Laws and Possibilities

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The initial part of this paper explores and rejects three standard views of how scientific laws might be systematically connected with physical necessity or possibility. The first concerns laws and their consequences, the second concerns the so-called counterfactual connection, and the third concerns a possible worlds construction of physical necessity. The remaining part introduces a neglected notion of possibility, and, with the aid of some examples, illustrates the special way in which laws reduce or narrow down possibilities.

1. Introduction. There are three influential theories that try to show that there is something distinctive and insightful about scientific laws—their modal character. The first attempts to show that scientific laws have nomic necessity, the second claims that laws have a modality that is revealed in the connection between laws and their associated counterfactual conditionals, and the third proposes that scientific laws are those generalizations that are true in all physically possible worlds. The aim of this study is to review those theories, set them aside as not having much promise, and describe a new way of connecting laws with possibilities.

The first proposal was supposed to show that laws, unlike ordinary generalizations, have nomic or *physical necessity*. Something is physically necessary if and only if it follows from the collection of all laws Equivalently, something is *physically possible* if and only if it is compatible with the set of all laws. It follows immediately that every law is physically necessary since it is among the set of all laws, from which it follows logically. That's a little too facile a conclusion. Though the definition of "physically possible" is logically very economical, relying just on the notions of logical implication and scientific law, nevertheless this particular account of physical necessity is a mixed blessing. On the one hand, it's economically constructed. On the other, it is a very strange modal, especially as a kind of necessity, since every iteration of it is false. For

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example, the physical necessity of something will fail to be physically necessary: If L is physically necessary, then it is a consequence of the set of scientific laws, but that truth is itself not a consequence of scientific laws. Therefore L's being physically necessary is not physically necessary. So this proposal for physical necessity is in worse shape than one could imagine. There's the simplicity of its construction on the one hand, and its pathology as a modal of necessity on the other. Moreover, this account of the physical necessity of laws tells us nothing significant about them, because no specific information about laws went into this definition of "physical necessity." You don't have to know anything about laws to know that they are physically necessary: All you have to know is the elementary fact about implication that the set of all laws logically implies each and every one of its members. That's not much to learn, and it's not a deep truth specific to laws. On balance, this first attempt to connect laws with physical possibilities seems to me very unpromising.

One final note: This particular notion of physical necessity might be used to try to separate laws from initial conditions or boundary conditions. I think that it would be a mistake. Uppermost in the motivation of this view of physical necessity is the justification of the view that laws are physically necessary, so that a non-Humean account of laws is needed. It isn't clear that, on this account, only laws are physically necessary. In fact, on this account of physical necessity, anything that follows from physical necessities is also physically necessary (a closure condition). As there are many consequences of laws that aren't laws, it follows that being physically necessary would not distinguish laws from nonlaws. Nor was it intended to do so. However there is a second strategy that relates laws to possibilities by means of a counterfactual connection.

2. The Counterfactual Connection. There is supposed to be a connection between laws and their corresponding counterfactuals that is often used as the hallmark for distinguishing laws from other generalizations (so-called "matter of fact" generalizations or "accidental" generalizations). What has seemed to have gone relatively unnoticed is that this connection also shows how laws are related to a special kind of modal necessity or possibility—indeed that expressed laws just are modal statements of a special kind. The familiar connection is simply that laws are distinguished from other generalizations in that they imply their corresponding counterfactuals. There are at least two readings which are not usually distinguished in the literature. On the first reading, the claim is that

> (1) If any A is a law, then A implies its corresponding counterfactual $CC(A)$.

Where "$CC(A)$" refers to the counterfactual corresponding to A, when

"A" is a generalization. Let "$L(A)$" be the statement "It is a law that A," so that (1) says that if $L(A)$, then $A \Rightarrow CC(A)$.

On the second reading,

(2) The statement that A is a law, "$L(A)$," implies that $CC(A)$.

That is, $L(A) \Rightarrow CC(A)$. Thus the counterfactual corresponding to the generalization "All *F*s are *G*s" is "If anything were an *F*, then it would be a *G*," and the counterfactual corresponding to a counterfactual generalization is just that counterfactual. To complete the characterization, in the case when A is not a generalization, one can take "$CC(A)$" to be just A itself.

