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ARTICLES

ON THE "HOT POTATO" EFFECT OF INFLATION: INTENSIVE VERSUS EXTENSIVE MARGINS

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Conventional wisdom is that inflation makes people spend money faster, trying to get rid of it like a "hot potato," and this is a channel through which inflation affects velocity and welfare. Monetary theory with endogenous search intensity seems ideal for studying this. However, in standard models, inflation is a tax that lowers the surplus from monetary exchange and hence reduces search effort. We replace search intensity with a free entry (participation) decision for buyers—i.e., we focus on the extensive rather than intensive margin—and prove buyers always spend their money faster when inflation increases. We also discuss welfare.

Keywords: Search, Inflation, Velocity, Free Entry

> The public discover that it is the holders of notes who suffer taxation [from inflation] . . . and they begin to change their habits and to economize in their holding of notes. They can do this in various ways . . . [T]hey can reduce the amount of till-money and pocket-money that they keep and the average length of time for which they keep it, even at the cost of great personal inconvenience . . . By these means they can get along and do their business with an amount of notes having an aggregate real value substantially less than before.

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> In Moscow the unwillingness to hold money except for the shortest possible time reached at one period a fantastic intensity. If a grocer sold a pound of cheese, he ran off with the roubles as fast as his legs could carry him to the Central Market to replenish his stocks by changing them into cheese again, lest they lost their value before he got there; thus justifying the prevision of economists in naming the phenomenon "velocity of circulation"! In Vienna, during the period of collapse . . . [it] became a seasonable witticism to allege that a prudent man at a cafe ordering a bock of beer should order a second bock at the same time, even at the expense of drinking it tepid, lest the price should rise meanwhile.
>
> —Keynes (1924, p. 51)

1. INTRODUCTION

Conventional wisdom has it that when inflation or nominal interest rates rise people try to spend their money more quickly—like a "hot potato" they want to get rid of sooner rather than later—and this is a channel through which inflation potentially affects velocity and welfare. For the purpose of this paper, this is our definition of the "hot potato" effect: when inflation increases, people spend their money faster. Search-based monetary theory seems ideal for studying this phenomenon, once we introduce endogenous search intensity, as in standard job-search theory [Mortensen (1987)]. This is done by Li (1994, 1995), assuming buyers search with endogenous intensity, in a first-generation model of money with indivisible goods and indivisible money along the lines of Kiyotaki and Wright (1993). One cannot study inflation directly in this model, of course, but Li proxies for it with taxation. Among other results, his model predicts that increasing the inflation-like tax unambiguously makes buyers search harder and spend their money faster, thus increasing velocity, and actually improving welfare.

His results may appear natural, but they do not easily generalize to relaxing the assumption of indivisible goods and money, which was made for convenience and not meant to drive substantive conclusions. Why? People cannot in general *avoid* the inflation tax by spending money more quickly—again like a "hot potato," buyers can only *pass it on* to sellers. Sellers are not inclined to absorb the incidence of this tax for free. Once we relax the restriction of indivisible goods and money, the terms of trade adjust with inflation, and the net outcome is that buyers reduce rather than increase their search effort. Intuitively, as a tax on monetary exchange, inflation reduces the return to this activity; when the return falls, agents invest less; and this means in the models that buyers search less and end up spending their money more slowly. The prediction that search effort increases with inflation depends on the terms of trade not being allowed to adjust.[1]

Lagos and Rocheteau (2005) prove these results using the search-based model in Lagos and Wright (2005) with divisible goods and money, which allows the terms of trade to be determined by bargaining, and allows one to introduce inflation directly rather than proxy for it by taxation. They show an increase in inflation reduces the surplus from monetary trade and hence buyers' incentive to search, so they spend their money less, not more, quickly. Lagos and Rocheteau go on to show one can get buyers to search more with inflation in a model with price posting as in Rocheteau and Wright (2005), rather than bargaining, for some parameter values. The trick is this: even though the total surplus falls with inflation, if buyers' share of the surplus goes up enough, which is possible under posting if parameters are just right, they may get a higher net surplus and hence increase search effort. This is clever, but not especially robust, in that one might think the "hot potato" effect was so natural it ought not to depend on extreme parameter values or on the pricing mechanism (posting vs. bargaining).

There is much additional work on the problem. Ennis (2008) assumes sellers have an advantage over buyers in terms of the frequency with which they can access a centralized market where they can offload cash (like Keynes's cheese merchant in the epigraph). Thus, inflation increases buyers' incentive to find sellers, because sellers can get money to the centralized market faster.[2] Nosal (2008) assumes buyers meet sellers with different goods and have to decide when to make a purchase. They use reservation strategies, and as inflation rises their reservation values fall, increasing the speed at which they trade. Dong and Jiang (2009) present a similar analysis in a model based on private information. Previously, Shi (1998) endogenized search intensity in the Shi (1997) model, and showed that it can increase with inflation, due to general equilibrium effects, for some parameter values.

All of this is fine, but we want to propose a new approach. Our idea is to focus on the extensive rather than the intensive margin—i.e., on how many buyers are searching, rather than on what any particular buyer does. The idea is obvious, once one sees it, but we think it is nonetheless interesting. If one will allow us to indulge in the Socratic method, for a moment, consider this. The goal is to get buyers to trade more quickly when the gains from trade are reduced by inflation. What kind of theory of the goods market would predict that buyers spend their money faster when the gains from trade are lower? That would be like a theory of the labor market that predicted firms hire more quickly when we tax recruiting. What kind of model of the labor market could generate that?

The answer is the textbook model of search and recruiting in Pissarides (2000). It does so because it focuses on the extensive margin—a free-entry or participation decision by firms. When recruiting is more costly, and thus less profitable, in that model, some firms drop out, increasing the hiring rate for those remaining through a standard matching technology. Of course firms hire faster when we tax them, because that is the only way to keep profit constant! The same logic works for the goods market. Of course people spend their money faster when inflation rises, because that is the only way to satisfy the analogous participation condition for consumers. This corresponds well to the casual observation that people are less likely to participate in monetary exchange when inflation is high, perhaps reverting to barter, home production, etc. Our results are also robust, in the sense that they do not depend much on parameters or pricing mechanisms.

There are at least two reasons for being interested in search behavior, along either the intensive or extensive margin. One concerns welfare and optimal policy: we want to know if there is too little or too much search, and how policy might correct any inefficiency. The other concerns positive economics. As mentioned, if buyers spend their money faster when inflation rises, this is one (if not the only) channel through which velocity depends on inflation and nominal interest rates. Understanding how velocity depends on monetary policy is important, because this is basically the same as understanding how money demand, or welfare, depends on monetary policy, as discussed, e.g., by Lucas (2000). In the simplest models, velocity and search intensity are identically equal. In more complicated models, velocity depends on several effects, but the speed with which agents spend their money is still one of the relevant effects.

In Section 2 we begin by presenting the data to confirm the conventional wisdom that velocity is increasing in inflation or nominal interest rates.[3] We then move to theory. In Section 3 we consider models with indivisible money in order to introduce some assumptions and notation, and to review the results in Li (1994, 1995). In Section 4 we consider models with divisible money, and show the following: with an endogenous search intensity decision (the intensive margin), the speed with which agents spend their money falls with inflation, as in Lagos and Rocheteau (2005); but with a participation decision (the extensive margin), the speed with which agents spend their money, and also velocity, always increase with inflation. We also discuss welfare implications, and show that with an endogenous participation decision for buyers, the Friedman rule might not be optimal—positive inflation or nominal interest rates can be desirable. In Section 6 we conclude.

2. EVIDENCE

We use quarterly U.S. data between 1955 and 2008 and Canadian data between 1968 and 2006. Figure 1a shows for the United States the behavior of inflation π and two measures of the nominal rate i, the government bond (T-Bill) rate and the Aaa corporate bond rate. Figure 2 shows a similar series for Canada.[4] Dotted lines are raw data and solid lines are HP trends. The models below satisfy the Fisher equation, $1 + i = (1 + \pi)/\beta$, where β is the discount factor. As one can see, this relationship is not literally true but not a bad approximation to the data. Figures 1b and 2b show velocity, $v = PY/M$, for the United States and Canada, where P is the price level, Y real output, and M the money supply, for three measures of money, $M0$, $M1$, and $M2$. We call the three velocity measures $v0$, $v1$, and $v2$. Obviously, v is lower for broader definitions of M. Also, although v has relatively small deviations between raw data and trend, there are interesting trend movements in $v0$ and $v1$.

