Non-seperability, Non-supervenience, and Quantum Ontology*

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An argument to the effect that quantum mechanics commits us to the existence of non-supervenient relations, and therefore that we should admit such relations into our quantum ontology as fundamental entities, has been given by Teller and reformulated by French. This paper aims, first, to explicate and evaluate that argument; second, to extend its premises in order to assess its relevance for other interpretations of quantum mechanics; and, third, to clarify its implications for holism and individuation in quantum ontology.

1. Ontology After Hobbes and Leibniz. One of the chief metaphysical prejudices of modernity is that real, physical relations (what Russell called 'internal' relations) between individuals do not exist as such. But, because we observe manifest relationality in the world, this leaves us with a "metaphysical puzzle": "How could the mere exemplification of *genuinely non-relational* attributes possibly give rise to an *appearance* of relatedness?" (Cleland 1984, 21) Two chief historical sources of the modern view are Hobbes and Leibniz. Hobbes denied the existence of genuine relations

*Received November 2002; revised May 2003.

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‡The material of this paper is derived from my Ph.D. dissertation, "Ontological Commitments and Theory Appraisal in the Interpretation of Quantum Mechanics" (University of Notre Dame, 1998). Financial support for this work was provided by the John J. Reilly Center for Science, Technology and Values and the Zahm Research Travel Fund, University of Notre Dame. Initial research for this paper was conducted while a Visiting Scholar in the Department of History and Philosophy of Science at the University of Cambridge during the Lent term 1997. My thanks to Steven French, Michael Esfeld and Yuri Balashov for helpful and stimulating discussions concerning the subject of this paper. My thanks also to several anonymous referees of earlier drafts of this paper for their critical and insightful comments and suggestions for improving the paper.

Philosophy of Science, 70 (October 2003) pp. 791–811. 0031-8248/2003/7004-0008$10.00

between individuals at every ontological level—from primitive material bodies (i.e., atoms) to persons. Any relation between material bodies in such an atomism is merely a comparison of the likeness or unlikeness, equality or inequality of individuals (Hobbes 1905, 82–83). In short, there are no relations as such, only juxtapositions of monadic properties or attributes. Nature, according to Hobbes, does not unite and harmonize, but rather divides and disperses, so that individuals are held together collectively not by any inherent relationality, but only by artificial, external constraint—whether atoms by forces in a material body or subjects by fear of the sovereign will in political society. Leibniz (as he is often read) claimed that relations, and spatial relations in particular, are "ideal" in the sense that they exist solely in the mind as a result of abstraction from real monadic properties; all physical relations are reducible (logically, at least) to monadic terms. And any *appearance* of relationality in the physical world is due not to efficient or physical causality, but rather to a "pre-established harmony" between the orders of efficient and final causation. Thus, in a manner not unlike Hobbes, it is God that harmonizes individuals, not nature.

Teller (1986, 1989) has recently characterized such a metaphysical view, which he has variously called 'local physicalism' and 'particularism,' as follows: first, the world is composed of numerically diverse physical individuals that possess non-relational (i.e., monadic) properties; and, second, all relations existing among such entities supervene upon the non-relational properties of the relata. In contrast, 'relational holism' admits the possibility of 'inherent' or 'non-supervenient' relations, that is, relations that do not supervene upon the non-relational properties of the relata. If one can give a precise characterization of the sufficient conditions for non-supervenient relations, then to the extent that one can show that a given contemporary physical theory satisfies those conditions, and thereby commits its adherents to the existence of such relations, one will have gone a long way toward overcoming our Hobbesian atomist inheritance. And, to the extent that such relations are embedded in the physics of the world, one will have done so without introducing a Leibnizian pre-established harmony by which to explain away the manifest relationality in the world.

An argument to the effect that quantum mechanics commits us to the existence of non-supervenient relations, and therefore that we should admit such relations into our quantum ontology as fundamental entities, has been given by Teller (1986) and reformulated by French (1989). This paper aims: first, to explicate and evaluate that argument; second, to extend its premises in order to assess its relevance for other interpretations of quantum mechanics; and, third, to clarify its implications for holism and individuation in quantum ontology.

2. Relations, Non-supervenience and Non-separability. First, we need to introduce the idea of 'relation' in the context of physical theory. Roughly, a 'physical relation' is formally represented as a quantity or magnitude that refers to two (or more) individuals that are distinguished prior to and independently of that quantity; and a 'non-supervenient' physical relation is a physical relation that cannot be completely represented formally in terms of physical quantities or magnitudes that each refer respectively to only one individual. Taking physical quantities or magnitudes that refer to only one individual as non-relational or 'monadic' properties, we can say that non-supervenient relations are not reducible to or determinable by the non-relational properties of the relata (cf. Teller's 'inherent relation'). Cleland (1984, 25) formulates these notions in modal terms:

A dyadic relation R is *supervenient* upon a determinable non-relational attribute P if and only if

1. $(\forall x,y) \supseteq [R(x,y)$ and there are no determinate attributes P_i and P_j of determinable kind P such that $P_i(x)$ and $P_j(y)]$;
2. $(\forall x,y)\{R(x,y) \supset$ there are determinate attributes P_i and P_j of determinable kind P such that $P_i(x)$ and $P_j(y)$ and $(\forall x,y)[(P_i(x)$ and $P_j(y)) \supset R(x,y)]\}$.

French (1989, 9–10) explicates her supervenience conditions (1) and (2) as follows: let R be a dyadic relation and P be a determinable non-relational attribute; then,

If R is genuinely supervenient on P then (1) implies that R cannot possibly appear in the absence of each of its relata instancing the requisite reductive property P, whereas (2) says that there must exist one or more pairs of determinate monadic properties (of kind P) whose exemplification alone is sufficient to guarantee the appearance of R.

He then characterizes 'strong non-supervenience' as the failure of both (1) and (2) and 'weak non-supervenience' as the failure of (2) alone. These notions, of course, can be explicated precisely only given an interpreted physical-theoretical formalism.