According to (1), if "All *F*s are *G*s" is a law, then "All *F*s are *G*s" implies that if anything were an *F* then it would be a *G*," and according to (2), "It is a law that all *F*s are *G*s" implies that if anything were an *F*, then it would be a *G*.

I shall also assume the usual condition that the counterfactual corresponding to a generalization also implies that generalization: "If anything were an *F* then it would be a *G*" implies that all *F*s are *G*s. Consequently, counterfactual conditionals obey the rule of Modus Ponens.

The two ways of reading the counterfactual connection are each interesting. Both can be used rather effectively to show that straightforward generalizations like "all the coins in Nelson Goodman's pockets are dimes" are not laws; the second version even offers a rather simple way of showing that the prefix "It is a law that . . ." is *factive* (that is, "It is a law that A" logically implies that A).

Both versions imply that there is a connection between laws and modality, though a different connection in each case. Here's why. It is relatively easy to show that any counterfactual conditional, $A > B$ (If A were the case, then B), can be proved equivalent to a statement having the form "$\Box_A(B)$", where "$\Box_A$" stands for a modal operator indexed to A, and "$\Box_A(B)$" is the result of applying the modal operator "$\Box_A$" to "B". If this is correct, then it explains why a simple account of such conditionals is hard to come by. Not only is there a modal operator explicitly present in counterfactuals (it's the prefix "If A were the case, then it would be the case that . . ."), but the modal operator can vary, depending upon the antecedent of the counterfactual.

Consider the two versions of the counterfactual connection in turn. According to (1), if it is a law that A, then A implies its corresponding counterfactual $CC(A)$. However, given that $CC(A)$ implies A, it follows that (1) yields the very implausible conclusion,

If it is a law that A, then A is equivalent to $CC(A)$ [$A \Leftrightarrow CC(A)$].

That is, all scientific laws are *equivalent* to counterfactual conditionals.

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With this result, the modal status of laws becomes evident with a vengeance.

The defects of such a view are many, but I shall defer that discussion for another time. Suffice it to say that those who thought they were adopting a relatively modest claim with (1) have gotten more than they bargained for. There is an additional consideration which makes the equivalence of laws with their corresponding counterfactuals look way off the mark. Counterfactuals are in general not equivalent to their contrapositives (generally they don't even imply them). However, laws that are expressed as conditionals have equivalent contrapositives. For example, "If there are no forces acting upon a body, then it will not accelerate" and its contrapositive, "If a body accelerates, then there are some forces acting upon it," were taken rightly by Maxwell to be equivalent forms of Newton's first law of motion.

Lastly, the suggestion that laws are counterfactuals is not convincing unless we can make the case for understanding how we can reason and evaluate arguments that involve them. But if the observation that counterfactuals are modal is correct, then the proper logic for them is some kind of quantified modal logic, and it's clear that the deep questions of which quantified modal theory to adopt, and which semantic theory is appropriate, are still very much unsettled matters. Thus I take it that (1), the first reading of the counterfactual connection, is a dead end.

Of course, we might then turn to the second reading of the counterfactual connection, according to which it is not the law *A*, but the statement "$L(A)$" ("It is a law that *A*") that implies the corresponding counterfactual. This interesting claim, however, goes no way towards showing that the law itself has modal status. The reason is that even if some *P* implies a necessary statement (whether it is "*A*", or "$L(A)$"), it does not follow that *P* itself is necessary. For example, in the modal system S5, every *P* (modal and nonmodal) implies that it is necessarily the case that it is possible that *P* ($P \Rightarrow \Box\Diamond(P)$); it doesn't follow that every statement is modal.

One version of the counterfactual connection, namely (1), led to the conclusion that all laws are counterfactuals. That is, I believe, an untenable view of laws, but it does guarantee some modal status for them. The second version of the counterfactual connection is of greater interest, but it doesn't support the conclusion that laws themselves have modal status.