Figure 3 shows scatterplots for the U.S. raw data on all three measures of v versus π and v versus i (we show only T-bill rates, but the picture looks similar for Aaa rates). Figure 4 shows scatterplots after higher-frequency movements in the

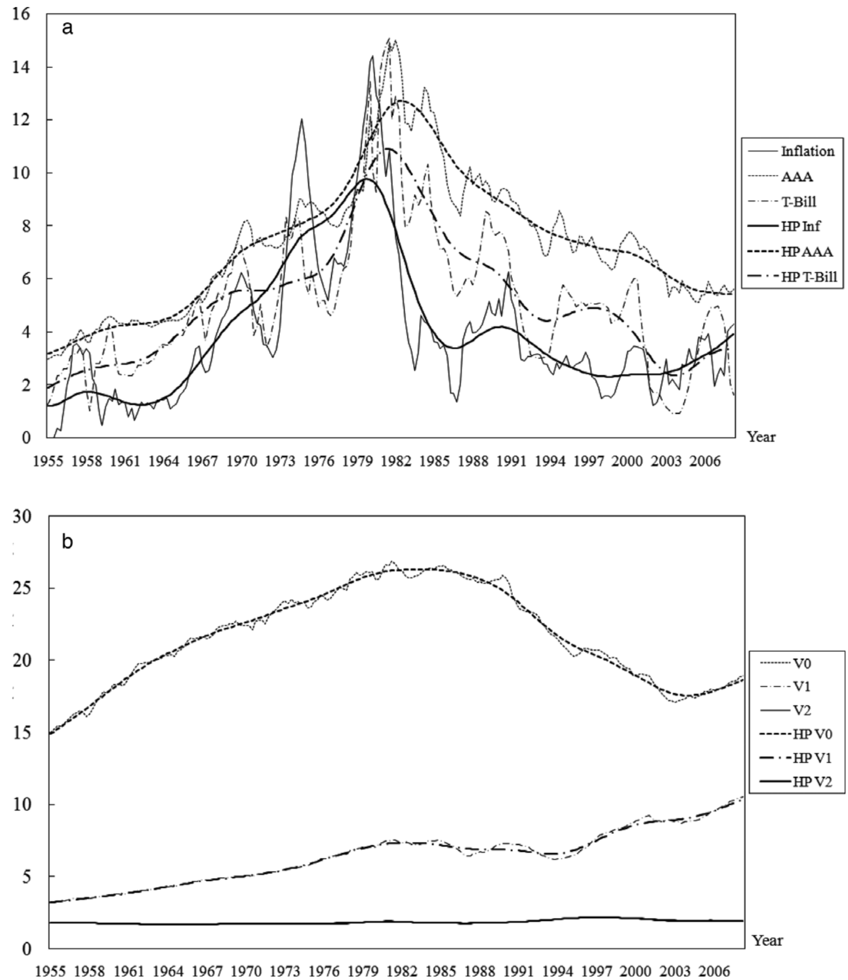

FIGURE 1. (a) Inflation and nominal interest rates, United States. (b) Velocity, United States.

series are filtered out, i.e., scatterplots of the HP trends. Figure 5 shows something similar after the low-frequency movements are filtered out, i.e., scatterplots of the deviations between the data and trends.[5] Table 1 gives the correlations. From the figures or the table, one can see that for the U.S. data $v1$ and especially $v0$ move together with π or i in the raw data, whereas $v2$ does not. However, $v2$ is strongly positively correlated with π or i at high frequencies, whereas the correlations for $v0$ are driven mainly by the low-frequency observations, and the correlations for $v1$ are positive at both high and low frequencies. Similar observations prevail in the Canadian data, with some interesting differences that we do not have time to dwell on.

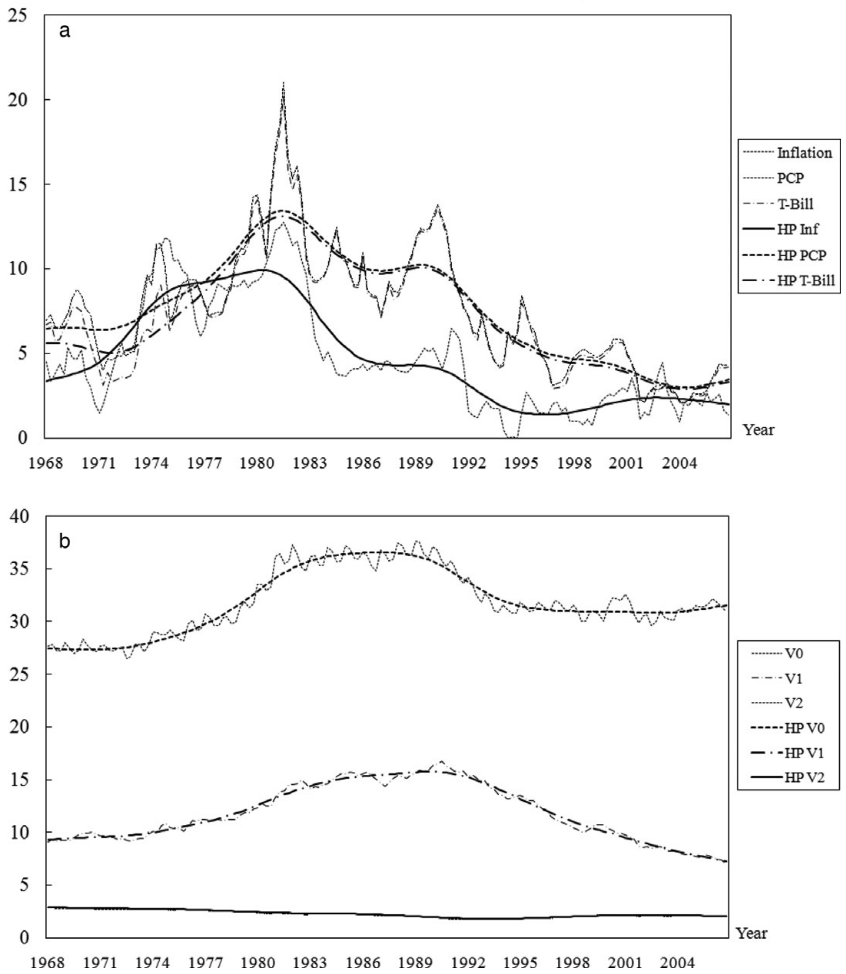

FIGURE 2. (a) Inflation and nominal interest rates, Canada. (b) Velocity, Canada.

There appears to be a structural break in velocity in the U.S. data, especially $v1$. Informally, looking at the charts, one might say that sometime in the early 1980s interest rates began to drop while $v1$ stayed flat. Or one might argue that the big change was in the mid-1990s, when π and i continued to fall but $v1$ started upward. To control for this in a simple way, Table 1 also reports the correlations for the United States when we stop the sample in 1982 (results are similar when we stop in 1995). We find that $v0$ moves about as strongly with π or i, but now both $v1$ and $v2$ move much more with π or i, at both high and low frequency. We conclude from all of this that the preponderance of evidence indicates that all measures of v move positively with π or i, although for some measures this is mainly at high frequency and for others at low frequency.

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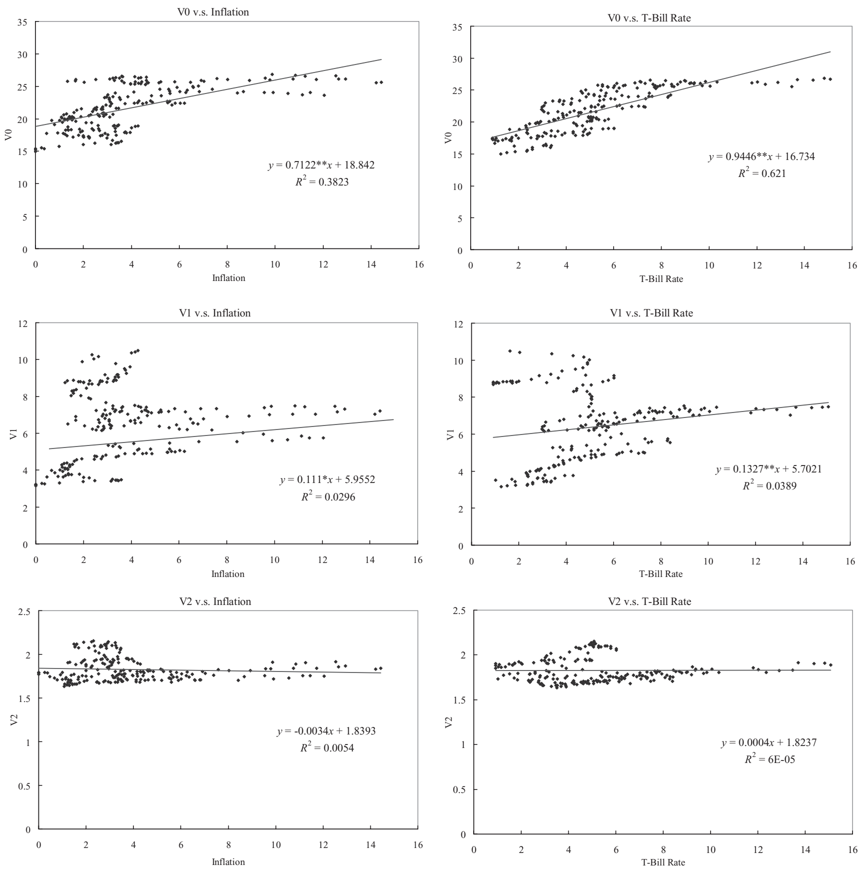

FIGURE 3. Scatterplots of raw data, United States.

