

## **Air exchange through doorways. The effect of temperature difference, turbulence and ventilation flow**

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### SUMMARY

Analytical expressions have been derived for the exchange of air across doorways or similar apertures, in terms of the temperature difference between the spaces on both sides of the opening and the net volume of air flowing through this as a result of unbalanced air supply or extract. A simple allowance for turbulence which gives reasonable correspondence with observation is included. The formulae, which assume complete air mixing on both sides of the doorway up to the plane of the aperture, predict outflows from the warmer side, when there is an excess air supply to this side, which are progressively smaller than those observed as the temperature difference rises above 2–3 °C and the volume of excess air supply increases to produce an averaged outflow velocity greater than 0.1–0.15 m/s. This seems to be due to lack of mixing of the warm outflowing air with the cooler air mass. A correction factor for this can be deduced as a function of the pressure difference due to the excess air supply. The limiting magnitude and general form of this function are compatible with a simple theoretical model of the air flow patterns involved.

### INTRODUCTION

In previous papers in this journal (Foord & Lidwell, 1975*a, b*; Lidwell, 1975) the transfer of air, and the consequent transfer of airborne particulates, from one room to another within a building has been discussed. A mathematical model has been described and compared with experimental observations made within a modern air-conditioned hospital (The Greenwich District General hospital south-east of London). The formulae proposed include terms for the air flows in and out of the rooms involved but, although experimental values were given for these in the particular situation, no theoretical model by which the values might be deduced generally was suggested.

Shaw & Whyte (1974) have reported the results of an extensive series of measurements made in a Glasgow hospital. They were able to vary experimentally the temperature differences between the air within the room studied and the lobby outside. The air supply and extract rates could also be adjusted at will. The same data are given in slightly varied form and, in some respects, in more detail by Whyte & Shaw (1972) and by Shaw (1976).

The observed rates of air exchange across the doorway could, however, only be reconciled with those calculated from the formulae deduced by Shaw (1972) by

the introduction of an arbitrary constant which varied between zero and unity according to the magnitude both of the temperature difference across the doorway and of the net air flow through it. As will be seen below these formulae contain inherent inconsistencies. An alternative model is therefore proposed which gives good agreement with their experimental observations, and with those from the Greenwich hospital.

#### SHAW'S FORMULA

The air flow through the doorway is assumed to be ideal, i.e. wholly non-viscous, so that the Bernoulli equation can be applied without modification. It is also assumed that the air temperature is uniform within each of the spaces separated by the aperture, i.e. that mixing of the air is effectively complete on both sides of the doorway up to the plane of the aperture. The first assumption is probably correct within the accuracy required; the second is manifestly an approximation as is clear from the difficulties, referred to below, in defining the temperature difference across the doorway.

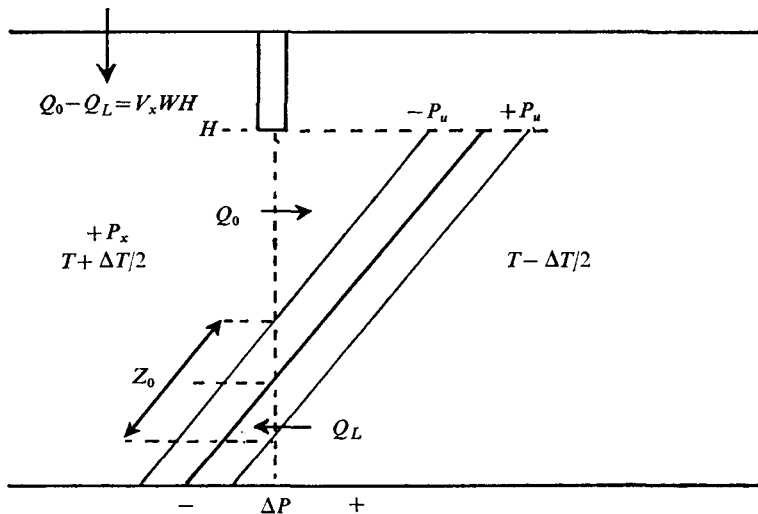


Fig. 1. The distribution of pressure with height in the plane of a doorway. Excess air supply to the left-hand side, which is at a higher temperature than the space to the right-hand, results in a positive pressure excess on the left-hand side and the pressure scale is shown as positive in this sense. The two thinner lines on each side of the central sloping line show the pressure shifts resulting from turbulent effects in the two directions. The symbols used are defined in the text.

Attention was principally directed to the effectiveness of an excess air supply to a room at a higher temperature than the space outside the doorway in reducing or eliminating inflow from this space into the room. The formulae are, however, symmetrical with respect to both the direction of any unbalanced air supply and the sign of the temperature difference across the doorway so that they are equally applicable to all the possible permutations of these factors.

The pressure differences and consequent flows through an open doorway are shown schematically in Fig. 1. By equating the excess pressure,  $P_x$ , within the room required