To illustrate these notions abstractly, consider the case of two physical systems S_1 and S_2 that jointly comprise the composite system S_{12}; and suppose P_1, P_2 and P_{12} represent properties of determinable kind P. Here, P_1 and P_2 each refer independently to S_1 and S_2, respectively, and thus are non-relational properties in the sense that P_1 could be exemplified even if P_2 were not and even if there exists no system other than S_1, and vice-versa regarding P_2; and P_{12} is clearly a composite relational property or relation, for its exemplification refers explicitly to two entities and is possessed by neither of those entities alone. Now, on the one hand, P_{12} would be genuinely supervenient on P if its exemplification implies that its relata S_1 and S_2 each exemplify a non-relational property P_1 and P_2, respectively, and further if the exemplification of P_1 and P_2 alone is sufficient to guarantee to the exemplification of P_{12}. On the other hand, P_{12} would be weakly non-supervenient on P if the exemplification of P_1 and P_2 alone is *not* sufficient to guarantee the exemplification of P_{12}, and would be strongly non-supervenient on P if it is possible that P_{12} is exemplified without either S_1 or S_2 exemplifying P_1 or P_2, respectively.

The question to be considered is whether quantum-mechanical systems can exhibit such non-supervenient relations in either of these senses. One might think that systems in superposed quantum states of the form $|\psi\rangle = \Sigma_i c_i |u_i\rangle$—which states have been taken by some to be *the* defining characteristic of the "quantum strangeness" (e.g., Dirac)—would surely exhibit non-supervenient relational properties if any quantum-mechanical systems do. But, as Teller (1986, 78) rightly points out, superposition alone yields only another monadic property of one and the same particle, not a relation between particles. Thus, a superposed quantum state by itself is not sufficient for the appearance of relations in quantum-mechanical systems. In order to find quantum-mechanical relations one must consider many-particle systems prepared in 'non-separable' quantum states.

We should next formulate a strict sense for the term 'non-separable' for our purposes that is "disentangled" from the many varied uses of that term in the literature. Suppose the quantum state of an N-particle system is represented by a vector in an N-fold tensor product Hilbert space $|\psi\rangle \in H = H_1 \otimes H_2 \otimes \cdots \otimes H_N$, such that the set of all vectors of the form $|u_i\rangle_1 \otimes |v_j\rangle_2 \otimes \cdots \otimes |w_k\rangle_N$ comprise a complete orthonormal basis for H, where $\{|u_i\rangle\},\{|v_j\rangle\}, \ldots ,\{|w_k\rangle\}$ are sets of basis vectors for $H_1, H_2, \ldots, H_N$, respectively. Then $|\psi\rangle$ is said to be *separable* if it can be represented by a single basis vector in H —

$$|\psi\rangle = |u_i\rangle_1 \otimes |v_j\rangle_2 \otimes \cdots \otimes |w_k\rangle_N,$$

for some choice of $i,j, \ldots, k$, and *non*-separable if it cannot. In general, a *non*-separable quantum state will be of the form

$$|\psi\rangle = \sum\nolimits_{i,j,\ldots,k} c_{i,j,\ldots,k} |u_i\rangle_1 \otimes |v_j\rangle_2 \otimes \cdots \otimes |w_k\rangle_N.$$

For the $N = 2$ case, there always exists a (not necessarily unique) set of orthonormal basis vectors $\{|u_i\rangle_1 \otimes |v_j\rangle_2\}$ for $H = H_1 \otimes H_2$ and a set of (non-zero) complex numbers $\{c_k\}$ such that $|\psi\rangle = \Sigma_k c_k |u_k\rangle_1 \otimes |v_k\rangle_2$ (bi-orthogonal decomposition theorem); but such quantum states are still non-separable.

Thus, from a mathematical point of view at least, there is nothing at all mysterious about non-separable quantum states. The "strangeness" arises only when one attempts to interpret non-separable quantum states as referring in some sense to properties possessed by systems prepared in such states. And this, as Teller (1986, 77–78) himself emphasizes, is the first interpretive assumption required by any argument for ontological commitments to relations, non-supervenient or otherwise, namely, that the quantum state be interpreted *ontologically,* as opposed to instrumentally, that is, as representing in some (perhaps incomplete) way the physical reality of quantum-mechanical systems and not merely as a mathematical tool for statistical prediction.

3. Non-supervenience in Quantum Mechanics. This brings us to the Teller-French argument that 'entangled' systems prepared in non-separable quantum states do in fact exhibit non-supervenient relations. The following is an elaboration of French's (1989) reconstruction of Teller's (1986) original version.

Consider a two-particle system—composed of 'particle-1' and 'particle-2'—prepared in a quantum state represented in terms of the eigenstates of a two-valued observable represented by the Hermitian operator O having distinct eigenvectors $|\alpha\rangle$ and $|\beta\rangle$ — $O|\alpha\rangle = \alpha|\alpha\rangle$ and $O|\beta\rangle = \beta|\beta\rangle$. Possible two-particle quantum states for the composite system, represented in terms of the eigenstates of the composite system observable represented by $O_{12} = O_1 \otimes O_2$, are as follows:

(i) $|\alpha\rangle_1|\alpha\rangle_2$,
(ii) $|\beta\rangle_1|\beta\rangle_2$, and
(iii) $\frac{1}{\sqrt{2}}(|\alpha\rangle_1|\beta\rangle_2 \pm |\beta\rangle_1|\alpha\rangle_2)$.

The states (i–iii) will be recognized to be the (anti-)symmetric eigenvectors of O_{12} having corresponding eigenvalues α^2, β^2 and $\alpha\beta$, respectively, and the state (iii) will be seen to be non-separable. The operators O_1 and O_2 represent single-particle monadic, non-relational properties of particle-1 and particle-2, respectively, and the operator O_{12} represents a relation between particle-1 and particle-2.

Now assume the following: a physical system possesses a definite property represented by a Hermitian operator *if and only if* the system is prepared in a quantum state represented by the associated eigenvector of that operator, in which case the property is represented by the corresponding eigenvalue. This, of course, is just the Eigenvector-Eigenvalue Property Rule (EE) that is common to the so-called 'orthodox' interpretations of Dirac and von Neumann. Given this rule, the following analysis results: If the two-particle system were prepared in any one of the above quantum states (i–iii) there would be a relation between the two particles represented in terms of the eigenvalues of O_{12}—namely $O_{12} = \alpha^2$, $O_{12} = \beta^2$, and $O_{12} = \alpha\beta$, respectively. Moreover, if the two-particle system were prepared in either of the quantum states (i) or (ii), then particle-1 and particle-2 would each be in a single-particle quantum state represented by an eigenvector of O—$|\alpha\rangle_1$ and $|\alpha\rangle_2$ for case (i), $|\beta\rangle_1$ and $|\beta\rangle_2$ for case (ii)—so that each would possess a corresponding monadic, non-relational property—$O_1 = \alpha$ and $O_2 = \alpha$ for case (i), $O_1 = \beta$ and $O_2 = \beta$ for case (ii). However, if the two-particle system were prepared in quantum state (iii), then neither of the particles would have a single-particle quantum state represented by an eigenvector of O. This is because, first, the quantum states (i–iii) are all distinct in the sense that each belongs to a mutually orthogonal sub-space, so that a system prepared in state (iii) is never in either (i) or (ii); and, second, because the states $|\alpha\rangle_1|\beta\rangle_2 \pm |\beta\rangle_1|\alpha\rangle$ belong to neither of the subspaces spanned by $|\alpha\rangle_1|\beta\rangle_2$ or $|\beta\rangle_1|\alpha\rangle_2$, a system in state (iii) is never simply in one or the other of the latter two states. Rather, the single-particle states of each would be represented by the same mixed-state density matrix—$\rho_1 = \frac{1}{2}(\,|\alpha\rangle\langle\alpha\,| \pm |\,\beta\rangle\langle\beta\,|\,) = \rho_2$; and because the state of neither particle would be represented by an eigenvector of O, neither would possess *any* single-particle monadic, non-relational property represented by O.