We want it to be clear we are not suggesting that these observations constitute a *puzzle*—i.e., that they are inconsistent with existing theories. Many models, including those with some but not all goods subject to a cash-in-advance constraint, as well as most recent search models, and many other models, can in principle match these data. In fact, because v is the inverse of M/PY, and it is common to take M/PY as a measure of money demand, any model where money demand decreases with i should be at least roughly consistent with the evidence on v. The purpose of this empirical digression is this: one reason for being interested in search behavior is that it contributes to the relationship between inflation and velocity, and we simply want to document what this relationship is. We now move to theory.

3. INDIVISIBLE MONEY AND GOODS

A [0, 1] continuum of agents meet bilaterally and at random in discrete time. They consume and produce differentiated nonstorable goods, leading to a

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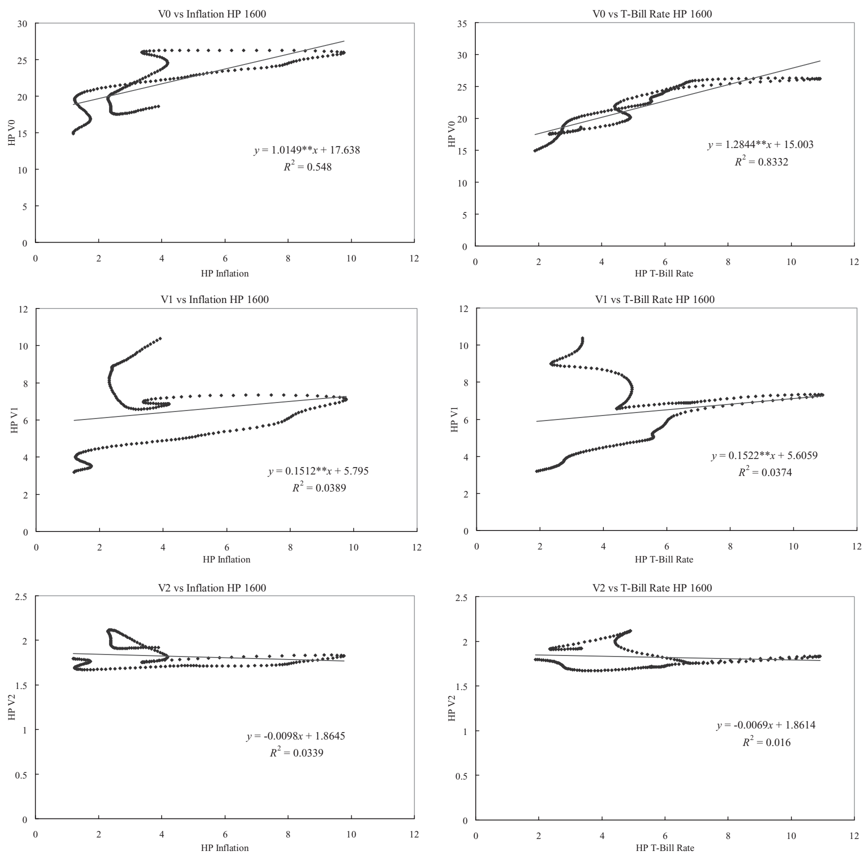

FIGURE 4. Scatterplots of HP trends, United States.

standard double coincidence problem: x is the probability that a representative agent wants to consume what a random partner can produce. As agents are anonymous, credit is impossible, and money is essential. So that we can review earlier results, for now goods and money are indivisible, and there is a unit upper bound on money holdings. Given M total units of money, at any point in time there are M agents each with $m = 1$ unit, called buyers, and $1 - M$ with $m = 0$, called sellers. Only sellers can produce, so if two buyers meet they cannot trade (one interpretation is that, after producing, agents need to consume before they produce again). Only buyers can search, so sellers never meet (one interpretation is that they must produce at fixed locations). Hence, every trade has a buyer giving 1 unit of money to a seller for $q = 1$ units of some good; there is no direct barter.

Each period, a buyer meets someone with probability α. The probability that he or she meets a seller who produces what he or she wants, a so-called *trade meeting*,

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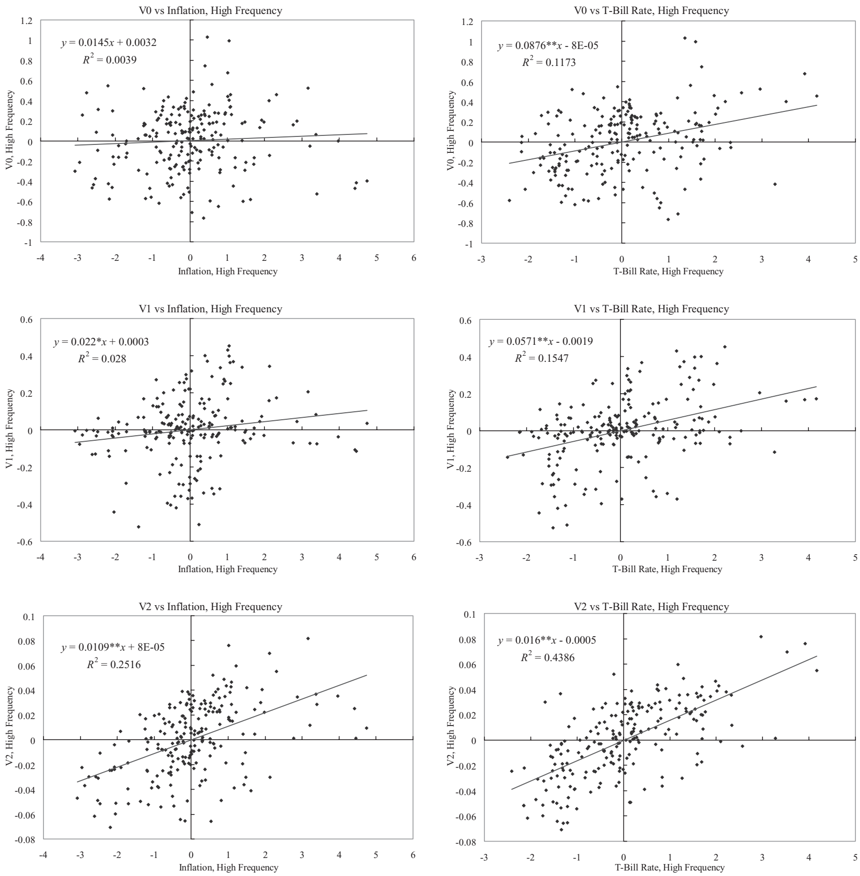

FIGURE 5. Scatterplots of deviations, United States.

is $\alpha_b = \alpha(1-M)x$. This is also velocity: $\alpha_b = v = PY/M$ because $PY = M\alpha_b$. The probability of such a meeting for a seller is $\alpha_s = \alpha_b M/(1-M) = \alpha M x$. Buyers choose search intensity. Given M and x, they can choose either α or α_b. We adopt the convention that they choose α_b, and we write search cost as $k(\alpha_b)$, where $k(0) = k'(0) = 0$, $k'(\alpha_b) > 0$, and $k''(\alpha_b) > 0$ for $\alpha_b > 0$.[6] Policy is modeled as a tax on money holdings, but because it is indivisible, rather than taking away a fraction of cash we take it all with probability τ each period (one interpretation is that buyers, in addition to meeting sellers, also meet government agents with confiscatory power). To focus on steady states we keep M constant by giving money to a seller each period with probability $\tau M/(1-M)$. This tax proxies for inflation.

Although for now q is indivisible, in general $u(q)$ and $c(q)$ are utility from consumption and disutility from production, where $u(0) = c(0) = 0$, $u'(q) > 0$,

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TABLE 1. Correlations

	U.S. 1955Q1–2008Q2			U.S. 1955Q1–1982Q4			Canada 1968Q1–2006Q2		
	V0	V1	V2	V0	V1	V2	V0	V1	V2
Raw Data									
Inflation	0.6205	0.1722	−0.0736	0.7866	0.8473	0.5805	0.0851	0.1580	0.5170
AAA	0.8390	0.4068	0.1094	0.8718	0.9377	0.5564	0.5251	0.6289	0.1884
T-Bill	0.7880	0.1971	0.0075	0.7980	0.8596	0.6296	0.6049	0.6758	0.0963
Trend (Low Freq)									
Inflation	0.7403	0.1971	−0.1840	0.9025	0.9524	0.5261	0.0386	0.1619	0.6147
AAA	0.8854	0.416	0.0805	0.9230	0.9813	0.5680	0.5554	0.7116	0.2173
T-Bill	0.9128	0.1933	−0.1266	0.9148	0.9740	0.5701	0.6500	0.7577	0.0984
Deviation (High Freq)									
Inflation	0.0603	0.1674	0.5016	−0.0570	0.1360	0.6027	0.3703	0.2857	−0.0816
AAA	0.0128	0.3235	0.4292	−0.1101	0.2188	0.4587	0.4684	0.5110	0.2131
T-Bill	0.3425	0.3933	0.6623	0.2642	0.3093	0.6121	0.4778	0.5182	0.2138