3. The Possible Worlds Connection. Let the metaphysically necessary statements be those that are true in all possible worlds, while the metaphysically possible statements are those that are true in some possible world. The standard way of treating variations of necessity and possibility (following S. Kripke) is to characterize them using relativized (or restricted) quantifiers. By this we mean that the quantifier "all" in "all possible worlds" is now a relativized universal quantifier with respect to some property Q, as follows: The "absolute" quantifier version is given by "$(\forall w)$("A" is true in w)" ("A" is true in all worlds w), and the "relativized" version is given by "$(\forall w)(Q(w) \rightarrow$ "A" is true in w)" ("A" is true in all Q-worlds). The relativized version of A's metaphysical possibility is that there is a world w such that $Q(w)$ and "A" is true in w.

One very familiar view of physical necessity (and possibility) is that it is just a special case of relativized necessity, for the special case when the property Q is taken to be "is a physically possible world". Then

> A is physically necessary if and only if it is true in all physically possible worlds.

It is assumed that this general scheme of relativising to some property Q will always result in a modal operator. If that is so, then one might cap this definition of the modal of physical necessity with a familiar definition of scientific law:

> A scientific law is any generalization that is physically necessary.

There's no denying the neatness and compactness of this strategy for connecting laws with necessity (and possibility). However it has several serious problems worth considering: There is the problem of ensuring modality. Some properties will yield a modal operator if you relativize with respect to them, and others will not. Not any old property, even any old plausible property, will work. To see why there's a problem lurking here, we should recall how the successful cases of relativization worked. The usual strategy requires an accessibility relation Rwv (a binary relation on possible worlds) such that the modal operator "$\Box$" satisfies the following condition, call it "(**N**)": "$\Box(A)$" is true in world w if and only if "A" is true in all worlds v that are accessible from w. That is,

> (**N**) "$\Box$(A) is true in world w if and only if $(\forall v)$ [$Rwv \rightarrow$ "A" is true in v].

The worry about which Qs will result in relativized modals, and which will not, arises from the fact that not every binary relation can be used as an accessibility relation. Some binary relations do not yield necessities via (**N**). For example if Rwv is taken to be the identity relation $w = v$, then there is no modal which satisfies (**N**). [The argument: $(\forall v)[w = v \rightarrow$ ("A" is true in v)] if and only if "A" is true in w. So, by (**N**), "$\Box(A)$" is true in world w if and only if "A" is true in world w (for all possible worlds w). Therefore "$\Box(A)$" is equivalent to "A" for all statements A.] This is the disaster that in modal logic is known as the collapse of the modal—or, as I have described it elsewhere, the box just isn't a modal operator (Koslow 1992, 246). There's a more general result. If any accessibility relation Rwv is a function (that is, if Rwv and Rwv^*, then $v = v^*$), then (using (**N**)) it will follow that "$\Box$" will not be modal, and conversely.[1] The moral is very simple: One has to be careful when specifying accessibility relations for relative necessities, or the result may not be a modal necessity.

Here is an example of the kind of thing that looks initially promising, but is far off the mark. Assume some concept of being a law of a possible world w, and some notion of closeness of possible worlds. We might try to introduce a notion of physical necessity by using the following binary relation as an accessibility relation:

> Rwv (v is accessible from w) if and only if v is the closest world to w (other than w itself) in which all the laws of w are true.

This would not yield a modal, because the relation R is a function (no w can stand in the relation R to two different worlds). The moral of this rather long story is that you cannot just assume, as some writers do, that we can get a notion of physical necessity just by relativizing the quantifiers over possible worlds.

