ON GRAVOTHERMAL OSCILLATIONS

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ABSTRACT: An isolated conducting gas sphere (an idealized model of a globular cluster) can have non-singular postcollapse similarity solutions for appropriate energy sources. Solutions have been found for a source mimicking three-body binaries; they are parametrized by N, the number of stars in the cluster, and are linearly stable if $N < 7000$, overstable if $7000 < N < 40,000$, and unstable if $N > 40,000$. The nonlinear development is always oscillatory and produces a typically lower central concentration than in the "equilibrium".

Previous work (e.g. Larson 1970, Hachisu et. al. 1978, Lynden-Bell and Eggleton 1980 [LBE], and Heggie 1984) has shown that models of conducting gas spheres are in good agreement with Fokker-Planck results for the pre- and post-collapse evolution of star clusters. Adding a local energy-generation rate per unit mass of the form (ρ = density, σ = velocity dispersion)

$$C\rho^a\sigma^b \tag{1}$$

to the structure equations laid out by LBE, Bettwieser and Sugimoto (1984 [BS]) found that post-collapse gas spheres undergo large variations in the central density (ρ_c), which they called "gravothermal oscillations" [GTOs].

Desiring to study GTOs further, I sought an "equilibrium" postcollapse solution that could be subjected to a linear stability analysis. Previously known self-similar postcollapse models (e.g. Hénon 1961, 1965; Inagaki and Lynden-Bell 1983) are all unstable, and with infinite growth rates, because of their infinite central densities. A fuller account of the work reported here will appear in Goodman (1987).

It is well known (Hénon 1965) that the half-mass radius, r_h, of an isolated postcollapse cluster expands as the two-thirds power of time, t. A non-singular postcollapse similarity solution must have a core radius, r_c, with the same scaling. It can be shown that self-similar solutions are possible if

$$6a + b = 5. \qquad (2).$$

I assume that $a > 1/2$ so that most of the energy is generated in the core.

Energy-generation by three-body binaries satisfies (2) with $a = 2$ and $b = -7$. Solutions based on this particular energy source have been constructed. The solutions are indexed by a single dimensionless parameter, Γ, which is related to N by

$$\Gamma \approx \frac{2.4}{N^2}\ , \tag{3}$$

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although the coefficient here conceals a logarithmic dependence on N and uncertainties in the binary efficiency. The structure of the solutions around the half-mass radius and beyond is almost independent of Γ; the solutions are nearly isothermal inside the half-mass radius. The age, t, of the self-similar solution is always approximately $11t_{rh}$. But the smaller is Γ the more extreme are the central conditions:

$$
\begin{aligned}
r_c/r_h &\approx 4.4/N^{2/3}, \\
t_{rc}/t_{rh} &\approx 40./N^{4/3},
\end{aligned} \tag{4}
$$

where t_{rc} and t_{rh} are the central and half-mass relaxation times as defined by Spitzer and Hart (1971). Three-body binary creation becomes less efficient compared to two-body relaxation as N grows, whence a balance between energy production in the core and the overall cluster expansion can be achieved only by increasing ρ_c.

When described by the appropriate comoving variables, the solutions appear static, so that their stability can be analyzed in the same way as that of a true equilibrium; modes evolve however as t^s rather than as $\exp(st)$. The models are stable for $\log\Gamma > -7.3258$, or (by (3)), for $N < 7000$. But for $7000 < N < 40,000$ they are overstable, and for $N > 40,000$ they are unstable.

For very large N, the growth rate approaches $(\sim 1/(570t_{rc})$, and a separate calculation shows that an infinite but non-singular isothermal sphere without an energy source has the same growth rate. This and other considerations lead me to agree with BS that, asymptotically at least, the instability leading to gravothermal oscillations is the same as that discovered by Antonov (1962). The physical details in the overstable regime, however, are still obscure.

When non-linear, the instabilities become chaotic relaxation oscillations. By far the largest part of a cycle is spent near minimum density, since $1/t_{rc}$ and the evolution rate are then greatest. Consequently, if observed at a random time in its postcollapse evolution, a cluster with $N \gg 40,000$ would have a much lower central concentration than would the self-similar equilibrium for the same N. Comparison of these results with the King sequence suggest $\log(r_{tidal}/r_c) \sim 2 - 3$ independently of N if N is large, but this result is tentative as tidal limitation and other important physical complications have been omitted from the present models, and because a thorough exploration of parameter space has not been made.

Instability is also likely for other energy sources that are inefficient enough so that $r_h/r_c > 10^2$ at late times. This appears to be the case for tidal binaries (Statler et. al. 1986).

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