$c'(q) > 0, u''(q) < 0, c''(q) \geq 0, u'(0)/c'(0) = \infty$, and q^* solves $u'(q^*) = c'(q^*)$. Let $\beta = 1/(1+r)$ be the discount rate. Let V_b and V_s be the value functions for buyers and sellers. Given that sellers are willing to trade goods for money, which we check below, these satisfy the Bellman equations:

$$(1+r)V_b = -k(\alpha_b) + \tau V_s + \alpha_b[u(q) + V_s] + (1 - \tau - \alpha_b)\, V_b, \qquad \textbf{(1)}$$

$$(1+r)V_s = \frac{\tau M}{1-M} V_b + \alpha_s[-c(q) + V_b] + \left(1 - \frac{\tau M}{1-M} - \alpha_s\right) V_s. \qquad \textbf{(2)}$$

In (1), e.g., τ is the probability of a buyer's money being taxed away, α_b is the probability of a trade meeting, and $1 - \tau - \alpha_b$ is the probability of neither event.[7]

As we said, for now we take $q = 1$ as fixed, as in first-generation money-search models, and write $u = u(1)$ and $c = c(1)$, assuming $c < u$. Also, for now we ignore the constraint $\alpha_b \leq 1 - \tau$, and return to it later. Then the necessary and sufficient FOC for α_b is

$$k'(\alpha_b) = u + V_s - V_b. \qquad \textbf{(3)}$$

Solving (1) and (2) for V_s and V_b, and inserting these plus $\alpha_s = \alpha_b M/(1-M)$ into (3), we can reduce this to

$$T(\alpha_b) = [r(1-M) + \tau + M\alpha_b]\, u - M\alpha_b c + (1-M)k(\alpha_b)$$
$$- [r(1-M) + \tau + \alpha_b]\, k'(\alpha_b) = 0. \qquad \textbf{(4)}$$

It is easy to show that $T(0) > 0$ and $T(\bar{\alpha}_b) < 0$, where $\bar{\alpha}_b = (1-M)x$ is the natural upper bound, assuming $k'(\bar{\alpha}_b) = \infty$. Hence, there exists $\alpha_b^e \in (0, \bar{a}_b)$ with $T(\alpha_b^e) = 0$. Although T is not monotone, in general, a sufficient condition for uniqueness is $k''' > 0$, because this makes T concave. To show that α_b^e is an equilibrium, we have only to check that sellers want to trade, $c \leq V_b - V_s$, which holds iff

$$(1-M)\alpha_b u - [(r + \alpha_b)(1-M) + \tau]\, c - (1-M)k(\alpha_b) \geq 0. \qquad \textbf{(5)}$$

Assuming this holds with strict inequality at $\tau = 0$ (see below), monetary equilibrium exists for all $\tau \leq \bar{\tau}$, where $\bar{\tau} > 0$ satisfies (5) at equality. In terms of the effects of policy, given that equilibrium is unique, the key result in Li follows immediately: $\partial\alpha_b^e/\partial\tau > 0$. Thus, a higher tax rate (read higher inflation) increases search intensity α_b^e, and hence velocity v.

In terms of optimality, in this model, average welfare $MV_b + (1-M)V_s$ is proportional to $\alpha_b(u-c) - k(\alpha_b)$. Hence the optimal α_b^* satisfies $k'(\alpha_b^*) = u - c$. Comparing this with equilibrium condition (3), $\alpha_b^e = \alpha_b^*$ iff $c = V_b - V_s$. Hence, the optimal τ is the maximum feasible $\bar{\tau}$, which implies that sellers get no gains from trade. This is a version of the standard Hosios (1990) condition saying, in this case, that buyers should get all surplus because they make all the investment in search effort. To put it another way, buyers equate the marginal cost of search to their private benefit, but unless they get all the gains from trade, sellers also get some benefit that is not internalized.

Also, given that $\tau = \bar{\tau}$ implies $\alpha_b^e = \alpha_b^*$, we can rearrange (5) at equality as

$$\bar{\tau} = \frac{1-M}{c}[\alpha_b^*(u-c) - k(\alpha_b^*) - rc], \tag{6}$$

where α_b^* is given by $k'(\alpha_b^*) = u - c$. Hence, $\bar{\tau} > 0$ iff $rc < \alpha_b^*(u-c) - k(\alpha_b^*)$. Finally, up to now we ignored the constraint $\alpha_b \leq 1 - \tau$. Because α_b^e is increasing in τ, with $\alpha_b^e = \alpha_b^*$ at $\tau = \bar{\tau}$, this will be valid in all equilibria as long as $\alpha_b^* \leq 1 - \bar{\tau}$. In conclusion, in this model, monetary equilibrium exists iff $\tau \leq \bar{\tau}$; α_b^e and v are increasing in τ; and the optimal policy $\tau = \bar{\tau}$ maximizes α_b^e and v. The main substantive result is that buyers spend their money faster when the inflation-like tax increases.[8]

We want to know if these substantive results are robust, and how they generalize. There are several directions one could go in this endeavor, and obviously allowing the terms of trade to be something other than a one-for-one swap of money for goods is desirable. Ultimately we want to consider the most recent search-based models where goods and money are divisible. There are several versions one could use, including those that build on Shi (1997), Green and Zhou (1998), or Molico (2006). We will use the model in Rocheteau and Wright (2005), which has the convenient feature that there are always two types of agents in the economy, buyers and sellers, which correspond well to the two types in the Li model. Before we go to the case where goods and money are divisible, however, it is useful to consider the case where they are indivisible but we incorporate some other elements of the setup to be analyzed below.[9]

An important element of the models below is an alternating market structure: each period there convene a decentralized market, DM, like the one analyzed above, and a centralized market, CM, without frictions. The population is again [0, 1], but it now consists of two permanently different types, called buyers and sellers. Assume the measure of buyers is N, with $N > M$, so that money is scarce. Types are defined as follows: buyers always want to consume but cannot produce in the DM; sellers can always produce but do not want to consume in the DM. One cannot have two such types in models with only a DM, because sellers will not produce for money if they never get to be buyers in some future DM. But in this model, sellers may value money even if they never get to be buyers in a future DM because they can spend it in the CM.

Let W_m^b and V_m^b be the value functions for buyers in the CM and DM, respectively, where $m \in \{0, 1\}$ indicates whether they have money or not. For sellers, replace the superscript b by s. In the CM, all agents trade money, labor, and a consumption good X different from the goods traded in the DM. Given a production function $x = H$, the real wage is 1, and we denote by ϕ the price of m in terms of X. Assuming there is discounting between the CM and DM, but not between the DM

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and CM, for an agent of type $j \in \{b, s\}$ we have

$$W_m^j = \max_{X,H,\hat{m}} \left\{U(X) - H + \beta V_{\hat{m}}^j\right\}$$

$$\text{s.t. } X = H + \phi(m - \hat{m}), \ \hat{m} \in \{0, 1\},$$

where $U(X)$ is a utility function satisfying the usual assumptions, and utility over H is linear.[10]

As is standard, it is easy to see that the choices of X and $\hat{m}$ are independent of m, that $X = X^*$, where $U'(X^*) = 1$, and that $W_1^b - W_0^b = \phi$. In terms of $\hat{m}$, it should be obvious that sellers have no incentive to take money out of the CM, so they set $\hat{m} = 0$. Indeed, sellers are somewhat passive in this model, and the only thing we have to check (as in the previous model) is that they are actually willing to produce the indivisible DM good for a unit of money; this requires $c \leq \phi$. For buyers, because $M < N$, in equilibrium some take $\hat{m} = 1$ out of the CM and others take $\hat{m} = 0$; this requires that they are indifferent between the two options,

$$\phi = \beta\left(V_1^b - V_0^b\right). \tag{7}$$

Given this, and continuing for now to use taxation as in Li's model, the DM value functions for buyers are

$$V_1^b = \tau W_0^b + \alpha_b \left(u + W_0^b\right) + (1 - \alpha_b - \tau)\, W_1^b - k(\alpha_b), \tag{8}$$

$$V_0^b = W_0^b. \tag{9}$$

Subtracting these and using (7), we have

$$\phi = \frac{\beta\,[\alpha_b u - k(\alpha_b)]}{1 - \beta(1 - \alpha_b - \tau)}. \tag{10}$$

Furthermore, any buyer with money in the DM chooses α_b to solve

$$k'(\alpha_b) = u - \phi. \tag{11}$$

Combining (10) and (11), we get the analog of (4) from the previous model:

$$T(\alpha_b) = (1 - \beta + \beta\tau)\, u + \beta k(\alpha_b) - [1 - \beta + \beta(\alpha_b + \tau)]\, k'(\alpha_b) = 0.$$

Again, $T(0) > 0$ and $T(\bar{\alpha}_b) < 0$, where $\bar{\alpha}_b$ is a natural upper bound. Moreover, $T'(\alpha_b) = -[1 - \beta + \beta(\alpha_b + \tau)]k''(\alpha_b) < 0$. Hence, there exists a unique $\alpha_b^e \in (0, \bar{a}_b)$ such that $T(\alpha_b^e) = 0$. It only remains to check the participation condition $c \leq \phi$ for sellers at the equilibrium value of ϕ, which holds iff

$$\beta\,[\alpha_b u - k(\alpha_b)] - [1 - \beta(1 - \alpha_b - \tau)]\, c \geq 0. \tag{12}$$

Monetary equilibrium exists for all $\tau \leq \bar{\tau}$, where $\bar{\tau} > 0$ satisfies (12) at equality.[11]

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We can easily differentiate to get $\partial \alpha_b^e / \partial \tau > 0$, so that a higher tax rate (read higher inflation) increases search intensity. Velocity is slightly more complicated here, because of the two-sector structure. Nominal spending is $PY = PX^* + M\alpha_b$, where $P = 1/\phi$ is the nominal price level, the first term is CM spending, and the second is DM spending. Hence,

$$v = \frac{X^*}{M\phi} + \alpha_b = \frac{X^*}{M\left[u - k'(\alpha_b)\right]} + \alpha_b, \tag{13}$$

by virtue of (11). Therefore $\partial v / \partial \tau > 0$ iff $\partial \alpha_b / \partial \tau > 0$, and so v also increases with τ. In terms of optimality, as before, $\alpha_b^e = \alpha_b^*$ iff sellers get no surplus, which here means $c = \phi$. Again, the optimal policy is the maximum feasible tax $\bar{\tau}$. This model with an alternating CM–DM structure therefore delivers the same basic results as the Li model. Hence, it is a good framework to use when we relax the assumption of indivisible money.