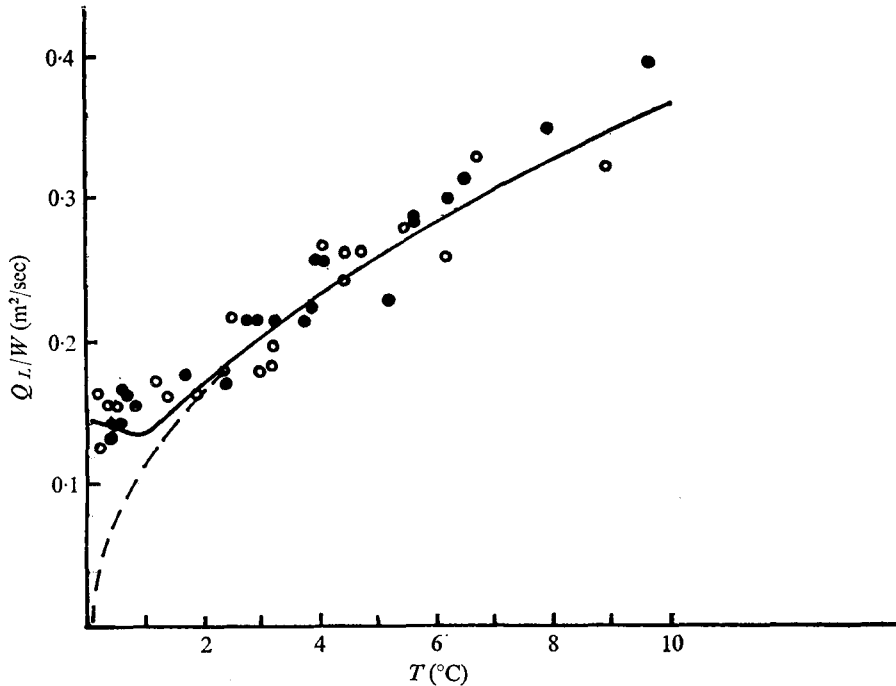


Fig. 2. Air exchange across the doorway as a function of the temperature difference on either side of the aperture. Data from Shaw (1976), (Fig. 4.10; see also Shaw & Whyte (1974; Fig. 9). The open circles are from experiments with door openings 0.9 m wide and the filled circles from openings 0.5 m wide. The broken line shows the exchange calculated from Shaw's formula, equation (1), with no allowance for turbulence, the full line gives the results obtained by the use of equations (8) and (9), which assume turbulent velocities in both directions normal to the door aperture of 0.14 m/s.

to produce a given net outflow with the velocity head corresponding to the mean outflow velocity,  $V_x$ , over the whole of the doorway Shaw obtained an expression for the inflow to the room

$$Q_L = \frac{2}{3} C \cdot \frac{W}{a} \left\{ \frac{aH}{2} - V_x^2 \right\}^{\frac{3}{2}}. \quad (1)$$

The volumetric outflow from the room is similarly given by:

$$Q_0 = \frac{2}{3} C \cdot \frac{W}{a} \left\{ \frac{aH}{2} + V_x^2 \right\}^{\frac{3}{2}}. \quad (2)$$

Where  $W$  is the width of the doorway of height  $H$  and  $a = 2g(\Delta\rho/\rho) = 2g(\Delta T/T)$  with  $g$  the constant of gravity,  $\rho$  the mean density of the air at a mean temperature  $T$ ,  $\Delta\rho$  and  $\Delta T$  the differences in density and temperature of the air on the two sides of the opening and  $C$  the coefficient of discharge. Throughout this paper the symbols, where possible, are those employed by Shaw.

There is some difficulty in practice in deciding on the appropriate value of  $\Delta T$ . Use of the difference between the temperatures at the top and bottom of the opening leads to good agreement with equation (1), when there is no excess air supply to the room, i.e.  $V_x = 0$ , for values of  $\Delta T$  between 3° and 10 °C, with  $C$  approximately

equal to 0.65 (see Fig. 2). If  $\Delta T$  is taken as the difference between the temperatures measured at mid height in the two rooms, at the side of the opening in a position uninfluenced by the air flowing through it, then the agreement is slightly inferior and  $C$  must be taken as approximately equal to 0.80, i.e. the temperature difference at mid height was about  $\frac{2}{3}$  that between the top and bottom of the door opening.

Owing to turbulent air movements the volume of air exchanged across the doorway does not tend to zero with  $\Delta T$  so that equation (1) fails at low values of  $\Delta T$ .

More seriously the net volume exchange calculated from equations (1) and (2) is not equal to  $V_x WH$ . The net air-flow, outward, is given by

$$Q = Q_0 - Q_L = \frac{2}{3} C \frac{W}{a} \left[ \left( \frac{aH}{2} + V_x^2 \right)^{\frac{3}{2}} - \left( \frac{aH}{2} - V_x^2 \right)^{\frac{3}{2}} \right]. \quad (3)$$

This expression can be reduced to:

$$Q = V_x WH \cdot \frac{C}{3\alpha} [(1 + \alpha^2)^{\frac{3}{2}} - (1 - \alpha^2)^{\frac{3}{2}}]$$

by putting

$$V_x = \alpha \left( \frac{aH}{2} \right)^{\frac{1}{2}}.$$

For values of  $\alpha$  between 0 and 1, which correspond to values of  $V_x$  between 0 and that needed to reduce  $Q_L$  to zero,  $Q$  lies between zero and  $0.61 V_x WH$ , for  $C = 0.65$ .

In deducing equation (1) the coefficient of discharge was not applied to the net outflow velocity  $V_x$ . If this is done then  $V_x$  in equation (1) must be replaced by  $V_x/C$ . The calculated net outflow then approaches closer to the true value when  $\alpha$  approaches unity, i.e.  $Q_L = 0$ , the limiting value being  $0.94 V_x WH$ .

This modification, however, does nothing to correct the second serious failing in the formula, namely the unsatisfactory representation given of the form of the relation between  $V_x$ ,  $Q_L$  and  $Q_0$  (see Figs. 3, 4). This is most clearly seen in the initial rate of change of  $Q_L$  as  $V_x$  is increased from zero.

From equation (1),

$$\frac{dQ_L}{dV_x} = -2C \frac{W}{a} \left( \frac{aH}{2} - V_x^2 \right)^{\frac{1}{2}} \cdot V_x,$$

i.e.  $dQ_L/dV_x$  is zero when  $V_x$  is zero, as is also  $dQ_0/dV_x$ . Inspection of the experimental results, however, shows that, at  $V_x = 0$ ,  $dQ_L/dV_x \simeq W$  for the doorways studied.