So, we have the following results for the three cases:

(i) particle-1 possesses $O_1 = \alpha$, particle-2 possesses $O_2 = \alpha$, and the two-particle composite possesses $O_{12} = \alpha^2$;
(ii) particle-1 possesses $O_1 = \beta$, particle-2 possesses $O_2 = \beta$, and the two-particle composite possesses $O_{12} = \beta^2$;
(iii) neither particle-1 nor particle-2 each possess an O-property, but the two-particle composite possesses $O_{12} = \alpha\beta$.

In cases (i) and (ii), both of the supervenience conditions (1) and (2) are clearly satisfied, for in each case both particles exhibit a non-relational O-property and those properties are jointly sufficient to guarantee that the two-particle composite exhibits the O-relation that it does; hence, the O-relation in each case genuinely supervenes upon the respective pairs of non-relational O-properties. In case (iii), however, both of the supervenience conditions (1) and (2) fail to be satisfied; for not only does the two-particle composite exhibit an O-relation that is not guaranteed by the exemplification of pairs of non-relational O-properties, thus violating supervenience condition (2), but neither of the relata of that relation exemplify *any* non-relational O-property at all, thus violating supervenience condition (1). Therefore, in case (iii) the O-relation exhibited by the two-particle composite is *strongly non-supervenient* on O.

Thus, ‘entangled’ many-particle quantum systems prepared in non-separable quantum states do in fact exhibit non-supervenient relations. The question, then, is whether we ought to admit such relations into our quantum ontology as fundamental entities. French (1989, 18) argues that we should. But, *must* one make an ontological commitment to the existence of non-supervenient relations? For sure, *only if* one accepts all the argument's premises, both explicit and implicit.

One can block the argument's conclusion, and thus avoid commitment to non-supervenient relations in quantum-mechanical systems, in (at least) either of two ways. First, of course, one can simply deny EE, in particular, the 'only if'-clause, which is crucial for the inference in case (iii) that neither particle possesses a non-relational O-property. And, inasmuch as the 'only if'-clause is one of the notoriously problematic assumptions that generates the infamous 'measurement problem,' there is perhaps good reason for rejecting it. But, if one wishes to maintain the 'only if'-clause of EE and yet still avoid commitment to non-supervenient relations, then one must deny that the operator O_{12} represents a genuine relation between particle-1 and particle-2. Here one might say instead that O_{12} represents merely the single-particle non-relational properties that are jointly exhibited when the states of both particles are measured together. French counters such a move by emphasizing that the operator O_{12} admits eigenvectors of type (iii) that express statistical *correlations* between the outcomes of joint measurements of the observable represented by O on both particles and claims that such statistical correlations "clearly express a relation between . . . the particles" (French 1989, 12). Implicit in French's claim is the assumption that the existence of such a statistical correlation between measurement outcomes is *sufficient* for the existence of a real, physical relation between the systems the measurement outcomes for which are correlated. Such an assumption is plausible *if* one takes, say, a 'disposition' or 'propensity' interpretation of probability, according to which quantum-mechanical probabilities themselves would be 'objective' in the sense of being primitive predicates in their own right that directly represent (dispositional) particle properties.

Along the lines of such thinking, one could replace EE with a weaker (i.e., less restrictive) property rule. This would extend the scope of the Teller-French argument by expanding the possible supervenience base for quantum-mechanical relations (i.e., the set of single-particle non-relational properties). But, introducing a weaker premise does *not necessarily* eliminate non-supervenient relations. Consider what might be called the Disposition Property Rule (D), which can be understood as a "widening" of EE that includes EE as a special case. This rule takes marginal probabilities to represent monadic or non-relational single-particle properties and joint probabilities to represent multi-particle relational properties (cf. French and Redhead 1988; Butterfield 1993); such an interpretation of quantum probabilities would support French's claim above. Suppose, then, that we have an N-particle system in a non-separable quantum state $|\psi\rangle$; N-particle relations would be represented by the composite-system density matrix $\rho_{12\,\ldots\,N} = |\psi\rangle\langle\psi|$, and single-particle non-relational properties would be represented by the respective single-particle density matrices $\rho_i = Tr_{j,k, \ldots \neq i}\, \rho_{12 \ldots N}$. Because $|\psi\rangle$ is non-separable, we have $\rho_{12 \ldots N} \neq \rho_1 \otimes \rho_2 \otimes \cdots \otimes \rho_N$; that is, the respective single-particle density matrices taken together are *not sufficient* to determine uniquely the composite system density matrix. Invoking D, it follows that the exemplification of the respective single-particle non-relational properties alone is not sufficient to guarantee the exemplification of the *N*-particle relational properties. Thus, while supervenience condition (2) is clearly violated here, supervenience condition (1) is still satisfied. In that case, the relations represented by $\rho_{12 \ldots N}$ are *weakly non-supervenient* upon the non-relational properties represented by the respective ρ_i's. So, even on the interpretation of quantum-mechanical probabilities in terms of dispositional properties, one is still committed to weakly non-supervenient relations.