4. DIVISIBLE MONEY

It is desirable to allow $m \in \mathbf{R}_+$, not only because $m \in \{0, 1\}$ is restrictive in a descriptive sense, but also because we can then determine the terms of trade in a nontrivial way, and we can analyze inflation directly instead of proxying for it with taxation. Although we ultimately allow both goods and money to be divisible, it facilitates the presentation to start with the case where the DM $q = 1$ is still indivisible but $m \in \mathbf{R}_+$. An additional virtue of divisible money is that now we can endogenize search on the extensive margin—determining the number of buyers who go to the DM—whereas with $m \in \{0, 1\}$ this was pinned down by the exogenous M.

Now assume that aggregate money supply grows as $\hat{M} = (1 + \gamma)M$. In steady state with real balances ϕM constant, the gross inflation rate is $\hat{P}/P = \phi/\hat{\phi} = \hat{M}/M = 1 + \gamma$. Let $z = \phi m$ denote the real balances that an agent brings to the CM, and $\hat{z} = \hat{\phi}\hat{m}$ be the amount he or she takes out of this market and into next period's DM. Let $W^j(z)$ and $V^j(z)$ be the CM and DM (time-invariant) value functions, for any $z \in \mathbf{R}_+$. The CM problem becomes

$$W^j(z) = \max_{X,H,\hat{z}} \{U(X) - H + \beta V^j(\hat{z})\}, \tag{14}$$
$$\text{s.t. } X = H + z - (1 + \gamma)\hat{z} + \phi\gamma M, \ \hat{z} \in \mathbf{R}_+,$$

where $\phi\gamma M$ is a lump-sum money transfer. Again $X = X^*$, and now $\partial W^j / \partial z = 1$. Sellers still choose $\hat{z} = 0$, but here the choice of $\hat{z}$ for buyers is slightly more complicated, because we cannot just take the FOC due to the fact that $V^b(\hat{z})$ may not be differentiable. In particular, a seller is willing to trade in the DM iff a buyer's real balances are enough to cover his cost c. Hence, there exists a cutoff z^*, characterized below, such that trade occurs iff $z \geq z^*$.

The DM value function for a buyer is the following: first, if $z \geq z^*$, then

$$V^b(z) = -k(\alpha_b) + W^b(z) + \alpha_b[u + W^b(z-d) - W^b(z)], \quad \textbf{(15)}$$

where d denotes the amount of real balances exchanged, and the term in brackets is his surplus from a DM trade, which reduces to $u - d$ using $\partial W^b/\partial z = 1$. Second, if $z < z^*$, then $V^b(z) = W^b(z)$. The seller's surplus is $-c + d$, and so $z^* = c$. Although there are several ways to determine the terms of trade, in much of this paper we use bargaining. However, in this version of the model, with indivisible goods, because a buyer in the DM cannot pay more than he or she has, he or she can effectively commit to pay not more than z^* by not bringing more, and in this way he or she can capture the entire surplus.[12]

Of course, as always, for this to be an equilibrium a seller has to be willing to trade, but this is true by definition of z^*. Additionally, we now have to check that buyers are willing to participate and bring $\hat{z} = z^*$, rather than $\hat{z} = 0$, because there is an ex ante cost of participating in the DM, which is the cost of acquiring the real balances in the previous CM. It is a matter of algebra to check that they are willing to bring z^* iff $\alpha_b(u-c) - ic - k(\alpha_b) \geq 0$, where i (the nominal interest rate) satisfies $1 + i = (1+\gamma)/\beta$, and α_b is the choice of search intensity given by $k'(\alpha_b) = u - c$. Note that $\alpha_b = \alpha_b^*$ does not depend on i, because the cost of bringing money to the DM in the first place is sunk when buyers choose α_b. In any case, we have the result that a monetary equilibrium exists iff $i \leq \bar{\imath}$, where $\bar{\imath} = [\alpha_b^*(u-c) - k(\alpha_b^*)]/c$.

Interestingly, in this model, from $k'(\alpha_b) = u - c$ we immediately get $\partial\alpha_b^*/\partial i = 0$; inflation has *no* effect on equilibrium search. It also has no effect on velocity, which turns out to be $v = X^*/Nc + \alpha_b^*$, where N is the measure of buyers. This is an artifact of indivisible goods, however, as we will soon see. Before getting into that analysis, we can preview the results to come along the extensive margin. Suppose instead of a search cost $k(a_b)$ we assume buyers have to pay a fixed cost k_b to participate in the DM, but once they are in, α_b is not their choice. Let σ_b be the fraction of buyers who choose to participate (we assume the number of participants is less than the total number of buyers N). Because sellers get in for free, all $1 - N$ of them participate. In equilibrium, assuming again an interior solution, the buyers' participation decision implies that $\alpha_b(u-c) - ic = k_b$.[13] Hence, $\partial\alpha_b^*/\partial i > 0$, so more inflation increases the probability of a trade meeting for buyers. Velocity in this case is given by $v = X^*/\sigma_b c + \alpha_b^*$. As σ_b is decreasing in α_b, more inflation also increases v.

So the extensive margin looks promising, and to analyze this in detail, we now move to the case of divisible DM goods. The CM problem is still given by (14), except that now $V^j(\hat{z})$ is differentiable, given the way we determine the terms of trade using bargaining. That is, in the DM, the pair (q, d) is now determined by generalized Nash bargaining, with threat points equal to continuation values and bargaining power for the buyer denoted θ. The key difference is that now by bringing more $\hat{z}$ the buyer can get more q. One can show, exactly as in Lagos and

Wright (2005), that in any equilibrium, if buyers bring $\hat{z}$ then $d = \hat{z}$ and q solves $g(q) = \hat{z}$, where[14]

$$g(q) \equiv \frac{\theta c(q)u'(q) + (1-\theta)u(q)c'(q)}{\theta u'(q) + (1-\theta)c'(q)}. \tag{16}$$

This implies that $\partial q/\partial \hat{z} = 1/g'(q) > 0$; again, bringing more money implies geting more stuff, unlike the case of indivisible goods.

Given all of this, we have

$$V^b(\hat{z}) = -k(\alpha_b) + W^b(\hat{z}) + \alpha_b[u(q) - \hat{z}],$$

where for now we return to the intensive margin, with α_b an individual choice. Thus, it satisfies

$$k'(\alpha_b) = u(q) - g(q), \tag{17}$$

after the bargaining solution $\hat{z} = g(q)$ is inserted. To determine $\hat{z}$ and hence $q = g^{-1}(\hat{z})$, consider the FOC for $\hat{z}$ for buyers in the CM:

$$1 + \gamma = \beta \frac{\partial V^b}{\partial \hat{z}} = \beta \left[\alpha_b \frac{u'(q)}{g'(q)} + 1 - \alpha_b \right].$$

Using $1 + i = (1+\gamma)/\beta$, we reduce this to

$$\frac{i}{\alpha_b} = \frac{u'(q)}{g'(q)} - 1. \tag{18}$$

Equilibrium now is a pair (q, α_b) solving (17) and (18).

Several remarks can be made about this model. For example, setting $\theta = 1$ implies that $g(q) = c(q)$ and then (17) guarantees that search effort is efficient; this is again the Hosios condition. Given $\theta = 1$, one can show that $q < q^*$ for all $i > 0$, where q^* is the efficient q, and $q = q^*$ iff we follow the Friedman rule $i = 0$. Hence the Hosios condition and the Friedman rule in combination define the efficient α_b^* and q^*. In any case, (17) and (18) define two curves in (q, α_b) space that we call the SE and BS (for search effort and bargaining solution). As shown in Figure 6, both curves start at (0, 0); SE increases as q increases from 0 to q^* and then decreases to $\alpha_b = 0$ when $q = \hat{q}$, where $\hat{q} > 0$ solves $u(\hat{q}) = c(\hat{q})$; BS increases to $(\tilde{q}, 1)$, where $\tilde{q} \in (0, q^*]$ solves $u'(\tilde{q}) = (1+i)g'(\tilde{q})$. They could potentially intersect at multiple points, but it is easy to check that the SOC for the buyer's choice of q and α only holds when BS intersects SE from below.