The variation in both inflow and outflow with increasing rate of air supply to the room are shown in Fig. 3.

#### VELOCITY SUMMATION

In deducing equation (1) the velocity of air flow at any height was derived by addition of the thermal pressure difference at that height to the velocity head attributed to the mean outflow velocity,  $V_x$ . An alternative is to add the velocity derived by the Bernoulli equation from the thermal pressure difference to the mean outflow velocity  $V_x$  itself.

The velocity inwards at height  $Z$  is then given by

$$C \left[ a \left( \frac{H}{2} - z \right) \right]^{\frac{1}{2}} - V_x.$$

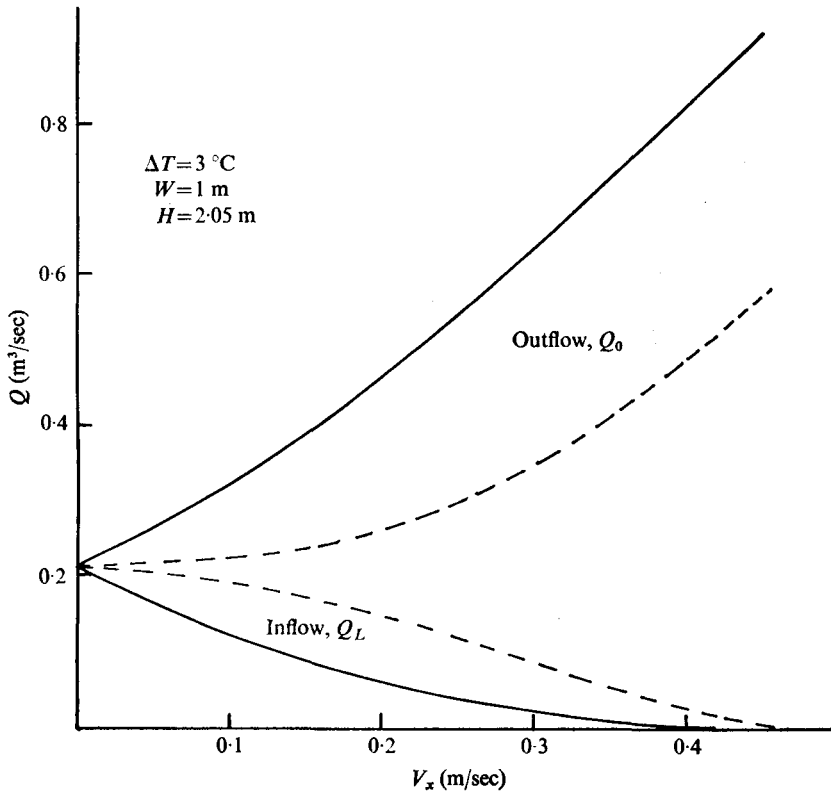


Fig. 3. Inflow and outflow through a doorway with excess air supply. Full lines show the experimentally observed values derived from the data given by Shaw (1976; Fig. 4.15); see also Shaw & Whyte (1974; Fig. 11). The broken lines are those calculated from Shaw's formula, equation (1), without the introduction of his 'coefficient of fictitious velocity'. The limiting value of  $V_x$  needed to reduce  $Q_L$  to zero has been taken from Whyte & Shaw (1972, Fig. 2.8).

from which it may be deduced that

$$Q_L = \frac{2}{3} C \frac{W}{a} \left[ \left( \frac{aH}{2} \right)^{\frac{3}{2}} - \frac{3aH}{4} \cdot \frac{V_x}{C} + \frac{1}{2} \cdot \left( \frac{V_x}{C} \right)^3 \right]. \tag{4}$$

From the method of derivation the net outflow must equal  $V_x WH$  and differentiation of equation (4) leads to

$$\frac{dQ_L}{dV_x} = -\frac{WH}{2}$$

which, for the door height of 2.05 m, is equal to  $-1.025 W$ . The observational data, shown in part in Figs. 4 and 5, lead to a mean value of  $dQ_L/dV_x$ , at  $V_x = 0$  and for  $W = 1$ , of approximately 1.03.

The form of the relation between  $V_x$  and  $Q_L$  calculated from equation (4) is shown in Fig. 4, and is clearly similar to that observed experimentally although it appears to underestimate the magnitude of the average excess outflow velocity,  $V_x$ , required for a given reduction in the rate of air inflow,  $Q_L$ .