A further implicit premise in the Teller-French argument and the extension thereof just given is that the quantum states (i–iii) are *complete* physical descriptions. By 'completeness' is meant here that *all* information necessary to determine a physically (and not merely statistically or empirically) complete assignment of (possible or actual) properties to a physical system is contained in the quantum state *alone,* i.e., that *all* single-particle and composite properties are to be assigned *solely* on the basis of the quantum state, so that the quantum state *by itself* is, via some specifiable rule, *exhaustive* of the properties assignable to a physical system, whether simple or composite. Teller (1986, 79) has claimed, though, that even if one denies the completeness of the quantum state-description, one can *not* avoid commitment to non-supervenient relations because hypothetical "hidden"-variable states could not provide a supervenience basis for the relations exhibited by systems in non-separable quantum states such as (iii). French (1989, 12) disagrees with Teller's claim and appeals to a particular example —namely, the Bohm-Vigier stochastic 'sub-quantum ether' theory—as evidence in favor of the possible reduction of such relations to non-relational single-particle properties in a hidden-variable state-description. I agree with French's conclusion on this point, though I would state the reason why more strongly. Both the 'only if'-clause of EE and D are warrantable, I would argue, *only* on the *presupposed* completeness of the quantum state-description. Therefore, if one denies the completeness assumption, then the 'only if'-clause of EE and D are also rejected; thus, the conclusion of the argument would be undercut without the need to appeal directly to any particular hidden-variable theory. Of course, this leaves open the question of whether or not there exists a hidden-variable theory that is consistent with the pertinent quantum-mechanical correlations but does not carry commitment to non-supervenient relations. We'll return to this question below.

The upshot of the above discussion is this. From the point of view of the Teller-French argument, at least, one can avoid commitment to non-supervenient relations in quantum-mechanical systems provided one denies any of the following assumptions: that the quantum state-description is physically complete, that preparation in a quantum eigenstate is both a necessary and sufficient condition for a system to possess a definite property, or that the existence of statistical correlations is sufficient for the exemplification of a real, physical relation.

4. Non-supervenience in Other Interpretations.

4.1. Non-supervenience in "Hidden"-Variable Theories. Let's consider hidden-variable theories, which deny at least the completeness assumption. While denying the completeness assumption leaves ontological commitment to non-supervenient relations undecided on the basis of the Teller-French argument, it is not obvious that assuming the incompleteness of the quantum state-description by itself is sufficient to avoid such commitment altogether. As mentioned above, Teller has made the claim that hidden-variable theories not only do not necessarily avoid ontological commitment to non-supervenient relations, but cannot in fact avoid such commitment. For, his argument runs, due to the Kochen-Specker theorem, the variables in any hidden-variable state-description must be contextual, "which," he claims, "comes to the same thing as being relational" (Teller 1986, 79, n. 1). But, whereas the former claim that hidden variables must be contextual is correct (with non-trivial qualification), the latter claim identifying 'contextual' with 'relational' is at least questionable if not simply incorrect.

'Contextuality' in the Kochen-Specker theorem refers to the following: if all the projection operators on a Hilbert space (of dimension greater than two) are to be simultaneously and consistently assigned definite and unique (eigen)values that are compatible with the quantum-mechanical expectation values of all the Hermitian operators on that space for an arbitrary quantum state, then such values must be 'contextual' in the sense that which value is assigned to any given projection operator can *not* be independent of which complete set of projection operators it is taken to be a member; and this implies that the value assigned to any Hermitian operator on the Hilbert space as a function of those projection-operator values (according to the spectral theorem) will likewise be contextual so that the expected value of any given quantum-mechanical observable will depend upon the set of mutually commuting observables together with which it is measured, that is, its measurement context. But, the Kochen-Specker theorem shows only that the *quantum-mechanical* observables *must* be contextual in a hidden-variable state-description, while the hidden variables themselves will be contextual *if* they are identified with the projection operators, which need not be the case (as, e.g., with Bohm's theory, where the hidden variables are actual positions in configuration space).

Thus, Teller's claim that hidden-variable theories cannot avoid ontological commitment to non-supervenient relations because hidden variables must be contextual appears to be incorrect for two reasons. First, as just pointed out, only certain classes of hidden-variable theories are constrained by the Kochen-Specker theorem to have contextual hidden variables; hence, even if 'contextual' here were to just mean 'relational,' *not all* hidden-variable theories would be committed to non-supervenient relations on the basis of the Kochen-Specker theorem alone. Second, the sense of 'contextual' relevant to the Kochen-Specker theorem has no clear connection to the sense of 'relational' relevant to the Teller-French argument. For 'contextual' in the Kochen-Specker theorem refers at most to a "relation" between a projection operator and a complete set of projection operators, or between a quantum-mechanical observable and its measurement context, *not* to a physical magnitude or quantity predicated jointly or compositely of several individuals, which is what 'relation' refers to specifically in the Teller-French argument. I thus agree with French (1989, 13) that contextual hidden variables are not equivalent to relations. So, it seems clear that even those hidden-variable theories whose hidden variables are constrained to be contextual by the Kochen-Specker theorem are not committed thereby to non-supervenient relations.

Let's now consider the question of non-supervenience in one particular hidden-variable theory, viz., Bohm's theory. Whether 'entangled' many-particle systems described by non-separable quantum states exhibit non-supervenient relations in Bohm's theory depends crucially upon how one chooses to interpret the quantum state and particle properties within the theory, over which there is considerable ongoing debate (cf. Cushing et al. 1996, Belousek 2003). At the risk of oversimplifying that debate, we will consider here three broad proposals for such an interpretation.

What all interpretations of Bohm's theory have in common is that the quantum state is taken to be an incomplete description of the physical reality of quantum systems, i.e., that by itself the quantum state is insufficient to exhaustively determine all (possible or actual) particle properties. In particular, Bohm's theory supplements the quantum state description with independent variables—viz., actual single-particle positions. And it is on the basis of the single-particle position, taken together with the quantum state, that all other particle properties (however extensive that set may be taken to be) are defined and assigned; primary among these other particle properties is the actual single-particle velocity or momentum, which is defined as the gradient of the phase of the quantum state and is assigned by evaluating the phase at the actual particle position. Bohm's theory also denies the other two assumptions under question—viz., the 'only if'-clause of EE and that correlations imply dispositional properties; the actual single-particle positions need not be eigenvalues of the position operator, and correlations are explained via the non-local effects of the quantum state dynamics rather than taken as primitive properties of 'entangled' systems.

