); (3) Iyetomi et al. [16]; (4) Baus et al. [17]; (5) Boercker et al. [18].

the static structure factors are simplified to

$$S_{ee}(k) = 1 - \frac{r_D^2}{r_{De}^2(1 + k^2\lambda_{ee}^2)(1 + k^2 r_D^2)}, \tag{4.2}$$

$$S_{ii}(k) = 1 - \frac{r_D^2}{r_{Di}^2(1 + k^2\lambda_{ii}^2)(1 + k^2 r_D^2)}, \tag{4.3}$$

$$S_{ei}(k) = \frac{r_D^2}{r_{De} r_{Di}(1 + k^2\lambda_{ei}^2)(1 + k^2 r_D^2)}, \tag{4.4}$$

with $r_D^{-2} = r_{De}^{-2} + r_{Di}^{-2}$.

In the second Sonine polynomial approximation, the normalized conductivity σ^* of a plasma can be written down [16], in the form

$$\sigma^* = \frac{\sigma}{\omega_{pe}} = 1.93 \left(\frac{3\pi}{2}\right)^{1/2} \frac{1}{4\pi\Gamma^{3/2} L}, \tag{4.5}$$

where $\omega_{pe} = (4\pi n_e e^2/m_e)^{1/2}$ is the electron plasma frequency and L is the generalized Coulomb logarithm. In a memory-function formalism coupled with the Green–Kubo formula, the generalized Coulomb logarithm is related to the static structure factors $S_{ab}(k)$ via the simple formula [17]

$$L = \int_0^\infty \frac{dk}{k} \frac{\left[S_{ee}(k)S_{ii}(k) - S_{ei}^2(k)\right]}{(1 + k^2\lambda_{ei}^2)^2}. \tag{4.6}$$

Formulae (4.1) and (4.6) provide a simple calculation scheme for the generalized Coulomb logarithm L and the electrical conductivity (4.5) of a dense high-temperature plasma.

Substituting (4.2)–(4.4) into (4.6), one can derive the following simple expression for L:

$$L = \ln\left(\frac{r_D}{\lambda_{ei}}\right) - \frac{1}{2}. \tag{4.7}$$

In Figs 1 and 2 we plot the electrical conductivity (4.5) of a hydrogen plasma with

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Table 1. The normalized electrical conductivity σ^* of a hydrogen plasma based on (4.6) and the expressions for the static structure factors (4.1).

$r_s = 0.4$		$r_s = 0.5$		$r_s = 0.6$	
Γ	σ^*	Γ	σ^*	Γ	σ^*
0.59	3.1835	0.69	2.5823	0.79	2.1814
0.60	3.1824	0.70	2.5815	0.80	2.1809
0.61	3.1818	0.71	2.5810	0.81	2.1806
0.62	3.1817	0.72	2.5808	0.82	2.1805
0.63	3.1822	0.73	2.5809	0.83	2.1806
0.64	3.1831	0.74	2.5813	0.84	2.1809
0.65	3.1844	0.75	2.5820	0.85	2.1814

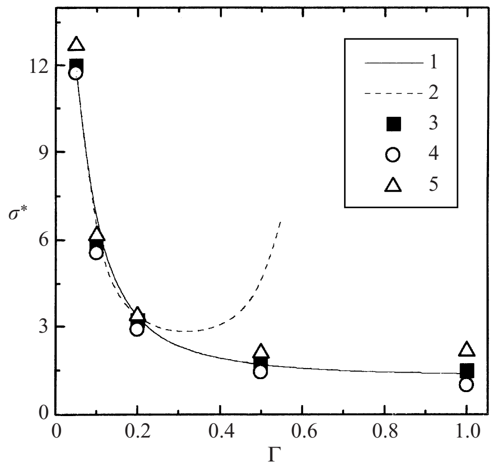

Figure 2. Normalized electrical conductivity σ^* of a hydrogen plasma at $r_s = 1.0$: (1) (4.5) with (4.1); (2) (4.5) with (4.7); (3) Iyetomi et al. [16]; (4) Baus et al. [17]; (5) Boercker et al. [18].

the generalized Coulomb logarithm (4.6) and compare it with the results of other methods. From Figs 1 and 2, it is clear that the simple analytical expression for the electrical conductivity obtaining from formula (4.7) fits very well for small Γ (Table 1).

Boercker et al. [18] considered electrons to be a classical subsystem, and therefore the results of this work are invalid for $\theta \leqslant 1$. If the Fermi degeneracy effect is weak ($\theta > 1$), then the electrical conductivity, evaluated in [18] shows fairly good agreement with the results presented here. From Figs 1 and 2, one can see that for $\theta \leqslant 1$ (or $\Gamma \geqslant 1$), the electrical conductivity (4.6) with the structure factors (4.1) matches with the data from [16], where the quantum density-response formalism has been used and the exchange effects are included through the Fermi distribution function of electrons.

It is easy to see that a minimum appears in the curve of the electric conductivity. Analogous behaviour of the normalized electric conductivity has been observed by Iyetomi et al. [16] and Boercker et al. [18]. The occurrence of a minimum in the curve of the electric conductivity has the following clear physical sense. It is well known that the scattering of electrons from ions plays an essential role in determining the plasma conductivity. In electron–ion interaction, the electrostatic forces,

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which in a plasma have a characteristic radius of action r_D (the Debye radius), are attractive forces, whereas quantum effects, which have a characteristic radius of action λ_{ei}, lead to the appearance of repulsive forces in complete accord with (3.1). The minimum location is thus determined by the equating of characteristic radii of action of the repulsive and attractive forces, $r_D \approx \lambda_{ei}$, which ultimately leads to the following condition for the dimensionless parameters:

$$\Gamma_{\min} \approx \sqrt{\tfrac{1}{3}\pi r_s}. \tag{4.8}$$

It follows from the analysis of Table 1 that the estimate (4.8) fits better for smaller density parameters r_s, because then the quantum effects play more important role.

5. Conclusions

In this paper, the electrical conductivity of high-temperature two-component plasmas has been investigated, and the results have been compared with other methods. It has been shown that the present results agree fairly well with the data of Iyetomi et al. [16], where the quantum density-response formalism was used. Unlike the work of Iyetomi et al., in which the degeneracy effects were taken into account via the Fermi distribution function of electrons, we have taken into consideration these effects in the interparticle potential (3.2)–(3.5). The analytical expression for the generalized Coulomb logarithm has been found for separable scales of action of the quantum and screening effects. It should be emphasized that the simple analytical expression for the electrical conductivity achieving from (4.7) fits very well with the more general formula (4.6) for small Γ. The location of the minimum in the curve of the electrical conductivity has been found, and its appearance is determined by the equality of the electron–ion de Broglie wavelength and the Debye radius.

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