When we increase the inflation rate γ or equivalently the nominal interest rate i, BS rotates up, which means that at any point where BS intersects SE from below q and α_b both fall. More formally, differentiate (17) and (18) to get

$$\frac{\partial q}{\partial i} = -\frac{k''}{D} \text{ and } \frac{\partial \alpha_b}{\partial i} = -\frac{u' - g'}{D},$$

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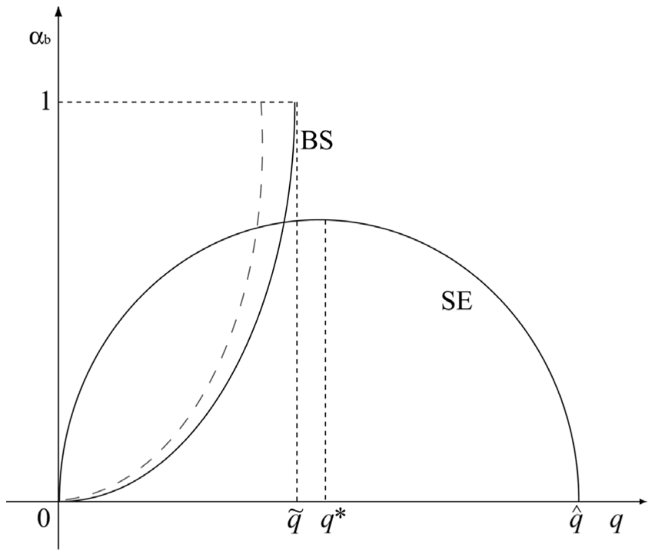

FIGURE 6. Equilibrium under endogenous search intensity.

where $D = -\alpha_b \ell' k'' - (u' - g')(\ell - 1)$, with $\ell = \ell(q) \equiv u'(q)/g'(q)$. The SOC is $D > 0$, and because $u' > g'$ for all $i > 0$ by (18), we conclude that q and α_b fall with i. This is the result in Lagos and Rocheteau (2005): inflation makes buyers spend their money less and not more quickly, because it reduces the buyers' surplus, which makes them less willing to invest in costly search.

At this point we move to study the extensive rather than the intensive margin of search—i.e., instead of search intensity, we return to a free entry decision by buyers.[15] To this end, we now assume a standard matching function $n = n(\sigma_b, \sigma_s)$, where n is the number of trade meetings, and now we interpret σ_b and σ_s as the measures of buyers and sellers in the DM (and not the measures in the total population, as some may not go to the DM). An individual's probability of a trade meeting is $\alpha_j = n(\sigma_b, \sigma_s)/\sigma_j$ for $j = b, s$. Assume n is twice continuously differentiable, homogeneous of degree one, strictly increasing, and strictly concave. Also, $n(\sigma_b, \sigma_s) \leq \min(\sigma_b, \sigma_s)$ and $n(0, \sigma_s) = n(\sigma_b, 0) = 0$. Define the buyer–seller ratio, or market tightness, by $\delta = \sigma_b/\sigma_s$. Then $\alpha_b = n(1, 1/\delta)$, $\alpha_s = n(\delta, 1)$ and $\alpha_s = \delta\alpha_b$. Also, $\lim_{\delta\to\infty} \alpha_b = 0$ and $\lim_{\delta\to 0} \alpha_b = 1$.

Participation decisions are made by buyers, who have to pay a fixed cost k_b to enter, whereas sellers get in for free and so all of them participate. We focus on the situation where the total measure of buyers N is sufficiently large that some but not all go to the DM, which means that in equilibrium they are indifferent. Of course this means buyers get zero expected surplus from participating in the DM, although those who actually trade do realize a positive surplus [just like the firms in Pissarides (2000)]. If one does not like this, it is easy enough to assume that all buyers draw a participation cost at random from some distribution $F(k)$ each

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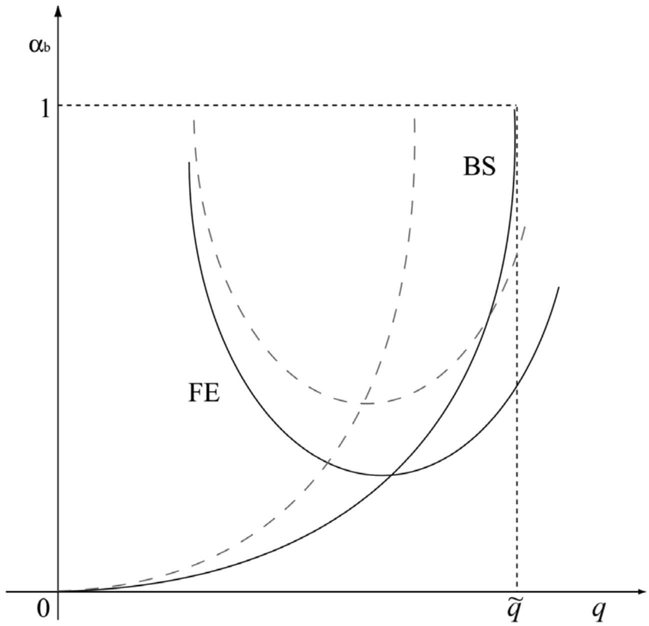

FIGURE 7. Equilibrium under free entry by buyers.

period. Then instead of all buyers being indifferent, there will be a marginal buyer with cost k^* who is indifferent about going to the DM, whereas all buyers with $k < k^*$ strictly prefer to go because they get a strictly positive expected surplus. Given that this is understood, for ease of presentation we focus on the case where k is the same for all buyers.

For a buyer who does not go to the DM, $X = X^*$ and $\hat{z} = 0$. For one who does, he pays cost k_b next period, but he has to acquire $\hat{z}$ in the current CM. Algebra implies that he wants to go iff $-(1+\gamma)\hat{z} + \beta\left[-k_b + \alpha_b u(q) + (1-\alpha_b)\hat{z}\right] \geq 0$. Using (16) and inserting the nominal rate i, this can be written as

$$-ig(q) - k_b + \alpha_b\left[u(q) - g(q)\right] \geq 0. \quad \textbf{(19)}$$

There are two costs of participating in the DM: the entry cost k_b and the cost of bringing real balances $ig(q)$. The benefit is α_b times the surplus. In equilibrium, (19) holds at equality:

$$\alpha_b = \frac{ig(q) + k_b}{u(q) - g(q)}. \quad \textbf{(20)}$$

Given q, this determines α_b. Then one gets the measure of buyers σ_b from $\alpha_b = n(1, \sigma_s/\sigma_b)$, with $\sigma_s = 1 - N$. A monetary equilibrium is a solution (q, α_b) to the free entry and bargaining conditions (20) and (18), defining the FE and BS curves in Figure 7.

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Restricting attention to the relevant region of (q, α_b) space, $(0, \tilde{q}) \times [0, 1]$, it is routine to verify the following: the curves are continuous, and BS is upward-sloping and goes through $(0, 0)$, whereas FE is downward (upward)-sloping to the left (right) of the BS curve, hitting a minimum where the curves cross.[16] Hence, there is a unique equilibrium, and at equilibrium we have $\partial\alpha_b/\partial i > 0$ and $\partial q/\partial i < 0$. To see this, note that as i increases the BS and FE curves both shift up, so α_b increases. To see what happens to q, rewrite the model as two equations in q and α_b/i by dividing (20) by i. This new version of FE satisfies the same properties as before: it is downward (upward)-sloping to the left (right) of the BS curve. But now as i increases the FE curve shifts down, whereas the BS curve does not shift (q as a function of α_b/i does not change when i changes). Hence q falls.

Now consider $v = Y/\phi M$. Total real output is $Y = Y_C + Y_D$. Real CM output is $Y_C = X^*$ as always, and real DM output is $Y_D = n(\sigma_b, \sigma_s)\phi M/\sigma_b = \alpha_b \phi M$, because M/σ_b is total cash per buyer participating in this market. Thus,

$$v = \frac{X^* + \alpha_b \phi M}{\phi M} = \frac{X^*}{\sigma_b g(q)} + \alpha_b,$$

using $M = \sigma_b g(q)/\phi$. Because $\partial\alpha_b/\partial i > 0$, we have $\partial\sigma_b/\partial i < 0$, and we already know that $\partial q/\partial i < 0$. Therefore we conclude that $\partial v/\partial i > 0$. Hence, this model unambiguously predicts that velocity increases with i. And, again, it predicts the "hot potato" effect $\partial\alpha_b/\partial i > 0$, for the following intuitively plausible reason: an increase in inflation or nominal interest rates must lead to buyers spending their money more quickly, because this is the only way to satisfy the free entry condition.