The first interpretive proposal I'll call 'minimalist' (e.g., Dürr et al. 1996). In this view, the only properties a Bohmian particle really possesses (in addition to its state-independent classical properties, mass and charge) are its actual position and velocity; all quantum properties or 'observables' represented by Hermitian operators, such as 'spin,' are regarded as merely fictions or as constructions that only catalogue possible position measurement outcomes and are thus eliminated from the theory's ontology (because they add neither empirical content nor explanatory power to the theory). On this view one might further interpret the quantum state itself as having only an 'instrumental' significance for statistical predictions, as merely representing a convenient summary of the possible motions of a system, and thus as being an abstract mathematical entity in configuration space having no concrete existence in physical space (as do Dürr *et al.*). On such an interpretation, there is simply no room for non-supervenience to arise in the first place, even when the quantum state is non-separable.

Consider an N-particle system represented in the $3N$-dimensional configuration space by the quantum state $\psi(q_1, \ldots, q_N)$ and the actual system configuration of single-particle positions $Q = (Q_1, \ldots, Q_N)$. The system velocity is simply $v^{\psi}(Q) = (v_1^{\psi}, \ldots, v_N^{\psi})$, where $v_i^{\psi} = dQ_i/dt = (\hbar/m)\mathrm{Im}(\nabla_i\psi/\psi)$. Now, for an 'entangled' system in a non-separable quantum state, each respective actual single-particle velocity v_i^{ψ} will depend upon the actual configuration for the *entire* system and not just on the respective single-particle position, i.e., will not be a function solely of the coordinates of the ith particle. But, such non-independence is not non-supervenience. There is here only a case of non-local state-dependence of each single-particle *velocity* on *positions* of the other particles in the system, where the actual system configuration, taken together with the quantum state, is sufficient to determine all of the single-particle velocities; thus, though the single-particle velocities are themselves (non-locally) relational quantities, there are, strictly speaking, no non-supervenient relations between the respective single-particle velocities themselves and, hence, no non-supervenience of either kind. One should not conflate either non-locality or (as will be emphasized further below) state-dependence with non-supervenience.

A second interpretive proposal agrees with the 'minimalist' view that the only real properties possessed by Bohmian particles are position and velocity, but, contrary to the previous view, seeks to interpret the quantum state itself 'realistically,' i.e., as in some sense representing a concrete reality in physical space (for methodological reasons such as explanation and the classical limit). Here there are two options. On the first option, only the phase S

of the quantum state, where $\psi(q_1, \ldots, q_N) = R(q_1, \ldots, q_N)e^{iS(q_1, \ldots, q_N)/\hbar}$, is interpreted dynamically as a 'guiding field' or 'pilot wave' that causally directs the motions of particles in a system according to the 'guidance condition' for the single-particle velocities, $v_i = (1/m_i)\nabla_i S$, which is mathematically equivalent to the above velocity formula (e.g., Valentini 1996). On this view the quantum state represents a real entity existing in configuration space over and above the particles themselves and their trajectories in physical space. Here no question of the non-supervenience of the 'pilot wave' on particle positions even arises because the former is posited from the beginning as an ontologically separate entity subsisting independently of the particles. On the second option, the modulus R of the quantum state is also interpreted dynamically in terms of the 'quantum potential,' $U = (-\hbar^2/2m)(\nabla^2 R/R)$. On this view, there are two further possibilities: either the quantum potential is interpreted as determining direct, non-local, many-particle 'quantum forces,' via a modified Newton's Second Law in analogy with classical mechanics, $F = -\nabla U$, that act on the particles in a system in addition to the local, two-particle classical forces arising from the classical potential (e.g., Bohm and Hiley 1993); or the quantum potential can be interpreted in terms of physical relations holding between the particles and having an existence over and above the particle postions. On the one hand, if the quantum potential is taken to represent direct forces, there is also no question of non-supervenience; for, again, although F would be non-local and state-dependent, neither of these by themselves imply non-supervenience. If, on the other hand, the quantum potential is taken to represent relations, then one would have non-supervenience.

For an N-particle system the quantum potential is given by $U = \Sigma_i (-\hbar^2/2m)(\nabla_i^2 R/R)$. Here, one might take $U_i = (-\hbar^2/2m)(\nabla_i^2 R/R)$ to represent the 'single-particle quantum-potential' of the ith particle. But, even though one would have simply $U = \Sigma_i U_i$, U would *not* supervene on the respective U_i when the quantum state is non-separable. In that case, R is irreducibly relational, referring to the coordinates of *all* particles in the system; and because U_i would depend upon R for the *total* system, each U_i would *not* be a *monadic* 'property' of the ith particle alone, but rather is already a *relational* property. Thus, although the N-particle quantum potential is just the sum of the single-particle quantum potentials, it is nonetheless *strongly non*-supervenient in the case of a non-separable quantum state: the relational N-particle quantum potential does not supervene upon non-relational single-particle quantum potentials because there simply are no such *non*-relational single-particle quantum potentials to begin with, in which case supervenience condition (1) fails. Insofar, then, as one takes the quantum potential to be in some sense a 'property' of a system, it can be interpreted as a (strongly) non-supervenient relational property of the N-particle system itself or as a system of N-place (strongly) non-supervenient relations holding among the particles themselves. Again, one should not conflate state-dependence with non-supervenience here: the quantum potential is state-dependent, but that the quantum potential is non-supervenient is *not because* it is state-dependent.

A third interpretive proposal, which is *prima facie* compatible with any of the above interpretations of the quantum state, I'll call 'expansive.' It holds that, in addition to the classical properties of position and velocity (or momentum) that are attributed to Bohmian particles in the 'minimalist' view, one should (for methodological reasons beyond prediction and explanation, e.g. the classical limit) expand the range of particle properties to include at least some of the usual quantum properties or 'observables,' such as 'spin,' that are represented by Hermitian operators (e.g., Holland 1993). Here, such operators are to be reconstructed out of the quantum state and particle position, but would nonetheless (again for methodological reasons) be interpreted as having an independent ontological status. Each such property would always be assigned a definite value, determined by the actual configuration of the system, whether or not the quantum state is an eigenstate of the Hermitian operator representing that property (i.e., the 'only if'-clause of EE is rejected). Thus, by the Kochen-Specker theorem, such definite-value assignments will be contextual in general (for operators on Hilbert spaces of dimension greater than two); but, as argued above, the inevitable contextuality of such properties does not necessarily imply non-supervenience. Whether or not such properties do in fact entail non-supervenient relations will depend precisely upon how those properties are defined.