Lastly, there is a problem with how physical necessity and metaphysical necessity are supposed to be related. Suppose that there is a property P such that a possible world has P if and only if it is a physically possible world. Then "A" is metaphysically necessary, $\Box(A)$, if and only if $(\forall w)$("A" is true in w), and "A" is physically necessary, $\Box_p(A)$, if and only if $(\forall w)(P(w) \rightarrow$ "A" is true in w). It follows that $\Box(A) \Rightarrow \Box_p(A)$. That is, anything that is metaphysically necessary must also be physically necessary. For example, if it is metaphysically necessary that cats are animals, then it must also be physically necessary that cats are animals. This looks plainly incorrect. If we were also to agree that any generalization that is physically necessary is a scientific law, it would follow that all generalizations that are metaphysically necessary are scientific laws. It looks as if one grand dream of metaphysics has come to fruition. Every metaphysical generality is a scientific law. Well that's the dream part; sometimes metaphysical inquiry will pay off in scientific laws. On the other hand, any metaphysical necessity will be a physical necessity, and combined with the claim that all physical necessities are consequences of scientific laws, it follows that any metaphysical necessity is a consequence of scientific laws. And that's a metaphysical nightmare.

This use of possible worlds has nothing plausible to say about any connection between laws and possibility. It seems reasonable then to look

1. In the more general case, the necessity and possibility operators become equivalent, which is another kind of modal collapse.

at some other way of thinking about physical necessity and possibility, other than the views I have just canvassed.

4. Starting Over: A New Kind of Possibility. I think it is possible to show that laws do narrow down, or reduce, possibilities. They do so if both the possibilities and the specific way of narrowing them down are different from what is usually understood.

4.1. Natural Possibilities. I begin with some indication of the broad variety of types of cases, which I shall gather up into a minitheory of natural possibilities. These include simple and complex cases of possibilities which can be linguistic or not, concrete or abstract, simple or richly structured, and mathematical or physical. Here are some familiar cases that are customarily and naturally described as possibilities:

1. A die is thrown and there are, as we say, six *possibilities*.
2. In sample spaces generally, the members of the space are usually described as *possibilities.*
3. For most physical theories there is an associated notion of the states of that theory. The totality of these, the state space of the theory, is commonly described as constituting the physical *possibilities* for those systems under study by the theory.
4. Suppose that in a state space of some physical theory, two points A and B are distinguished. Usually it's said that there is a family of paths connecting A and B (including the actual one) which are described as the *possible* routes or paths from A to B. So, rather than the theoretical states, sometimes it may be the trajectories or paths which are the natural possibilities.

4.2. Serious Possibilities. Why are these various examples serious modal *possibilities* rather than just ways of merely speaking with the vulgar? The short story is that all our examples are cases of a kind of structure which I have elsewhere called a *natural implication structure* (Koslow 2003, 169–183). Such a structure of natural possibilities is a set N which satisfies three conditions:

1. N has at least two members;
2. any two members of N don't overlap, intersect, or have anything in common, and are 'mutually incompatible' in some sense of that term; and
3. being an N is in a sense the widest, most inclusive possibility under consideration.

Given the wide variety of the examples I have in mind, there does not

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seem to be any particular interpretation of the terms 'overlap', 'intersect', or 'mutually incompatible' which would show that all of them were examples of a set of natural possibilities. Nevertheless there is a simple theory of these natural possibilities that provides a clear and uniform sense for the conditions 1–3, and which will also allow for there to be negations, conjunctions, conditionals, and quantifications of possibilities. In short, it enables there to be a logic of these natural possibilities.

The idea is to take all the subsets of N, $\wp(N)$, as a structure of the N-possibilities, where the unit sets $\{x\}$ for each member of N are now taken as the elementary or natural possibilities of the structure. These genuine possibilities of the structure can be things like paths, orbits, or the states of a theory. The field of sets of a probability space is a familiar example of this kind of structure, where the unit sets of the members of the sample space are the natural possibilities.

We assume that the modal operators on such a structure can be defined as special kinds of functions which map the structure to itself. And we can show that necessity ($\Box$) and possibility ($\Diamond$) operators can be defined which represent the possibilities and the necessity elements of the structure. These operators are very close to the necessity and possibility operators of S5, and yet very different.[2] That is, necessity is a normal modal satisfying the necessitation condition as well as the conditions that are characteristic axioms for the modal systems T, K4, and S5. The difference lies with the relation between necessity and possibility. Our necessity implies not-possibly-not (but not conversely), and our possibility implies not-necessarily-not (but not conversely), and this reflects the fact that the necessity modal is a T-modal, but the possibility modal is not.

