Foundations of Modern Analysis, by J. Dieudonné. Academic Press Inc., New York, 1960. xiv + 361 pages. $ 8.50.

This very interesting book develops what, in the author's judgment, every young mathematician should know of the indispensable background of analysis, a controlling limitation being the amount of material that can be covered in a single year's course for first year graduate students.

A rough idea of its contents may be gathered from the chapter headings, which are as follows: Elements of the theory of sets; Real numbers; Metric spaces; Additional properties of the real line; Normed spaces; Hilbert spaces; Spaces of continuous functions; Differential calculus; Analytic functions (Appendix: Application of analytic functions to plane topology); Existence theorems; Elementary spectral theory. Of these eleven chapters and one appendix, five are long, averaging roughly 50 pages (Chapters 3, 8, 9, 10, 11), and the remaining seven are short, averaging about 15 pages. The book is completely self-contained, except for an assumed familiarity with standard calculus and some vector space theory.

The first four chapters are relatively standard. The set theory is informal and the real numbers are presented axiomatically. The necessarily long chapter on metric spaces discusses compactness, connectedness and completeness, and the fourth chapter treats the special order properties of the real numbers. Although little out of the ordinary has been covered here, there are many interesting points of detail, such as the author's non-diagonal proof of the uncountability of the real number system, and his direct construction of the logarithm function.

The special flavor of the book is due, it seems to the reviewer, to two aspects of the remaining chapters. The first is the delicate job of tightrope walking the author does in Chapters 5-7 in order to present enough, but not too much, Banach space theory. And the second is the consistency with which the differential mathematics in Chapters 8-10 is presented in a Banach space setting.

General measure and integration theory is beyond the scope of the book. Nor does it discuss the completion of a metric space or the duality theory of Banach spaces. The $L^p$ spaces, including $L^2$, are thus not available as objects of study and application. However, the author does have naturally available certain Banach spaces: the real number system by hypothesis and, consequently, the spaces of



https://doi.org/10.1017/S0008439500025959 Published online by Cambridge University Press

bounded continuous real or complex-valued functions; also, the $l_2$ model of a Hilbert space. These spaces are briefly studied, along with a certain amount of standard general theory in Chapters 5-7. Many of the "meaty" theorems of Banach space theory (Hahn-Banach, closed graphs, etc.) will not be found here; but neither are they needed for the applications the author has in mind.

Chapter 8 develops the differential calculus in the spirit of modern differential geometry. The setting is consistently that of Banach spaces where, of course, the differential of a map can be viewed as between the same pair of spaces as the map itself, and is simply the linear map of closest fit. Also discussed in Chapter 8 is the theory of integration of Banach space valued functions of one real variable, to the extent necessary for complex integration, Taylor's formula with remainder, and one or two other applications. The integral is simply the extension to the completion of S under the uniform norm of the obviously defined integral on the space S of step functions.

Chapter 9 presents the elementary theory of analytic functions--Banach space valued, of course. All of the standard theory is included: power series, analytic continuation, Cauchy integral formula, Laurent series, residues. Moreover, the treatment is for functions of several complex variables as far as possible. The appendix applies these analytic tools to plane topology, and includes a proof of the Jordan curve theorem.

The power of the abstract method is displayed by the applications discussed in the final two chapters. In Chapter 10 the differential theory is applied to various contexts of the elementary contraction fixed point theorem to obtain a whole array of existence theorems of local type, including the implicit function theorem, the basic theory of ordinary differential equations, (including dependence on parameters and initial values, and analyticity results), and the Frobenius theorem on total differential systems.

Chapter 11 caps the book with a beautiful set of applications tailor-made to the stand the author has taken on Banach space theory in Chapters 5-7, namely, the theory of compact operators and its application to the Fredholm equation with continuous kernel and the solution thereby of the Sturm-Liouville problem.

In conclusion it need only be said that the book is overflowing with stimulating, testing problems, generally arranged in groups which work up to significant additions to the textual material.

Lynn H. Loomis, Harvard University



https://doi.org/10.1017/S0008439500025959 Published online by Cambridge University Press