Consider, for example, the following proposed definition for the state-dependent z-component of the single-particle 'spin' (cf. Dewdney 1992): $s_z = \frac{\hbar}{2}(\psi^* \sigma_z \psi / \psi^* \psi)$. For a two-particle system, the joint spin-component would be $s_z^{(12)} = \frac{\hbar^2}{4}(\psi_{12}^* \sigma_z^{(1)} \sigma_z^{(2)} \psi_{12} / \psi_{12}^* \psi_{12})$. Now, in the case of a separable quantum state, $\psi_{12} = \psi_1 \cdot \psi_2$, the joint spin-component will be determined by the respective single-particle spin-components, $s_z^{(12)} = s_z^{(1)} \cdot s_z^{(2)}$, satisfying supervenience condition (2); and, because the single-particle spin-components $s_z^{(1)}$ and $s_z^{(2)}$ would depend upon ψ_1 and ψ_2, respectively, they would refer only to the respective coordinates of each particle separately and, hence, be monadic properties, satisfying supervenience condition (1). Thus, the joint spin-component relation genuinely supervenes upon the non-relational single-particle spin-components. In the case of a non-separable quantum state, $\psi_{12} \neq \psi_1 \cdot \psi_2$, however, one will have $s_z^{(12)} \neq s_z^{(1)} \cdot s_z^{(2)}$, in violation of supervenience condition (2); and, because the single-particle spin-components $s_z^{(1)}$ and $s_z^{(2)}$ would both depend upon ψ_{12}, and thus refer jointly to the coordinates of *both* particles, each is already a *relational* property, in violation of supervenience condition (1). In this case, then, because there are no monadic single-particle spin-component properties upon which the joint spin-component relation could supervene, in violation of both supervenience conditions, the joint spin-component relation would be strongly non-supervenient.

So, one has several interpretive options within Bohm's theory, two of which entail a commitment to non-supervenient relations (for a comparative appraisal of the various interpretive options in Bohm's theory according to normative criteria such as coherence and explanation, see Belousek 2003).

4.2. Non-supervenience in a "Radical" Interpretation. Let's reconsider ontological commitment to non-supervenient relations within an interpretation that takes the quantum state-description to be complete and accepts either the (strong) EE or the (weak) D property rule. What we aim to exhibit here is a possible interpretation of non-separable states that bears no commitment to non-supervenience in any sense and thus presents an interesting case in this context regardless of its ultimate plausibility. As we saw earlier, one can avoid commitment to non-supervenient relations from an 'orthodox' view by denying that the operator $O_{12} = O_1 \otimes O_2$ represents a genuine relation exhibited by the two-particle composite system. But, as mentioned above, even if one denies the D property rule, EE itself seems to imply that O_{12} does indeed represent a relation existing between particle-1 and particle-2 in the state $|\psi\rangle = \frac{1}{\sqrt{2}}(|\alpha\rangle_1|\beta\rangle_2 \pm |\beta\rangle_1|\alpha\rangle_2)$, where $|\alpha\rangle$ and $|\beta\rangle$ are eigenvectors of O. Thus, to avoid commitment to non-supervenient relations altogether—which commitment follows evidently from the assumptions of completeness and either the EE property rule (strongly non-supervenient relations) or D (weakly non-supervenient relations)—one must re-interpret the non-separable quantum state $|\psi\rangle$ itself in such a way that the above inferences to non-supervenience are rendered non sequitur. And the most direct way to do so is to deny the very conditions which make such inferences to non-supervenient relations possibly valid, namely, that $|\psi\rangle$ represents the quantum state of two, numerically distinct entities whose prior and independent distinguishability is necessary for the existence of any relation represented by operators of which $|\psi\rangle$ is an eigenvector.

To motivate such a view, one might propose that the quantum-mechanical state-description be taken strictly and literally as it stands as ontologically primary, that is, that one should let the quantum state itself suggest how to parse the world and thereby "read off" the ontology of the world directly from its representation. The present suggestion is this: First, assume the quantum state-description to be complete and take whatever physical system is referred to by a (pure) quantum state itself as a single 'simple quantum entity.' Second, define a 'composite quantum entity' as any system referred to by a quantum state $|\Psi\rangle$—whether pure or mixed—that admits 'proper parts,' that is, sub-systems whose own state is itself a pure quantum state, where the state of a sub-system is obtained by partial tracing over the appropriate degrees of freedom of the composite-system density matrix; if a system admits of no 'proper' sub-systems, then it is a simple, not composite, quantum entity. One would have, then, only two cases of 'composite quantum entities' having 'proper parts': (i) a physical system whose quantum state is represented by a single product of (pure) quantum states—$|\Psi\rangle = |\varphi\rangle \otimes |\chi\rangle \otimes |\xi\rangle \otimes \cdots$ or $\rho = |\varphi\rangle\langle\varphi| \otimes |\chi\rangle\langle\chi| \otimes |\xi\rangle\langle\xi| \otimes \cdots$; and (ii) a physical system whose quantum state is represented by a mixture (i.e., non-trivial convex sum) of (pure) quantum sates—$\rho = \Sigma_i |c_i|^2\ \rho_i$, where each $\rho_i = |\Psi\rangle\langle\Psi|$ is a pure-state density matrix of type (i). In case (i), each vector in the product quantum state would itself represent a single simple 'quantum entity'; and in case (ii), each pure-state density matrix would itself represent a single simple 'quantum entity,' while the sum would represent a mixture having $|c_i|^2$ proportion of each simple 'quantum entity.' The individuation of such simple 'quantum entities' would be grounded upon distinction among quantum states themselves: two distinct quantum states refer to two distinct simple 'quantum entities'; and quantum states are distinct if they differ by more than a phase factor. In this way, quantum states are defined by the rays of a Hilbert space such that there is in some sense an ambiguity here in referring to basic quantum entities; but, such referential ambiguity is founded upon a precise mathematical description subject to an unequivocal criterion for distinguishing referents and thus is unproblematic.