An allowance for turbulence can be made by superimposing a turbulent velocity,

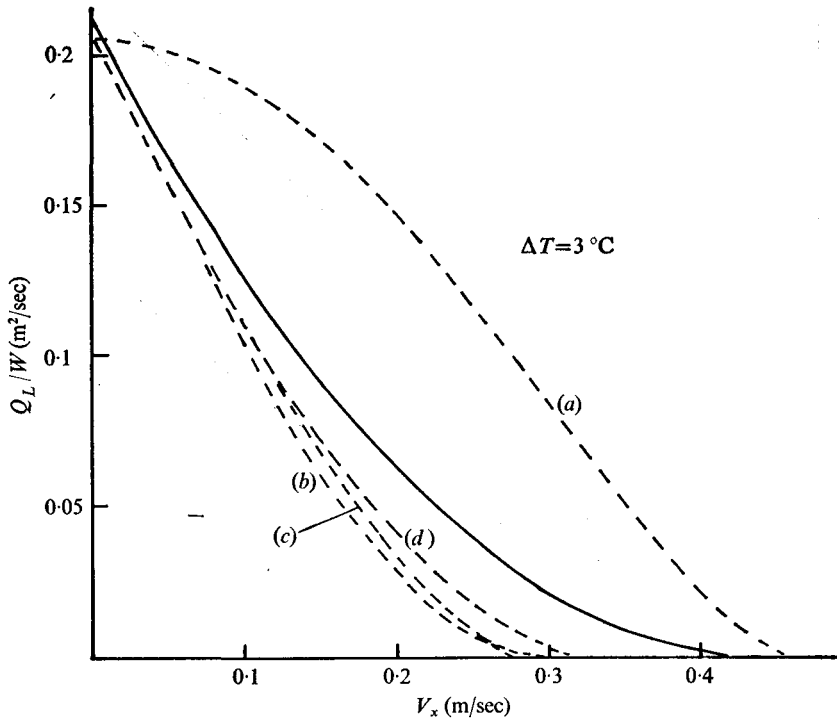


Fig. 4. Comparison of the inflow with excess air supply calculated according to several models. The full line gives the experimental results. (a) Calculated according to Shaw's formula, equation (1). (b) Calculated by the method of velocity summation, equation (4). (c) Calculated by pressure summation without any allowance for turbulence, i.e. from equations (8) and (9) with  $p_u$  taken as zero. (d) Calculated from equations (8) and (9) with allowance for turbulence as in Fig. 2.

Table 1. *Excess air supply required for total exclusion*

Method of calculation	Values of $V_x$ (m/s) at which $Q_L$ falls to zero ( $\Delta T$ , °C)							
	0	0.5	1.0	2	3	5	7	10
Shaw, equation (1)	—	0.19	0.26	0.37	0.46	0.59	0.70	0.83
Shaw, modified (see text)	—	0.12	0.17	0.24	0.30	0.38	0.45	0.54
Velocity summation, equation (4)	—	0.12	0.17	0.24	0.29	0.38	0.45	0.54
Velocity summation, with turbulence	0.14	0.26	0.31	0.38	0.44	0.52	0.59	0.68
Pressure summation, equations (8) and (9), zero turbulence	—	0.11	0.16	0.23	0.28	0.36	0.43	0.51
Pressure summation, with turbulence	0.10	0.17	0.21	0.27	0.31	0.39	0.45	0.53
Pressure summation, coherent outflow	0.10	0.18	0.22	0.31	0.41	0.61	0.78	0.95
Observed values	0.10	0.16	0.24	0.34	0.42	—	—	—

Observed values from Whyte & Shaw (1972; Fig. 2.8); average turbulence taken as 0.14 m/s.

$v$ , on the outflow velocity  $V_x$  and assuming this to act in each direction over half the aperture. As will be seen from (Table 1), however, this does not give a very good representation of the mean outflow velocity  $V_x$  required to reduce  $Q_L$  to zero.

PRESSURE SUMMATION

Although the procedure of velocity summation described above gives a reasonable account of the relation between the air exchange volumes and the excess air supply the concept is rather an artificial one.

By equating the excess pressure due to the air supply to that required to give the necessary net outflow, equations can also be derived which correspond well with observation. At the same time an approximate method has been introduced to allow for the effects of air turbulence. This has been assumed to be equivalent to the addition of a 'turbulence pressure'  $p_u$ , operative in one direction over half the area of the door opening and in the opposite direction over the other half. The resulting pressure distributions are also shown in Fig. 1.

The neutral level,  $Z_0$ , is given by

$$p_x \pm p_u = a \left( \frac{H}{2} - Z_0 \right)$$

or

$$Z_0 = \frac{H}{2} - \frac{p_x \pm p_u}{a} \tag{5}$$

(where the positive sign for  $p_u$  refers to turbulence pressure outwards, i.e. in the same direction as  $p_x$  and  $p_x, p_u$  are written for  $2P_x/\rho$  and  $2P_u/\rho$  respectively, with  $\rho$  equal to the mean density of the air).

If  $H > Z_0 > 0$  then

$$Q'_L = \frac{1}{2} CW \int_0^{Z_0} (az)^{\frac{1}{2}} dz \tag{6}$$

and

$$Q'_0 = \frac{1}{2} CW \int_0^{H-Z_0} (az)^{\frac{1}{2}} dz, \tag{7}$$

where the apostrophe refers to that part of the doorway where the turbulence pressure is outward or inward, according to the sign of  $p_u$ . Whence

$$Q'_L = \frac{1}{6} CWH \cdot \left( \frac{aH}{2} \right)^{\frac{1}{2}} \left( 1 - \frac{2(p_x \pm p_u)}{aH} \right)^{\frac{3}{2}} \tag{8}$$

and

$$Q'_0 = \frac{1}{6} CWH \cdot \left( \frac{aH}{2} \right)^{\frac{1}{2}} \left( 1 + \frac{2(p_x \pm p_u)}{aH} \right)^{\frac{3}{2}}. \tag{9}$$

If  $Z_0$  is apparently less than zero, i.e.  $p_x + p_u > aH/2$ . Then

$$Q'_L = 0$$

and

$$Q'_0 = \frac{1}{2} CW \int_{-Z_0}^{H-Z_0} (az)^{\frac{1}{2}} dz$$

$$= \frac{1}{6} CWH \left( \frac{aH}{2} \right)^{\frac{1}{2}} \left[ \left( \frac{2(p_x + p_u)}{aH} + 1 \right)^{\frac{3}{2}} - \left( \frac{2(p_x + p_u)}{aH} - 1 \right)^{\frac{3}{2}} \right]. \tag{9a}$$

If  $Z_0$  is apparently greater than  $H$ , i.e.

$$p_u - p_x > \frac{aH}{2}$$

Then

$$Q'_0 = 0$$

and

$$Q'_L = \frac{1}{2} CW \int_{z_0-H}^{z_0} (az)^{\frac{1}{2}} . dz$$

$$= \frac{1}{6} CWH \left(\frac{aH}{2}\right)^{\frac{1}{2}} \left[ \left(\frac{2(p_u - p_x)}{aH} + 1\right)^{\frac{3}{2}} - \left(\frac{2(p_u - p_x)}{aH} - 1\right)^{\frac{3}{2}} \right]. \tag{8a}$$

The total inward and outward flows are obtained by summing the values of  $Q'_L$  and  $Q'_0$  for the two directions of turbulence pressure and by equating the net outflow to the excess air supply, i.e. putting

$$Q_0 - Q_L = V_x WH.$$

In principle  $p_x$  can now be eliminated by combining this expression with equations (8) and (9). This cannot, however, be conveniently done in explicit form, but, by inserting selected values of  $p_x$  in these equations, corresponding values of  $Q_L$ ,  $Q_0$  and  $V_x$  are easily obtained.