These results are natural, and they are quite robust, at least as long as we maintain the assumption that buyers are the ones who face a DM participation choice.[17] They are robust in the sense that in our baseline model the results do not depend on parameter values. They also do not depend much on the pricing mechanism. The same qualitative results hold with proportional rather than Nash bargaining [as used in money models by Aruoba et al. (2009)], and with Walrasian price taking [as used by Rocheteau and Wright (2005)]. We also tried price posting with directed search. Recall that Lagos and Rocheteau (2005) could get agents to spend their money faster under this pricing mechanism for some parameter values in their intensive-margin model. In our extensive-margin model, under price posting and directed search, agents might or might not spend their money faster with inflation, depending on parameters. So in both models, the results are ambiguous under price posting and directed search. But Lagos and Rocheteau can *only* get the desired "hot potato" effect for very low inflation; we get it for all parameters *except possibly* very low inflation.

Finally, we analyze welfare, which was part of our original motivation for this study. As in most related models, the Friedman rule $i = 0$ plus the Hosios condition $\theta = 1$ is necessary and sufficient for $q = q^*$. But given q^*, we may not get efficiency in terms of entry, because there is a search externality at work: participation by buyers increases the arrival rate for sellers and decreases it for other buyers. As is often the case in models with entry, there is a Hosios condition for efficient participation, which sets the elasticity of the matching function with respect to the number of buyers equal to their bargaining power θ. But this conflicts in general with the condition $\theta = 1$ required for $q = q^*$.

Formally, the planner's problem is to choose sequences for $\{\sigma_{bt}, q_t^b, q_t^s, X_t, H_t\}$ to maximize

$$\sum_{t=0}^{\infty} \beta^t \left\{ n(\sigma_{bt}, \sigma_s) \left[u\left(q_t^b\right) - c\left(q_t^s\right) \right] - \sigma_{bt} k_b + U(X_t) - H_t \right\},$$

subject to $q_t^b \leq q_t^s$ and $X_t \leq H_t$. Optimality requires for all t

$$\frac{u'(q)}{c'(q)} = 1, \tag{21}$$

$$n_1(\sigma_b, \sigma_s)\,[u(q) - c(q)] = k_b. \tag{22}$$

We want to compare this with the equilibrium conditions under Nash bargaining, which we repeat here for convenience:

$$\frac{u'(q)}{g'(q)} = 1 + \frac{i}{\alpha_b}, \tag{23}$$

$$\alpha_b\,[u(q) - g(q)] = i g(q) + k_b. \tag{24}$$

Clearly $i = 0$ and $\theta = 1$ achieve $q = q^*$, and given this, entry is efficient iff $n_1(\sigma_b, \sigma_s) = \alpha_b$, which is equivalent to saying the elasticity of the matching function with respect to σ_b is 1.

In general, we do not get efficiency if this elasticity is not 1. For instance, if the matching function is Cobb–Douglas we can assign any value $\eta \in (0, 1)$ to this elasticity. With $\eta < 1$ and $\theta = 1$, the number of buyers in the DM is necessarily too high. An important implication is that for a given θ, and especially for a relatively high θ, the Friedman rule $i = 0$ may not be optimal. When the number of buyers is too high, a small increase in the nominal rate from $i = 0$ entails a welfare cost, because it reduces q, but it also brings the number of buyers closer to the efficient level. When $\theta = 1$, for i near 0, the welfare consequence of the effect on q is of second order because q is near q^*, and therefore the net gain is positive and the optimal policy is $i > 0$. It is not hard to construct explicit examples to this effect.[18]

We are not ready to take a stand on the definitive quantitative analysis in this paper, but for the sake of illustration, consider the following. Assume the relatively

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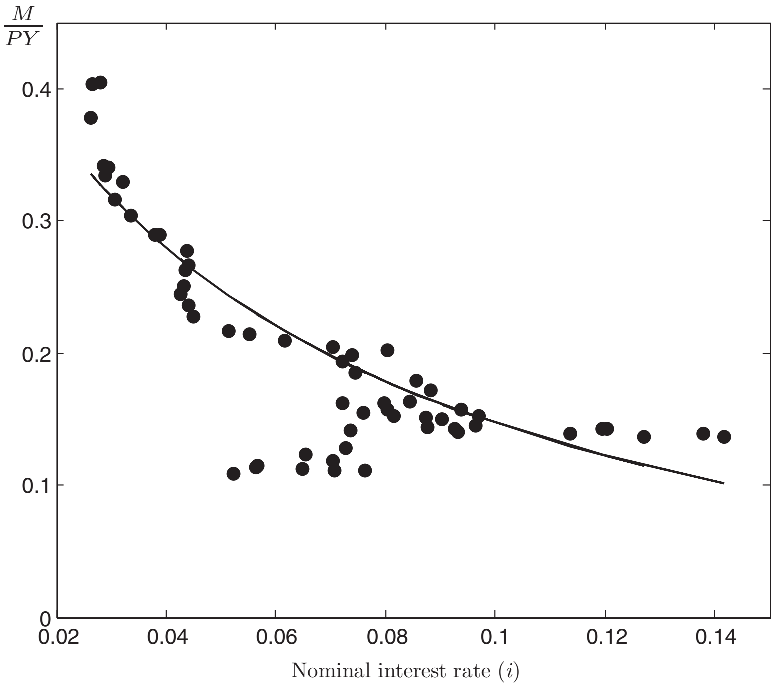

FIGURE 8. Money demand: model and data.

standard functional forms:

DM utility:	$u(q) = \frac{(q+b)^{(1-\mu)} - b^{(1-\mu)}}{1-\mu}$,
DM cost of production:	$c(q) = q$,
CM utility:	$U(X) = A \ln X - H$,
Matching function (Kiyotaki–Wright):	$n(\sigma_b, \sigma_s) = \frac{\sigma_b \sigma_s}{\sigma_b + \sigma_s}$.

Set $\beta = 1/1.03$ and $b = 0.0001$, and normalize $\sigma_s = 1$. Then calibrate the remaining parameters as follows. Set A to match average M/PY, and μ to match the interest elasticity of M/PY, in the annual U.S. data (1948–2005), as shown by the model's implied "money demand" curve in Figure 8. Finally, set entry cost k so that the DM contributes 10% to aggregate output, and θ so that the DM markup is 30%.[19]

The welfare cost of inflation is measured using the standard method: we ask how much total consumption agents would be willing to give up to reduce inflation to the Friedman rule. In general, bargaining power θ plays a key role in these calculations. Figure 9 shows the cost of inflation as it ranges up to 20%, under different values of θ. For our benchmark value of $\theta = 0.671$, optimal inflation is above the Friedman rule, but still negative. At the optimal policy, with $\theta = 0.671$,

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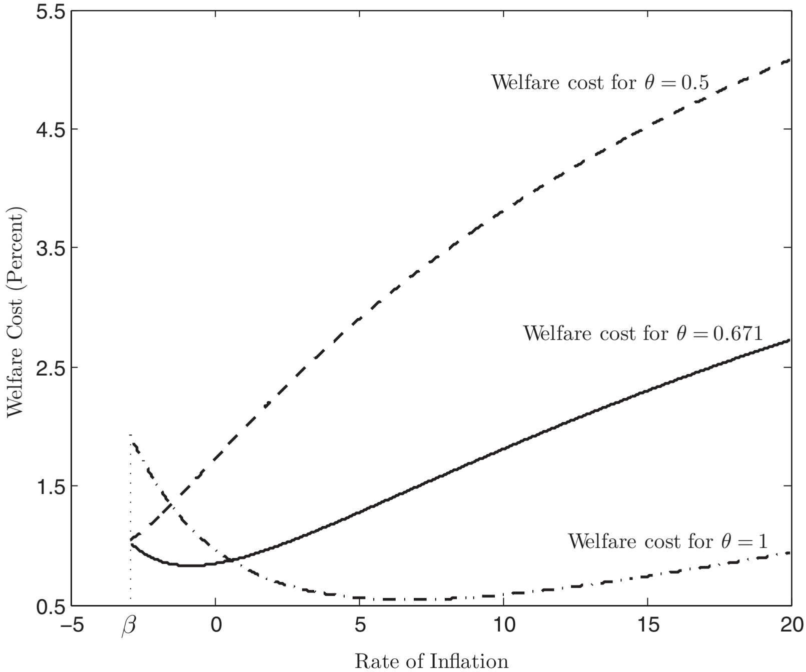

FIGURE 9. Welfare cost of inflation under diffrent values of θ.

welfare in consumption units is 0.2% above what it would be at the Friedman rule. Also shown is the case $\theta = 1$, where optimal inflation is well over 5%, and at the optimal policy welfare is nearly 2% above what it would be at the Friedman rule. And finally, for $\theta = 1/2$, the Friedman rule is optimal—positive nominal interest rates are not always efficient, but they could be. Again, these results are not meant to be definitive, and are certainly sensitive to parameter values, but they clearly indicate to us that extensive-margin models are worth further study.