5. Laws and the Narrowing down of Possibilities. Thus far, I have explained why I think of various natural possibilities as serious modals. I can now consider the special way in which laws narrow down or rule out possibilities. That is,

(**LP**) Laws narrow down possibilities.

Several things should be said immediately. I am not suggesting that this is the only thing to be said about laws, nor is the narrowing down of possibilities some feature that distinguishes laws from nonlaws (as the planetary example below shows, laws can narrow down a set of possibilities, but other conditions which enter into explanations with those laws can narrow down that set even further).

There is also an ambiguity of scope. A *wide* scope version requires that

2. The details of the construction can be found in Koslow 2003.

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there is a set of possibilities such that if any *A* is a law, then *A* narrows down that set. On the *narrow* scope version, if *A* is a law, then there is a set of possibilities such that *A* narrows them down. The narrow scope version allows that the set of possibilities could differ from law to law, and will usually differ with the theoretical setting for that law. Wide scope requires a super set of possibilities, which gets reduced by every law. The narrow version yields a more refined and more accurate account of the way things go scientifically, and this is the version I wish to defend.

The basic idea of the narrowing down of possibilities involves several assumptions whose full support I must postpone for another occasion.

1. To each scientific law *L*, there is associated a certain non-empty set of possibilities $\wp_L$ that satisfies the following conditions. The assumption is that there is at least one such set of possibilities; there may, however, be several. I don't insist on the uniqueness or even the maximality of each of these sets.

2. The set of possibilities can be narrowed down in the sense that some of the possibilities are 'ruled out' or excluded by the law *L*. Clearly, a sense has to be provided for a law's excluding or ruling out such possibilities as orbits or states. Consequently, the suggestion that *A* rules out or excludes *B* by implying the negation of *B*, will not work. What would the negation of a trajectory or a state be, and how would it be implied by laws?

3. Each law involves certain physical quantities or magnitudes. I shall not say much at present about physical magnitudes. For the present purpose it is enough to note that some physical quantities (e.g., mass, length, velocity, density, kinetic energy, temperature, charge, etc.) are functions that map physical entities or structures of physical entities to elements of some mathematical structure (e.g., a real number, vector, matrix, tensor, etc.) This, however, is not the whole story. An important group of physical magnitudes are *functionals*. They figure prominently in laws of classical mechanics, electromagnetism, thermodynamics, and a whole range of contemporary field theories (relativistic, as well as quantum theoretical) and they are mappings from functions to mathematical structures, rather than mappings from physical entities to those mathematical structures. The point is that certain functionals are physical magnitudes. The final assumption is:

4. To each scientific law there is not only some set of possibilities $\wp$ associated with that law, but there is also a special kind of property Σ_Φ, which depends on a functional magnitude Φ and which holds or fails to hold of the possibilities in $\wp$. We shall refer to the property Σ_Φ as a *functional property*. So here is the idea as to how it is that laws exclude or narrow down possibilities:

(**LRP**) (i). For each law L there is some property Σ_Φ (expressed with the aid of a functional Φ) that holds or fails to hold for the members of a certain set of possibilities, and (ii) L implies that some possibilities fail to have that property.

According to (ii), if L is a scientific law, then for some possibility α of a set of possibilities $\wp$, $L \Rightarrow \neg\Sigma_\Phi(\alpha)$. It may also happen that some law excludes all but one of the possibilities which are associated with it, though that is not generally so for all laws. Furthermore, a law L may actually guarantee that some of the possibilities have the functional property Σ_Φ, that is, for some possibility β, $L \Rightarrow \Sigma_\Phi(\beta)$.

There is little space to show in any detail how typical laws reduce possibilities. Any law which can be expressed as a special case of Hamilton's Principle of Least Action easily falls under the concept expressed by (**LRP**). The reason is that such applications proceed by specifying a particular Lagrangean ($T - U$) for some physical system, where T is the kinetic energy and U the potential energy of that system. A set of curves f, g, . . . is specified between two points in a state space. Think of these curves as the possibilities, and then there is a functional called the *action* for the particular Lagrangean L, given by

$$\Psi[f] = \int_{t_0}^{t_1} L_f(q,\ dq/dt,\ t)dt$$

(where the integral is taken along the curve f). With the help of the functional Ψ, we can define a functional property Σ_Ψ of the possibilities (curves f, g, . . .) as follows:

Σ_Ψ holds of the curve f if and only if f is a curve for which Ψ, the action, is an *extremal*—that is, $\Psi(f)$ is either a maximum or a minimum.