In particular the values of  $V_x$  required to reduce  $Q_L$  to zero are obtained from the limiting pressure needed to give total exclusion,  $Z_0 = 0$ , with  $p_u$  inward, i.e. negative. Then from equation (5)

$$p_x - p_u = \frac{aH}{2}.$$

The two components of  $Q_0$  then give, via equations (9) and (9a), with  $p_u$  negative and positive respectively,

$$Q_0 = A \left[ 2^{\frac{3}{2}} + \left(\frac{2(aH/2 + 2p_u)}{aH} + 1\right)^{\frac{3}{2}} - \left(\frac{2(aH/2 + 2p_u)}{aH} - 1\right)^{\frac{3}{2}} \right],$$

where

$$A = \frac{1}{6} . CWH \left(\frac{aH}{2}\right)^{\frac{1}{2}},$$

and since  $Q_0 = V_x WH$  then, when  $Q_L = 0$ ,

$$V_x = \frac{1}{3} C(aH)^{\frac{1}{2}} \left[ 1 + \left(1 + \frac{2p_u}{aH}\right)^{\frac{3}{2}} - \left(\frac{2p_u}{aH}\right)^{\frac{3}{2}} \right], \tag{10}$$

which reduces to  $\frac{2}{3} C(aH)^{\frac{1}{2}}$  when  $p_u$  is vanishingly small. When the temperature difference is zero  $a = 0$ , and equations (6) and (7) cannot be directly evaluated.

However, in the absence of any vertical pressure gradient, the flows in both directions can be immediately written down as

$$Q_L = \frac{1}{2} CWH(p_u - p_x)^{\frac{1}{2}}, \tag{11}$$

$$Q_0 = \frac{1}{2} CWH(p_u + p_x)^{\frac{1}{2}}. \tag{11a}$$

The initial rate of change of  $Q_L$  with increasing  $V_x$  can also be readily obtained for

$$\frac{dQ_L}{dV_x} = \frac{dQ_L}{dp_x} \cdot \frac{dp_x}{dV_x} = \frac{dQ_L}{dp_x} \frac{WH}{\frac{dQ_0}{dp_x} - \frac{dQ_L}{dp_x}}$$



And if  $H > Z_0 > 0$  then from equations (8) and (9)

$$\frac{dQ'_L}{dp_x} = \frac{3}{2}A \left(1 - \frac{2(p_x \pm p_u)}{aH}\right)^{\frac{1}{2}} \cdot \frac{-2}{aH}$$

and

$$\frac{dQ'_O}{dp_x} = \frac{3}{2}A \left(1 + \frac{2(p_x \pm p_u)}{aH}\right)^{\frac{1}{2}} \cdot \frac{2}{aH}$$

Summing for both signs of  $p_u$  at  $p_x = 0$

$$\frac{dQ_L}{dp_x} = -\frac{3A}{aH} \left[ \left(1 + \frac{2p_u}{aH}\right)^{\frac{1}{2}} + \left(1 - \frac{2p_u}{aH}\right)^{\frac{1}{2}} \right],$$

$$\frac{dQ_0}{dp_x} = \frac{3A}{aH} \left[ \left(1 + \frac{2p_u}{aH}\right)^{\frac{1}{2}} + \left(1 - \frac{2p_u}{aH}\right)^{\frac{1}{2}} \right] = -\frac{dQ_L}{dp_x},$$

whence

$$\frac{dQ_L}{dV_x} = -\frac{WH}{2}.$$

If, however,  $Z_0$  ( $p_u$  positive) is apparently less than zero at  $p_x = 0$  then  $Z_0$  ( $p_u$  negative) must be greater than  $H$ , and we then have

$$\frac{dQ_L}{dp_x} = -\frac{3A}{H} \left[ \left(\frac{2p_u}{aH} + 1\right)^{\frac{1}{2}} - \left(\frac{2p_u}{aH} - 1\right)^{\frac{1}{2}} \right], \quad \text{from equation (8a)}$$

and

$$\frac{dQ_0}{dp_x} = \frac{3A}{aH} \left[ \left(\frac{2p_u}{aH} + 1\right)^{\frac{1}{2}} - \left(\frac{2p_u}{aH} - 1\right)^{\frac{1}{2}} \right], \quad \text{from equation (9a)}$$

whence again

$$\frac{dQ_0}{dp_x} = -\frac{dQ_L}{dp_x}$$

and

$$\frac{dQ_L}{dV_x} = -\frac{WH}{2}.$$

As Fig. 2 shows, the introduction of a term for air turbulence in this way gives reasonable agreement with the volumes of air exchanged across the doorway in the absence of any net inflow or outflow if  $p_u$  is taken as approximately 0.047 (m, s units), which corresponds to an average turbulent air velocity across each half of the aperture area of 0.14 m/s (approximately 28 ft/min).

The calculated values show a slight fall in the inflow volumes at small values of  $\Delta T$  which is not apparent in the experimental results. This discrepancy, which disappears at values of  $\Delta T$  greater than 2–3 °C, when the effects of turbulence become vanishingly small, may arise from the variability of the experimental results or from deficiencies in the simplified model used to derive the equation.