Now, were the quantum state of a system represented by a coherent superposition of (pure) product states of type (i),

$$|\Psi\rangle = \sum\nolimits_{i,j,k,\ldots} c_{i,j,k,\ldots} |\varphi_i\rangle \otimes |\chi_j\rangle \otimes |\xi_k\rangle \otimes \cdots,$$

that is, by a non-separable quantum state, the 'quantum entity' referred to by such a state would have no definable 'proper parts'; for every state obtained by partial tracing over $\rho = |\boldsymbol{\Psi}\rangle\langle\boldsymbol{\Psi}|$ would represent an 'improper' mixture, which could not be interpreted here as referring to a simple 'quantum entity.' By denying that 'improper' mixed states have any proper reference and, hence, ontological significance, one effectively stipulates that the non-separable quantum state $|\boldsymbol{\Psi}\rangle$ refers to a single simple 'quantum entity,' a single basic subject of quantum-predication, one individual physical system. So, the answer to our question—namely, whether or not the two-particle O-relation exhibited by a system in a quantum state represented by $|\psi\rangle = \frac{1}{\sqrt{2}}(|\alpha\rangle_1 |\beta\rangle_2 \pm |\beta\rangle_1 |\alpha\rangle_2)$ supervenes upon non-relational single-particle O-properties—is that not only are there no non-relational O-properties upon which the O-relation could supervene, but there simply are no simple 'quantum entities' other than that referred to by $|\psi\rangle$ itself that could even possess such properties; that is, there is no relation at all in this case because, strictly speaking, there are no relata in the first place! On this view, then, instead of taking the operator $O_{12} = O_1 \otimes O_2$ as representing a relation between *two* simple 'quantum entities,' it should rather be taken as representing a monadic property possessed by a single simple 'quantum entity.' And in the case where $|\psi\rangle = |\alpha\rangle_1|\alpha\rangle_2$, or $|\psi\rangle = |\beta\rangle_1|\beta\rangle_2$, etc., $|\psi\rangle$ would refer to a composite 'quantum entity' according to rule (i) and O_{12} would represent a relation exhibited by the composite that genuinely supervenes upon the non-relational properties represented by O_1 and O_2 and possessed by the simple 'quantum entities' whose respective quantum states are $|\alpha\rangle_1$ and $|\alpha\rangle_2$, etc. Therefore, on this ontological interpretation of the quantum state-description, one is not committed to non-supervenient relations in either sense.

One might, of course, think such an interpretation to be ad hoc, invented simply to avoid a certain mode of ontological commitment. Such a criticism, though, would fail to appreciate the point of view that motivates the proposal. The interpretive stance does *not* seek to avoid the question of ontological commitment, but rather to 'naturalize' it in a radical way. One may very well disagree with such a project on other methodological or on metaphysical grounds, but it is surely not ad hoc. Another possible objection to such a view is that it appears to be guilty of equivocating upon the ontological significance of the operator $O_{12} = O_1 \otimes O_2$: does it represent a relation or a non-relational property? The interpretation proposed above seems to answer 'both,' and thus is evidently incoherent. But, again, such a criticism would fail to see the point. The proposed interpretation takes the quantum state *itself* as the basic entity in the ontology, not the operators; and doing so is motivated by taking seriously the notion that the quantum state is a *complete* description the physical reality of quantum-mechanical systems and represents *directly* the physical systems themselves. Thus, the quantum state *itself* is ontologically *primary,* and operators only secondary, the significance of the latter being founded upon that of the former. Whether a given operator represents a relation or a non-relational property depends upon the quantum state concerned, and the rules here for deciding whether a given quantum state refers to a simple or composite 'quantum entity'—and, hence, whether operators of the form $O = O_1 \otimes O_2 \otimes O_3 \otimes \cdots$ represent relations or non-relational properties—are unequivocal. So, the charge of incoherence fails.

However, at least two potential interpretive difficulties do immediately present themselves. First, if a non-separable quantum state refers to a *single* simple 'quantum entity,' how are the "particle" labels in the representation of the state to be understood? On the one hand, one might say here that they merely refer to the "internally correlated" degrees of freedom of one and the same system. On that view, one would have to interpret the outcomes of correlation experiments (such as in the EPR-Bell arguments) likewise in terms of the impinging of one and the same simple 'quantum entity' upon two space-like separated detectors, which would explain (non-locally, for sure) the observed statistical correlations well enough but leave one with space-like extended entities in one's ontology (cf. Howard 1989). On the other hand, it is well known that one can do quantum mechanics of systems of many 'identical particles' in a label-free representation (e.g., in a Fock-space, rather than a Hilbert-space, representation); and such an approach here would eliminate this interpretive difficulty altogether, where label-free 'occupation number' states could be taken as referring to so many non-individual 'quanta' (cf. Teller 1995). Second, what happens when *two* simple 'quantum entities', each prepared in a pure quantum state, interact in such a way that results in a non-separable quantum state for the "composite" system (as is the case in general)? By the above interpretation, there is post-interaction only a single simple, not composite, 'quantum entity'. Thus, there is here a clear loss of self-identity of the original simple 'quantum entities.' Still, this is in itself nothing logically absurd, for there is no a priori reason why the number of simple 'quantum entities' must be conserved in interactions (as is not the case, e.g., in some interpretations of states in quantum field theory). Of course, this view would engender a measurement problem with a vengeance so that one might not want to consider it further. Nonetheless, while this interpretation may not be compelling enough to take seriously because of its high collateral commitments, it at least provides a possible interpretation that does not bear any ontological commitment to non-supervenient relations and which provides an unequivocal criterion for individuating its basic entities.

We may summarize the above results for the various interpretive options regarding ontological commitment to non-supervenient relations as follows:

A: Assume Eigenvector-Eigenvalue Property Rule (EE) and completeness ⇒ strongly non-supervenient relations
B: Assume Dispositional Property Rule (D) and completeness ⇒ weakly non-supervenient relations
C: Assume either EE or D and completeness, but deny ontological reference of 'improper' mixed states and ontological status to 'improper' sub-systems ⇒ no non-supervenient relations (if any relata!)
D: 'Minimalist' Bohm theory, with only position and velocity as real particle properties ⇒ single-particle velocities genuinely supervene on single-particle positions
E: 'Minimalist' Bohm theory with 'pilot wave' ⇒ no non-supervenient relations
F: 'Minimalist' Bohm theory with 'quantum potential' interpreted as non-local, many-particle forces ⇒ no non-supervenient relations

G: ‘Minimalist’ Bohm theory with ‘quantum potential’ interpreted as relational system property or as system of N-place relations ⇒ N-particle quantum potential strongly non-supervenes upon relational single-particle quantum potentials

H: ‘Expansive’ Bohm theory, with some quantum observables as real particle properties alongside position and velocity ⇒ contextual quantum relations that are not necessarily non-supervenient, depending upon choice of definition for such properties

One thus sees quite clearly that the question of non-supervenience in quantum ontology turns crucially upon one’s initial choice of interpretive assumptions. Deciding one’s ontological commitment, then, depends upon the choice of an interpretation and, hence, upon an overall appraisal of the several interpretations with respect to normative criteria of theory choice.