5. CONCLUSION

We studied the relationship between inflation or nominal interest rates, on the one hand, and the speed with which agents spend their money, on the other. We are mainly interested in what we call the "hot potato" effect: when inflation increases, people spend their money faster. We also discussed the effects of inflation or interest rates on velocity and on welfare. We first presented some evidence on velocity, showing that it tends to increase with inflation and nominal interest rates. We then discussed theory. With indivisible money and goods, as in Li (1994,1995), we generate a positive relationship between the variables in question: with higher inflation or interest rates, people spend their money faster, because they increase search intensity. But this is an artifact of indivisibilities. To emphasize this, we

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rederived results from Lagos and Rocheteau (2005), in a slightly different model, showing that with divisible goods and money search, intensity decreases with inflation.

Then we changed the framework by focusing on the extensive rather than the intensive search margin—i.e., on how many buyers are searching, rather than on what any given buyer is doing. This is the main contribution of the paper. Now the model unambiguously predicts that a rise in inflation leads to an increase in the speed with which agents spend their money, which is the "hot potato" effect we set out to capture. Although undoubtedly both margins can be relevant, in reality, we think focusing on the extensive margin is interesting for a variety of reasons, including the implications for welfare. In particular, it is not hard to get inflation above the Friedman rule to be the optimal monetary policy in this framework. Also, although we do not have direct evidence on this, the predictions of the model are consistent with the casual empirical observation that people are less inclined to participate in cash-intensive market activity during periods of higher inflation.

In terms of methodology, we also think the exercise makes the following useful point. Many times, when one strives to do monetary economics with relatively explicit microfoundations, one hears the following critical question: "Why did we need a search or matching model, when similar insights could be developed and similar predictions derived with a reduced-form model, say one that simply assumes money in the utility function or imposes cash in advance?" Well, in this paper, the issues are all about search and matching. We are interested in the speed with which buyers spend their money. The relevant arrival rates are determined either on the intensive margin using search intensity, or on the extensive margin using a matching function and endogenous participation. It is not only for aesthetic reasons that one might like search-and-matching theory; it is exactly the right tool for the job in many applications, including the one under consideration here.

NOTES

1. This is reminiscent of Gresham's law: good money drives out bad money when prices are fixed, but not necessarily when they are flexible. See Friedman and Schwartz (1963, fn. 27) for a discussion and Burdett et al. (2001, Section 5) for a theoretical analysis of this idea.

2. This is reminiscent of the model of middlemen of Rubinstein and Wolinsky (1987), where there are gains from trade between sellers and middlemen because the latter meet buyers more quickly than the former meet buyers.

3. Although it would be nice to have direct evidence on the speed with which agents spend their money, we do not; hence we look at velocity.

4. Instead of the Aaa corporate rate for Canada we use the Prime Corporate Paper rate, which is a weighted average of rates posted for 90-day paper by major participants in the Canadian market.

5. Scatterplots for the Canadian data look similar and are omitted.

6. This is how search is assumed to operate in Li (1994, 1995). Here is a physical environment consistent with the specification. There are some number of agents N_A and locations $N_L > N_A$. Each period a seller occupies a location. Then each buyer samples a location, in a *coordinated* manner—say, they sample sequentially, and no one samples the same location as a previous buyer (to avoid the coordination friction emphasized in the directed search literature). The number of sellers is $N_A(1-M)$, and each produces a buyer's desired good with probability x. Hence, the buyer's probability of a trade

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meeting is $\alpha_b = \alpha(1 - M)x$ with $\alpha = N_A/N_L$. The key to this specification is this: when a buyer chooses a search effort, it affects his or her probability α_b, but not that of other buyers, although it does affect α_s for sellers. Lagos and Rocheteau (2005) use a different setup, starting with an underlying matching technology giving the number of meetings as a function of total search effort by buyers and the number of sellers, $n(M\bar{e}, 1 - M)$, where $\bar{e}$ is average buyer effort. The probability that a given buyer meets a seller is $en(M\bar{e}, 1 - M)/\bar{e}M$, where e is his or her own effort. In this setup a buyer's search effort affects this probability for other buyers. This complicates the analysis but does not affect the results. In any case, we return to general matching functions below.

7. We assume that payoffs $-k(\alpha_b)$, $u(q)$, and $c(q)$ are all received next period, which is why the value functions V_b and V_s discount everything on the right; this affects nothing of substance, but makes for an easier comparison to models with divisible money. Also, as we only consider steady states, value functions are always time-invariant.

8. In terms of technical details, notice that for τ near the optimum $\bar{\tau}$ we have $T' < 0$, and hence uniqueness follows even without the restriction $k''' > 0$. And of course we know that $\alpha_b^e < \alpha_b^*$ for all $\tau < \bar{\tau}$ in any equilibrium, even if we have multiple equilibria. Finally, all this takes M as given, but it is a simple exercise to optimize over M as well as τ.

9. A different approach is to keep $m \in \{0, 1\}$, but make q divisible, determined using bargaining as in Shi (1995), Trejos and Wright (1995), or Rupert et al. (2001). Assuming buyers make take-it-or-leave-it offers, to reduce the algebra, bargaining implies

$$c(q) = \frac{\alpha_b u(q) - k(\alpha_b)}{r + \alpha_b + \tau/(1 - M)}.$$

Equilibrium is a pair (q, α_b) solving this plus the FOC for effort, $k'(\alpha_b) = u(q) - c(q)$. The first relation defines a curve in (q, α_b) space we call BS; it looks like a loop starting at (0, 0) because for small α_b there are two solutions, say q^H and q^L, and for large α there are none. The FOC defines a curve we call SE; it is strictly concave, goes through (0, 0), and is maximized at $q = q^*$, where $u'(q^*) = c'(q^*)$. One can show that BS and SE cross where BS is vertical in (q, α_b) space, and an increase in τ shifts BS left along the SE curve. Because it is not clear that there is a unique intersection (although this seems to be true in examples), consider the equilibrium with the highest q. Then an increase in τ reduces q, but whether this reduces α_b depends on whether q is above or below q^*, so the results are ambiguous in general. As we show, this is an artifact of indivisible m. And even with indivisible m, one could say it is an artifact of not allowing *lotteries*, as in Berentsen et al. (2002), because in that model we never get $q > q^*$.

10. Quasi-linear utility is necessary to keep the model tractable once we allow divisible money, but it is easy to generalize many other elements of the model (b and s can have different CM utility functions U^b and U^s, we can have firms in the CM with nonlinear production functions, and so on).

11. As in the preceding section, this assumes $\alpha_b < 1 - \bar{\tau}$, which is valid if $c \geq \beta[\alpha_b u - k(\alpha_b)]$.

12. We do not dwell on this issue, because we soon move to models with divisible goods and money. See Jean et al. (2010) for an extended discussion.

13. This is explained in more detail below for the model with divisible goods (as well as divisible money).

14. The surplus for a buyer is $\Sigma_b = u(q) - \hat{z}$, and the surplus for a seller is $\Sigma_s = \hat{z} - c(q)$. Given that buyers do not bring more money than they spend, insert $d = \hat{z}$ into the generalized Nash product $\Sigma_b^\theta \Sigma_s^{1-\theta}$, take the first-order condition with respect to q, and rearrange to get (16).

15. This is in a sense opposite to the approach in the literature on limited participation in both reduced-form models [e.g., Alvarez et al. (2008) or Khan and Thomas (2007)] and search models [Chiu and Molico (2007)] of money. Those models assume that agents have to pay a cost to access something analogous to our CM, sometimes interpreted as a financial sector.

16. Proof: The properties of BS are obvious. The slope of FE is given by $\partial\alpha_b/\partial q \simeq (u - g)ig' - (ig + k_b)(u' - g')$, where $\simeq$ means "equal in sign." Eliminating k using (20) and simplifying, $\partial\alpha_b/\partial q \simeq i + \alpha_b - \alpha_b u'/g'$. From the CM problem, the derivative of the objective function $1 + \gamma = \beta\partial V^b/\partial\hat{z}$ can be rewritten in terms of q as $-(i + \alpha_b) + \alpha_b u'(q)/g'(q) = -\partial\alpha_b/\partial q$. There is a unique solution

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to this maximization problem; $\partial\alpha_b/\partial q$ is positive (negative) as q is less (greater) than the solution that is given by (18). Hence the FE curve is decreasing (increasing) to the left (right) of the BS curve.

17. For the record, we also studied the model where all buyers enter but sellers have to pay $k_s > 0$, and the model where each agent can choose to be a buyer or seller. In those models, the results are ambiguous, and v can increase or decrease with i in examples.

18. Similar results can be found in Nosal and Rocheteau (2009), although there it is slightly easier, because they assume that buyers who do not participate become sellers in the DM, whereas we assume they simply sit out. Hence, in their model, when the number of buyers is too high, inflation can, by reducing the number of buyers and increasing the number of sellers, actually increase the number of DM trades. For us inflation always reduces the number of DM trades because it decreases the number of buyers and the number of sellers is fixed. But it can still increase welfare.

19. The parameter b is here for purely technical reasons, so that $u(0) = 0$, but is set close to 0 so that DM utility displays approximately constant relative risk aversion. The 10% DM share is targeted so that the results are easily comparable to Lagos and Wright (2005). The 30% DM markup target is discussed in Aruoba et al. (2009). The results of the calibration are $(A, \mu, k, \theta) = (2.709, 0.373, 0.147, 0.671)$.

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