In the Greenwich Hospital the temperature differences across the doorways were always very small, less than 0.4 °C, and the perceptible air movement was less than that at Glasgow. The mean turbulent air velocity, measured with a katathermometer, was about 15 ft/min (0.08 m/s).

Two calculated curves are shown in Fig. 5.  $H(a)$  assumes zero temperature difference across the doorway and a value of  $p_u$  of 0.019, corresponding to turbulent velocities of 0.09 m/s (about 18 ft/min).  $H(b)$  is deduced for a temperature difference of 0.1 °C and a value of  $p_u$  of 0.015, or turbulent velocities of 0.08 m/s (about 16 ft/min). The latter corresponds better to the form of the observed data, given

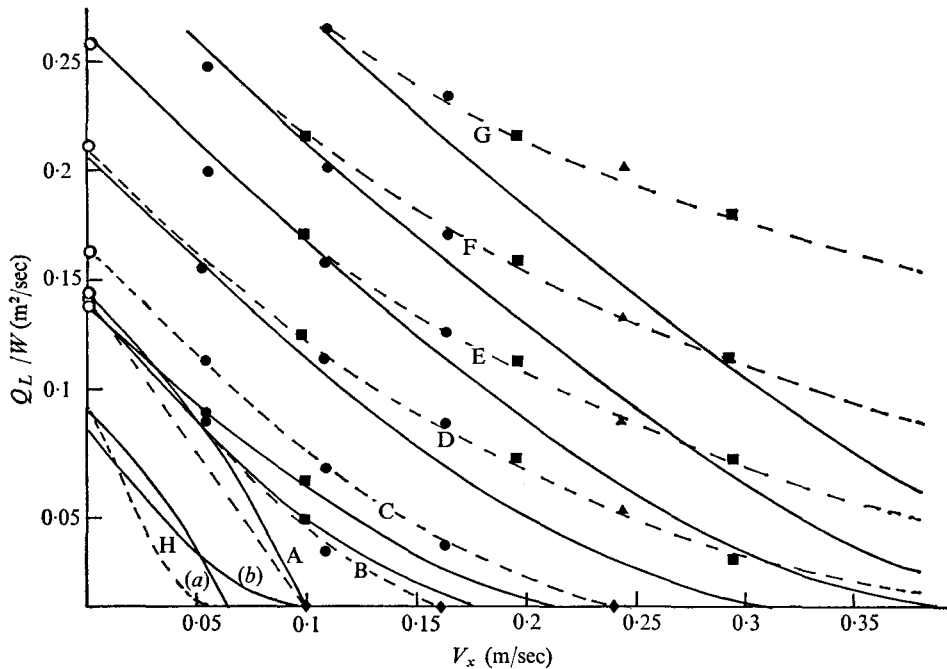


Fig. 5. The relation between inflow and excess air supply over a range of temperature differences. The full lines show the values calculated from equations (8) and (9) and the broken lines the experimentally observed results. The pairs of lines A–G relate to the data obtained by Shaw & Whyte (1974) with temperature differences across the doorway of 0, 0.5, 1.0, 3.0, 5.0, 7.0 and 10.0 °C respectively. The broken lines have been drawn to indicate the form of the relation between inflow and excess air supply indicated by the values obtained from Shaw (1976; Fig. 4.15); see also Shaw & Whyte (1974; Fig. 11). The circles correspond to observations made with a width of aperture equal to 0.9 m, the squares those made with a width of 0.5 m and the triangles those made with a width of 0.1 m. These points do not represent original experimental values but are those derived for the chosen values of  $\Delta T$  from the smoothed curves given by Shaw (or Shaw & Whyte). The limiting values of  $V_x$  needed to reduce  $Q_L$  to zero, diamonds, have been taken from Whyte & Shaw (1972; Fig. 2.8). The group of lines labelled H are related to the data obtained by Foord & Lidwell (1975*a*). The broken line shows the experimental results. The full line (a) has been calculated from equations (8) and (9) assuming no temperature difference across the doorway and turbulent velocities normal to the plane of the aperture of 0.09 m/s. The full line (b) has been calculated similarly for a temperature difference of 0.1 °C and turbulent velocities of 0.08 m/s.

by the broken line, but predicts a substantially greater value for the excess outflow velocity,  $V_x$ , needed to reduce inflow to zero. Fig. 5 also shows that the correspondence between the observed and the computed values becomes progressively less satisfactory as both  $\Delta T$  and  $V_x$  increase. The excess air supply required to reduce the inward air flow becomes much greater than that predicted by any of the methods of calculation described above. This is also shown by the figures in the Table which give the observed and calculated values of  $V_x$  required to reduce this inflow to zero.

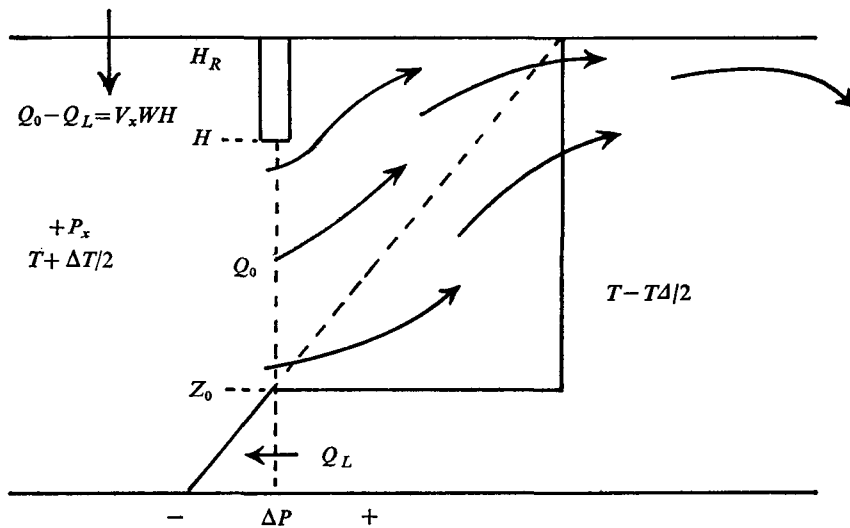


Fig. 6. Pressure and air flow distribution across a doorway with large temperature differences and excess air supply to the left-hand side. The variation of pressure difference in the plane of the doorway is indicated by the full line. The diagonal broken line shows the pressure difference at the boundary between the two air masses. The arrows show the general pattern of air flow.