In all these 'Hamiltonian' cases, possibilities are ruled out or excluded if and only if it is implied that they fail to satisfy the functional property Σ_Ψ, that is, they fail to be extremals.

Here's a simple example of this kind of case: Newton's first law of motion.[3] For a free particle moving in Euclidean three space, the potential U is 0, so that the Lagrangean for the free particle is $L = T = m(d\mathbf{r}/dt)^2/2$. If one uses generalized coordinates, then

$$L = m/2[(dq_1/dt)^2 + (dq_2/dt)^2 + (dq_3/dt)^2].$$

The Lagrange-Euler equations then yield that the generalized momentum

3. This example is indebted to Arnold 1978, 60.

$p = \partial L/\partial q_i$ is constant ($dp/dt = d(\partial L/\partial q_i)/dt = 0$), since the Lagrangean is not a function of the generalized coordinates q_i. This simple result can be expressed by saying that straight lines are the extremals of the action of free particles. The law of inertia, in this formulation, rules out any possible path that is not a straight line.

There are other familiar cases when laws narrow down sets of possibilities. The Newtonian law of gravitation is one such. I omit the well-known details and hope that a very truncated description will suffice. The assumption that the gravitational force on a body is central, attractive, and inverse square implies that the orbit (in polar coordinates) is given by $r(\theta) = \lambda(1 + \varepsilon)/[1 + \varepsilon \cos(\theta - \theta_0)]$, the focal equation of a conic section, with eccentricity ε, where the constant λ is defined as $|L|^2/[m\alpha(1 + \varepsilon)]$, L is the total angular momentum of the planetary body, and α is Gm_1m_2. If the possible orbits are taken to be at least differentiable curves $X(\theta)$, we have the functional $\Phi = \lambda(1 + \varepsilon)/[1 + \varepsilon \cos(\theta - \theta_0)]$, and we define a functional property Σ_Φ such that $\Sigma_\Phi(X(\theta))$ if and only if $X(\theta) = \Phi$. Since Newton's Law of Gravitation implies that $r(\theta) = \Phi$ (that is, that $r(\theta)$ is a conic),[4] it follows that the law implies $\neg\Sigma_\Phi(X(\theta))$, for any curve $X(\theta)$ that is not a conic.

The gravitational case shows in a clear way that although laws can narrow down a set of possibilities, they needn't narrow them to just one. In the gravitational case, several possibilities (i.e., the various conics) are left, and in the case of some laws, the so-called impossibility laws, all of the possibilities may be ruled out. An *explanation* of the elliptical orbit of Mars narrows down the possible orbits even further from the conics to the ellipse. This further reduction beyond what the Law of Gravitation yields, is obtained with the use of further information included in the explanation—that $\lambda > 0$ and $0 < \varepsilon < 1$.

REFERENCES

Arnold, Vladimir I. (1978), *Mathematical Methods of Classical Mechanics*. New York: Springer Verlag.

Barger, Vernon, and Martin Olsson (1973), *Classical Mechanics: A Modern Perspective*. New York: McGraw Hill.

Corben, Herbert Charles, and Philip M. Stehle (1957), *Classical Mechanics*. New York: John Wiley.

Koslow, Arnold (1992), *A Structuralist Theory of Logic*. New York: Cambridge University Press.

——— (2003), "Laws, Explanations and the Reduction of Possibilities", in Hallvard Lillehammer and Gonzalo Rodriguez-Pereyra (eds.), *Real Metaphysics: Essays in Honour of D. H. Mellor*. London: Routledge, 169–183.

4. For details of this familiar example, both Barger and Olsson 1973 and Corben and Stehle 1957 are very lucid.