5. Non-supervenience, Holism, and Individuation. Much has already been written concerning ‘holism’ in connection with both non-separability and non-supervenience in quantum-mechanical systems (e.g., Teller 1986, 1989; Healey 1990). Rather than adding to this literature, we seek here only to add a few cautionary notes regarding the characterization of a ‘quantum holism’ in terms of non-supervenience.

First, it should be emphasized here that non-supervenient relations are *not ubiquitous* in quantum-mechanical systems, but rather involve only state-dependent relations and non-relational properties. The state-independent relations or composite properties of many-particle quantum-mechanical systems are genuinely supervenient upon the state-independent properties of the individual particles, just as in classical mechanics. To illustrate this point, consider the composite state-independent properties—mass, charge and ‘spin-type’—of a nucleus of atomic number Z(= number of protons) and mass number A (= number of nucleons, i.e., number of protons and neutrons together). The nuclear charge is simply $\sum_{i=1}^{Z} e_i = Ze$, where e is the magnitude of the electronic charge. And the nuclear mass, though not the mere sum of the single-nucleon masses, is nonetheless determined (to a very close approximation, at least) as a function of the nucleon masses, the atomic number and the mass number via the so-called ‘semi-empirical mass formula’:

$$M(Z, A) = Zm(^1H) + Nm_N - B(Z, A)/c^2,$$

where $m(^1H)$ is the hydrogen nuclear mass (i.e., proton mass), $N(= A - Z)$ is the neutron number, m_N is the neutron mass, and $B(Z, A)$ is the nuclear binding energy (cf. Krane 1988, chap. 3). Similarly, the ‘spin-type’ of a nucleus, which determines its statistical behavior, genuinely supervenes on the 'spin-types' of the respective nucleons (or particles): e.g., a deuterium nucleus, composed of two spin-1/2 nucleons, behaves compositely as a spin-1 system obeying Bose-Einstein statistics.

Second, not every case of non-supervenience listed above is peculiar to quantum mechanics. In particular, a sort of 'correlational' non-supervenience similar to that in interpretive option B, due to the Dispositional property rule (D), is found in classical statistical mechanics. There the N-particle correlation function on the phase space for an N-particle system similarly cannot be recovered from the single-particle correlation functions; i.e., the marginal or single-particle probability distributions are insufficient to uniquely determine the joint or N-particle probability distribution. And this classical 'correlational' non-supervenience arises for precisely the same reason as it does in quantum statistics for N-particle systems interpreted according to D, viz., that the single-particle functions or states are obtained by integrating over or 'tracing out' degrees of freedom pertaining to all other particles, which in both cases effectively averages out all the multi-particle correlations contained in the particle function or composite state (cf. Huang 1987, chap. 3). Thus, such 'correlational' non-supervenience would seem to reflect more the choice of definition for the single-particle states rather than an inherent physical holism.

Third, regardless of one's interpretive stance, even the quantum-mechanical world is manifestly particularistic in many of its characteristic features. This is illustrated, for example, by the granular "exposure specks" that comprise the 'interference' pattern generated on a photographic plate by a double-slit experiment, the singular detector events that register the radiative decays of unstable nuclei, and the singular space-like-separated detector events that comprise the empirical data that confirm the very correlations and 'entanglement' under study. Thus, any adequate quantum ontology capable of accounting for the manifest features of characteristic quantum phenomena must exhibit both a well-defined particularism as well as a relationalism (which would seem to rule out the "radical" interpretive option C). The ontology of the quantum world is better described as a 'relational particularism' or 'paricularist relationalism' rather than Teller's 'relational holism.'

The issue of the particularity and singularity of the phenomena raises the problem of individuation in quantum ontology. Quantum non-separability, interpreted ontologically, presents us with properties of 'entangled' composite systems that can*not* be attributed *solely* to the respective single components of such systems. The implications of this for the problem of individuation in quantum ontology can be understood in either of two mutually exclusive ways: either as indicating the lack of individuation of those single components, i.e., as denying the ontological separateness of what are formally non-separable; or as suggesting the existence of real physical relations among entities that are already individuated or ontologically separate, though not formally separable. The former view, to be convincing and conclusive, must make two key presuppositions: first, the contentious assumption (in light of, e.g., the measurement problem) that the quantum state is a complete description of the physical reality of such systems; and, second, the chief metaphysical prejudice of modernity that all real physical properties are strictly monadic, properly attributable only to single entities. The denial of individuation on the basis of formal non-separability, moreover, would seem to undercut any ontological explanation for the particularity and singularity of the very empirical phenomena that experimentally confirm the existence of quantum non-separability (noted above). Non-supervenience challenges this view and its metaphysical prejudice by presenting the option of interpreting non-separability in terms that not only need not deny the possibility of individuation but actually presuppose it: a necessary condition for the possibility of the very existence of a real relation is the (ontologically) prior and independent physical distinguishability of the relata; a real relation must be grounded in a physical, not merely numerical, distinction. It is thus clear that the failure of formal separability *by itself* does not necessarily imply the failure of the possibility of individuation or ontological separation; for insofar as non-separability can be interpreted in terms that presuppose individuation, formal separability cannot itself be taken to be a *necessary* condition of individuation (cf. French 1989, 4, 7–8).

However, it also becomes clear that the coherence of the interpretation of non-separability in terms of non-supervenience requires the articulation of a precise sense in which the quantum-mechanical entities between which such relations exist are physical individuals, i.e., one must provide a physical principle of individuation adequate to such an interpretation of non-separability. Thus, one needs to examine the possibilities for particle individuation under each interpretation that admits non-supervenient relations. The primary question here will be to what extent the possibility of individuation within quantum mechanics, and hence ontological commitment to non-supervenience, necessitates the assumption of the incompleteness of the quantum state (cf. Belousek 1999, 2000a), which would constrain the range of possibilities for interpreting non-separability in terms of non-supervenience. For sure, if one assumes the incompleteness of the quantum state, which is common to each of the interpretive options within Bohm's theory, then one can give a precise physical characterization of the individuation of quantum particles within the theory itself in terms of unique, physically distinguishable space-time trajectories and thus provide the requisite ontological underpinning for non-supervenient relations (cf. Belousek 2000b); and, by choosing an interpretive option under Bohm's theory that admits non-supervenience relations, one would have a coherent quantum ontology that naturally provides a relatively clear explanation of the 'relational particularism' characteristic of quantum phenomena. In any case, coherent ontological commitment to non-supervenient relations must be underpinned by an account of the physical individuation of quantum-mechanical systems that characterizes such individuation as prior to and independent of the existence of such relations.

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