COHERENT OUTFLOW AND FAILURE OF MIXING

The fundamental presupposition of all the formulae presented so far is that mixing is complete and produces uniform air temperatures on both sides of the doorway up to the plane of the aperture. This can only be an approximation, which is likely to fail progressively as increased temperature differences and unbalanced air flows extrude large masses of air, which are not easily dispersed. The plane of the doorway is then no longer the boundary between two air masses and the distribution of pressure between top and bottom of the doorway is no longer linear. The resulting pattern is illustrated in Fig. 6. In addition to the virtual elimination of the pressure gradient over the upper part of the doorway above the neutral level the extrusion of a coherent mass of warm air removes the effect of the lintel on the total thermal pressure difference, which is now determined by the ceiling height rather than the door height.

When this pattern is fully developed

$$Q_0 \rightarrow CW p_0^{\frac{1}{2}} (H - Z_0)$$

and  $p_0 \rightarrow a(H_R - Z_0)$ , where  $H_R$  is the height of the ceiling and  $P_0 = \frac{1}{2} p_0 p$  is the pressure across the aperture above the neutral level. The limiting value of the outflow is then given by

$$Q_0 = CW a^{\frac{1}{2}} (H_R - Z_0)^{\frac{1}{2}} (H - Z_0). \tag{12}$$

By integration of equation (7), neglecting turbulence,

$$Q_0 = \frac{2}{3} CW a^{\frac{1}{2}} (H - Z_0)^{\frac{3}{2}}.$$

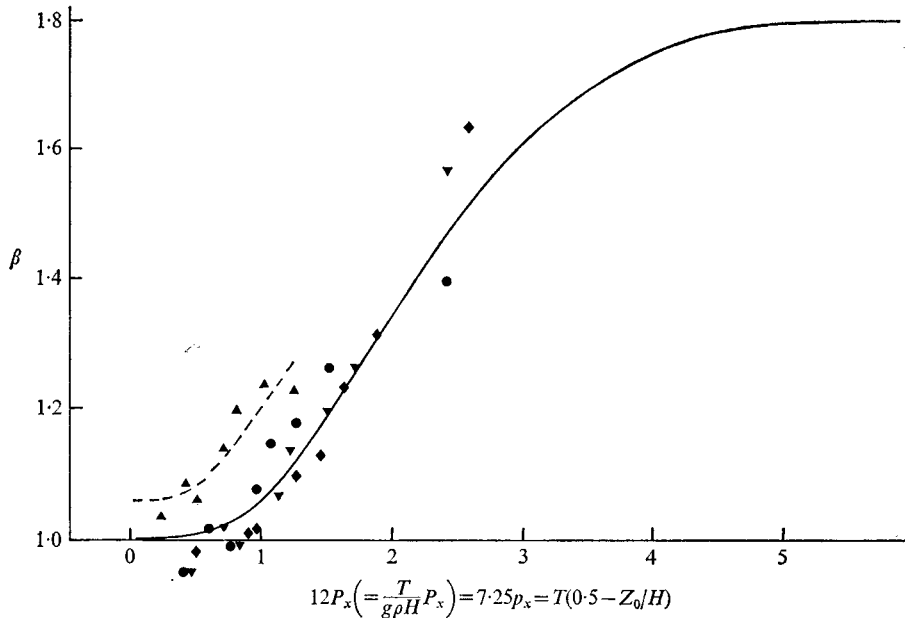


Fig. 7. The ratio between the observed outflow at high temperature difference and that calculated from equations (8) or (9) for the same volume of inflow,  $Q_L$ , i.e. at the same level above the floor,  $Z_0$ , of the neutral line, shown as a function of the excess pressure or the equivalent combination of excess temperature and height of the neutral line. The upward-pointing triangles correspond to observed outflow values  $Q_0$  at a temperature difference of 3 °C, the circles at a temperature difference of 5 °C, the downward-pointing triangles a temperature difference of 7 °C and the diamonds those observed at a temperature difference of 10 °C. The full line has been drawn to give a good representation of the variation of inflow,  $Q_L$ , with increasing excess air supply as shown in Fig. 8. The broken line shows the slightly higher values of  $\beta$  associated with the experimental results for  $\Delta T = 3$  °C.

The effect of the coherent outflow when fully developed is then to increase the magnitude of  $Q_0$  by a factor  $\beta$  which reaches a value of

$$\beta = \frac{3}{2} \left( \frac{H_R}{H} \right)^{\frac{1}{2}}, \quad (13)$$

when  $Q_L$  is reduced to zero, i.e.  $Z_0 = 0$ . At the Glasgow hospital  $H$  was equal to 2.05 m and  $H_R$  approximately 2.95 m leading to a limiting value of  $\beta$  of approximately 1.8.

It has not been possible to derive any explicit formula for the transition between the situation corresponding to effectively complete air mixing up to the plane of the aperture, when  $\beta$  is unity, and the condition of fully developed coherent outflow. However, it is clear from the good correspondence between the calculated and experimental figures shown in Fig. 5 when  $V_x$  is less than 0.1 m/s, i.e.  $Z_0$  is only slightly less than  $0.5H$ , and the magnitude of the divergence at values of  $\Delta T = 10$  °C, which correspond to values of  $Q_0$  approaching 1.8 times the calculated values at  $V_x$  values above 0.5 m/s, that the change must take place between these limits.

Values of  $\beta$  have been calculated corresponding to the experimental values

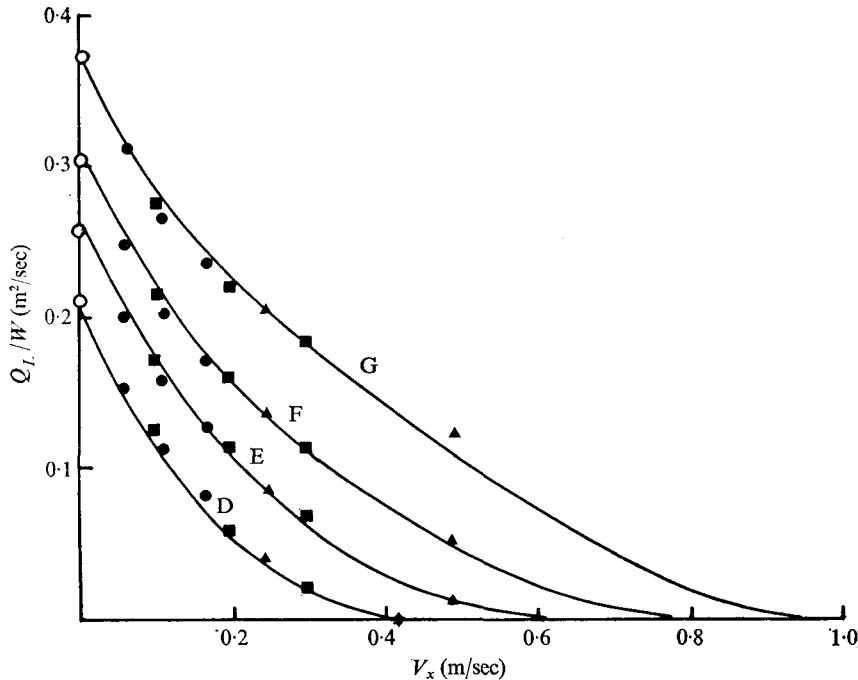


Fig. 8. The relation between inflow,  $Q_L$ , and excess air supply at high temperature differences with allowance for the development of coherent outflow. The full lines D–G are the values calculated from equations (8) and (9) modified by the coherent outflow factor  $\beta$ . D corresponds to a temperature difference of 3 °C, E one of 5°, F one of 7° and G one of 10 °C. The experimental values taken from Shaw (1976; Fig. 4.15) – see also Shaw & Whyte (1974; Fig. 11) – are indicated on the Figure. The filled circles relate to observations with a doorway width of 0.9 m, the squares to a doorway width of 0.5 m and the triangles to a width of 0.1 m. The limiting value of  $V_x$  indicated by a diamond is taken from Whyte & Shaw (1972; Fig. 2.8). The open circles are averaged values for all three door widths.

derived from the data given by Shaw (1976; Fig. 4.15) for values of  $\Delta T$  of 3°, 5°, 7° and 10 °C. Some of these are the values shown in Fig. 5. These values of  $\beta$  have been plotted in Fig. 7 as a function of the product of the temperature difference and the extent to which the neutral level falls below the mid-level, i.e.  $\Delta T (0.5 - Z_0/H)$ . This product is proportional to the excess pressure,  $P_x$ , needed to produce the corresponding outflow. The relation is similar for all temperature differences except that all the values of  $\beta$  for  $\Delta T = 3^\circ$  are high by some 5–8%. This discrepancy is apparent in Fig. 2 for the exchange volumes with no excess outflow and the initial difference persists as  $V_x$  increases up to the point at which inflow is reduced to zero. This may be due to the method used for deriving the smoothed curves of fig. 4.15 (Shaw, 1976) from which the values shown in Fig. 7 have been derived. If  $\beta$  is to vary between 1, at  $Z_0 = 0.5H$  or greater, and 1.8 as  $Z_0$  nears zero, i.e.  $P_x$  becomes large, then the form of the relation cannot be greatly different from that indicated in Fig. 7.

When the values of  $\beta$  indicated in the figure are applied to the air flows calculated

from equations (8) and (9) good agreement with the observed values of  $Q_L$  is obtained over the whole range of values of  $\Delta T$  and  $V_x$  explored (Fig. 8). The data were obtained for doorway widths of 0.1, 0.5, 0.9 m and, within this range, the relation between  $V_x$  and  $Q_L/W$  appeared to be independent of the value of  $W$ .

The situation illustrated in Fig. 6 is not symmetrical with respect to interchange of  $Q_0$  and  $Q_L$  owing to the one-sided effect of the lintel. For massive intrusion of cooler air, i.e. large values of  $Q_L$  and  $V_x$  negative, the maximum value of  $\beta$  would be  $\frac{3}{2}$ .

#### CONCLUSION

The formulae developed in this paper, together with a semi-empirical relation for the transfer from a condition of complete mixing up to the plane of the aperture to a condition where coherent flow of one of the air masses removes the temperature boundary to one side of this, are able to give a satisfactory quantitative account of the volumetric transfer of air across a doorway with temperature differences up to 10 °C. The levels of air turbulence in the experimental situation were in the area of 0.09–0.15 m/s. Much higher values might delay the onset of coherent outflow. At much lower values this might develop at lower temperature differences. All the observations were made with doorways of about 2 m height. Substantial differences in this value might also result in differences in the flow patterns in relation to temperature differences and net outflow volumes. In occupied spaces, however, doorway heights vary little and the range of turbulent air velocities found is rarely greatly different from those experienced in the series of observations discussed here so that the values derived from these data are probably applicable to the majority of hospital and office situations.

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